The Timing of High-Speed Regenerative Repeaters

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A simplified method for determining the behavior of the timing portion of a chain of regenerative repeaters is presented. The method applies to self-timed as well as separately timed systems. The effect of random noise, as modified by the timing circuit bandwidth, is calculated. Defects other than noise are discussed. System requirements for the satisfactory transmission of a number of types of signals are determined.

I. SCOPE OF PAPER

Technical advances in the microwave art, and particularly in connection with waveguide transmission, have made pulse code modulation with the use of regenerative repeaters a plausible means for broadband transmission over distances of thousands of miles. Such transmission can be successful only if the signal can be satisfactorily retimed in a chain of one to a few hundred repeaters.

The problem of retiming has been considered, chiefly in connection with a particular type of repeater, by E. D. Sunde.[1] Extensive mathematical treatments of this problem appear in accompanying articles by W. R. Bennett[2] and H. E. Rowe[3] of Bell Telephone Laboratories. This paper points out some of the problems encountered in timing and presents a less mathematical method for determining the behavior of the timing portion of a chain of regenerative repeaters. It also considers the behavior in connection with the requirements for the satisfactory transmission of broadband signals through 100 repeaters.

The method used here is based on the fact that the behavior of the timing part of a chain of self-timed binary repeaters is very similar to that of a chain for which timing information is transmitted over a separate analog channel. Both of these systems, therefore, can be analyzed by methods which are known for analog transmission. The method is applied to determine the effects of random noise upon a single repeater and it is then extended to include a chain of repeaters. The effect of timing circuit bandwidth is determined. The method yields numerical answers which agree with experimental results, as pointed out in the following article.[4] It is found that, although noise effects accumulate along a chain of repeaters, the rate of accumulation is low.

Effects other than those due to noise are discussed. In a realistic case these effects appear to be more severe than those of noise. Since they result from equipment deficiencies, it should be possible to reduce them to tolerable values by employing improved timing circuits. In particular, this can be accomplished by means of a few slightly more complicated timing circuits distributed throughout a system employing simple tuned circuits as timing filters.

We conclude that, although noise and effects resulting from changes of pulse pattern preclude the possibility of regenerating PCM signals an indefinite number of times, it is possible to regenerate the few hundred times required for a long system without having to resort to complicated timing equipment or provide a signal-to-noise ratio much greater than that imposed by system requirements other than those involved in timing.

II. GENERAL CONSIDERATIONS

The main advantage of transmitting information in binary-pulse form results from the fact that such pulses can be regenerated at each repeater (see Ref. 5). The process of regeneration removes most of the signal distortion produced by noise, bandwidth limitations, etc., during transmission. If the distortion is not allowed to become excessive between regenerators the signal at the end of a system with many repeaters might have little more distortion than the signal at the input to the chain of repeaters.

The regeneration of binary pulses involves two functions: (1) the removal of undesirable amplitude effects; (2) the restoration of each pulse to its assigned position in time. An ideal repeater provides a perfect train of timing pulses to perform the second of these functions. In a practical repeater, these pulses may be displaced from their correct positions as a result of noise or other disturbances. The fundamental timing problem is that of deriving suitable timing pulses in the presence of such disturbances. There are two basic methods of providing timing information at a repeater: in the first method, this information is transmitted over a separate channel, and the system is said to be "externally timed"; in the second, it is derived from the incoming train of signal pulses to provide a "self-timed" system.

In either of the above systems the timing wave will be recovered as an approximately sinusoidal wave through a narrow-band filter of some kind. This sine wave is converted to a train of short pulses, the times of occurrence of which are determined by the phase of the sine wave. Although the timing function is performed only by the train of pulses, we shall use the term "timing wave" to indicate either this train or the sine wave from which it is derived. Along with the sine wave at the output of the filter there will be undesired frequency components due to any disturbances which may exist. The resultant wave is caused to vary in phase and amplitude as shown for a single interfering component by Fig. 1(a). Limiting can be employed to remove the amplitude variations but the phase deviations, as shown by Fig. 1(b), are of the most concern in a timing system. The amount of phase deviation is determined by the ratio of timing signal power to timing noise power. We shall, therefore, direct a considerable amount of attention to the calculation of this ratio.

When the disturbing power entering the timing circuit consists of several components or of a large number spread over some frequency range, the resultant effect is equal to the sum of the effects of the individual components if we limit consideration to the case where the

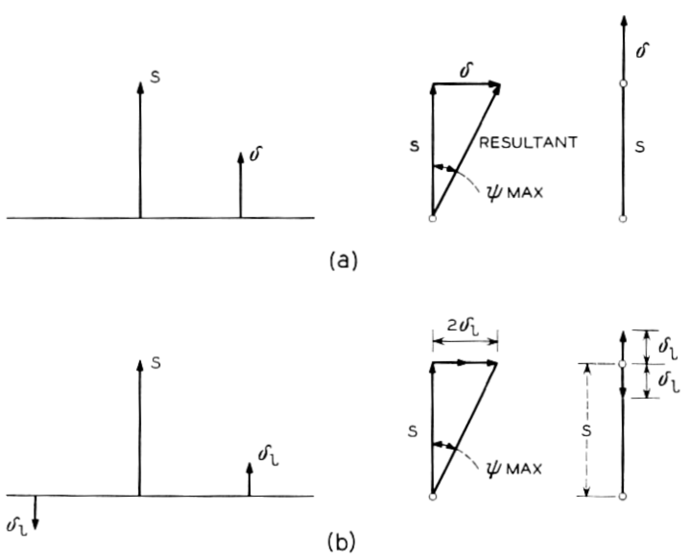

Fig. 1 — Effect of a single interfering frequency (a) before limiting and (b) after limiting.

signal-to-noise ratio is large. It is obvious that, for the distributed spectrum, the narrower the band of the timing filter, the fewer the number of components it passes and the smaller the phase deviation will be. From a practical standpoint, stability considerations place a limit on how narrow this band can be made; i.e., the narrower the band of the circuit, the more subject it becomes to slight detunings brought about by temperature variations, etc.

III. EXTERNALLY TIMED SYSTEMS

3.1 *Straight-Through Timing Wave Repeater*

Fig. 2(a) shows an externally timed system with one timing channel supplying two signal channels. Fig. 2(b) shows, in greater detail, a

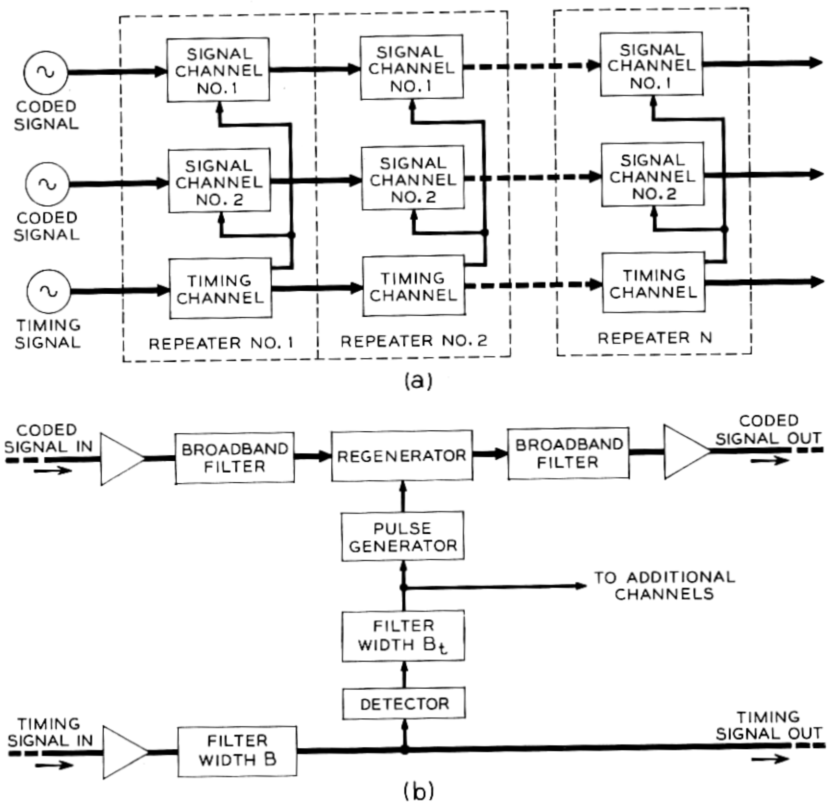

Fig. 2 — (a) Externally timed system with one timing channel supplying two signal channels; (b) single repeater—straight-through timing channel.

single repeater of such a system. The straight-through timing channel shown on Fig. 2(b) appears to provide the simplest form of timing circuit. Here a sine wave at pulse-repetition frequency f_r is amplitude modulated* on an RF carrier of frequency f_c and transmitted straight through the system. At each repeater this RF signal is simply amplified, filtered and sent on to the next repeater. Each RF circuit, including filters, is assumed to have a frequency characteristic which is flat over a total band of width $B = 2f_r$ so as to pass the amplitude modulation sidebands. The characteristic is assumed to cut off sharply outside of this range. It is assumed that all noise in the system originates at the input to each repeater, that the RF signal-to-noise ratio is always large and that there is unity gain from repeater input to repeater input. The timing channel of Fig. 2(b) is seen to be a straightforward analog system in which, at the end of N repeaters, the noise power will be N times that at a single repeater. The RF signal-to-noise ratio at any repeater is given by

$$\left(\frac{W_s}{W_n}\right)_{\mathrm{RF}} = \frac{P_s}{NW_0B}, \tag{1}$$

where P_s is the mean RF power at the peak of an envelope cycle† and W_0 is the noise power density, in watts per cycle of bandwidth, contributed by a single repeater.

