A Reflection Theory for Propagation Beyond the Horizon*

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(Manuscript received January 9, 1957)

Propagation of short radio waves beyond the horizon is discussed in terms of reflection from layers in the atmosphere formed by relatively sharp gradients of refractive index. The atmosphere is assumed to contain many such layers of limited dimensions with random position and orientation. On this basis, the dependence of the propagation on path length, antenna size and wavelength is obtained.

INTRODUCTION

It was pointed out several years ago[1] that power propagated beyond the radio horizon at very short wavelengths greatly exceeds the power calculated for diffraction around the earth. This beyond-the-horizon propagation has stimulated numerous experimental and theoretical investigations.[2] Booker and Gordon,[3] Villars and Weisskopf[4] and others have developed theories based on scattering of the radio waves by turbulent regions in the troposphere. This paper proposes a theory in which uncorrelated reflections from layers in the troposphere are assumed responsible for the power propagated beyond the horizon.

In developing this theory, some arbitrary assumptions of necessity have been made concerning the reflecting layers since, at the present time, our detailed knowledge of the atmosphere is insufficient. However, calculations based on the theory are found to be in good agreement with reported measurements of beyond-the-horizon propagation.

Measurement of the dielectric constant of the atmosphere[5] has shown that relatively sharp variations in the gradients of refractive index exist in both the horizontal and vertical planes. Although the geometrical structure of the boundaries formed by the gradients is not well known, one may postulate an atmosphere of many layers of limited extent and

* This material was presented at the I.R.E. Canadian Convention, Toronto, Canada, October 3, 1956.

arbitrary aspect.* The number and size of the reflecting layers, as well as the magnitude of the discontinuities in the gradient of dielectric constant which form them, influence the received power.

The reflecting properties of the layers are discussed first. Next, an expression for the received power is obtained by summing the contributions of many layers in the volume common to the idealized patterns chosen to represent the transmitting and receiving antenna beams.† This expression is then used to calculate the effect on received power of changes in such parameters as the orientation of the antennas, wavelength, distance, and antenna size.

The MKS system of units is used throughout.

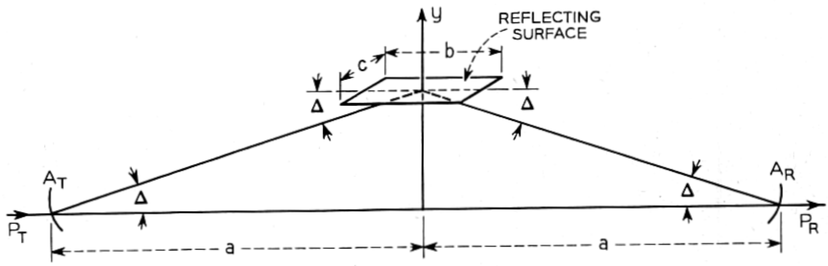

Fig. 1 — Reflection by a layer.

EFFECT OF LAYER SIZE

Propagation from a transmitting antenna of effective area A_T to a receiving antenna of effective area A_R by means of a reflecting layer is illustrated in Fig. 1. The ray from transmitter to receiver grazes the layer at angle Δ. The reflection from the layer depends on the amplitude reflection coefficient, q, which is a function of the grazing angle, and on the dimensions of the layer relative to the dimension of a Fresnel zone.‡ Three cases, depending on the layer dimensions, will be considered.

* After this paper was submitted for publication, a report was received giving some measurements of sharp variations in dielectric constant gradient and estimates of the horizontal dimensions of layers in the troposphere. J. R. Bauer, The Suggested Role of Stratified Elevated Layers in Transhorizon Short-Wave Radio Propagation, Technical Report No. 124, Lincoln Laboratory, M.I.T., Sept., 1956.

† Under some conditions, layers outside the volume common to the antenna beams may contribute appreciably to the received power. Phenomena such as multiple reflections and trapping mechanisms are not considered in this study.

