The Effects of Mode Filters on the Transmission Characteristics of Circular Electric Waves in a Circular Waveguide

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Departures from perfect geometry in a multi-mode circular waveguide used for circular electric wave transmission will cause coupling between the TE_{01} *signal mode and the other propagating modes. Such coupling causes serious degradation of signal fidelity after long travel distances. These effects have been studied in a long 5-inch-diameter guide in the 9000-mc band, for effective pulse travel distances up to 12 miles. Mode filters have been developed which suppress all spuriously generated modes in this guide. It is found that the insertion of these mode filters at reasonable intervals along the waveguide reduces pulse distortion to a negligible level and smooths the variations in the loss versus frequency characteristic.*

I. INTRODUCTION

A circular waveguide operated in the TE_{01} or circular electric mode is very attractive for use as a long-distance transmission medium.[1, 2] The attenuation coefficient of the TE_{01} mode decreases monotonically with increasing frequency in a given waveguide so that, in theory, one can transmit information through such a system with as low a loss as desired merely by operating at a sufficiently high frequency. In practice, in order for one to take advantage of this low-loss property, the frequency must be so high that the waveguide is operating in the multimode region. This introduces the probability that coupling will occur between the low-loss TE_{01} signal mode and the other propagating modes if there are any deviations from perfect circularity and straightness in the guide. The coupling resulting from a series of imperfections, with the resultant transfer of power back and forth between TE_{01} and other modes, can cause serious degradation of the transmission properties of a long waveguide. These mode conversion and reconversion effects present one of the most serious problems to the use of waveguide as a practical long-distance transmission medium.

Much research has already been done on this and other aspects of TE_{01} transmission properties. Discussions of earlier contributions are included in a paper by Miller[2] which views the over-all problems and possibilities of TE_{01} transmission systems and discusses in detail many specific problems, including mode conversion.

Several ways of combating mode conversion effects have been proposed. Good results can be obtained over narrow frequency bands by cancelling one imperfection against another by such actions as orienting individual pipe sections or adjusting the joints between sections. However, this approach destroys one of the most attractive features of waveguide as a transmission medium, its enormous bandwidth capabilities. An obvious solution is to eliminate the mode conversion by obtaining as nearly perfect a waveguide as possible. Unfortunately, the tolerances required to accomplish this in a practical waveguide appear to be unattainable. A more reasonable approach is to provide high loss in the waveguide to all modes except the desired TE_{01} mode, thereby attenuating the spuriously generated modes before reconversion to TE_{01} can occur. Ideally, this loss would be provided continuously along the waveguide, perhaps by using a modified guide structure such as a helix waveguide.[2, 3] However, considerable improvement can be achieved in the performance of a regular solid copper guide by inserting discrete mode filtering structures at intervals along its length.

The purpose of the experiments described in this paper was (1) to investigate the seriousness of mode conversion effects in a long waveguide which had been erected as carefully as possible, (2) to develop a discrete filter which would be effective against the spurious modes generated from TE_{01} in this waveguide and (3) to study the TE_{01} transmission properties of the line with these filters inserted at various spacings.

II. OBSERVATIONS OF MODE CONVERSION EFFECTS

The waveguide used in these experiments was the 4.73-inch I.D., 5.00-inch O.D. installation at Bell Telephone Laboratories in Holmdel, N.J. It was constructed in 1946 for operation at 9000 mc, where it will support 40 modes. The theoretical TE_{01} attenuation constant at 9000 mc is about 2 db per mile. It is the same line used and described in earlier papers.[1, 2]

The shuttle-pulse equipment developed by A. C. Beck[1] was used for the transmission tests. This system is shown in Fig. 1. A 0.1 microsecond pulse is introduced into the line and bounces back and forth between shorting pistons at each end. The transmitter and receiver coupling holes are so small that the loss through them is negligible compared to the heat loss of the line. By changing the variable delay, which controls the scope sweep, one can observe the pulse on the scope at any desired time after it has entered the line and, therefore, after any desired number of trips up and down the line. The coupling holes respond to many of the 40 propagating modes, but, because of the high attenuation of all but TE_{01}, only that remains after several trips.

This system provides good conditions for studying mode conversion-reconversion phenomena in the line because the pulse traverses the same guide many times. Since identical conversions and reconversions occur during each trip, their effects become very pronounced after many trips. Also, if no loss to unwanted modes is provided at the ends of the line, an imperfection which causes a conversion from TE_{01} to some other mode X as the signal pulse travels down the line will cause a reconversion of the same magnitude from X back to TE_{01} when the spurious pulse X travels back up the line. This reconverted TE_{01} pulse will, in general, not be in phase with the main TE_{01} signal pulse which is also traveling back up the line. The phase difference depends on the identity of mode X, the operating frequency and the distance between the imperfection and the reflecting piston. By moving the piston, one can change this phase difference and observe the period of the interference with piston motion, thereby identifying mode X. Since the same process occurs each trip, the magnitude of the interference increases

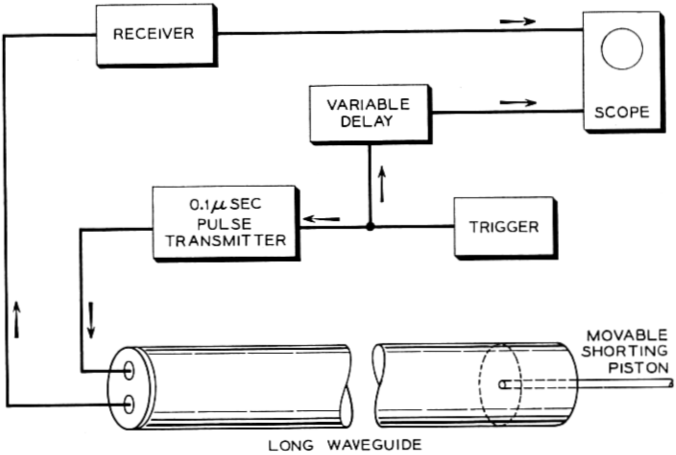


Fig. 1 — Block diagram of pulse transmission test equipment.