At each repeater some signal is taken off through a branch circuit where it is detected to recover the envelope, which is filtered through the narrow-band timing filter of bandwidth B_t to become the timing wave at that repeater. Although noise power is available over the entire band B, those components lying within two bands of width B_t and spaced by frequency f_r on either side of the RF carrier contribute most of the noise to the timing circuit.

From (23) of Appendix A we find the timing wave signal-to-noise ratio at the output of the narrow filter of the first repeater to be

$$\frac{W_s}{W_{na}} = \frac{P_s}{8W_0B_t}, \tag{2}$$

where W_s is the mean power of the sinusoidal timing wave and W_{na} is the mean noise power. From a consideration of the way noise accumulates in a system, it is evident that, for N repeaters, the timing signal-

* Although it should be possible to transmit timing information by other means, for example by frequency modulation, we shall limit consideration in this paper to amplitude modulation systems.

† Power is specified in this manner rather than in the more usual terms of RF carrier power since, in later discussions of pulses, peak power is the significant quantity.

to-noise ratio is

$$\left(\frac{W_s}{W_{na}}\right)_N = \frac{P_s}{8NW_0B_t}. \qquad (3)$$

3.2 *Remodulation Timing Wave Repeaters*

Fig. 3 shows a single repeater for another type of separate timing system. Here, at each repeater, the envelope of the RF wave is recovered by the detector, filtered through the narrow filter of width B_t and remodulated onto a locally generated RF wave for transmission to the next repeater. The timing signal-to-noise ratio at the output of the narrow-band filter of a remodulation repeater will be the same as that at the corresponding point in a straight-through repeater if both employ ideal, flat timing filters. This ratio for a chain of N repeaters is, therefore, given by (3).

If instead of using a local oscillator, the incoming signal is employed as a source of RF and, if the modulator is replaced by a time gate, the repeater of Fig. 4 evolves from that of Fig. 3. This configuration does not require a local oscillator but has the disadvantage that noise can be passed through the gate via a second path which does not include the timing filter. This difficulty can be overcome by replacing the gate with an ideal pulse regenerator. Such a regenerator produces an output which consists entirely of off-on pulses, the times of occurrence of which depend only upon the timing wave. Thus, no timing noise is passed through the second path except in the form of errors in the pulse pattern. These errors have negligible effect upon timing.

If the gate is replaced by a regenerator, the result is the repeater shown on Fig. 5. It is evident that this repeater can also serve as a self-timed repeater in a signal channel. We thus arrive at the same configura-

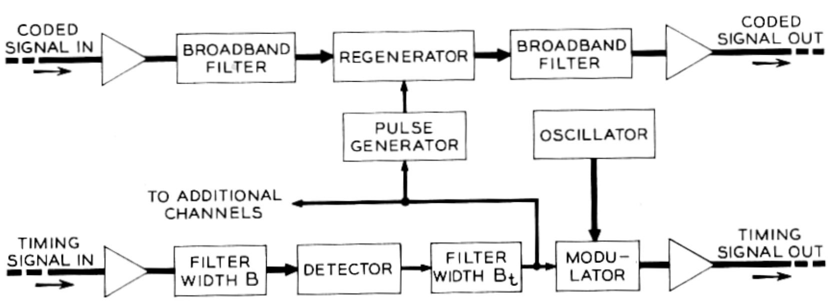

Fig. 3 — Single repeater—remodulation timing channel.

tion for a self-timed, signal-channel repeater and a desirable form of repeater for a timing channel. If we adopt this as a preferred form of timing-wave repeater (as we shall in this paper), the most significant difference between a timing channel and a chain of repeaters carrying a complete signal lies in the type of signal transmitted. For the timing channel, the signal envelope will be a sine wave at pulse-repetition frequency at most points in the system; for the complete channel, the signal envelope will consist of the varying pattern of pulses which results from coding of the information being transmitted. The timing portion of the repeater shown on Fig. 5 is seen to consist of a detector, a narrow-band timing filter, an amplifier, a limiter and a pulse generator. When the repeater is being used as part of a separate timing channel the timing wave for the signal channels can be taken off at the output of the limiter.

For the systems of Fig. 4 and Fig. 5 the signal envelope exists in the form of pulses at some points in the repeater; however, if the output of

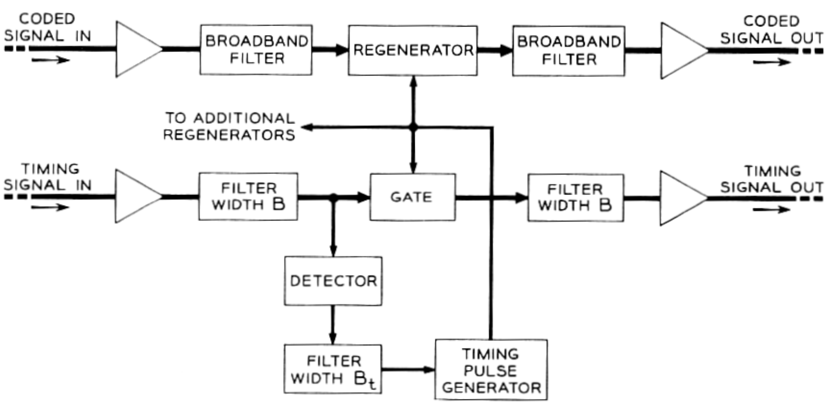

Fig. 4 — Single repeater—remodulation timing channel.

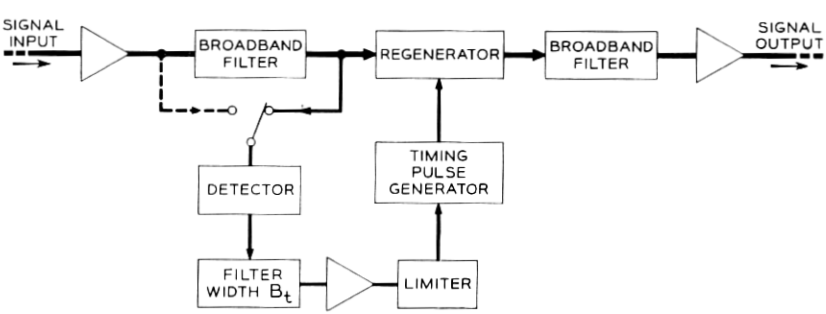

Fig. 5 — Self-timed repeater.

the gate or regenerator is passed through a suitable RF filter the envelope is reduced to a sine wave of frequency f_r when all pulses are present. In all cases involving an externally timed system, the signal input to the detector can thus be made to consist of an RF carrier 100 per cent amplitude modulated by a sine wave at pulse-repetition frequency, f_r [see Fig. 6(b)]. This is also true for a self-timed repeater when all pulses are present. In a later section of this paper the effect of RF bandwidth is discussed in more detail.