‡ The power received by reflection from the layer in Fig. 1 can be calculated approximately by assuming it to be the same as the power that would be received by diffraction through an aperture in an absorbing screen, the dimensions of the aperture being the same as the dimensions of the layer projected normally to the directions of propagation. The field at the receiver is calculated from the distribution of Huygens sources in the aperture. The received power, expressed in

Case 1. Large Layers

If the layer were a plane, perfectly reflecting surface of unlimited extent, the power at the terminals of antenna A_R would be the same as the power received under line-of-sight conditions,[6]

$$P_R = P_T \frac{A_T A_R}{4\lambda^2 a^2}$$

If the layer has an amplitude reflection coefficient, q, the received power is,

$$P_R = P_T \frac{A_T A_R}{4\lambda^2 a^2} q^2$$

This relation applies when the layer dimensions are large in terms of the wavelength and are large compared with the Fresnel zone dimensions; that is, $b > \sqrt{2a\lambda}/\Delta$ and $c > \sqrt{2a\lambda}$.

Case 2. Small Layers

When the dimensions of the layer are small compared with the Fresnel zone, but large compared with the wavelength, the received power is given by the "radar" formula,

$$P_R = P_T \frac{A_T A_R}{\lambda^4 a^4} c^2 (b\Delta)^2 q^2$$

This relation applies when $b < \sqrt{2a\lambda}/\Delta$ and $c < \sqrt{2a\lambda}$.

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terms of Fresnel integrals, is

$$P_R = P_T \frac{A_T A_R}{\lambda^2 a^2} [C^2(u) + S^2(u)][C^2(v) + S^2(v)]$$

where $u = \dfrac{c}{\sqrt{\lambda a}}$ and $v = \dfrac{b\Delta}{\sqrt{\lambda a}}$

When u and v are very large, we have, approximately,

$$C(u) = S(u) = C(v) = S(v) = \tfrac{1}{2}$$

and the expression for P_R reduces to that given for Case 1 above, except for the factor q^2.

When both u and v are very small, we have approximately,

$$C(u) = u \qquad C(v) = v$$
$$S(u) = o \qquad S(v) = o$$

and the expression for P_R reduces to that given for Case 2.

When u is large and v is small the expression for P_R given in Case 3 results.

Case 3. Layers of Intermediate Size

If the layer dimensions are such that c is large but $b\Delta$ is small, compared with the Fresnel zone dimension, the received power is given by

$$P_R = P_T \frac{A_T A_R}{2\lambda^3 a^3} (b\Delta)^2 q^2$$

In the atmosphere, c and b are likely to be about equal, on the average, and we have for this case, $\sqrt{2a\lambda} < b < \sqrt{2a\lambda}/\Delta$.

All three of these cases may be present at various times, since the structure of the atmosphere changes from day to day. However, for the purpose of the present study, Case 3 is considered most prevalent and is assumed in all the calculations to follow.

Many of the numerous layers that are assumed to contribute to the received power are not necessarily horizontally disposed, they may be oriented in any direction. Therefore, reflection in the direction of the receiver can take place from layers located both on and off the great circle path. If there are N contributing layers per unit volume in the region V common to the radiation patterns of the transmitting and receiving antennas, then for Case 3,

$$P_R = P_T \frac{A_T A_R N b^2}{2\lambda^3 a^3} \int_V \Delta^2 q^2 \, dV \tag{1}$$

In this relation it has been assumed that the layer size and the number of layers per unit volume remain sensibly constant throughout the common volume.

The integration process requires expressions for the reflection coefficient q and the grazing angle Δ of the layers in the common volume. These quantities are derived in the following sections.

REFLECTION COEFFICIENT OF A LAYER

The reflection coefficient of a plane boundary (Fig. 2) separating two media whose dielectric constants, relative to free space, differ by an increment $d\varepsilon$ is given by Fresnel's laws of reflection. For both polarizations, the plane wave reflection coefficient of the boundary is

$$q = d\varepsilon/4\Delta^2$$

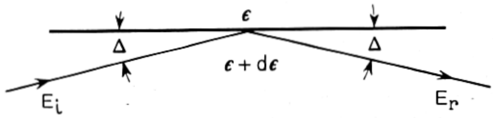


Fig. 2 — Reflection at a boundary between two homogeneous media.

provided $1 \gg \Delta^2 \gg d\varepsilon$. This reflection coefficient for an incremental change in dielectric constant can be used to calculate the reflection from discontinuities in the gradient of the dielectric constant of the atmosphere such as those shown for a stratified medium at $y = 0$ and $y = h$ in Fig. 3(b).

Such variations of dielectric constant are assumed to be representative of discontinuities in gradient as they exist in the physical atmosphere. The variations form the reflecting layers.