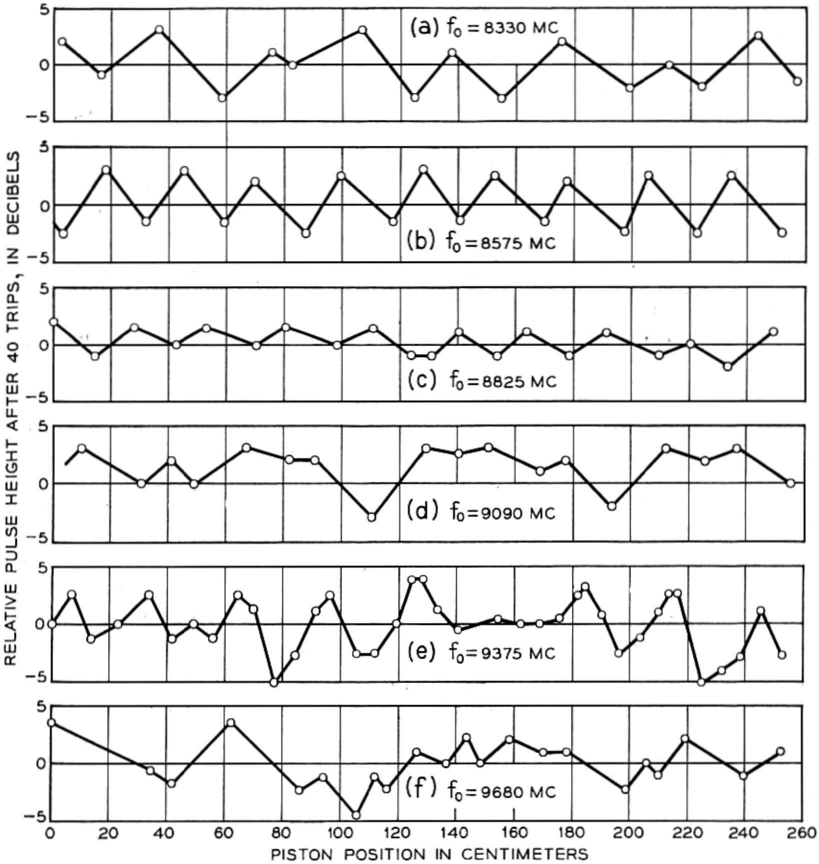


Fig. 2 — Measured variation of TE_{01} pulse height with piston position after 40 round trips in a 425-foot waveguide with no filters.

with the number of round trips the signal pulse has traveled, so very small effects become visible after many trips. Of course, this process occurs for each imperfection in the line with the magnitude and piston period characteristic of that imperfection and the modes it generates. Thus, for a line with many random imperfections, we can expect a very complicated fluctuation of observed pulse height with piston motion when no mode filters are in the line. If effective filters are inserted in front of the piston, power which is converted on the way down the line will be absorbed and can no longer reconvert to TE_{01} on the way back. Therefore, the fluctuations with piston motion should disappear when mode filters are inserted. The same general argument explains the effects observed when the operating frequency is changed and the piston is held stationary, except that, in this case, the electrical length of the waveguide is changed. Therefore, the effect of changing frequency is similar to that of linearly stretching or shrinking the guide at constant frequency.

The effect on the TE_{01} pulse of traveling many round trips in a waveguide with no filters is shown in Figs. 2 and 3. The guide was about 425 feet in length to the central piston position. Fig. 2 shows the measured variation of the pulse height with piston position after 40 trips at various frequencies. The available piston travel was about eight feet. Points were taken only at the extremes of the variations, so fine-grain accuracy was not obtained. Although some of the curves show regularities, the variation in general is more or less random, as is expected for a line with random imperfections. Photographs of the oscilloscope traces were taken at several frequencies at the piston positions giving the highest and the lowest observed pulse heights. Some of these photographs are shown in Fig. 3. Each trace shows three consecutive trips, the center trip having traveled the distance noted at the left of each pair of photographs. The receiver sensitivity has been adjusted in each case so that the pulses are the same size. Of course, the attenuation constant is actually much higher at the unfavorable piston positions, in some cases by a factor of 1.5.

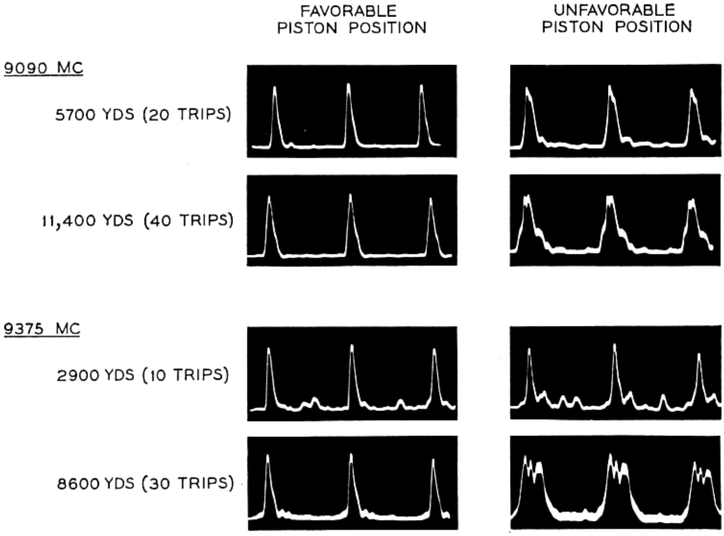


Fig. 3 — Oscillograms of pulses at good and bad piston positions for various travel distances in a 425-foot waveguide with no filters.

It is clear that the distortion of the pulse becomes very severe after many trips with the piston at the unfavorable position. This distortion occurs because the reconverted TE_{01} pulses, having traveled for a while in spurious modes, occur at different times and with different phases from those of the signal pulse. The small pulses along the baseline in the 9375-mc pictures at 10 trips are mostly modes other than TE_{01} which were introduced by the transmitting coupler and have not yet died out. However, all distortions of the pulses at larger distances are caused by mode conversion-reconversion interference.

Obviously, mode conversion has important effects on the transmission characteristics of this line. The next step in the investigation was to identify the spurious modes present and to discover a mode-filtering structure which would provide high loss to them.