In comparing the various systems we see that, for the straight-through timing system of Fig. 2, noise is passed from repeater to repeater unchanged in form and builds up over the entire RF band B, as shown on Fig. 7(a). For the remodulation system of Fig. 3 the noise existing over a narrow band of width B_t is passed along a chain of repeaters by being amplitude-modulated on the RF carrier along with the timing wave. The resultant RF sidebands occupy a narrow band above the carrier

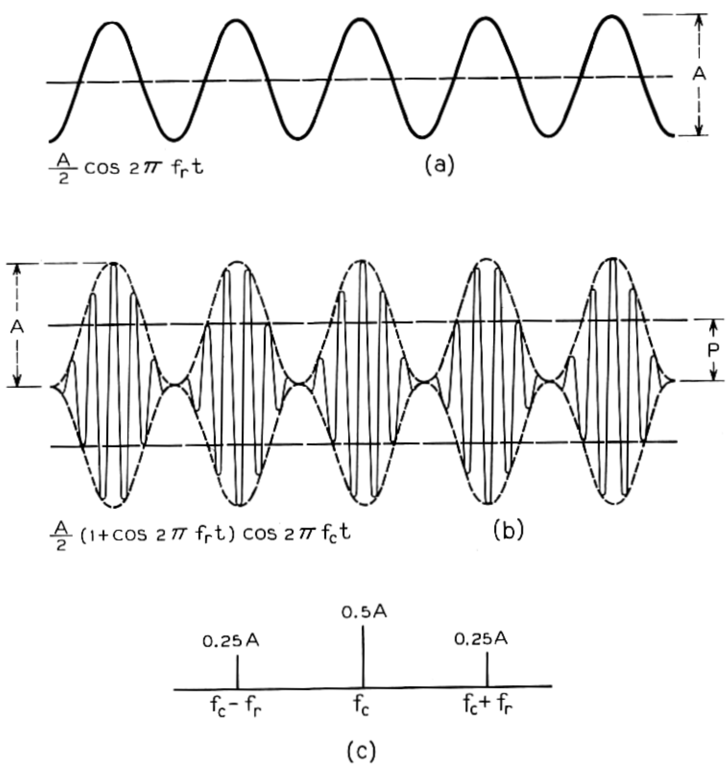

Fig. 6 — Timing signal: (a) envelope; (b) RF signal; (c) frequency spectrum.

and an equal band below the carrier, and build up as indicated in Fig. 7(b). The sidebands above the carrier are correlated with corresponding ones below the carrier, since they originate from the same noise source. For the system of Fig. 5, noise components existing throughout the narrow band B_t time-modulate the envelope of the signal out of a repeater. After filtering, the noise output of one of these repeaters is the same, from a qualitative standpoint, as the noise out of a remodulation repeater [see Fig. 7(b)].*

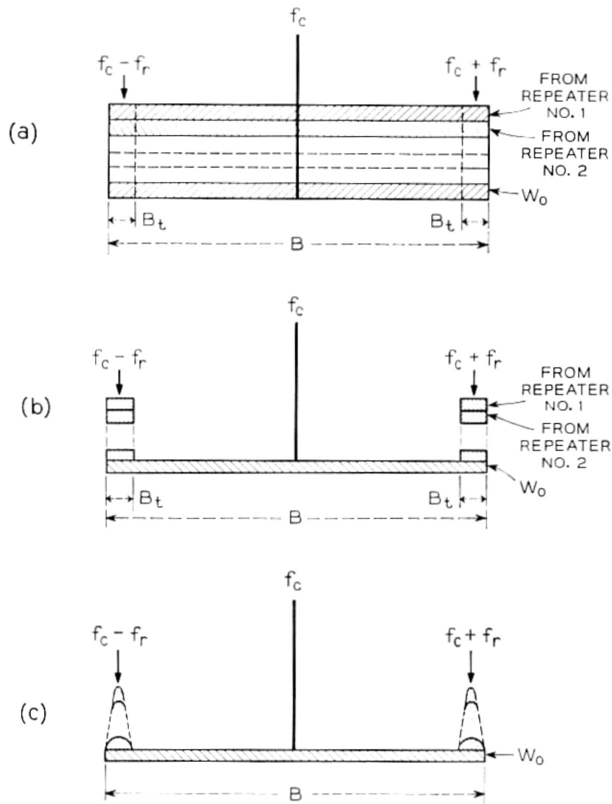

Fig. 7 — Buildup of noise in a chain of repeaters: (a) straight-through repeaters; (b) remodulation repeaters with flat filters; (c) remodulation repeaters with tuned circuits.

* This figure is somewhat idealized in that it assumes a clean, noise-free carrier at each repeater. In an actual system the carrier will be accompanied by noise components, so that at the end of the system there will be some noise power distributed throughout the band B.

3.3 *S/N Ratio in the Presence of Random Noise — Flat Filters*

At the output of the timing filter of the first repeater of the type shown on Fig. 5 the signal-to-noise ratio is the same as for the straight-through repeater or the remodulation repeater and is given by (2):

$$\frac{W_s}{W_{na}} = \frac{P_s}{8W_0B_t}. \tag{4}$$

In the process of limiting, the components of noise in phase with the timing sine wave are removed so that the signal-to-noise ratio becomes

$$\left(\frac{W_s}{W_n}\right)_1 = \frac{P_s}{4W_0B_t}. \tag{5}$$

This is the ratio of mean timing wave power to the mean power carried by the phase modulation sidebands resulting from noise. The mechanism of phase modulation and the sidebands which exist after the signal and a single-frequency noise component have been limited are shown by Fig. 1(b). The timing pulses derived from the phase-modulated sinusoidal timing wave will deviate in time as a result of this noise modulation. Since the time of occurrence of a pulse out of the time gate, or regenerator, is determined by the time of occurrence of the corresponding timing pulse, it follows that the envelope of the signal out of a repeater will deviate in time in the same way as does the timing wave in that repeater.

If the second repeater of a chain were free of local noise its timing wave would follow phase excursions of the incoming signal and be phase-modulated to the same extent as the timing wave at the first repeater. (If both repeaters have flat timing filters the noise sideband pattern will be transmitted unaltered from the first repeater to the second.) Since the two repeaters are assumed to be identical, locally generated noise acting alone will produce the same average phase deviation in the second repeater as is produced in the same way in the first repeater. The average noise sideband power arising from local noise is therefore equal to that resulting from time deviations of the incoming signal. Since there is no correlation between the sidebands resulting from these two sources of noise they add on a random, or power, basis. For flat filters the timing signal-to-noise ratio at the second repeater is therefore one-half that at the first repeater and, at the Nth repeater, is

$$\left(\frac{W_s}{W_n}\right)_N = \frac{P_s}{4NW_0B_t}. \tag{6}$$

3.4 *S/N Ratio in the Presence of Random Noise — Tuned Circuits*

For the system of flat filters discussed above, the bandwidth of the total timing system is the same as the bandwidth of a single filter. With tuned circuits, or other peaked filters, the system bandwidth decreases continuously as the number of repeaters (and hence the number of filters) is increased. With tuned timing filters, noise builds up as shown by Fig. 7(c). At the first repeater the noise input to the detector is the same as for the repeater with a flat filter. The input to the detector at the second repeater differs from the corresponding input for a repeater with a flat filter in that the components from the first repeater have a different frequency distribution. If we now consider the second repeater to be free of locally generated noise, it is no longer true that the timing wave out of its filter is deviated to the same extent as the wave at the corresponding point in the first repeater. In this case, some of the sidebands about the timing wave suffer attenuation in the second filter and similarly at succeeding repeaters. The noise power which originates in the first repeater is filtered by N circuits before it reaches the end of the system; that which originates in the second repeater is filter by $(N - 1)$ circuits and so on, as shown on Fig. 7(c).

In Appendix B it is shown that, for a system of N repeaters of this type, the timing signal to-noise ratio becomes

$$\left(\frac{W_s}{W_n}\right)_N = \frac{P_s Q}{7.1\sqrt{N}\,W_0 f_r}, \tag{7}$$

where Q is the "quality factor" of the tuned circuits and the other quantitites are as specified previously. Equation (7) indicates that, with tuned circuits, noise power increases as the square root of the number of repeaters. For flat timing filters the noise power increases directly as the number of repeaters, as shown by (6). This advantage of tuned circuits over flat filters seems to have been first pointed out by H. E. Rowe.[3]

3.5 *Timing Displacement*

Having obtained the timing wave signal-to-noise ratio for a chain of repeaters, it is fairly simple to determine the rms magnitude of the phase deviations of the timing wave. First, consider the noise to be replaced by a single-frequency interference having the same mean power. From Fig. 1(b) it is evident that the following approximate relationships hold:

$$\Psi_{\max} = \frac{2\delta_l}{s} \qquad \text{and} \qquad \Psi_{\text{rms}} = \frac{\sqrt{2}\,\delta_l}{s},$$

where δ_l is the magnitude of one of the sidebands of the amplitude limited timing signal and s is the amplitude of the sine wave. Returning to the equivalent noise we have

$$\frac{\sqrt{2}\,\delta_l}{s} = \sqrt{\frac{W_n}{W_s}}, \qquad \text{or} \qquad \Psi_{\text{rms}} = \sqrt{\frac{W_n}{W_s}}. \tag{8}$$

Using the value of signal-to-noise ratio given by (7) for a chain of repeaters with tuned circuits and the above expression for Ψ_{rms} yields

$$\Psi_{\text{rms}} = 2.66N^{1/4} \sqrt{\frac{W_0 f_r}{P_s Q}}. \tag{9}$$

It should be noted that, although the rms phase deviation produced by noise is the same as the rms deviation produced by the equivalent single frequency, the peak deviations will be different for the two types of interference. Therefore to determine the peak phase deviation the appropriate peak factor must be applied to (9).