The method of calculating the reflection coefficient of such a stratified medium is due to S. A. Schelkunoff[7] and is illustrated schematically in Fig. 3 in which the medium has been subdivided into incremental steps. Consider the reflected wave from a typical incremental layer, dy, situated a distance y above the lower boundary of the layer, 0. From Fig. 3(a) it is clear that the phase of this wave is $4\pi y/\lambda \sin \Delta$ relative to that of a wave reflected from the lower boundary. The incremental reflection coefficient is $d\varepsilon/4\Delta^2 = -K\, dy/4\Delta^2$, where K is the change in gradient of the dielectric constant at the boundaries of the layer. The field reflected by layer dy is therefore,

$$dE_R = -E_i \frac{K}{4\Delta^2} e^{-j(4\pi y/\lambda) \sin \Delta}\, dy$$

One now obtains the complete reflected field by summing the reflections from all increments within the layer of thickness h.

$$E_r = \int_0^h dE_r = jE_i \frac{K\lambda}{16\pi\Delta^2 \sin \Delta} [1 - e^{-j(4\pi h/\lambda) \sin \Delta}]$$

This relation shows that the layer is equivalent to two boundaries at

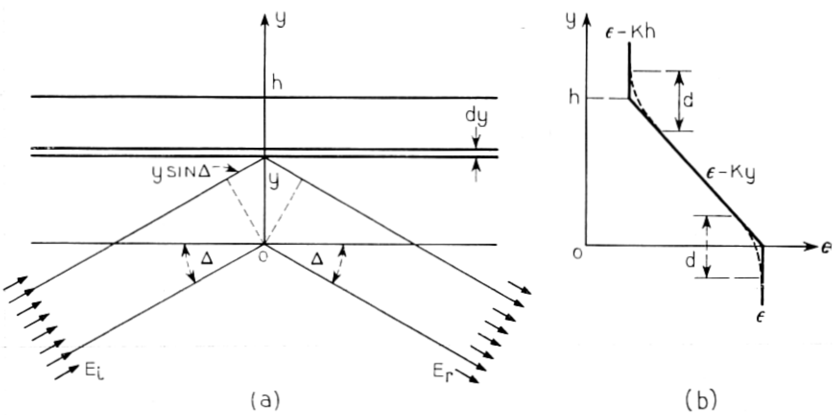


Fig. 3 — Plane-wave reflection at an incremental layer dy within a stratified medium extending from $y = 0$ to $y = h$.

$y = 0$ and $y = h$, each with reflection coefficient

$$q = \frac{K\lambda}{16\pi\Delta^3} \tag{2}$$

If the abrupt change in slope, the solid line in Fig. 3(b), is replaced by a gradual change as indicated by the dotted lines, (2) still holds provided $d < \lambda/4\Delta$. For more gradual changes, $d = n\lambda/4\Delta$ where $n > 1$, the reflection coefficient is

$$q = \frac{K\lambda}{16\pi\Delta^3} \cdot \frac{\sin \frac{\pi n}{2}}{\frac{\pi n}{2}}$$

and q varies with n between $q = 0$ and

$$q = \frac{K\lambda}{16\pi\Delta^3} \cdot \frac{2}{\pi n}$$

Smoothing of the boundaries reduces the value of q.

It will be assumed in all the calculations to follow that reflection from layers in the troposphere is described by (2).

VARIATION OF STRENGTH OF LAYERS WITH HEIGHT

The formula for the reflection coefficient includes the factor K, which represents the change in the gradient of the dielectric constant at the boundaries of the layer. A dielectric constant profile constructed of many randomly positioned gradients is shown schematically in Fig. 4. The variations are shown as departures from the standard linear gradient. Measurements[5] indicate that the fluctuations of the dielectric constant normally decrease with height above ground. The changes in the dielectric constant gradients associated with these fluctuations probably vary in a similar manner so that K is some inverse function of the height above the earth. However, to simplify the computation of received power, to be described later, we have adopted the cylindrical coordinate system shown in Fig. 5, and it is convenient, then, to let K be a function of ρ, the distance from the chord joining the transmitter and receiver to the point in question.