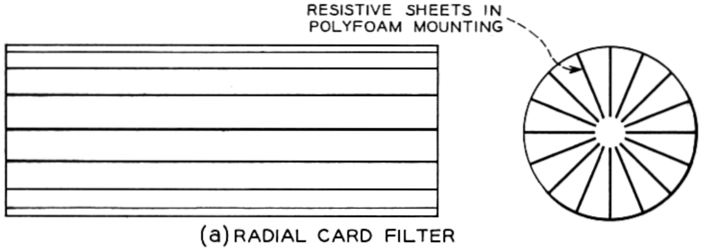
III. DEVELOPMENT OF AN EFFECTIVE MODE FILTER FOR THE 5-INCH LINE

Earlier work by A. P. King and others at Bell Laboratories has resulted in several mode filter designs.[2] Some of these are illustrated in Fig. 4. The radial card filter consists of resistive sheets mounted in foamed polystyrene along diametral planes of the guide. The TE_{0n} E-lines, being concentric circles, are always normal to the sheets. All other modes have either a radial or longitudinal component (or both) of electric field and therefore suffer loss in such a device. The spaced-ring line and helical line are structures which provide good guide-wall conductivity only in the circumferential direction, which is all that the circular electric wave requires, and poor conductivity in the longitudinal direction. Since all modes except the circular electric modes have wall current in the longitudinal direction, they suffer increased loss in such a structure.

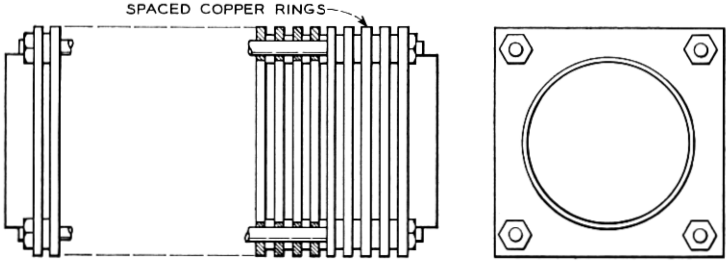
It was felt that the spaced-ring and helical structures would be best suited to the purposes of these experiments, since they should have lower TE_{01} loss than the radial card filter. The polyfoam mounting raises the TE_{01} loss sufficiently so that frequent insertion of radial card filters in a long line is not practical.

Since the important spurious modes in the 5-inch line were not as yet identified, it was decided to perform preliminary studies on the effectiveness of spaced-ring and helical filters, using modes which were readily generated with available apparatus. Therefore, an experiment was performed to measure the TE_{11} and TM_{11} losses and the TE_{11}-TM_{11} conversions in these structures. The TM_{11} mode is known to be a serious problem in TE_{01} transmission around bends.[2, 4]

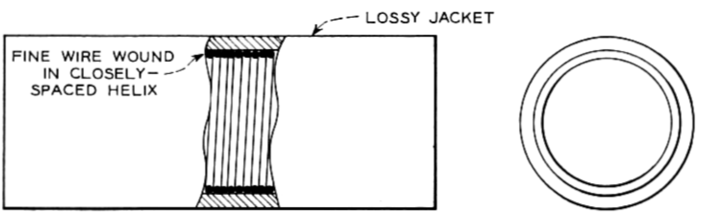
The experimental setup is sketched in Fig. 5. The pulse transmitter delivers a short pulse 0.006 microseconds in length at a 100-kc rate. Both this transmitter and the associated receiving equipment were developed and built by A. C. Beck and G. D. Mandeville of the Laboratories.[5] The filter under test is mounted at one end of about 300 feet of the 5-inch line, and the movable shorting piston is located inside the filter. Test pulses are then transmitted and received from the other end of the line by means of pure mode transducers.[6] With the setup as shown, the transmitted and received pulses can be in either the same or different modes. The short pulse gives enough time resolution for the modes of interest to be separated completely after 300 feet of travel



(a) RADIAL CARD FILTER



(b) SPACED RING LINE



(c) HELICAL LINE

Fig. 4 — Several kinds of discrete mode filters.

because of their different group velocities. Changing the position of the piston changes the exposed length of the filter under test. Because of the reflection in the piston, the effect of a filter of twice the length of the exposed section is measured. Normalization of the data to zero filter length is easily done by placing the piston in front of the filter.

The results of measurements of TM_{11} and TE_{11} losses and of TE_{11}-TM_{11} conversion versus filter length in a spaced-ring line are plotted in Fig. 6. The filter used consisted of $\frac{1}{16}$-inch-thick copper rings separated by $\frac{1}{16}$-inch air spaces. It can be seen that the interactions are fairly complicated. Recent work by Morgan and Young on the theory of helix waveguide[3] indicates that a set of noncoupled propagating modes can exist in a spaced-ring or helical structure, but that their field configurations are quite different from those of the modes found in a solid circular guide. Therefore, one expects the insertion of such a filter in a solid guide to produce conversions at the first guide-to-filter boundary between the incident solid-guide mode to several of the helix-guide modes, as well as conversions at the second boundary from these helix-guide modes back to several solid-guide modes, including the incident mode. Each of the

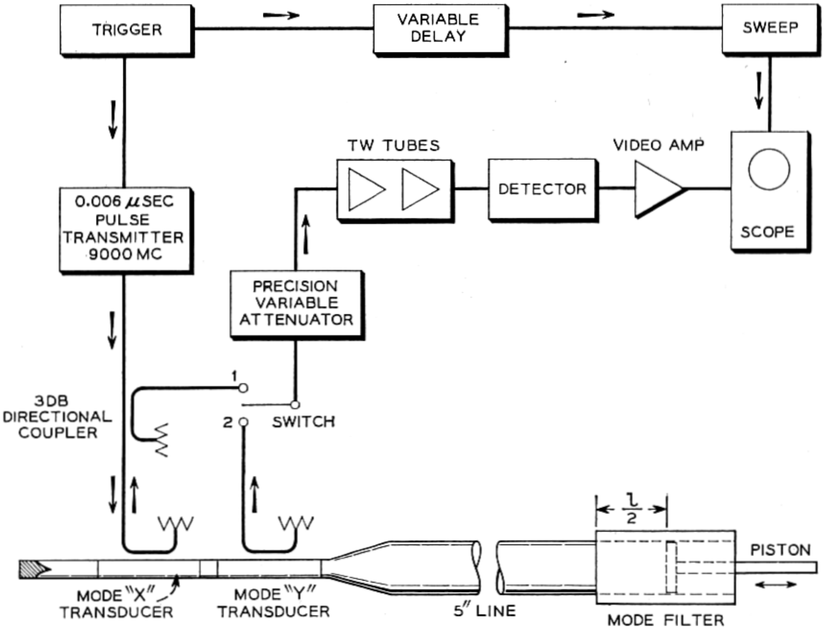


Fig. 5 — Block diagram of equipment for mode losses and conversions in mode filters, using the 9000-mc short pulser.

solid-guide modes departing from the filter will consist of several components, each produced from a different one of the modes inside the filter and with a relative phase that can be changed by changing the length of the filter. Therefore, an involved picture of loss versus filter length for the various modes is to be expected. A much simplified analysis of this interaction, using only two modes in the solid guide and two in the filter and disregarding reflections, is outlined in the Appendix. This solution gives fairly good agreement with the observed curves, and has been used to obtain the approximate attenuation constants noted on Figs. 6 and 7.