IV. SELF-TIMED SYSTEMS

Since we have chosen the same repeater configuration for externally timed and self-timed systems, the main difference between the two lies in the types of signals each transmits. When a chain of repeaters is used as a separate timing channel for one or more signal channels it transmits a fixed pulse pattern or a sinusoidal timing wave, whereas when used as a self-timed system the chain transmits the varying pulse pattern which results from coding a baseband signal. For a self-timed system there are gaps in the train of signal pulses applied to the timing circuit filter. Improved timing signal-to-noise ratio is obtained if the circuit is so arranged that no noise is applied to the filter during these gaps. For a baseband system this can be accomplished by inserting a nonlinear device, such as a peak amplifier, ahead of the filter and adjusting it so that only voltages exceeding some threshold value at the input will be passed to the filter. This is discussed in greater detail in the following article. If, in a carrier system, a square-law detector is employed to recover the pulse envelopes which supply the input to the timing circuit, this detector can be made to perform much the same function as the nonlinear device mentioned above. This is pointed out by Rowe.[3] Because of its advantage over the linear detector in this respect, the square-law detector is the only type which will be considered here. For repetitive patterns in which only one pulse position out of M possible positions is occupied, it follows from a consideration of the Fourier components involved that the component at pulse-repetition frequency, f_r, is reduced in amplitude by the factor $1/M$. The more general expression for timing wave power then becomes, from (20) and (21) of Appendix A:

$$W_s = \frac{\alpha^2 A^4}{32M^2}, \tag{10}$$

where α is the detection coefficient of the particular detector used and A is the peak RF amplitude.

If we assume that, because of the nonlinearity of the detector, the noise power applied to the timing filter is proportional to the number of pulses per unit time, the more general expression* for noise power becomes, from (22):

$$W_{na} = \frac{\alpha^2 A^2 W_0 B_t}{2M}. \tag{11}$$

Since it evaluates the noise in a single repeater, (11) applies to either flat timing filters or tuned circuits, as long as B_t is taken as the effective bandwidth.

From (10) and (11) we obtain the timing wave signal-to-noise ratio for a single repeater at the output of the timing filter, before amplitude limiting:

$$\left(\frac{W_s}{W_{na}}\right)_1 = \frac{A^2}{16W_0B_tM} = \frac{P_s}{8W_0B_tM}. \tag{12}$$

After limiting, it is

$$\left(\frac{W_s}{W_n}\right)_1 = \frac{P_s}{4W_0B_tM}. \tag{13}$$

Equation (13) differs from (5) only by the factor M in the denominator of (13). Similarly, from (7), the signal-to-noise ratio for a chain of repeaters with tuned timing filters becomes

$$\left(\frac{W_s}{W_n}\right)_N = \frac{P_sQ}{7.1\sqrt{N}\,MW_0f_r}, \tag{14}$$

and

$$\Psi_{\text{rms}} = 2.66N^{1/4}\sqrt{\frac{MW_0f_r}{P_sQ}}. \tag{15}$$

* Although this expression is only approximate, the approximation has been found to be sufficiently good from a practical standpoint.

4.1 *The Effects of Variations in Pulse Pattern*

The equations derived previously are based on repetitive pulse patterns, so that caution must be observed in applying them to a system for which the pulse pattern is changing. If we assume that the probability of a pulse being present in any pulse position is equal to one-half, the M of (15) has a long-time average value of 2. However, if no restrictions are placed on the pattern of pulses to be transmitted, there may be comparatively long periods during which there are no pulses present. Unless the timing filter has an infinite Q, the timing wave may decay to an unusable value during these periods. Fortunately, it is possible, with only a small sacrifice of information-handling capacity, to code messages in such a way as to guarantee that the longest gap will be only a few tens of pulse intervals. This is accomplished by omitting the one amplitude level corresponding to no pulses in the code group. Even with this type of coding it is possible with some signals, such as television, to have relatively long periods when considerably fewer than one-half of the pulses are present. During these periods M will have a value greater than 2 and the phase deviations will be correspondingly greater. Some other effects of changing pulse pattern will be discussed in a later section.

4.2 *The Effects of Radio-Frequency Bandwidth*

All of the analysis leading to (15) was based on consideration of a radio-frequency band of sufficient width to resolve adjacent pulses and thereby produce the wave of Fig. 6(b) when all pulses are present. Both theory and experiment indicate that a system should provide satisfactory amplitude regeneration with considerably less than this amount of bandwidth. Since conservation of bandwidth is always desirable, we proceed to study the effects upon timing of reducing the bandwidth to something less than that required to resolve adjacent pulses. Frequency distributions and a possible filtering arrangement for adjacent PCM channels are shown in Fig. 8, where two adjacent carriers are represented by f_{c1} and f_{c2}. The transmitting filter in each channel corresponds to the filter following the regenerator in Fig. 5. Returning to Fig. 8, we see that the transmitting filters have some loss to the sidebands spaced by the repetition frequency from the carriers. This loss is necessary to obtain the wave of Fig. 6(b) from the gated output and it can become as great as 6 db if the gating pulse is extremely short. The channels are shown separated by a guard band B_G. If components from one channel fall beyond this guard band they may interfere with an adjacent channel, and especially with its timing circuit. Furthermore, once these components enter the common medium, filtering is not effective against them, since the desired components would be reduced to very nearly the same extent as the undesired. This is evident from the receiving filter characteristic of Fig. 8 when one considers that all usable timing information, as well as timing disturbances, is contained in the bands of width B_t spaced either side of a carrier by the frequency f_r.

If the timing detector is connected as shown by the solid line in Fig. 5 the receiving filter, which follows the input amplifier, will attenuate timing signal and timing noise occurring in the narrow bands of width B_t to very nearly the same extent, as can be seen from Fig. 8. Then, from the standpoint of the timing signal-to-noise ratio, it makes little difference what characteristic this filter has, though some disadvantage would follow from too large a reduction in the absolute value of the timing wave. It should, therefore, be advantageous to connect the timing-wave

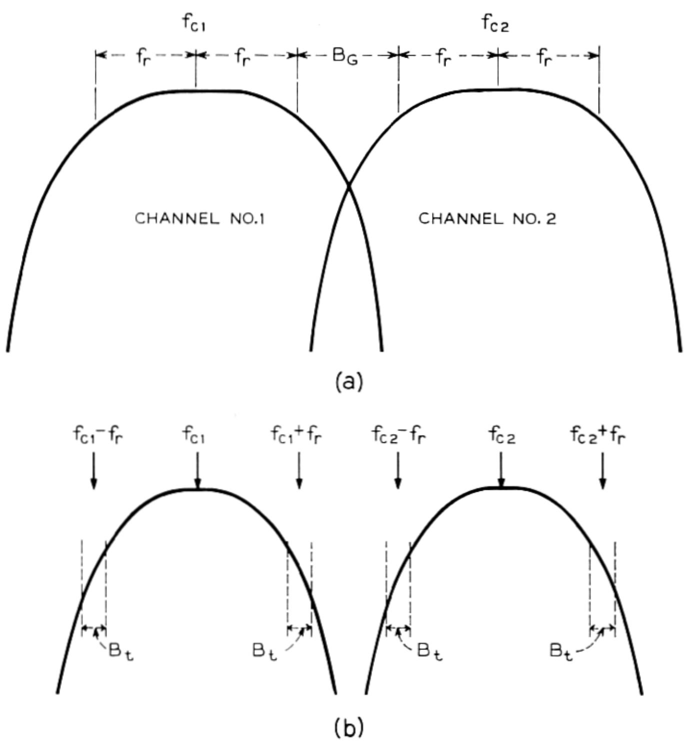

Fig. 8 — Filter characteristics: (a) transmitting filter; (b) receiving filter.

detector ahead of the filter, as shown by the dashed line of Fig. 5, in order to avoid timing as a consideration in the design of this RF filter. (For broadband RF amplifiers it would be desirable to retain some filtering ahead of the detector in order to maintain a large signal-to-noise ratio at the detector input.)

We conclude that, from the standpoint of timing, the transmitting RF filter at each repeater is the important one. For such filters with characteristics matched to the length of pulse out of the regenerator so as to produce the wave of Fig. 6(b), the results expressed by (15) apply. If the transmitting filter is made narrower than shown so as to have some additional loss* at frequencies spaced from the carrier by f_r, there will obviously be a reduction in the timing signal-to-noise ratio. Proper slicer operation can be obtained for bands considerably narrower than that required for resolution of adjacent pulses. It is therefore possible to make the receiving filter ahead of the regenerator considerably narrower, as indicated by the characteristic at the bottom of Fig. 8. Such filtering provides additional discrimination against noise and other interference which might affect the slicer.