We assume, therefore, that

$$K = \frac{3{,}200K_1}{\rho} \tag{3}$$

where K_1 is the change in gradient at point A in Fig. 5 which, for a typi-

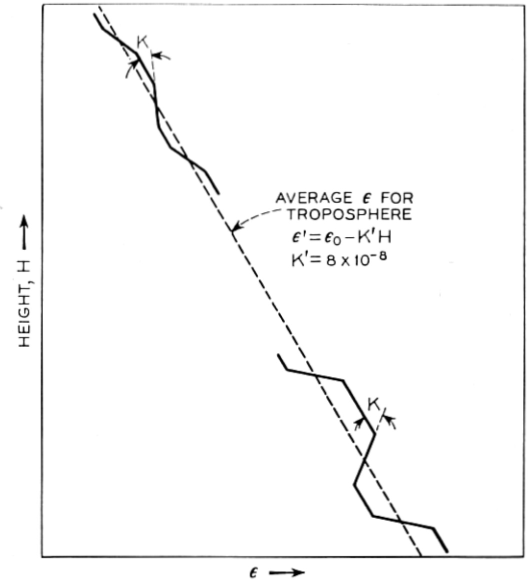


Fig. 4 — Schematic illustration of variation of the dielectric constant in the troposphere.

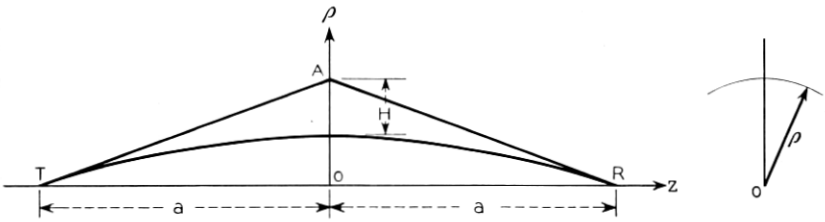


Fig. 5 — Coordinate system for a beyond-the-horizon circuit.

cal path length of 200 miles, is about 1,600 meters (1 mile) above the earth or, since $\rho = 2H$, 3,200 meters from the z axis.

Equation 3 is used in all the calculations to follow.

THE GRAZING ANGLE Δ

The grazing angle Δ at the slightly tilted layer shown in Fig. 6 is given by

$$\tan 2\Delta = \frac{2a\rho}{a^2 - z^2 - \rho^2}$$

Throughout the volume common to the antenna patterns, $\Delta \ll 1$, $\rho \ll a$ and $z < a/2$. Then

$$\Delta \approx \frac{\rho}{a} \tag{4}$$

It is evident that Δ is constant and equal to ρ/a when the point (ρ, z) is located on a cylinder with axis TR and radius ρ. It is this feature that

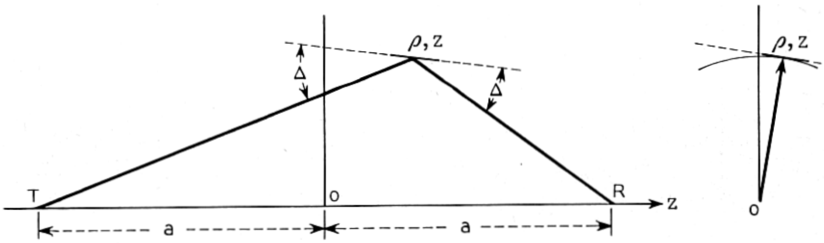


Fig. 6 — Grazing angle Δ at a layer.

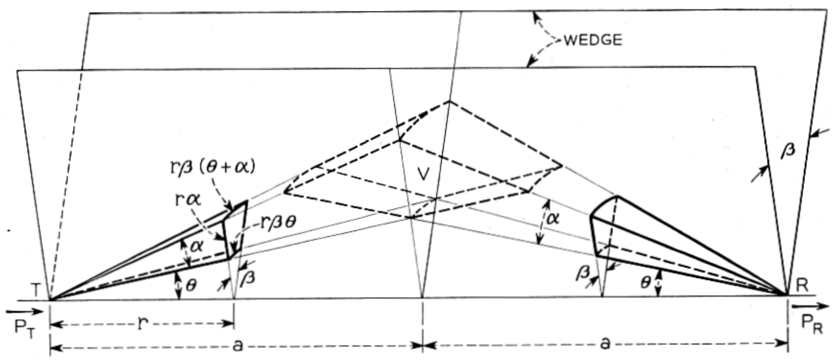


Fig. 7 — Idealized antenna patterns used in this study.

