Interchannel Interference Due to Klystron Pulling

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A source of interchannel interference in certain multichannel FM systems is the so-called "frequency pulling effect." This effect, which occurs in systems using a klystron oscillator, is produced by an impedance mismatch between the antenna and the transmission line feeding it. In this paper expressions are developed for the magnitude of the interference when the speech load is simulated by random noise.

INTRODUCTION

In a recent paper[1] the problem of interchannel interference produced by echoes in an FM system was treated. The mathematical development in that paper can be used to calculate the distortion that arises when a Klystron oscillator is connected to an antenna through a transmission line of appreciable length.

In the system we study, the composite signal wave (the "baseband signal") from a group of carrier telephone channels in frequency division multiplex is applied to the repeller of a Klystron and thereby modulates the frequency of the Klystron output wave. If the antenna does not match the transmission line perfectly, the output frequency is altered slightly by an amount proportional to the mismatch.

This effect, known as "pulling," results in intermodulation between the individual telephone channels. In this study, the composite signal will be simulated by a random noise signal of appropriate bandwidth and power. It is assumed that some particular message channel is idle; i.e., there is no noise energy in the corresponding frequency band (which is relatively narrow in comparison with the bandwidth of the composite signal). If the system were perfect, no power would be received in this idle channel at the output of the FM detector. In the following work, the intermodulation noise falling into this channel because of the "pulling effect" will be computed. This leads to "Lewin's integral," so called, which is tabulated herein.

PULLING EFFECT

In a perfect FM system the carrier wave can be written

$$E_0(t) = A \sin [pt + \varphi(t)] \tag{1}$$

where A is a constant and the signal is $S(t) = d\varphi/dt = \varphi'(t)$, measured in radians/second. As mentioned in the Introduction, we assume that when the FM oscillator is connected directly to a transmission line with a slightly mismatched antenna at the far end, its frequency is changed. The reactive component of the input impedance of the line "pulls" the frequency of the oscillator to its new value. When the antenna is perfectly matched, there is no change in oscillator frequency.

If the characteristic impedance of the line is Z_K and the impedance of the antenna is Z_R, the impedance Z looking into the line is

$$Z = Z_K \frac{Z_R + Z_K \tanh P}{Z_K + Z_R \tanh P}$$

$$= Z_K \frac{1 + \rho e^{-2P}}{1 - \rho e^{-2P}}$$

where ρ is the reflection coefficient

$$\rho = \frac{Z_R - Z_K}{Z_R + Z_K}$$

and P is the propagation constant of the line. If the loss of the line is negligible and the reflection coefficient is small, the input impedance is approximately

$$Z \doteq Z_K[1 + 2\rho(\cos \omega T - i \sin \omega T)] \text{ ohms}$$

where ω is the oscillator frequency in radians per second and T is twice the delay of the line.

It will be observed that the magnitude of the reactive component of Z oscillates as the phase angle ωT increases.

The dependence of the frequency of an oscillator upon the load reactance has been expressed by earlier workers as a "pulling figure." This figure is customarily defined as the difference between the maximum and minimum frequencies observed when the load reactance is varied over one cycle of its oscillation (the variation being accomplished, say, by increasing T). The load is taken to be such that it causes a voltage standing wave ratio of 1.5. This corresponds to a reflection coefficient of 0.20 and 14 db return loss.

In our work, we assume that the change in frequency is directly proportional to the reactive component of the input impedance. More precisely, we assume that the ideal transmitter frequency of $p + \varphi'(t)$ radians/sec is changed by the pulling effect to

$$p + \varphi'(t) + 2\pi r \sin [T(p + \varphi'(t))] \text{ radians/sec} \tag{2}$$

where r is given by

$$r = 2.5 \mid \rho \mid \times \text{(Pulling Figure in cycles/sec)}$$

POWER SPECTRUM OF INTERCHANNEL INTERFERENCE

The distortion produced by the pulling effect is given by the third term in (2). This distortion will be denoted by $\theta'(t)$:

