Design, Performance and Application of the Vernier Resolver*

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(Manuscript received May 29, 1957)

The Vernier Resolver is a precision angle transducer which, from the stand-point of performance, resembles a geared up synchro resolver, except that the step-up ratio between the mechanical angle and the electrical signal is obtained electrically.

Vernier resolvers with step up ratios of 26, 27, 32 and 33 have been designed and built.

The unit is a reluctance type, variable coupling transformer. By placing all windings on the stator, sliding contacts are eliminated. Both the stator and the rotor are laminated. Because of the averaging effect inherent in a laminated construction, the accuracy of the unit exceeds by many times the machining accuracy.

The performance of present experimental units is characterized by a repeatability of better than ±3 seconds of shaft angle, and a standard deviation error over one full revolution of less than 10 seconds of arc.

I. INTRODUCTION

The precise measurement of an angle is a basic operation in many technical fields. The observation of stars, mapping of land, machining in the factory are all operations which require angle measurements. Of course, an angle can be measured by reading a calibrated dial. However, in automatically controlled operations the angular position of a shaft has to be sensed electrically. The instrument which performs the conversion from a mechanical angle to an electrical output is called an angle transducer. One commonly used angle transducer is the synchro resolver.

Basically this is a variable coupling transformer with one primary winding and two output windings displaced 90 degrees from each other. The variable electrical coupling is accomplished by placing the primary

* The Vernier Resolver was developed under the sponsorship of the Wright Air Development Center.

winding on the rotor and the secondary windings on the stator or vice versa. The primary winding is excited from an alternating voltage source of, say, 400 cycles per second. The amplitudes of the induced secondary voltages of the synchro resolver are ideally proportional to the sine and cosine of the rotor orientation. These two induced, amplitude modulated voltages are the resolver output.

The accuracy of commercially available synchros is, at best, three minutes of arc. Certainly, this accuracy is sufficient for many applications. In the machining of precision parts and in field applications involving the measurement of elevation and azimuth of distant targets, however, accuracies down to 10 seconds of arc are required.

One might be tempted to try to meet this requirement by merely refining the present standard synchro. However, even if this refinement were possible, it still would be a difficult task to transmit this near-perfect synchro output and also to convert it into other analog forms without losing most of the added accuracy because of noise in the system. The transmission and conversion problem can be side-stepped by going to a so-called "two speed" or "vernier" representation of the angle. This representation is obtained by using two synchros; one, the low speed synchro, is positioned directly to the particular angle and the other, the high-speed synchro, is geared up with respect to the former. The angle is now represented by two synchro outputs. Assuming perfect gears the accuracy of this system is improved by the step-up ratio in the gearing.

This approach has been adopted in the past, but unfortunately, it has major disadvantages to it. First, precision gears of better than one minute of arc are expensive, relatively large and of limited life due to wear. Second, considerable torque is required to overcome the gear friction and the inertia effect of the high speed synchro. For these reasons, it is desirable to replace the geared-up synchro by a transducer which performs the step-up between input and output electrically. The vernier resolver is such an angle transducer.

The unit is a reluctance type, variable coupling transformer. By placing all windings on the stator, sliding contacts are eliminated. Both the stator and the rotor are built up of laminations. The step-up ratio is equal to the number of teeth on the rotor lamination. Prototype units have been built with step-up ratios of 26, 27, 32 and 33. The accuracy of these units is characterized by a standard deviation error of less than 10 seconds of arc. This high degree of accuracy is due largely to the averaging effect inherent in a laminated construction. The unit may be regarded, simply, as a device which senses the average orientation of all rotor laminations with respect to the stator. Because of the great number of laminations (one hundred in the present units) the effect of individual imperfections in laminations is greatly reduced.