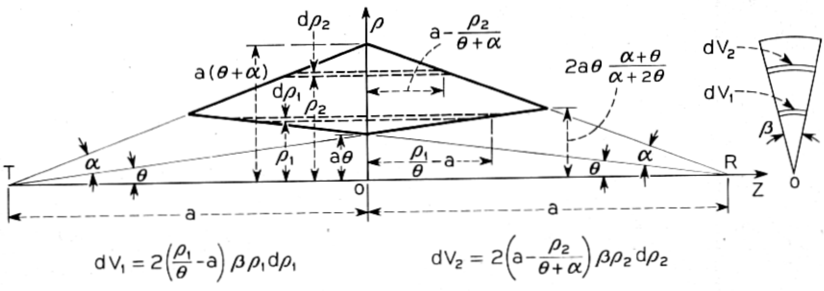


Fig. 8 — Integration over the common volume.

suggested the unusual idealized antenna patterns shown in Fig. 7, which are described in the next section.

CALCULATION OF THE RECEIVED POWER

Substituting (2), (3) and (4) in (1), one obtains for the received power,

$$P_R = P_T M a A_R A_T \lambda^{-1} \int_V \rho^{-6}\, dV \tag{5}$$

where

$$M \approx 2000 b^2 K_1^{\,2} N \tag{6}$$

To integrate over the volume common to actual antenna patterns would be difficult. We have, as mentioned before, replaced the actual patterns with the idealized patterns shown in perspective in Fig. 7 and in plane projection in Fig. 8. The patterns (Fig. 7) are bounded by side planes of the large wedge and by surfaces of cones with axis TR. The common volume is indicated by broken lines and is well defined. Since the grazing angle Δ is constant for the incremental cylindrical volumes dV_1 and dV_2 shown in Fig. 8, it is easy to integrate over the common volume V and we obtain

$$\int_V \rho^{-6}\, dV = \frac{\beta}{6\theta^4 a^3} f\left(\frac{\alpha}{\theta}\right) \tag{7}$$

$$f\left(\frac{\alpha}{\theta}\right) = 1 + \frac{1}{\left(1 + \dfrac{\alpha}{\theta}\right)^4} - \frac{1}{8}\left(\frac{2 + \dfrac{\alpha}{\theta}}{1 + \dfrac{\alpha}{\theta}}\right)^4 \tag{8}$$

The function $f(\alpha/\theta)$ is plotted in Fig. 9.

The gain of the idealized antennas is $G = 8\pi/\alpha\beta(\alpha + 2\theta)$ and the effective area is

$$A = \frac{2\lambda^2}{\alpha\beta(\alpha + 2\theta)} \tag{9}$$

The area of a cross section of the antenna pattern is bounded by two straight sides, $r\alpha$, and two curved sides $r\theta\beta$ and $r(\theta + \alpha)\beta$. The aspect ratio is defined as the ratio of the sum of the lengths of the curved sides to the sum of the lengths of the straight sides. It is equal to one when

$$\beta = \frac{2\alpha}{\alpha + 2\theta} \tag{10}$$

Substituting (10) in (9) gives the effective area of the idealized antenna

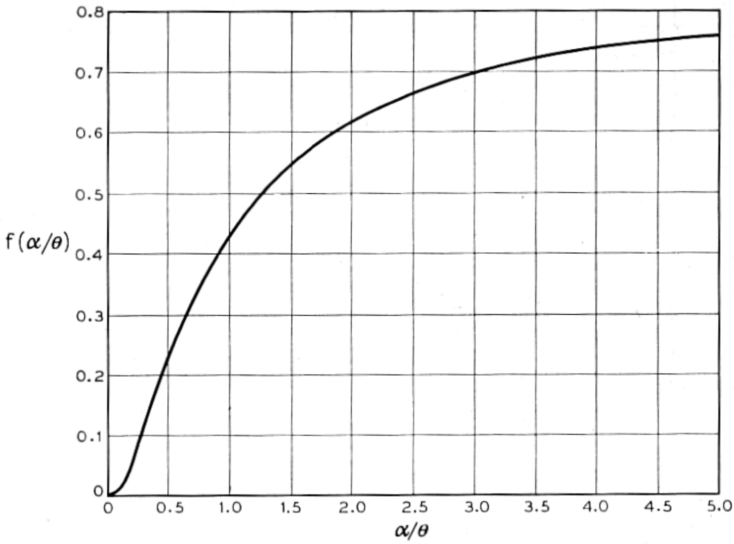


Fig. 9 — The function $f\ (\alpha/\theta)$.

with aspect ratio one,

$$A = \frac{\lambda^2}{\alpha^2} \tag{11}$$

Substituting (7), (10) and (11) in (5) gives for *identical transmitting and receiving antennas* with *aspect ratio one*,

$$P_R = P_T \frac{M}{3} \frac{\lambda^3}{a^3} \frac{1}{\theta^5 \alpha^2} \frac{1}{2 + \frac{\alpha}{\theta}} f\left(\frac{\alpha}{\theta}\right)^* \tag{12}$$

For actual antennas, α may be taken to be the half-power beam-width.