Spaced-ring lines of various spacings and ring thicknesses and with both air- and neoprene-filled spaces were tested in similar fashion. Helical lines wound of several wire sizes and with various jacket materials were also built and measured. Fig. 7 shows the results obtained with a helix wound of No. 30 Formvar-coated wire and coated with a carbon-loaded casting cement. This helix gave the best performance of the several filter designs tested. Therefore, several copies of it were made for use in the tests described below.

The effects on the TE_{01} pulse of the 425-foot line with several 18-

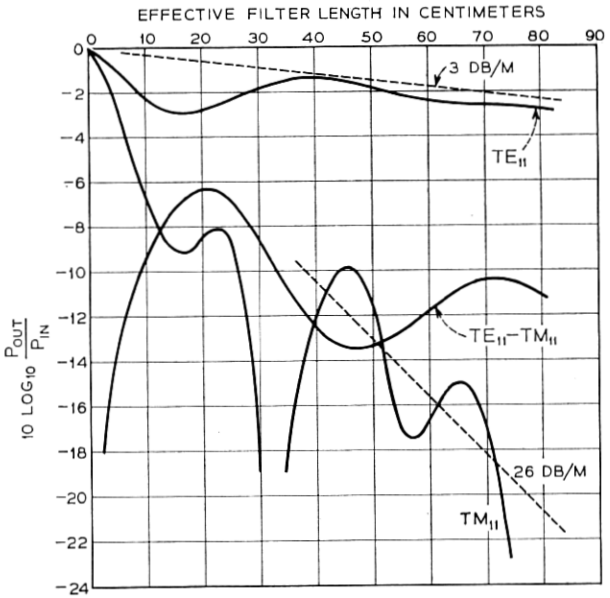


Fig. 6 — Measured TE_{11} loss, TM_{11} loss and TE_{11}-TM_{11} conversion vs. filter length in an air-spaced, spaced-ring mode filter.

inch helical filters spaced uniformly along its length are shown in Figs. 8 and 9. These measurements were taken in the identical fashion as the no-filter results given in Figs. 2 and 3.

Fig. 8 shows the measured pulse height variation versus piston position after 40 trips, with nine helices in the line. These curves were taken at the same frequencies and are plotted on the same scale as the curves of Fig. 2. The filters produce a marked reduction in the pulse height fluctuations at 8330, 8825 and 9090 mc. At 8575, 9375 and 9680 mc, the fluctuations are only slightly lessened and have become almost periodic. This regularity indicates mode conversion-reconversion interference between TE_{01} and a single other mode. The period of the variation at all these frequencies is exactly that to be expected from a TE_{01}-TE_{12} conversion-reconversion process.

Fig. 9 includes photographs of the pulses at the same frequencies and piston positions as Fig. 3, after eight helices were inserted in the line. The distortion is greatly reduced, and at 9090 mc, where the pulse height

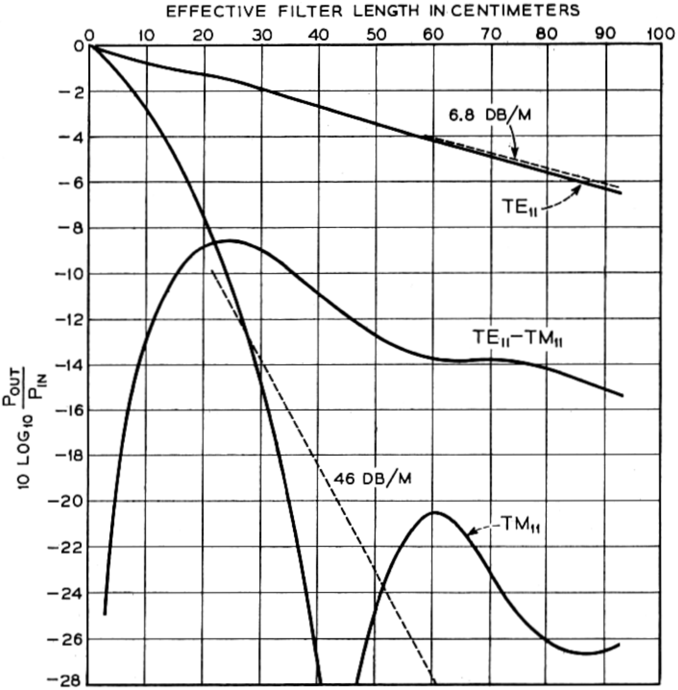


Fig. 7 — Measured TE_{11} loss, TM_{11} loss and TE_{11}-TM_{11} conversion vs. filter length in a helix mode filter constructed of No. 30 wire with a lossy jacket.

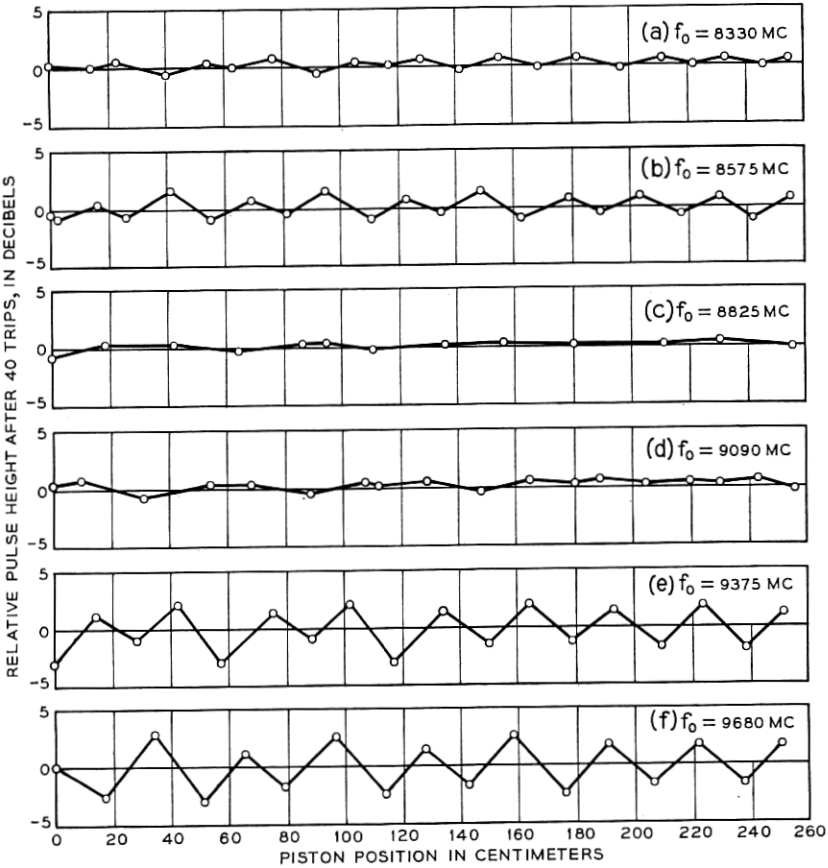


Fig. 8 — Measured variation of TE_{01} pulse height with piston position after 40 round trips in a 425-foot waveguide with nine helix mode filters.