$$\theta'(t) = 2\pi r \sin [pT + T\varphi'(t)] \tag{3}$$

Our problem is to compute the power spectrum of $\theta'(t)$. In particular, we are interested in the case where the signal $\varphi'(t)$ represents the composite signal wave from a group of carrier telephone channels in frequency division multiplex. All of the channels except one are assumed to be busy. Although the power spectrum of $\varphi'(t)$ is zero for frequencies in the idle channel, the same is not true for the power spectrum of $\theta'(t)$. In fact, the interchannel interference (as observed in the idle channel) is given by that portion of the power spectrum of $\theta'(t)$ which lies within the idle channel. We shall denote the corresponding interchannel interference power in the idle channel by $w_c(f)\,df$ where the idle channel is assumed to be of infinitesimal width and to extend from frequency $f - df/2$ to $f + df/2$. The function $w_c(f)$ will now be computed by using the procedure developed in Reference 1.

The first step is to assume the signal $\varphi'(t)$ to be a random noise current. In order to avoid writing φ' a great many times we shall set $\varphi'(t) = S(t)$, where now $S(t)$ stands for the signal. Then the autocorrelation function for the distortion $\theta'(t)$ is

$$\begin{aligned} R_{\theta'}(\tau) &= \text{avg}\,[\theta'(t)\theta'(t+\tau)] \\ &= (2\pi r)^2 \text{avg}\,[\sin (Tp + T\varphi'(t)) \sin (Tp + T\varphi'(t+\tau))] \\ &= (2\pi r)^2 \text{avg}\,[\sin (Tp + TS(t)) \sin (Tp + TS(t+\tau))] \\ &= \frac{(2\pi r)^2}{2} \text{avg}\,[\cos (TS(t) - TS(t+\tau)) \\ &\qquad\qquad - \cos (2pT + TS(t) + TS(t+\tau))] \\ &= \tfrac{1}{2}(2\pi r)^2 \{\exp [-T^2R_S(0) + T^2R_S(\tau)] \\ &\qquad\qquad - \cos (2pT) \exp [-T^2R_S(0) - T^2R_S(\tau)]\} \end{aligned} \tag{4}$$

where

$$R_S(\tau) = \int_0^\infty w_S(f) \cos 2\pi f\tau \, df \tag{5}$$

and $w_S(f)$ is the power spectrum of the applied signal $S(t)$. The last expression in (4) follows from the next to the last by analogy with equation (1.14) of Reference 1.

The dc component of the distortion $\theta'(t)$ is its average value $\overline{\theta'}$ which may be computed from

$$\overline{\theta'}^2 = R_{\theta'}(\infty) = \frac{(2\pi r)^2}{2} [e^{-T^2 R_S(0)}(1 - \cos 2pT)] \tag{6}$$

This follows from (4) since $R_S(\infty) = 0$.

The auto-correlation function of the distortion, excluding the dc component, is then

$$R_{\theta' - \overline{\theta'}} = \frac{(2\pi r)^2}{2} [e^{-T^2 R_S(0)}][(e^{T^2 R_S(\tau)} - 1) - (e^{-T^2 R_S(\tau)} - 1) \cos 2pT] \tag{7}$$

The interchannel interference spectrum is

$$w_c(f) = 4 \int_0^\infty R_c(\tau) \cos 2\pi f\tau \, d\tau \tag{8}$$

where, by analogy with equation (1.22) of Reference 1,

$$R_c(\tau) = \frac{(2\pi r)^2}{2} [e^{-T^2 R_S(0)}][(e^{T^2 R_S(\tau)} - T^2 R_S(\tau) - 1) - (e^{-T^2 R_S(\tau)} + T^2 R_S(\tau) - 1) \cos 2pT] \tag{9}$$

As mentioned before, the function $w_c(f)$ is of interest because

$$P_I = w_c(f) \, df \tag{10}$$

is the average interference power appearing at the receiver in an idle channel of width df centered on frequency f.