In the following sections, (12) will be used to derive some of the general properties of propagation beyond the horizon.

* K. Bullington has suggested that a useful form for equation (12) is

$$P_R = \left[\frac{P_T\lambda^2}{4a^2\alpha^4}\right]\left[\frac{4M\lambda}{3\theta^4}\right]\left[\frac{\frac{\alpha}{\theta} f\left(\frac{\alpha}{\theta}\right)}{2 + \frac{\alpha}{\theta}}\right]$$

where the first term in brackets represents the power that would be received in free space, the second term involves the characteristics of the troposphere, and the third term is a correction factor for narrow beam antennas.

RECEIVED POWER VERSUS ANGLE θ

The angle θ is the angle between the lower edge of the idealized antenna pattern and the straight line joining the terminals (see Fig. 7). The minimum value of θ is determined by the profile of the transmission path. If λ, α and a are constants, (12) can be used to calculate the ratio of the powers, P_{R1} and P_{R2}, received at two different angles, θ_1 and θ_2,

$$\frac{P_{R1}}{P_{R2}} = \left(\frac{\theta_2}{\theta_1}\right)^5 \frac{2 + \frac{\alpha}{\theta_2}}{2 + \frac{\alpha}{\theta_1}} \frac{f\left(\frac{\alpha}{\theta_1}\right)}{f\left(\frac{\alpha}{\theta_2}\right)} \tag{13}$$

Equation (13) shows the importance of having the angle θ as small as possible. For example, for $\theta_1 = \alpha$ and $\theta_2/\theta_1 = 1.25$, the power ratio is 3.4 (5 db). Thus in an actual circuit the antenna pattern should be close to the horizon plane.

RECEIVED POWER VERSUS WAVELENGTH

Consider a given path in which a and θ are specified. Equation (12) can be used to calculate the ratio of received powers, P_{R1} and P_{R2}, corresponding to two different wavelengths, λ_1 and λ_2.

$$\frac{P_{R1}}{P_{R2}} = \left(\frac{\lambda_1}{\lambda_2}\right)^3 \left(\frac{\alpha_2}{\alpha_1}\right)^3 \frac{2 + \frac{\alpha_2}{\theta}}{2 + \frac{\alpha_1}{\theta}} \frac{f\left(\frac{\alpha_1}{\theta}\right)}{f\left(\frac{\alpha_2}{\theta}\right)} \tag{14}$$

where α_1 and α_2 are the beamwidths of the antenna patterns at wavelengths λ_1 and λ_2 respectively.

Case I. Equal antenna gains at the two wavelengths.

For this case, $\alpha_1 = \alpha_2$ and equation (14) reduces to

$$\frac{P_{R1}}{P_{R2}} = \left(\frac{\lambda_1}{\lambda_2}\right)^3 \tag{15}$$

In free space the power ratio would be

$$\frac{P_{R1}}{P_{R2}} = \left(\frac{\lambda_1}{\lambda_2}\right)^2 \tag{16}$$

or

$$\frac{P_{R1}/P_{R2}\text{ (Beyond-Horizon)}}{P_{R1}/P_{R2}\text{ (Free Space)}} = \frac{\lambda_1}{\lambda_2} \tag{17}$$

Thus if 400 mcs and 4,000 mcs were propagated over the same path, the antenna gains being identical for the two systems, then, on the average, one would expect the received power relative to the free space value at 400 mcs to be 10 db higher than that at 4,000 mcs because of the characteristics of the troposphere.

Case II. Equal antenna apertures for the two wavelengths.