fluctuation was greatly reduced, the difference between favorable and unfavorable piston settings is practically eliminated. Note that, because of this improvement, we were able to observe the pulses after more trips than in Fig. 3.

We conclude that the helical filters used in this experiment were effective against all spuriously generated modes except TE_{12}. At frequencies where the net TE_{01}-TE_{12} conversion in the waveguide is small, presumably because the electrical length between conversion centers is unfavorable to TE_{12} build-up, the pulse height and shape vary little with piston setting after the addition of the helices to the line. At frequencies where the TE_{01}-TE_{12} conversion is strong, the pulse shape and height vary with piston position and with the TE_{01}-TE_{12} conversion period.

The mode TE_{12} arises from such defects in the waveguide as offset axes, tilted joints or curvature in the longitudinal direction. It has the second lowest loss of all the modes in the 5-inch pipe at 9000 mc, its attenuation constant being only $2\frac{1}{2}$ times that of TE_{01}. Its field is concentrated in the center of the guide and is but loosely coupled to the wall, producing relatively small longitudinal wall currents. For this reason, it is not affected as greatly by the helical line as are other modes with higher longitudinal currents. The TE_{12} loss in the helices described above was measured and found to be about 1 db per meter, or about 0.5 db per filter. The recent work of Morgan and Young indicates that this loss could have been increased by the use of different jacket materials.

To improve their TE_{12} performance, existing helical sections were modified by the addition of diametral resistance sheets. The resulting filter is shown in Fig. 10. The sheets are made by coating thin Mylar film on both sides with resistive material, then cutting from the film a section shaped like two isoceles triangles joined at their apexes. This section is twisted and mounted inside a helix in such a way that the plane of one triangle is perpendicular to the plane of the other, each plane being defined by the axis and a diameter of the guide. This twisted sheet provides loss to all polarizations of TE_{12}.

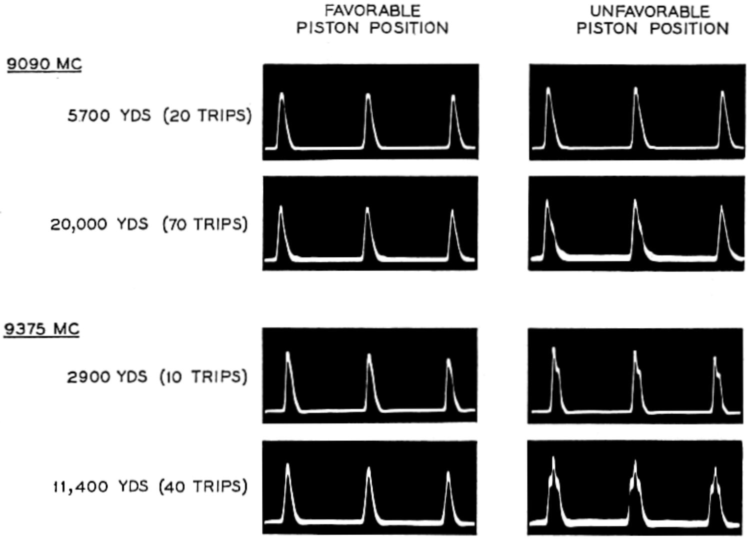


Fig. 9 — Oscillograms of pulses at good and bad piston positions for various travel distances in a 425-foot waveguide with eight helix mode filters.

Since it was impossible to run the piston inside the resistance-sheet helical filters, a slightly different measuring technique from that used with the spaced-ring and helical lines was employed to determine the optimum surface resistivity of the sheet. Instead of the test section being at the far end of the line (see Fig. 5), it was placed at the near end, just beyond the transducers. Using this system with the short-pulse equipment, modes produced at the filter by conversion from the incident mode are resolved in time by the trip down the line and back. On the return trip through the filter, each of these modes will again suffer various conversions, so that there will be pulses in several different modes at each of several times, each time corresponding to the travel time of a particular mode produced at the first trip through the filter. Therefore, by sending and receiving through pure-mode transducers and measuring travel times, one knows the transmitted mode, the mode in which the power traveled and the received mode — and quite an assortment of mode loss and conversion data can be obtained.

Fig. 11 shows the measured TE_{11} and TE_{12} losses and TE_{11}-TM_{11}, TE_{11}-TE_{12} and TE_{11}-TE_{31} conversions in a resistive-sheet helix versus the RF surface resistivity of the film. The TE_{11} and TE_{12} losses show definite maxima with resistivity. From these data it was decided that a film resistance of 200 to 300 ohms per square gave optimum performance. Accordingly, films were made in this range and mounted in the available helices.

A comparison of the effectiveness of the helical filters with that of the resistive-sheet helices at 9375 mc is shown in Fig. 12. With two regular helices in front of the movable piston, the 40th-trip TE_{01} pulse height varies about ±3 db with piston position, exhibiting the TE_{01}-TE_{12} period. With one resistive-sheet helix before the piston, the variations are reduced to ±0.5 db, which is within experimental error.

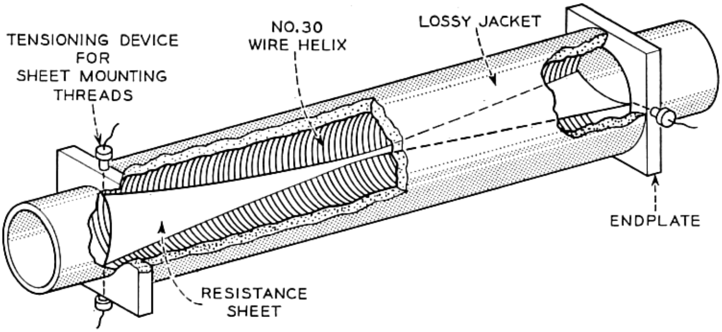


Fig. 10 — A resistance-sheet helix mode filter.