RATIO OF INTERCHANNEL INTERFERENCE TO SIGNAL POWER

The average signal power appearing in a busy channel of width df centered on the frequency f is

$$P_S = w_S(f) \, df \tag{11}$$

and hence the ratio of the interchannel interference power to the signal power is

$$\frac{P_I}{P_S} = \frac{w_c(f)}{w_S(f)} \tag{12}$$

We now obtain an expression for this ratio on the assumption that the random noise signal $S(t)$ (which is used to simulate the multichannel signal) has the power spectrum

$$w_S(f) = \begin{cases} P_0, & 0 < f < f_b \\ 0, & f > f_b \end{cases} \tag{13}$$

where P_0 is a constant. $S(t)$ is measured in radians/sec and $P_0 f_b$ is measured in (radians/sec)^2. $P_0 f_b$ is given by

$$P_0 f_b = \overline{S^2(t)} = \text{avg}\,[\varphi'(t)]^2 = (2\pi\sigma)^2$$

where σ is the rms frequency deviation of the signal measured in cycles/second. According to (5) this signal has the auto-correlation function

$$R_S(\tau) = \int_0^{f_b} P_0 \cos 2\pi f\tau \, df = P_0 \left[\frac{\sin 2\pi f\tau}{2\pi\tau}\right]_0^{f_b} = (2\pi\sigma)^2 \frac{\sin u}{u} \tag{14}$$

where

$$u = 2\pi f_b \tau$$

The interference power spectrum $w_c(f)$ corresponding to the $w_S(f)$ of (13) may be obtained by substituting (14) in (9) to get $R_c(\tau)$ and then using (8). The result is

$$\begin{aligned} w_c(f) = 4\frac{(2\pi r)^2}{2} e^{-b} \int_0^\infty & [(e^{bu^{-1}\sin u} - bu^{-1}\sin u - 1) \\ & - (e^{-bu^{-1}\sin u} + bu^{-1}\sin u - 1)\cos 2pT] \frac{\cos au}{2\pi f_b}\, du \end{aligned} \tag{15}$$

where u is the same as in (14) and we have set

$$a = f/f_b \qquad b = (2\pi\sigma T)^2$$

This integral may be expressed in terms of Lewin's integral which is studied in Appendix III of Reference 1. Thus

$$w_c(f) = \frac{(2\pi r)^2 e^{-b}}{2\pi f_b} [I(b, a) - I(-b, a)\cos 2pT](\text{radian/sec})^2/\text{cps} \tag{16}$$

where $I(b, a)$ and $I(-b, a)$ are tabulated for various values of a and b. Since we began the problem by dealing directly with $\theta'(t)$ which is a radian frequency, rather than $\theta(t)$ which is a radian phase, $w_c(f)$ has the

TABLE I—VALUES OF $e^{-b}I(b, a)$ FOR $b > 0$

b	e^b	$e^{-b}I(b, a)$					
		$a = 0$	0.25	0.50	0.75	1.00	1.25
0.0	1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.25	1.284	0.082	0.072	0.062	0.052	0.042	0.031
0.5	1.649	0.272	0.241	0.209	0.176	0.142	0.107
1.0	2.718	0.761	0.685	0.602	0.511	0.414	0.314
2.0	7.389	1.560	1.440	1.291	1.117	0.919	0.713
3.0	20.08	1.913	1.801	1.645	1.448	1.215	0.968
4.0	54.60	1.974	1.888	1.751	1.566	1.341	1.098
5.0	148.4	1.905	1.844	1.731	1.571	1.372	1.153
6.0	403.4	1.794	1.751	1.660	1.525	1.356	1.166
7.0	1097.	1.680	1.649	1.575	1.463	1.320	1.157
8.0	2981.	1.576	1.552	1.492	1.398	1.277	1.138