4.3 *Effects Other Than Noise*

Noise is not the only source of timing difficulty in a self-timed system. Assume, for example, a defect in the timing circuit such that the phase of the timing wave which it delivers is a function of the pattern of pulses being transmitted. Then any change of pulse pattern may result in a change of timing wave phase. Such defects exist in the form of amplitude-to-phase conversion in limiters and other nonlinear elements, phase changes due to the finite width of the pulses applied to the timing circuit and phase shifts due to detuning of the timing circuits. Although the results are somewhat similar to those produced by noise they differ in two important respects: (1) Except for detuning, it is very probable that the deviation will be the same at each repeater, so that deviations add directly rather than on a random basis. It is also possible, though highly improbable, for deviations due to detuning to add directly. (2) These defects are not inherent in timing circuits and might conceivably be reduced to a negligible value by proper design of timing equipment.

For a chain of repeaters having defects of this type, transmission

* It appears to be possible to devise systems which provide satisfactory timing even when the band is limited to such an extent that this loss becomes considable. Some extreme cases are discussed by Bennett.[2] Such systems do not lend themselves to simple analysis and therefore will not be dealt with here.

delay is a function of the pulse pattern, which, in turn, is a complicated function of the signal being transmitted. This is somewhat analogous to delay distortion in an analog system. The effect of changes of pulse pattern is not amenable to general treatment, since it is a function of the particular timing circuit being considered. The defect has been evaluated by Sunde[1] for a model based on one particular type of repeater. Our experience up to the present time leads us to believe that, unless some specific remedy is applied, this problem may be more serious than that posed by random noise. The frequency spectrum of the phase deviations resulting from changes of pulse pattern is determined by the rates at which the pattern changes and the bandwidth of the timing circuit in which the deviations are produced. The spectrum at the output of any repeater will be further modified by all succeeding repeaters. The solution to the changing pulse pattern problem therefore appears to be that of providing narrow-band high-stability filters in the timing circuits.

4.4 *Detuning*

Although detuning has already been considered along with the other phase-conversion effects, it warrants further consideration. All calculations up to this point have been based on the assumption that the timing circuits are always perfectly tuned to the pulse-repetition frequency, f_r. This requirement is difficult, if not impossible, to fulfill; we therefore attempt to determine the effect on timing of small amounts of detuning. We assume that each circuit was originally perfectly tuned and that delays were then adjusted to provide perfect alignment between signal pulses and timing pulses. Any subsequent change of tuning will result in a change of phase delay through the circuit and consequently a departure from alignment between signal and timing pulses. An excessive amount of alignment displacement can result in errors in pulse pattern. Fortunately, the steady-state phase displacements do not accumulate in a chain of repeaters except as a change of total delay through the system. Such changes of system delay do not degrade system performance, as long as they take place slowly.

W. R. Bennett has shown[2] that, in addition to the steady displacement discussed above, there is a dynamic displacement of timing-wave phase when the pulse pattern varies with time; i.e., there are additional variations of phase about the displaced position. When the pulse pattern varies in a random manner such that the probability of a pulse being present at any pulse position is one-half, the resultant phase fluctuations are very similar to those produced by random noise. He has determined that, under these conditions,

$$\epsilon = K\sqrt{\pi Q}, \tag{16}$$

where ϵ is the rms phase deviations resulting from detuning at a single repeater and K is the tuning error in cycles per second divided by the pulse-repetition frequency.

The manner in which dynamic phase deviations build up along a chain of repeaters is governed by the way the circuits are detuned. If the detunings are random, the phase deviation increases slowly as the number of repeaters is increased; however, if all of the circuits are detuned in the same way the phase deviations add directly from repeater to repeater.

Although there is some uncertainty as to how the dynamic deviations due to detuning build up along a repeater chain, we can determine approximately the maximum amount of detuning which can be tolerated under the most unfavorable conditions and still meet the most stringent system requirements. Although the calculations are based on some approximations regarding the spectral distribution of the phase modulation sidebands, the results are probably correct to within a factor of two or better and should prove useful in determining a bound on tolerable detuning. Assume that all circuits are detuned in exactly the same way and that the system must meet voice multiplex requirements as calculated in Section V. Then, for a chain of 100 repeaters with timing circuits having a Q of 100 at 160 mc, the maximum tolerable amount of detuning can be calculated from (16) to be about three parts in 10^4 if dynamic phase deviations are to be kept to the allowable value.* If the timing circuits in the various repeaters are detuned in a random manner — and this is the most probable condition — the amount of detuning which can be tolerated should be considerably greater than three parts in 10^4.

The steady-state alignment displacement, β, occurring at a single repeater can be expressed approximately as $\beta = 2QK$, where β is in radians and K is the tuning error as before. If K is made equal to 3×10^{-4} to meet the dynamic requirements as determined above we calculate a value of 0.06 radian for β. This is reasonably small, so that, under the specified conditions, the tuning requirements are determined by the dynamic effects; under different conditions, the requirements on tuning

* Although these deviations add in phase from repeater to repeater, the total deviation for 100 repeaters is not 100 times that for a single repeater because of the decreased bandwidth of the chain. By a method similar to that of Appendix B, it can be shown that the deviation is about 30 times that for a single repeater if we assume a flat spectrum.

accuracy may be set by limitations on static misalignment. If, to determine the limitations imposed by static displacement, we take 0.2 radian as the maximum allowable value of β and assume a circuit Q of 100, we calculate a value for K of 1×10^{-3}. We conclude that timing circuits with a Q of 100 should never be allowed to become detuned by as much as one part in 10^3, but that detuning by one part in 10^4 would be tolerable even under the most unfavorable conditions.

4.5 *Frequency Stability*

There are two types of variation of pulse-repetition frequency to consider: slow drifts, which take place at a rate less than about one cycle per second, and more rapid variations. A slow drift of frequency is equivalent to detuning all circuits in the same direction by an amount equal to the frequency departure. To keep alignment displacement and the phase deviations produced by changes of pulse pattern to within tolerable bounds these frequency drifts should be kept to less than about three parts in 10^4, as calculated for detuning.

Any change of pulse-repetition frequency results in a change of the total delay through a system, even with all other parameters fixed. The change of phase for a given change of frequency is given by

$$\Delta\Phi = 2NQ \frac{\Delta f_r}{f_r} \tag{17}$$

if the timing filters are simple tuned circuits and if the rate of change is low enough to be passed by all of these filters in tandem. Since there may be a range of rates low enough to be passed by the timing filters but high enough to interfere with the signal being transmitted, it is evident that frequency stabilization is indicated. The pulse-repetition frequency is completely determined at the transmitting terminal and therefore can and should be kept from varying by more than about one part in 10^6. The short-time stability should be better; this should eliminate frequency deviations from consideration.

V. REQUIRED SYSTEM PERFORMANCE

5.1 *General Requirements*

We are considering in this paper the possibility of transmitting broadband signals over long distances. For purpose of illustration, let us assume that we are interested in a system which is capable of transmitting two television channels on one RF carrier. It appears likely that such transmission would require a 20-mc sampling rate and eight digits per sample, resulting in a pulse-repetition frequency of 160 mc. For present purposes we need have only a moderately accurate estimate of such requirements, since any general conclusions we draw would be unaffected by reasonable deviations from the 160-mc figure. Such a system would be capable of transmitting other types of signal; for example, it might accept and deliver signals in the same form in which they are transmitted over the L3 coaxial cable. The L3 system, which is coming into use in the telephone plant, is described in Ref. 6.

In addition to the granularity noise resulting from quantization, the signal recovered at the end of a PCM system will contain noise resulting from errors in pulse pattern produced during transmission through the system. If the increment in time deviation at a single repeater, resulting from noise or other timing deficiencies, becomes large, there is a possibility that the resultant misalignment between signal pulses and timing pulses will result in errors in the transmitted pulse pattern. We have built an experimental system[4] employing simple timing circuits and found that the errors produced in this system by the effects of random noise on timing were entirely negligible in comparison with the number of errors produced by amplitude effects. Since this should be the case in general, we shall limit our consideration in this discussion to the effects of time deviations, or weave, upon the recovered signal.

5.2 *Over-all Effect of Time Displacements*

In operation, a PCM or other binary system transmits information sample by sample. Normally, these samples are taken at perfectly regular intervals and should be recovered in the same manner. Transmission through a system in which the timing wave suffers deviations in phase due to noise or other defects is equivalent to transmission through a system which includes a varying amount of delay. The spacing between recovered samples varies in accordance with the time deviations. If the rate of deviation is considerably less than the frequency of the recovered signal, the distortion takes the form of pure phase modulation of this signal. Fig. 9 shows the effect of time deviation upon a sine wave which has been coded, transmitted and decoded. The recovered wave is seen to be distorted. The effects of time deviations can best be determined by considering various types of signals separately. Frequency-division voice multiplex and color television appear to place the most stringent requirements on timing. They are the only types of signals which are considered in detail here. The requirements imposed by other types of signals can be calculated by similar procedures.