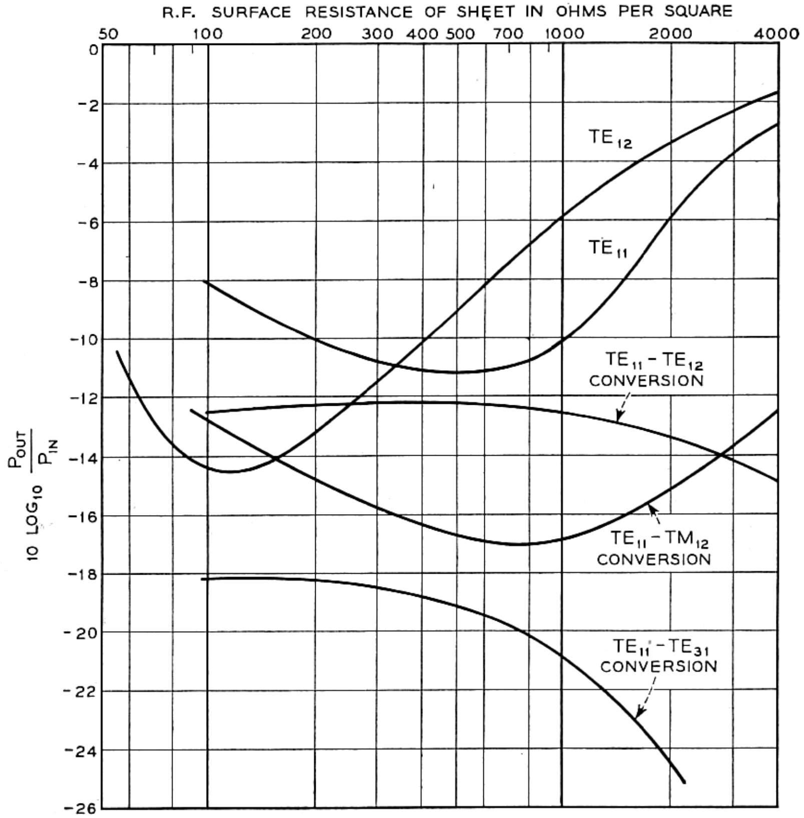


Fig. 11 — Measured losses and conversions vs. sheet surface resistance for several modes in resistance-sheet helix mode filters.

We conclude that the resistive-sheet helices are effective in suppressing all modes which are spuriously generated from TE_{01} in the 5-inch waveguide. They were used in the remainder of the work reported below.

IV. MEASUREMENTS OF ATTENUATION VERSUS FREQUENCY WITH VARIOUS FILTER SPACINGS

In a practical use of waveguide as a long-distance transmission medium, the effects of mode conversion-reconversion will become apparent not by moving pistons, but by variations in the attenuation versus frequency characteristic of the waveguide. For a broadband system, it is obviously desirable that this characteristic be flat over wide regions of the spectrum to be used. If mode conversion is present, one expects a more-or-less chaotic variation of the attenuation versus frequency, depending on the relative positions and phases of the converting centers, as described earlier.

The insertion of discrete mode filters at regular intervals in a long waveguide divides the line into a series of short independent sections, since the conversion occurring in one section cannot cause reconversion in another. At a given frequency, the attenuation constant of the line is the average of the attenuation constants of these independent sections. This average will vary about some mean value as the frequency is changed. By decreasing the filter spacing, and thereby increasing the number of independent sections in a given line, the average is taken over a larger sample and its variance about its mean should decrease. Also, since the maximum distance over which interactions can occur will be reduced, the average attenuation constant will vary less rapidly with frequency. Therefore, we expect that the attenuation versus frequency characteristic of a given line will show smaller and less rapid variations about a constant mean value as the filter spacing is decreased.

Measurements of this kind have been made in a 500-foot length of the 5-inch line. The results are shown in Fig. 13. Points were taken every 25 mc in frequency, except in the vicinity of 8800 mc, where a resonance in the sending apparatus at the TM_{42} mode cutoff frequency interfered with the measurement. The 0.1-microsecond pulse which was used has a width of about 20 mc, and the reproducibility of the pulse center frequency is ±5 or 10 mc; so the measurements are about as fine-grained as is possible with this equipment. The TE_{01} loss of the resistance-sheet

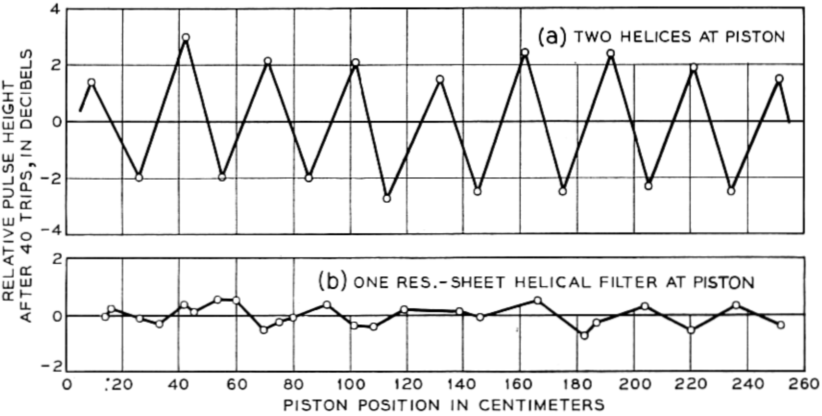


Fig. 12 — Comparison of effectiveness of helix and resistance-sheet helix mode filters at 9375 mc. Measured pulse height vs. piston position variations after transmission through two helices or one resistance-sheet helix.