TABLE II—VALUES OF $I(b, a)$ FOR $b < 0$

b	$I(b, a)$					
	$a = 0$	0.25	0.50	0.75	1.0	1.25
0.0	0.000	0.000	0.000	0.000	0.000	0.000
−0.25	0.092	0.080	0.068	0.057	0.045	0.034
−0.5	0.349	0.300	0.254	0.210	0.167	0.125
−1.0	1.25	1.06	0.885	0.723	0.576	0.432
−2.0	4.16	3.41	2.76	2.20	1.76	1.34
−3.0	8.03	6.37	4.97	3.88	3.14	2.46
−4.0	12.6	9.66	7.23	5.49	4.55	3.74
−5.0	17.8	13.2	9.40	6.89	5.93	5.19
−6.0	23.6	16.8	11.4	8.00	7.25	6.85
−7.0	30.0	20.7	13.1	8.71	8.48	8.78
−8.0	37.2	24.8	14.5	8.93	9.59	11.0

dimensions of $(\text{radians/sec})^2/\text{cps}$. The signal in the same dimensions is P_0 or $(2\pi\sigma)^2/f_b$. Therefore the ratio of the interchannel interference power to the signal power is:

$$\frac{P_I}{P_S} = \frac{1}{2\pi}\left(\frac{r}{\sigma}\right)^2 e^{-b}[I(b, a) - I(-b, a)\cos 2pT] \tag{17}$$

The quantity $e^{-b}I(b, a)$ for $b > 0$ is tabulated in Table I. The quantity $I(b, a)$ for $b < 0$ is given in Table II. These tables, which are also given in Reference 1, are repeated here for the convenience of the reader.

When the rms frequency deviation σ is so small that $b = (2\pi\sigma T)^2$ is small compared to unity, the approximation

$$I(b, a) \approx b^2\pi(2 - a)/4$$

leads to

$$\frac{P_I}{P_S} \approx (2\pi^2 r\sigma T^2)^2(2 - a)(1 - \cos 2pT)/2 \tag{18}$$

When σ and T are such that $b \gg 1$, the approximation

$$I(b, a) \approx (6\pi/b)^{1/2} \exp\left[b - \frac{3a^2}{2b}\right]$$

leads to

$$\frac{P_I}{P_S} \approx \left(\frac{3}{8\pi^3}\right)^{1/2} \frac{r^2}{\sigma^3 T} \exp\left[-\frac{3}{2}\left(\frac{a}{2\pi\sigma T}\right)^2\right]$$

Equation (17), when converted to decibels, breaks down conveniently into two terms which may be designated D_1 and D_2 :