5.2.1 *Color Television*

Color TV has, in addition to those of black-and-white TV, a requirement that transmission through the system must not cause any appreciable change in the phase relationship between the color carrier and the reference carrier derived from the color bursts. These two carriers are at the same frequency (3.58 mc). Although the color carrier and the color bursts would be subjected to the same type of time deviation during transmission, the results would be different in the final TV receiver. To obtain reductions in effects produced by noise or interference, the reference carrier in a TV receiver is derived from a circuit having a time constant so long that, from our standpoint, we can consider the phase of its wave to be fixed. The color carrier, on the other hand, must be derived from a circuit which allows rapid variations of phase. Therefore rapid variations of delay cause corresponding changes of the phase relationship between these two 3.58-mc carriers. It is con-

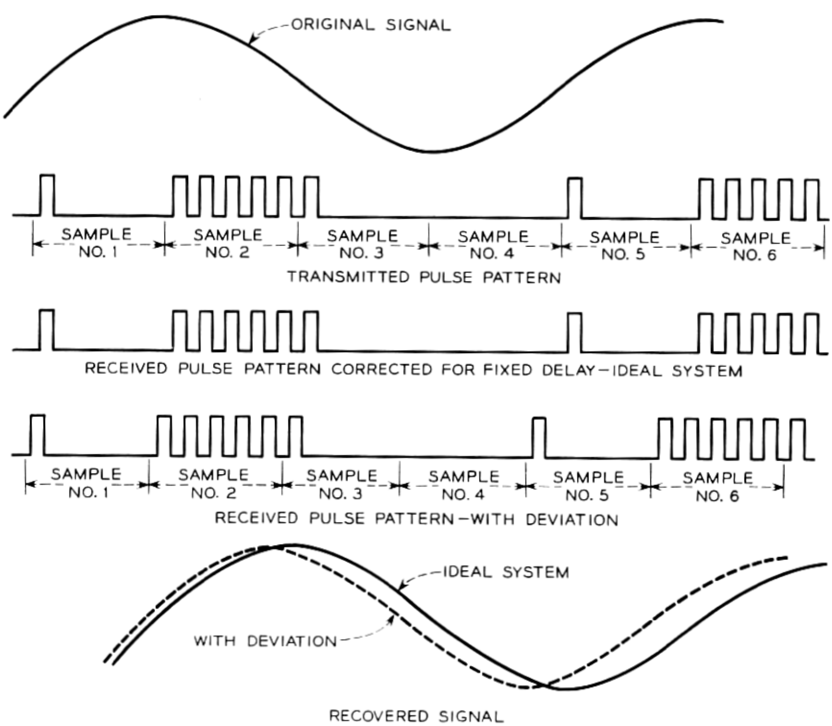

Fig. 9 — The effect of timing-wave phase deviations on a transmitted signal.

venient to express system requirements in terms of allowable phase deviations at pulse-repetition frequency. Change of delay, in millimicroseconds, is common to all frequencies being transmitted through the system, but resultant change of phase, in degrees, is proportional to frequency, so that

$$\frac{\Phi_r}{\Phi_{3.58}} = \frac{f_r}{3.58 \times 10^6},$$

where $\Phi_{3.58}$ is the change of phase of the 3.58-mc signal and Φ_r is the corresponding change of phase at pulse-repetition frequency. For a repetition frequency of 160 mc,

$$\Phi_r = 44.7\, \Phi_{3.58}.$$

In other words, for every degree of phase deviation permitted to the color carrier 44.7 degrees, or about 0.8 radian, are permitted to the 160-mc pulse carrier.

5.2.2 *L3 Frequency-Division Speech Multiplex*

After decoding and filtering, the recovered signal for this system has a spectrum extending approximately from 300 kc to 8 mc. This spectrum was originally derived by combining a large number of voice channels spaced 4 kc apart and each occupying a band of about 3 kc. To become useful signals, each of these bands must be filtered out of the complete assemblage and moved back down to the audible frequency range. For the present, our interest is in the composite signal before this final filtering [see Fig. 10(a)]. If there is time displacement of samples in the course

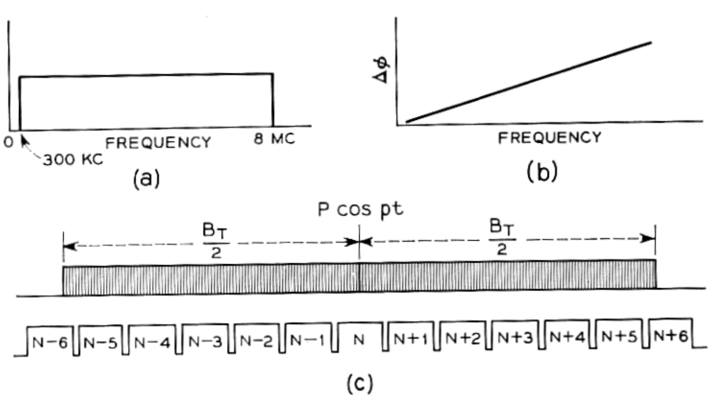

Fig. 10 — The L3 multiplex system: (a) baseband frequency range; (b) phase deviation produced by a given time deviation; (c) noise spectrum.

of transmission through the system, each component of the composite signal will be displaced in time by the same amount. Each component will thereby be subjected to a spurious phase modulation, the magnitude of which will depend linearly upon the frequency of the component; i.e., there will be a triangular distribution of phase modulation as shown by Fig. 10(b). Whether the resultant sidebands produce noise in the channel itself or crosstalk into adjacent channels depends upon their spacing from the signal. In Appendix C it is determined that the maximum rms phase deviation, Φ_r, which can be tolerated at a pulse-repetition frequency of 160 mc and still meet the assumed multiplex requirements is equal to 0.177 radian. W. R. Bennett[2] and E. D. Sunde have also analyzed this problem and arrived at substantially the same value of tolerable deviation.

We have seen that, for color TV, we can tolerate about 0.8 radian of timing-wave phase shift for every degree of allowable phase shift of the color carrier. For L3 voice multiplex we can tolerate only about 0.20 radian of timing wave phase fluctuation. Then, unless the color carrier phase must be held closer than one degree, it appears that color TV transmission requirements are much less stringent than those for voice multiplex. However, we cannot ignore the color TV requirements until we assure ourselves that this signal does not produce more phase deviation due to detuning, amplitude-to-phase conversion, etc., than is produced by the multiplex signal.

VI. ATTAINING REQUIRED SYSTEM PERFORMANCE

Since timing wave phase deviations accumulate along a chain of repeaters, it is evident that regeneration cannot be repeated indefinitely. The question then appears to be: "Is it practical to build equipment which will regenerate a few hundred times and still provide an output signal which will meet the rather stringent requirements discussed above?" Consider first the effects of random noise: To avoid an excessive number of errors due to purely amplitude effects it is necessary to maintain a signal-to-noise ratio of about 23 db* at each repeater for a chain of a few hundred repeaters. If we choose as a typical, practical system, one with a 320-mc RF bandwidth providing a 23-db S/N ratio and with timing circuits each having a Q of 200 at a pulse repetition frequency of 160 mc, the timing wave phase deviations due to random noise in 200 repeaters can be calculated from (15) to be $\Psi_{\text{rms}} = 0.056$

* This figure, which is the ratio of RF power at the peak of a pulse to rms noise power, is based on experimental results involving incomplete regeneration. An improved regenerator might require only a 19- to 20-db ratio.

radian, which is below the L3 voice multiplex requirements which we assumed.

We have seen that phase deviations may also arise as a result of changes of pulse pattern. Consideration of the effects of changes of pulse pattern leads us to believe that a simple timing circuit will meet system requirements if these effects exist alone. However, since detuning, amplitude-to-phase conversion and random noise will probably exist simultaneously, it may be highly desirable to obtain some improvement in performance over that provided by the simple circuits. This indicates a need for circuits which provide higher Q without sacrifice of stability. Fortunately such circuits exist and are not unduly complicated. For example, a phase-locked oscillator can be made to be the equivalent of a stable, high Q tuned circuit. Furthermore, only a limited number of these exalted-Q circuits need be employed in a system to obtain considerable advantage, as can be seen from the following example. As a reference system, assume 100 repeaters, each with a timing circuit having a Q of 100. If, at the output of the system, we insert one timing circuit with a Q of 10,000 we obtain an improvement in timing signal-to-noise ratio of about 10.5 db. If 10 of the narrow-band circuits are distributed throughout the system the improvement increases to approximately 16 db. If all repeaters are provided with the sharp circuit the total improvement is about 20 db. A practical arrangement appears to be to use a simple type of timing circuit at all unattended repeaters, with much narrower circuits at those which are attended.