In preparation for a close study of the vernier resolver we shall describe the performance of an ideal unit, and also introduce some technical terms. The output of the vernier resolver consists of two amplitude modulated voltages one of which is called the sine-voltage and the other the cosine voltage. The amplitudes of these voltages are proportional to the sine and cosine of "n"-times the rotor orientation. The factor "n" which, of course, is a function of the rotor configuration will be called the order of the resolver. We shall call the arctangent of the ratio of the secondary voltages — sine-voltage over cosine-voltage — the "signal-angle". Furthermore, to define a positive sense of rotation and to make the signal-angle definition unambiguous, we shall assume that, with continuous positive shaft rotation, the signal-angle runs through a sequence of cycles, each going from zero to 360°. Thus, one signal-angle cycle corresponds to a shaft rotation of $(1/n)$th; of one revolution. This angular interval is called the "vernier" interval.

II. DESIGN PRINCIPLES

2.1 *A Simplified Description*

Fig. 1 represents a simplified model of a third order vernier resolver. The unit consists of a laminated rotor with three equally spaced teeth and a laminated 4-pole-shoe stator. Each pole-shoe bears one exciting coil (not shown in the figure) and one output coil. Successive exciting coils are wound in opposite directions, connected in series, and energized from an ac source. Thus, successive pole-shoe fields alternate in phase. The two output windings each consist of two diametrical output coils connected in phase opposition.

If the rotor were a circular cylinder, the net voltage in either output winding would be zero. However, because of the three rotor teeth, the induced voltage of either output winding goes through three identical cycles per rotor revolution. Consequently, the amplitude E_c of the induced cosine-voltage e_c can be represented as a Fourier series of three times the shaft angle θ_m,

$$E_c = E_1 \cos (3\theta_m) + E_3 \cos [3(3\theta_m)] + \cdots , \tag{1}$$

where E_1, E_3 are the Fourier components of E_c with respect to $(3\theta_m)$.

The series is free of even harmonic terms because of the symmetry between positive and negative half-cycles. The expression for the amplitude E_s of the sine-voltage e_s is obtained by substituting $[\theta_m - (\pi/6)]$

for θ_m in (1);

$$E_s = E_1 \sin (3\theta_m) - E_3 \sin [3(3\theta_m)] + \cdots . \tag{2}$$

The magnitudes of the Fourier components depend on the design details of the unit. In a properly designed unit all higher order Fourier components are sufficiently small so that the induced signal voltages closely approximate those of an ideal third order resolver as expressed by the following equations:

$$E_c = E_1 \cos (3\theta_m), \tag{3}$$

$$E_s = E_1 \sin (3\theta_m), \tag{4}$$

$$\theta_S \equiv \tan^{-1} \frac{E_s}{E_c} = 3\theta_m , \tag{5}$$

where θ_S is the signal-angle.

2.2 *Analysis of Practical Case*

Fig. 2 shows an assembled unit, and typical stator and rotor laminations are illustrated in Figs. 3 and 4.

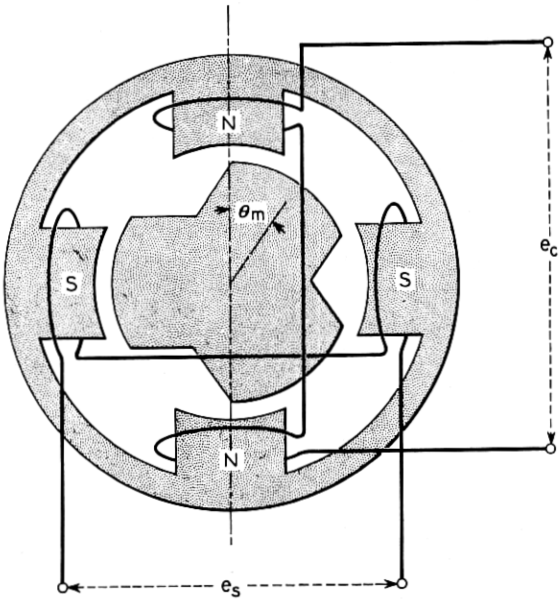


Fig. 1 — Schematic of a 3rd order vernier resolver.