$$10 \log P_I/P_S = D_1 + D_2$$

$$D_1 = 10 \log (r/\sigma)^2 \tag{19}$$

$$D_2 = 10 \log \frac{1}{2\pi} e^{-b}[I(b, a) - I(-b, a) \cos 2pT]$$

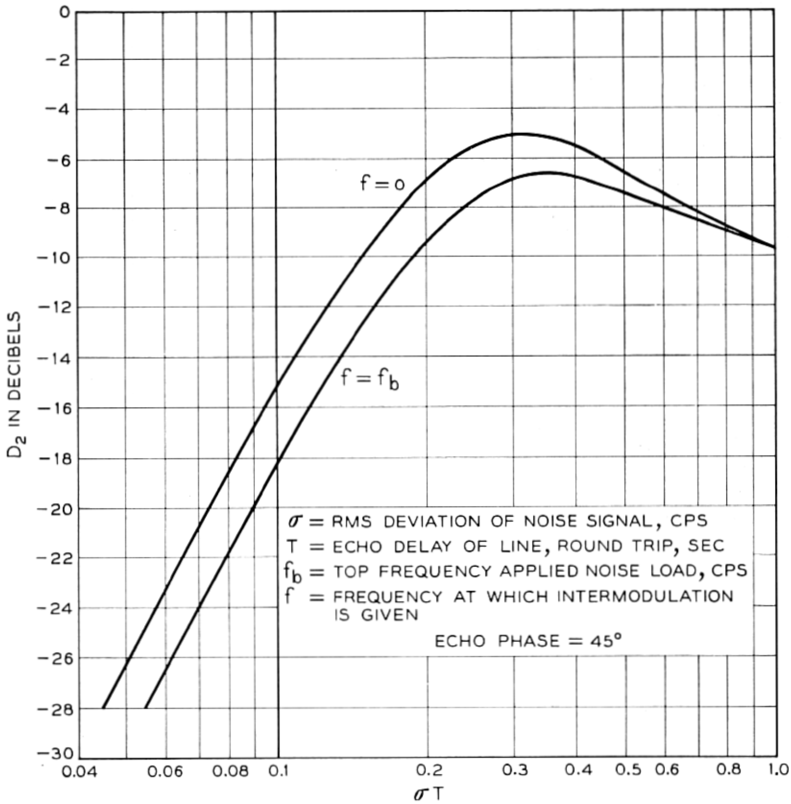

Fig. 1 — Plot of D_2 as a function of σT

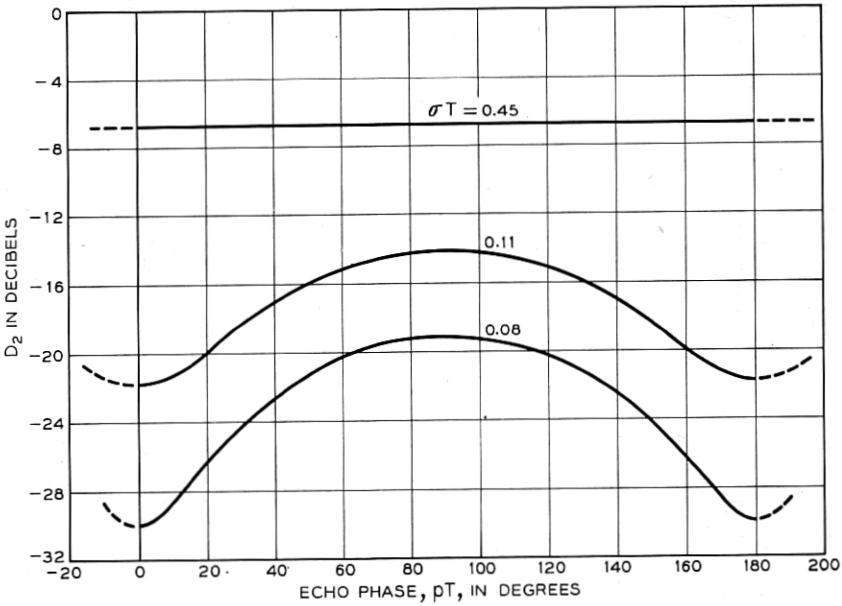

Fig. 2 — Plot of D_2 as a function of echo phase when $f = f_b$

The term D_2 depends only on the rms deviation σ, the round-trip echo delay T of the line, the ratio $a = f/f_b$, and the echo phase pT. The term D_2 is plotted in Fig. 1 as a function of σT for two channels, one at the top and the other at the bottom of the signal band. Since the carrier frequency may be expected to be very high, the carrier phase $2pT$ will be a very large number of radians even with very short wave guide runs. Hence the curves on Fig. 1 are plotted for the average value of cos $2pT$ which is zero.

The curves on Fig. 2 show how D_2 depends on pT and the parameter σT. In this case the channel is taken at the top of the signal band. When pT is an odd multiple of 90 degrees, it turns out that we have even order modulation products only; and when pT is a multiple of 180 degrees, odd order products only. The curves show that the distortion becomes less dependent on the echo phase as the quantity σT increases.

REFERENCE

1. W. R. Bennett, H. E. Curtis and S. O. Rice, Interchannel Interference in FM and PM Systems, B.S.T.J., **34**, pp. 601–636, May, 1955.