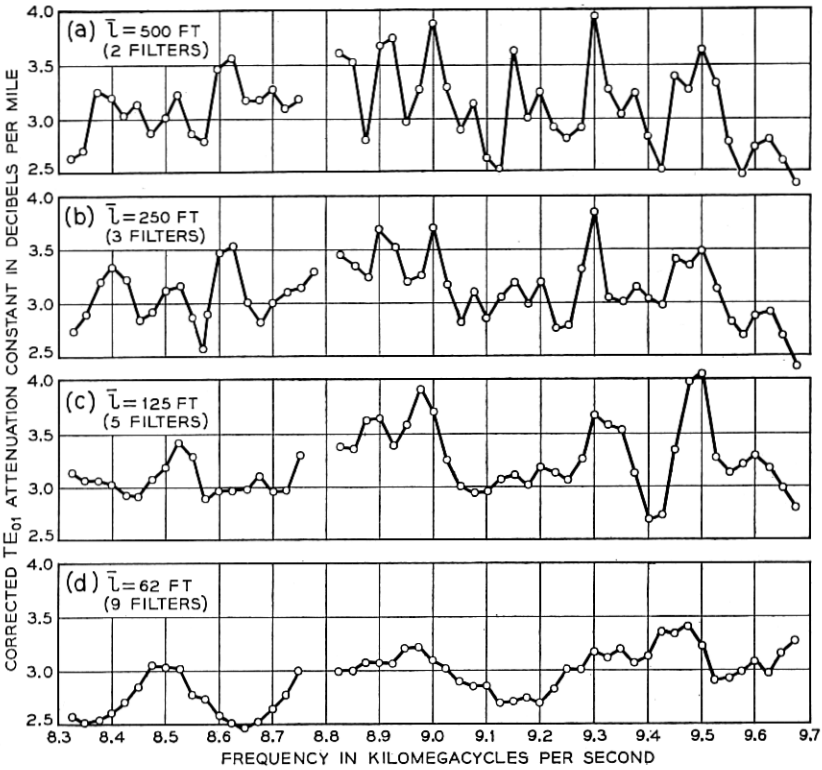


Fig. 13 — Measured TE_{01} attenuation constant vs. frequency in a 500-foot waveguide with resistance-sheet helix filters at various spacings.

helices, 0.027 db per filter, has been subtracted from these results to allow a direct comparison of the mode conversion effects.

Curve (a) of Fig. 13 shows the measured TE_{01} attenuation constant versus frequency with a resistance-sheet helical filter at each end of the 500-foot line. This arrangement decouples the mode conversion effects of one trip from the next trip and gives the true attenuation constant of the line as a whole. Curve (b) shows the measured TE_{01} attenuation constant versus frequency with a third mode filter inserted at the middle of the line, giving an average filter spacing of 250 feet. Curve (c) gives the results with an average spacing of 125 feet, and (d) gives those with an average spacing of 62 feet. As the filter spacing is decreased, the fluctuations in the attenuation constant versus frequency decrease in magnitude, as expected, and the peaks broaden out so that the curves become much smoother.

For curve (d), with an average filter spacing of 62 feet, the 500-foot line was divided into eight supposedly independent sections by the nine mode filters used. To determine whether these eight sections indeed acted independently, a further experiment was performed. The results are shown in Fig. 14. The line was divided in the middle and the two halves were measured separately. These measurements are shown as

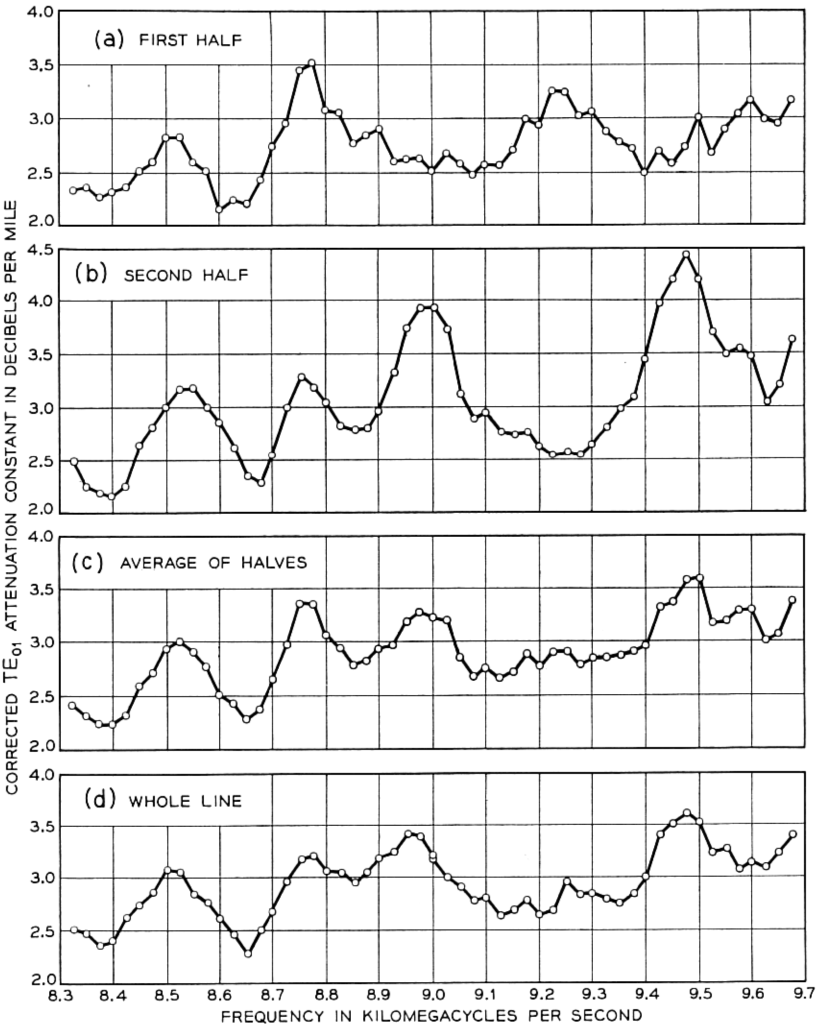


Fig. 14 — Measured TE_{01} attenuation constant vs. frequency for parts and for whole of 500-foot waveguide.

curves (a) and (b); they were made with a modified transmitting and receiving coupler so that the TM_{42} cutoff resonance at 8800 mc no longer appears. Although these curves are quite dissimilar, their average at each experimental frequency, which is plotted in curve (c), is in excellent agreement with the results obtained for the whole line, shown in curve (d). This agreement indicates that the individual line sections between filters do act independently and that the attenuation constant of the whole line is indeed the average of the attenuation constants of the individual sections.

These curves also illustrate the reduction in the magnitude of the fluctuations which is obtained by increasing the number of line sections. One expects that a much longer line with the same filter spacing would show much less variation of TE_{01} attenuation constant with frequency. It should be pointed out that this does not mean that the transmitted signal through the longer line will show less variation in amplitude. Elementary statistics tells us that the standard deviation of the attenuation constant should decrease as $1/\sqrt{n}$, where n is the number of equal independent line sections. Therefore, the standard deviation of the transmitted power level (expressed in db) should increase as $\sqrt{n}$.