VII. CONCLUSIONS

In the timing portion of a chain of regenerative repeaters (whether self-timed or externally timed) noise builds up in much the same manner as in an analog transmission system. For the purpose of analysis, this portion of a system can then be considered as a separate analog channel. The major disturbing effect of noise is the production of phase deviations of the timing wave, which in turn result in deviations of the times of occurrence of the output signal pulses.

External timing, when compared with self timing, has some definite advantages in certain systems but has disadvantages in others. Whether it is preferable to employ self timing or external timing in a particular system will depend upon many considerations beyond the scope of this paper, including the number of channels being transmitted and the delay stability of the transmission medium.

Indications are that, for long chains of repeaters, other effects, such as those due to changes of pulse pattern, may be more detrimental to timing than the effects of random noise. Although noise and the effects of changes of pulse pattern on timing limit the number of repeaters of a given design which can be operated in tandem, it appears practicable to extend systems to a length of several hundred repeaters. Although most system requirements might be met by employing simple tuned circuits as timing filters it may prove to be preferable to obtain the considerable improvement which results from addition of a relatively few circuits of a different type which provide decreased bandwidth without sacrifice of tuning stability.

VIII. ACKNOWLEDGMENTS

W. M. Goodall first pointed out the possibility of treating the timing part of a chain of repeaters as a separate analog channel. The writer wishes to thank H. T. Friis, W. R. Bennett and H. E. Rowe for the use as background of material which is not yet published as well as for the specific material mentioned here. He is also indebted to J. C. Schelleng and others for many helpful suggestions in the preparation of this paper.

APPENDIX A

Random Noise in a Single Repeater with a Square-Law Detector

As a starting point, we assume that all pulses are present and that the RF filtering is so adjusted that the RF signal into the detector can be expressed as

$$f(t) = \frac{A}{2}(1 + \cos 2\pi f_r t) \cos 2\pi f_c t + V_N . \tag{18}$$

The signal function is shown on Fig. 6(b). The corresponding envelope is indicated by Fig. 6(a) and the frequency spectrum by Fig. 6(c); V_N represents the voltage arising from noise which is assumed to have a uniform power density of W_0 watts per cycle of bandwidth. If we follow the procedure of S. O. Rice[7] and assume the detector to have a current-voltage characteristic given by

$$I = \alpha V^2, \tag{19}$$

we obtain the detector output currents by squaring (18) and multiplying by α.

The recovered timing wave (at frequency f_r) is given by

$$V_t = \frac{\alpha A^2}{4} \cos 2\pi f_r t, \tag{20}$$

and, for the timing wave power,

$$W_s = \frac{\alpha^2 A^4}{32}. \tag{21}$$

An expression for the noise spectrum at the output of the detector is given by Rice's equation (4.5–17) in Ref. 7. If we assume that our signal-to-noise ratio is large, we can neglect the cross-products among noise components in comparison to the cross-products between signal components and noise components. Since our timing circuit selects only those noise components which lie in a narrow band about f_r at the output of the detector, we need consider only the corresponding terms in Rice's equation. We find that the timing noise power at the output of the filter is

$$W_{na} = \frac{\alpha^2 A^2 W_0 B_t}{2}, \tag{22}$$

where B_t is the effective bandwidth of the timing filter. The timing wave signal-to-noise ratio is

$$\frac{W_s}{W_{na}} = \frac{A^2}{16 W_0 B_t}.$$

Since the RF signal power, P_s, at the peak of a pulse is equal to $A^2/2$, this equation can be written

$$\frac{W_s}{W_{na}} = \frac{P_s}{8 W_0 B_t}. \tag{23}$$

APPENDIX B

Random Noise — Chain of Repeaters with Tuned Circuits

From the standpoint of timing a chain of self-timed repeaters is equivalent to the chain of tuned circuits of Fig. 11. There will be unity gain, at midband, from one circuit to the next and noise arising at the input to each of the amplifiers shown.

H. T. Friis has shown that, for a chain of m such tuned circuits in tandem, the effective bandwidth is given by

$$B_{\text{eff}} = \frac{\pi f_r}{2Q} h_m, \tag{24}$$

where

$$h_m = \frac{(2m)!}{(2^m \times m!)^2} \quad \text{and} \quad Q \gg 1. \tag{25}$$

The first five values of h_m calculated from (25) are:

m	1	2	3	4	5
h_m	1	0.5	0.375	0.312	0.27

For large values of m,

$$h_m \doteq \frac{1}{\sqrt{\pi m}}. \tag{26}$$

Noise arising in the last amplifier, or repeater, of the chain will be filtered by one tuned circuit, that arising at the next-to-the-last amplifier will be filtered by two circuits and so on to noise at the input to the chain which will be filtered by all N circuits. The total noise power contributed by the N repeaters is given by

$$(W_{na})_N = \frac{\pi f_r}{2Q} \left(\sum_{m=1}^{m=N} h_m \right) d_0 , \tag{27}$$

where d_0 is the noise power density at the input to a single amplifier.

If we use the simpler expression for h_m as given in (26),

$$(W_{na})_N \doteq \frac{\pi f_r}{2Q} \left(\sum_{m=1}^{m=N} \frac{1}{\sqrt{\pi m}} \right) d_0 . \tag{28}$$

Since (26) gives a good approximation to h_m only for values of m greater than about five, we can increase the accuracy of (28) by subtracting out the first five terms and replacing them by the values of h_m obtained

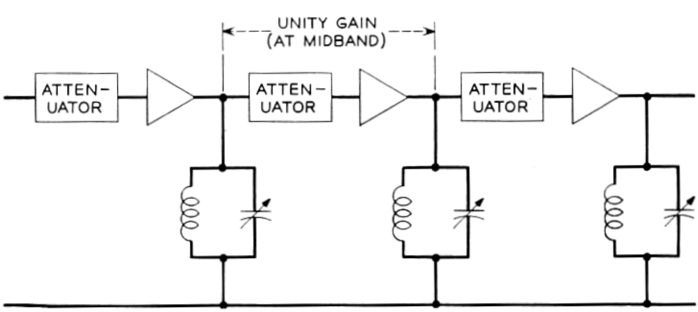

Fig. 11 — Simple tuned circuits in tandem.

from the above table. Equation (28) then becomes

$$
\begin{aligned}
(W_{na})_N &= \frac{\pi f_r}{2Q}\left[\frac{1}{\sqrt{\pi}}\left(\sum_{m=1}^{m=N}\frac{1}{\sqrt{m}} - \sum_{m=1}^{m=5}\frac{1}{\sqrt{m}}\right) + 2.46\right] d_0 \\
&= \frac{\pi f_r}{2Q}\left[\frac{1}{\sqrt{\pi}}\left(\sum_{m=1}^{m=N}\frac{1}{\sqrt{m}} - 3.23\right) + 2.46\right] d_0 .
\end{aligned}
\tag{29}
$$

Since

$$\sum_{m=1}^{m=N}\frac{1}{\sqrt{m}} = 2\sqrt{N} - 1$$

to a fair degree of accuracy, (29) becomes

$$(W_{na})_N \doteq \frac{\sqrt{\pi}\, f_r \sqrt{N}}{Q}\, d_0 = \frac{1.77 f_r \sqrt{N}}{Q}\, d_0 . \tag{30}$$

From (22) of Appendix A we see that the appropriate value of d_0 is

$$d_0 = \frac{\alpha^2 A^2 W_0}{2}.$$

Then,

$$(W_{na})_N = \frac{1.77\alpha^2 A^2 W_0 f_r \sqrt{N}}{2Q}. \tag{31}$$

The timing-wave power is the same at the end of the chain as for a single repeater. From (21) of Appendix A,

$$W_s = \frac{\alpha^2 A^4}{32},$$

and

$$\left(\frac{W_s}{W_{na}}\right)_N = \frac{A^2 Q}{2 \times 14.2 W_0 f_r \sqrt{N}} = \frac{P_s Q}{14.2 W_0 f_r \sqrt{N}}. \tag{32}$$

If we consider only the quadrature component of the noise,

$$\left(\frac{W_s}{W_n}\right)_N = \frac{P_s Q}{7.1 W_0 f_r \sqrt{N}}. \tag{33}$$

APPENDIX C

Required Performance for Voice Multiplex

In this section, the maximum tolerable phase deviation of the timing wave at the end of a chain of repeaters is calculated from parameters based on the L3 system.