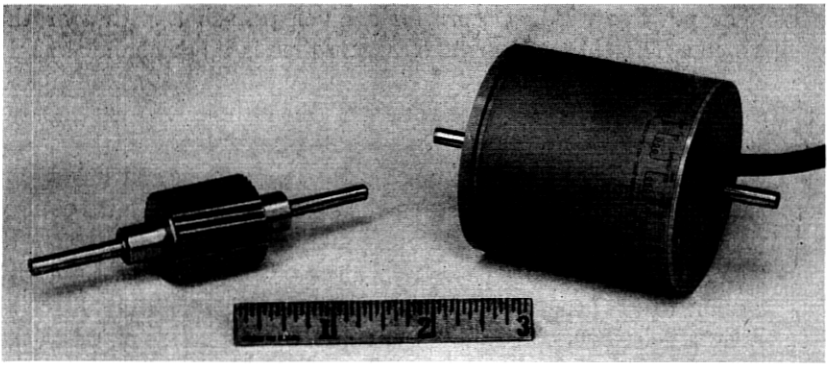


Fig. 2 — View of the assembled vernier resolver and its rotor.

The stator lamination is of ten pole-shoes, each pole-face having three teeth. All 30 teeth of the stator are equally spaced. The number of equally spaced teeth of the rotor lamination is equal to the order "n" of the resultant vernier resolver.

The exciting winding, as in the simplified model, produces ac magnetic fields alternating in phase from one pole-shoe to the next. Each of the two output windings is distributed over all ten pole-shoes.

To describe the turns distribution of these windings it is necessary to define the positive winding sense and the electrical angle of a pole-

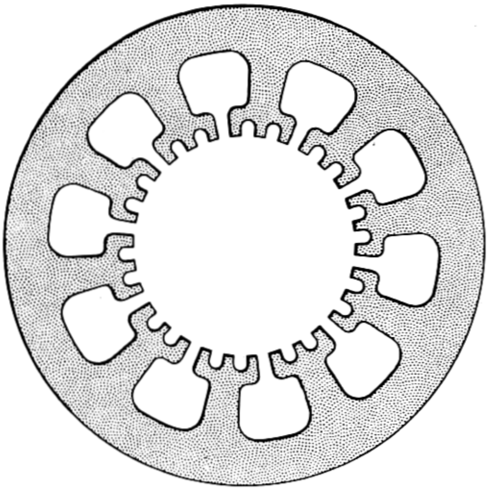


Fig. 3 — Stator lamination of the vernier resolver.

shoe. The positive winding sense for a given pole-shoe is that of the exciting coil. The electrical angle of a pole-shoe is its mechanical angle measured clockwise, with respect to a reference on the stator, multiplied by the number of rotor teeth (the order of this resolver).

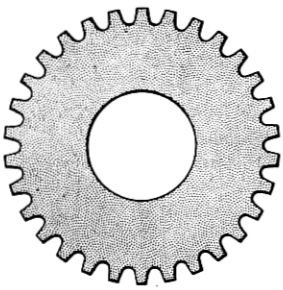
The winding which produces the cosine voltage, e_c, consists of ten coils, one on each pole-shoe, connected in series. Each coil has a different number of turns depending on the pole-shoe to which it belongs. Specifically, this number of turns is equal to a design constant "t" multiplied by the cosine of the electrical angle of the pole-shoe. Similarly, the winding which produces the sine-voltage, e_s, consists of ten coils, one on each pole-shoe. The number of turns of each coil is equal to the same constant "t" multiplied by the sine of the electrical angle of the particular pole-shoe.

With α_e being the electrical angle between adjoining pole-shoes and with the electrical angle of pole-shoe No. 0 equalling zero, the turns of the coils of the cosine-winding on pole-shoes No. 0 through 9 are:

$$\begin{aligned} t_{C0} &= t \cos (0) \\ t_{C1} &= t \cos (\alpha_e) \\ \vdots \quad & \quad \vdots \quad \vdots \\ t_{C9} &= t \cos (9\alpha_e) \,. \end{aligned} \tag{6}$$

Similarly the turns distribution, t_s, of the sine winding is:

$$\begin{aligned} t_{s0} &= \mathrm{t} \sin (0) \\ t_{s1} &= t \sin (\alpha_e) \\ \vdots \quad & \quad \vdots \quad \vdots \quad \vdots \\ t_{s9} &= t \sin (9\alpha_e) \end{aligned} \tag{7}$$



ORDER OF RESOLVER	NUMBER OF ROTOR TEETH
26	26
27	27
32	32
33	33

Fig. 4 — Rotor lamination of the vernier resolver.