V. CONCLUSION

These experiments have indicated that mode conversion-reconversion phenomena can have considerable effect on the TE_{01} transmission properties of a long multi-mode waveguide. It is very desirable to provide additional loss to modes other than TE_{01} by the use of mode filters. The mode TE_{12} is a particularly troublesome one because it is generated from TE_{01} in the types of imperfections likely to be encountered in a practical waveguide, has a low attenuation constant and is relatively difficult to suppress.

Mode filters have been built which are effective against all modes spuriously generated in a 5-inch waveguide at 9000 mc, including TE_{12}. They consist of a section of helix waveguide with a lossy jacket and diametral resistance sheets mounted inside.

With effective filters in a 500-foot waveguide, distortion of the TE_{01} signal pulse after it has traveled distances of the order of ten miles is negligible. However, the TE_{01} attenuation constant still fluctuates with frequency because of mode conversion-reconversion interactions in the line sections between filters. These fluctuations may be reduced considerably by decreasing the spacing between filters in the line. A spacing of about 60 feet was found to smooth the attenuation versus frequency curve of a 500-foot waveguide sufficiently for it to be reasonably flat over bandwidths of the order of 100 mc.

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APPENDIX

We shall discuss here the junction between a solid waveguide and a helix waveguide under the simplifying assumptions that (1) the solution can be written in terms of only two solid-guide modes and two helix-guide modes; (2) there are no reflections from the junction and (3) the junction is lossless and reciprocal.

Under these assumptions one can write a 2 × 2 transfer matrix $[T]$ to represent the transmission from solid to helix guide. Transmission from helix to solid guide is then given by $[\tilde{T}]$, the transpose of $[T]$. Therefore the transmission through a helix-guide section of length l inserted in a solid guide is given by the matrix equations

$$\begin{aligned}[O] &= [S(l)][E] \\ [S(l)] &= [\tilde{T}]\begin{bmatrix} e^{-\Gamma_x l} & 0 \\ 0 & e^{-\Gamma_y l} \end{bmatrix}[T],\end{aligned} \tag{1}$$

where $[O]$ and $[E]$ are column matrices representing the normalized output and input mode amplitudes, respectively, and Γ_x and Γ_y are the propagation constants in the helix guide of the two helix-guide modes.

When $l = 0$, we expect $[O] = [E]$, so that

$$[\tilde{T}][T] = [1]. \tag{2}$$

Conservation of energy requires that

$$[\tilde{T}][T^*] = [1], \tag{3}$$

where the asterisk denotes complex conjugate.

Equations (2) and (3) enable one to write the components of $[T]$ in the form

$$[T] = \begin{bmatrix} \sqrt{1 - b^2} & b \\ -b & \sqrt{1 - b^2} \end{bmatrix},$$

so that

$$[S(l)] = \begin{bmatrix} (1 - b^2)e^{-\Gamma_x l} + b^2 e^{-\Gamma_y l} & b\sqrt{1 - b^2}(e^{-\Gamma_x l} - e^{-\Gamma_y l}) \\ b\sqrt{1 - b^2}(e^{-\Gamma_x l} - e^{-\Gamma_y l}) & b^2 e^{-\Gamma_x l} + (1 - b^2)e^{-\Gamma_y l} \end{bmatrix}.$$

The absolute squares of the components of $[S(l)]$ represent the power transmission and conversions for the solid-guide modes resulting from the presence of the helix section. After some algebraic manipulation, they may be written

$$\begin{aligned} \mid S_{11}(l) \mid^2 &= 2b^2(1 - b^2)e^{-(\alpha_x+\alpha_y)l}\,[\cosh\,(\Delta\alpha l - \varphi) + \cos\,\Delta\beta l] \\ \mid S_{22}(l) \mid^2 &= 2b^2(1 - b^2)e^{-(\alpha_x+\alpha_y)l}\,[\cosh\,(\Delta\alpha l + \varphi) + \cos\,\Delta\beta l] \\ \mid S_{12}(l) \mid^2 &= 2b^2(1 - b^2)e^{-(\alpha_x+\alpha_y)l}\,[\cosh\,\Delta\alpha l - \cos\,\Delta\beta l] \end{aligned} \qquad (4)$$

where $\Gamma_x = \alpha_x + j\beta_x$, $\Gamma_y = \alpha_y + j\beta_y$,

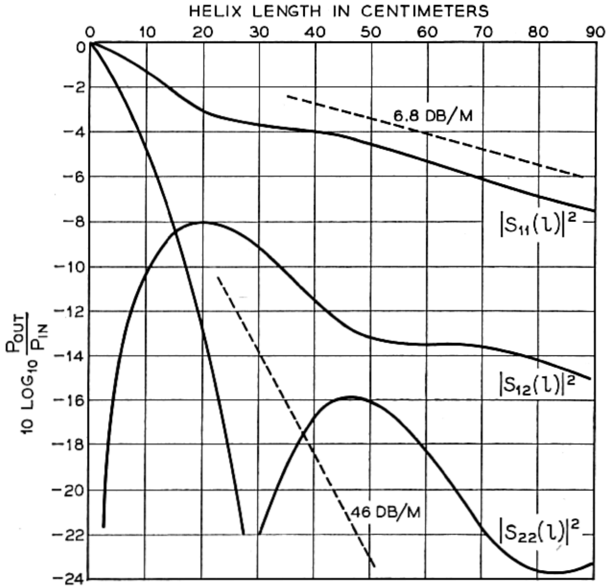

Fig. 15 — Plots of Equations (4) for $b^2 = 0.852$, $\alpha_x = 6.8$ db per meter, $\alpha_y = 46$ db per meter and $\Delta\beta = 10.6$ radians per meter.

and

$$\begin{aligned}\Delta\alpha &= \alpha_y - \alpha_x \\ \Delta\beta &= \beta_y - \beta_x \\ \varphi &= \log_e \frac{b^2}{1 - b^2}.\end{aligned}$$

Equations (4) are plotted in Fig. 15 for the values of the parameters which give the best fit to the TE_{11}-TM_{11} results for the helix of Fig. 7. A comparison of Figs. 7 and 15 shows good qualitative agreement. However, the lack of good quantitative agreement would indicate that the interaction is more complex than the simple case of the above analysis.

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