As an aid to understanding the effects of time deviations let us first consider the case where a single audio tone is being transmitted through one voice channel. After decoding and filtering, this tone will result in a single frequency lying somewhere in the range between 300 kc and 8 mc (see Fig. 10). If the samples from which this frequency is derived vary in time the component will be phase-modulated. The spacing of the resultant sidebands from the channel frequency is determined by the rate at which its phase is being deviated. Since we are interested in determining results for the voice channel which is most affected, we choose a channel at the upper end of the band but far enough inside the band that noise sidebands produced when it is active do not exceed 8 mc. Let the decoded signal component resulting from the tone be represented by $P \cos pt$. If the phase deviation is sinusoidal, as it will be if produced by a single interfering wave somewhere in the system, the decoded output will be:

$$B(t) = P \cos (pt + \Phi \cos qt), \tag{34}$$

where Φ is the magnitude of the phase deviation and q is the angular frequency at which this deviation takes place. By expanding (34) and limiting consideration to the case where the deviation is small, the approximate value of signal is:

$$V(t) \doteq P \cos pt - \frac{P\Phi}{2} [\sin (p + q)t + \sin (p - q)t]. \tag{35}$$

Thus, $V(t)$ is seen to consist of the signal, $P \cos pt$, plus two sidebands spaced from it by a frequency corresponding to q. The signal power in a single channel carrying the tone $P \cos pt$ is $P^2/2$. The power in the two sidebands expressed by (35) is

$$\left(\frac{P\Phi}{2}\right)^2 = \frac{P^2\Phi^2}{4}. \tag{36}$$

When the phase deviations are produced by noise or other random effects each signal will have sidebands distributed about it and extending over the band B_T, as shown by Fig. 10(c), where B_T is the effective bandwidth of all of the timing circuits in tandem. Fig. 10(c) shows the sideband power to be evenly distributed over H voice channels, where H is the number of voice bands encompassed by the timing-circuit band. Thus,

$$H = \frac{B_T}{B_s}, \tag{37}$$

where B_s is the bandwidth of a signal channel.

If a number of adjacent channels are active simultaneously, each will have a spectrum as shown by Fig. 10(c). These spectra will overlap in frequency, as shown in Fig. 12. For the sake of simplicity, we first assume that all channels are equally loaded. It is evident from Fig. 12

that a total of H channels will throw interfering power into each voice channel. With the interference from each channel spread over H channels and with H channels contributing to each other, the end result is the same as though all of the interfering power produced by a channel were concentrated in a single channel. If T is the maximum tone carrying capacity of a voice channel, the maximum signal-to-noise ratio is obtained by dividing the maximum signal power, $T^2/2$, by the noise sideband power $(P^2\Phi^2)/4$, as expressed by (36). We obtain

$$\frac{S}{N}\,(\text{audio}) = \frac{2T^2}{P^2\Phi^2}. \tag{38}$$

Let us assume that multiplex requirements call for a signal-to-noise ratio of at least 60 db or 10^6. Then:

$$\Phi^2 = \frac{2T^2}{10^6 P^2}, \qquad \text{or} \qquad \Phi = \frac{\sqrt{2} \times 10^{-3} T}{P}. \tag{39}$$

If all channels are loaded to capacity with tone, $P = T$ and $\Phi = \sqrt{2} \times 10^{-3}$. This is the peak value of the maximum tolerable sinusoidal deviation. The corresponding rms deviation, whether sinusoidal or random, is 1×10^{-3} radian.

When the channels are loaded with speech, as they normally will be, the interference will be less than when they are loaded with tone. For speech loading the interference produced by each channel will still be

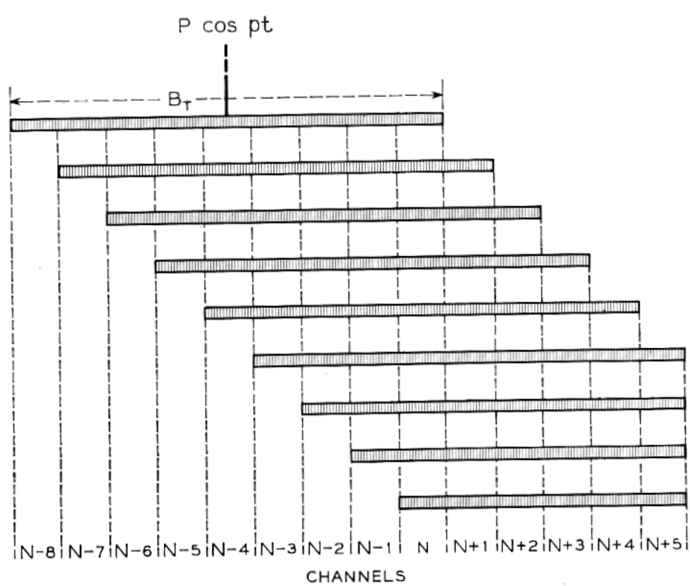

Fig. 12 — Multiplex system—overlapping noise spectra.

distributed over H channels, but the total interfering power will no longer be H times that produced in a single channel. This is the well-known speech multiplexing effect. At any instant the different channels will produce different amounts of disturbance, so that the total peak noise sideband power falling into a single channel is given by

$$\text{Peak noise power} = \frac{1}{H}\sum_{m=1}^{m=H}\frac{P_m^{\,2}\Phi^2}{2} = \frac{\Phi^2}{2H}\sum_{m=1}^{m=H}P_m^{\,2}. \tag{40}$$

To calculate H, we first determine the effective bandwidth of all of the timing channels in tandem. It is evident from (30) of Appendix B that, for the case of tuned circuits,

$$B_T = \frac{1.77f_r}{Q\sqrt{N}}\,. \tag{41}$$

As an example, consider a chain of 100 repeaters, each with a timing circuit Q of 100 and operating at a 160 mc repetition rate. For this system, $B_T = 284$ kc. For a voice-channel spacing of 4 kc, $H = 284/4 = 71$ channels. From the Holbrook-Dixon multiplex data[8] we find that 71 voice channels require approximately 10 db more peak power-handling capacity than is required by a single channel. Then,

$$\sum_{m=1}^{m=71} P_m^{\,2} = 10P^2,$$

$$\text{Total peak power noise} = \frac{\Phi^2 \times 10P^2}{2 \times 71}\,.$$

Using this expression for the noise power,

$$\frac{S}{N}\,(\text{peak audio}) = \frac{14.2T^2}{\Phi^2P^2}\,. \tag{42}$$

To obtain the rms signal-to-noise ratio, we must take into account the tone peak factor of 3 db and the speech peak factor of 13.6 db for 71 channels, as obtained from the Holbrook-Dixon data. Then,

$$\frac{S}{N}\,(\text{rms audio}) = \frac{156T^2}{\Phi^2P^2}\,. \tag{43}$$

As before in determining the required signal-to-noise ratio, we take $T = P$ and assume that the required power ratio is 60 db or 10^6. Setting the ratio as expressed by (43) equal to 10^6, we find $\Phi = 12.5 \times 10^{-3}$ radian at 8 mc, and

$$\begin{aligned}\Phi_{\text{rms}} &= 8.84 \times 10^{-3} \text{ radian at 8 mc}\\ &= 20 \times 8.84 \times 10^{-3} = 0.177 \text{ radian at 160 mc.}\end{aligned} \tag{44}$$

SYMBOLS

s = peak amplitude of sinusoidal timing wave.

W_s = mean power of the recovered sinusoidal timing wave.

δ = peak amplitude of a single interfering component whose frequency is near that of the timing wave.

δ_l = peak amplitude of each of the phase modulation sidebands resulting from limiting the timing wave and a single interference of peak amplitude δ.

W_{na} = mean noise power at the output of the timing filter and before limiting.

W_n = mean noise power at the output of the timing filter after limiting.

P_s = mean RF power at the peak of a signal pulse.

A = peak RF amplitude corresponding to P_s.

W_0 = RF noise power density in watts per cycle of bandwidth.

B_t = effective bandwidth of the timing circuit of a single repeater.

B_T = effective bandwidth of all of the timing circuits in a system operating in tandem.

B = bandwidth of RF section of repeater employing ideal flat filters.

M = the ratio of the total number of pulse positions contained in a given time interval to the number of pulses present in the same interval.

f_c = frequency of an RF carrier.

f_r = pulse-repetition frequency (usually considered to be 160 mc in this article).

N = number of repeaters in the chain of interest.

$\Psi_{\max}$ = maximum instantaneous phase excursion of the sinusoidal timing wave from its long time average value.

ϵ = rms phase deviations which result when a random pulse pattern excites a detuned circuit.

β = steady-state phase shift resulting from detuning of a tuned circuit.

k = ratio of the amount of detuning in cycles per second to the nominal resonant frequency of the circuit.

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