To obtain the voltage induced in the output coils, the flux amplitude for each pole-shoe must be established. Defining the electrical angle of the rotor, θ_e, as its mechanical angle multiplied by its number of teeth and choosing θ_e to be zero when the center of a rotor tooth lines up with the center of pole-shoe No. 0, one can write for the flux amplitudes ϕ_0 through ϕ_9 of pole-shoes No. 0 through No. 9:

$$\begin{aligned} \phi_0 &= A_0 + A_1 \cos \theta_e + A_2 \cos 2\theta_e + \cdots \\ \phi_1 &= A_0 + A_1 \cos (\theta_e - \alpha_e) + \cdots \\ \vdots & \quad \vdots \quad\quad \vdots \quad \vdots \quad \vdots \\ \phi_9 &= A_0 + A_1 \cos (\theta_e - 9\alpha_e) + \cdots \end{aligned} \tag{8}$$

where $A_0, A_1, A_2, \ldots$ are the Fourier Components of ϕ.

The amplitude, E_c, of the voltage induced in the cosine winding is the sum of the products of the pole-shoe flux, ϕ_ν, measured in [volt sec] and the coil turns, $t_{c\nu}$, multiplied by the exciting current frequency in radians per second, ω:

$$E_c = \omega \sum_{\nu=0}^{9} \phi_\nu t_{c\nu} . \tag{9}$$

Substituting the values of $t_{c\nu}$ and ϕ_ν from (6) and (7) and neglecting all higher order Fourier components of ϕ, one obtains

$$E_c = \frac{10}{2} \omega A_1 \cos \theta_e . \tag{10}$$

Similarly one obtains for the amplitude E_s of the sine voltage:

$$E_s = \frac{10}{2} \omega A_1 \sin \theta_e . \tag{11}$$

As required, the two induced secondary voltages are proportional to the cosine and sine of the electrical rotor angle.

As shown in the appendix, an analysis which takes into account the higher order Fourier components of the pole-shoe flux shows that a sinusoidally distributed winding is sensitive solely to the so-called slot-harmonics. The order "m" of these harmonics is given by the expression

$$m = kq \pm 1 \tag{12}$$

where "k" is any integral positive number and q is the number of pole-shoes divided by the largest integral factor common to the number of pole-shoes and the number of rotor teeth. For instance, in the case of a

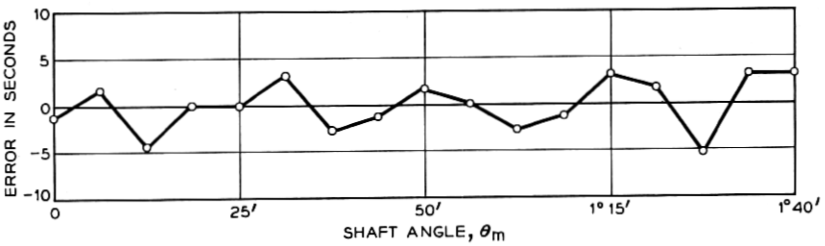


Fig. 5 — Error of a 27th order vernier resolver over $\frac{1}{8}$ vernier interval.

27th order vernier resolver this common factor is 1 and consequently the slot harmonics are of order: 9, 11, 19, 21, etc.

The effect of the slot harmonics can be reduced by the following means:

a) Selecting the dimensions as well as the number of rotor and stator teeth such as to keep the higher order flux components low.

b) Using a "skewed" rotor or stator, in which successive laminations are progressively displaced with respect to their angular orientation.

III. PERFORMANCE

Clifton Precision Products Co. built experimental resolver models of order 26, 27, 32 and 33 using the laminations shown in Figs. 3 and 4. The best results were obtained with 27th order resolvers. Their performance is described in the following sections.

3.1 *Repeatability and Accuracy*

The repeatability is better than ±3 seconds of shaft angle.

Figs. 5, 6 and 7 show the error curves taken on a 27th order vernier resolver after compensating with trimming resistors for the fundamental and second harmonic error with respect to the vernier interval. (In essence, the effect of these trimming resistors is either to add or to subtract a small voltage to one or both of the resolver signals.) Fig. 8 shows an error curve before trimming.*

3.2 *Temperature Sensitivity*

The error introduced by a temperature change of 70°C is less than 25 seconds of shaft rotation.

* The error curves really represent the combined error of the tested resolver itself plus that of the testing apparatus.

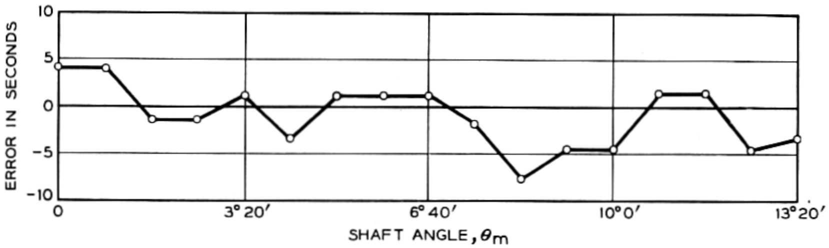


Fig. 6 — Error of a 27th order vernier resolver over one vernier interval.

It may be pointed out that the housing of the tested unit was of aluminum. A unit with a non-magnetic steel housing should be of lower temperature drift, because stator stack and housing would then have the same temperature coefficient of expansion.

3.3 *Transformation Ratio, Input and Output Impedances*

At maximum coupling the induced output voltage is 0.123 times the exciting voltage and is leading in phase by 6°.

The impedance of the input winding with the output windings open is $117 + j\,781$ ohms.

The impedance of the output windings with the primary winding shorted is $235 + j\,920$ ohms.

The effect of the rotor position on this impedance is hardly noticeable.

3.4 *Output Signal Distortions*

The harmonic content of the output signal at maximum coupling is:

fundamental 1.7 volts
2nd harmonic 0.2 mv
3rd harmonic 13.5 mv
5th harmonic 5.4 mv

The harmonic content of the output signal at minimum coupling (null voltage) is:

fundamental 1.6 mv
2nd harmonic 0.05 mv
3rd harmonic 2.0 mv

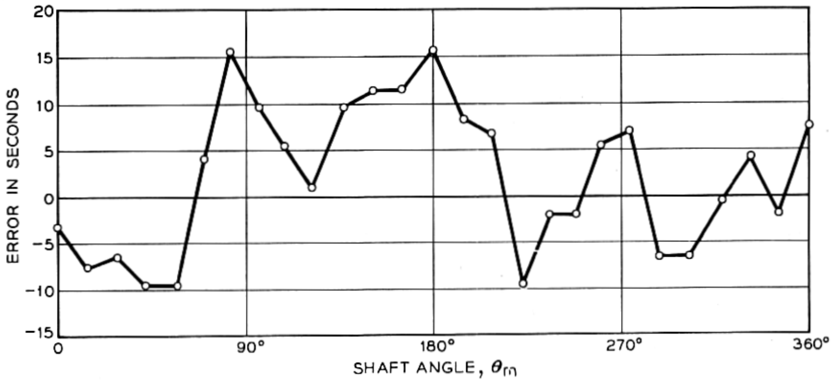


Fig. 7 — Error of a 27th order vernier resolver over one shaft revolution.

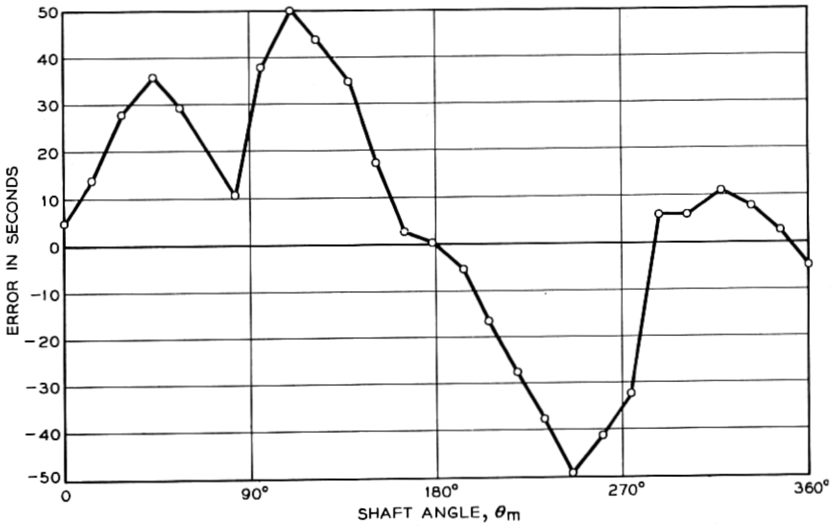


Fig. 8 — Error of a 27th order vernier resolver before trimming.

3.5 *Moment of Inertia and Friction Torque*

The moment of inertia of the rotor of a 27th order harmonic resolver is 63 gram cm sq.

The maximum break-away friction torque among five units was 0.027 in. oz. No change in this torque due to excitation of the unit could be detected.

IV. APPLICATION

In its application, the vernier resolver is usually directly coupled to a standard resolver or some other coarse angle transducer. Such a system which represents a variable, in this case the shaft angle, in two scales, coarse and fine, will be called a vernier system.

The following sections describe applications using the vernier resolver in an encoder, a follow-up system and an angle-reading system.

4.1 *Vernier Angle Encoder*

A vernier angle encoder converts a shaft angle into a pair of digital numbers, one being the coarse and the other being the vernier number. This type of encoder can be built by mechanically coupling a standard resolver directly to a vernier resolver. The outputs of the two resolvers, after encoding, represent the coarse and the vernier number.

The output of a resolver may be encoded, for instance, by the following method. The primary winding of the resolver is excited from an a-c source of, say, 400 cycles per second. The two induced secondary voltages are in phase with each other. Their amplitudes are proportional to the cosine and sine of the electrical rotor angle, θ_e .

These two amplitude modulated voltages are combined by means of two phase-shifting networks into two phase-modulated voltages. One network first advances the sine voltage by 90° and then adds it to the cosine voltage. The other network performs the same addition after retarding the sine voltage by 90°. The result is two constant amplitude voltages with relative phase shift of twice the electrical rotor angle. The time interval between the respective zero crossings of these two voltages is con-

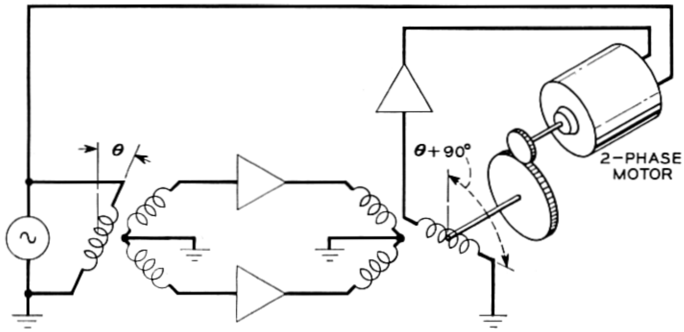


Fig. 9 — Resolver servo system.

verted into digital representation by means of an electronic stop-watch (time encoder).

4.2 *Vernier Follow-up System*

A vernier follow-up system can be built much like the present two speed synchro control-transformer system, except that the geared up synchros are replaced by vernier resolvers. Fig. 9 illustrates the vernier portion of this system. Since the output impedance of the vernier resolver is fairly large, it may be desirable to use amplifiers, as shown in Fig. 9, to energize that vernier resolver which plays the part of the control-transformer.

4.3 *Vernier Angle-Reading System*

A visual vernier angle-reading system as required to read the position of a rotary table can be built by using the output of a vernier resolver to position a standard resolver.

The coarse angle can be read as usual from calibration lines marked directly on the rotary table. The vernier angle reading is obtained by coupling a vernier resolver directly to the rotary table. The output of this resolver is used to position a standard control transformer. This control transformer will go through "n" revolutions for each revolution of the rotary table, where "n" is the order of the vernier resolver. The reading of a dial coupled either directly or through gears to the control transformer provides the vernier reading.

V. SUMMARY

Vernier resolvers of order 26, 27, 32 and 33 have been designed, built and tested.

The construction of the unit is very simple because all windings are located on the stator. The absence of brushes and slip rings makes the unit inexpensive in production and reliable in performance.

The performance of present experimental models is characterized by a repeatability of better than ± 3 seconds of arc and by a standard deviation error over one revolution of less than 10 seconds of arc.

Production units should be of even higher accuracy because better tooling fixtures would be used and minor design improvements would be incorporated.

The principal forseeable application of the resolver lies in vernier systems. Vernier encoder, vernier servo and vernier angle-reading systems are readily obtained by applying existing techniques to the vernier resolver.

ACKNOWLEDGEMENT

The development of the vernier resolver was undertaken under the sponsorship of the Wright Air Development Center. The work was encouraged and furthered by J. C. Lozier of Bell Telephone Laboratories. Valuable design contributions were made by J. Glass of Clifton Precision Products Co. All testing and evaluating of test results was done by T. W. Wakai of Bell Telephone Laboratories.

APPENDIX

Symbols

- E Amplitude of induced voltage
- p Number of pole-shoes
- n Number of rotor teeth
- θ_e Electrical rotor angle, equal to its geometrical angular position multiplied by n
- q p divided by largest integral factor common to n and p
- n' n divided by the same factor
- ν Pole-shoe number running from 0 to $(p - 1)$
- m Order of Fourier component representing the pole-shoe flux as a function of the electrical rotor angle, θ_e
- α_e Electrical angle between adjoining pole-shoes, equal to the geometrical angle multiplied by n
- k A number equal to zero or to any positive integer.

In accordance with equations (7) and (8) the voltage induced in the sine-winding coil on the νth pole-shoe by the mth flux harmonic is:

$$E_{sm\nu} = \omega[A_m \cos m\,(\theta_e - \nu\alpha_e)\,t \sin\,(\nu\alpha_e)]. \tag{13}$$

After trigonometric transformation:

$$E_{sm\nu} = \tfrac{1}{2}A_m t\omega\,[\sin\,(m\theta_e - (m-1)\nu\alpha_e) + \sin\,(-m\theta_e + (m+1)\nu\alpha_e)]\,. \tag{14}$$

The voltage, E_{sm}, induced in the sine winding is obtained by summing the expression of (14) over all values of ν. Since $(p\alpha_e)$ is a multiple of 2π the summing of all sine terms from $\nu = 0$ to $\nu = (p - 1)$ results in zero unless the angle $(m \pm 1)\,\alpha_e$ is an integral multiple, k, of 2π. This condition is spelled out in the following equation:

$$(m \pm 1)\, \alpha_e = k2\pi\,. \tag{15}$$

The electrical angle α_e being the mechanical angle between successive pole-shoes divided by the number of rotor teeth is:

$$\alpha_e = \frac{2\pi n}{p}. \tag{16}$$

Dividing p and n by the largest common integral factor, one can write

$$\alpha_e = \frac{2\pi n'}{q}. \tag{17}$$

Substituting this expression into (15) and solving for m, one obtains

$$m = \frac{kq}{n'} \pm 1 \tag{18}$$

where m and k are integers or zero. Consequently, (18) is satisfied for the following values of m:

$$m = 1; \qquad q \pm 1; \qquad 2q \pm 1; \cdots\,. \tag{19}$$

The amplitude of the voltage, E_{sm}, induced in the sine winding by flux harmonics of order m, where m is specified by (19), is

$$E_{sm} = \frac{p}{2} A_m t\omega \sin (m\theta_e). \tag{20}$$

Similarly one obtains for the voltages, E_{cm}, induced in the cosine winding

$$E_{cm} = \frac{p}{2} A_m t\omega \cos (m\theta_e). \tag{21}$$