For this case, $\alpha_1/\alpha_2 = \lambda_1/\lambda_2$ and (14) reduces to

$$\frac{P_{R1}}{P_{R2}} = \frac{2 + \frac{\alpha_2}{\theta}}{2 + \frac{\alpha_1}{\theta}} \; \frac{f\left(\frac{\alpha_1}{\theta}\right)}{f\left(\frac{\alpha_2}{\theta}\right)} \tag{18}$$

Experimental data for this case was obtained on the 150 nautical mile test circuit between St. Anthony and Gander in Newfoundland.[8] The antennas for both wavelengths were paraboloids 8.5 meters in diameter. Simultaneous transmission tests at $\lambda_1 = 0.074\ m$ and $\lambda_2 = 0.6\ m$ were conducted for a full year. For this circuit,

$$\theta = 0.94° \text{ (4/3 earth radius)}$$

$$\alpha_1 = 66\,\frac{0.074}{8.5} = 0.575°$$

$$\alpha_2 = 66\,\frac{0.6}{8.5} = 4.65°$$

Using these values in (18), we get for the ratio of received powers,

$$P_{R1}/P_{R2} = 1.01$$

For antennas of equal aperture in free space,

$$P_{R1}/P_{R2} = (\lambda_2/\lambda_1)^2 = 65.5$$

Therefore,

$$\frac{P_{R1}/P_{R2} \text{ (Beyond Horizon)}}{P_{R1}/P_{R2} \text{ (Free Space)}} = \frac{1}{65} = -\,18.1 \text{ db}$$

The Summary of Results, Sections 1 and 2 on page 1316 of Reference 8, gives -17 db for this ratio. The agreement between calculated and measured values is very good.

RECEIVED POWER VERSUS DISTANCE

If antenna size and wavelength are specified, (12) gives for two distances, a_1 and a_2,

$$\frac{P_{R1}}{P_{R2}} = \left(\frac{a_2}{a_1}\right)^7 \frac{2 + \dfrac{\alpha}{\theta_2}}{2 + \dfrac{\alpha}{\theta_1}} \; \frac{f\left(\dfrac{\alpha}{\theta_1}\right)}{f\left(\dfrac{\alpha}{\theta_2}\right)} \tag{19}$$

For $a_2 = 2a_1$, (19) gives for different values of α/θ_1

$\alpha/\theta_1 =$	0.5	1	2	4
$P_{R1}/P_{R2} =$	276 (24 db)	197 (23 db)	138 (21.5 db)	104 (20 db)

Fig. 1 in Bullington's paper,[9] which gives the median signal level in decibels below the free space value as a function of distance, shows an 18 db increase in attenuation when the distance is doubled. This corresponds to a ratio of received powers of 18 + 6 = 24 db. The examples in the table above give an average increase in attenuation of 22 db.

RECEIVED POWER VERSUS ANTENNA SIZE

Equation 14 can be used to calculate the effect on received power of changing simultaneously the size (and, hence, the beamwidths) of the antennas used for transmitting and receiving, the wavelength and distance remaining fixed.

$$\frac{P_{R1}}{P_{R2}} = \left(\frac{\alpha_2}{\alpha_1}\right)^3 \frac{2 + \dfrac{\alpha_2}{\theta}}{2 + \dfrac{\alpha_1}{\theta}} \; \frac{f\left(\dfrac{\alpha_1}{\theta}\right)}{f\left(\dfrac{\alpha_2}{\theta}\right)} \tag{20}$$

where P_{R1} and P_{R2} are the received powers corresponding to the antenna beamwidths α_1 and α_2 respectively.

As an example, let α_2 be constant and equal to 4° and let θ be 1°, corresponding to a 200-mile circuit. The table below gives the ratio P_{R1}/P_{R2} as α_1 is varied.

α_1	4°		2°		1°		0.5°		0.25°
P_{R1}/P_{R2} (db)	0		10		18.5		25.7		31.4
Change in db		10		8.5		7.2		5.7	

Since α is inversely proportional to the antenna dimensions, the table shows that continued doubling of the antenna dimensions results in less and less increase in output power. The increase varies from 10 to 5.7 db in the table. This is a characteristic feature of beyond-the-horizon propagation. In free space, doubling the antenna dimensions would result in a 12-db increase in output power.

Large antennas and high power transmitters are costly, and a proper balance between their costs requires careful studies which are outside the scope of this paper. In general, it is not believed worth while from power considerations to increase the antenna size much beyond the dimensions that correspond to a pattern angle, α, equal to angle θ.

Another factor to be considered, however, is the effect of antenna size on delay distortion in beyond-the-horizon circuits. From simple path length considerations, one concludes that the delay distortion decreases when the beamwidths of the antennas are made smaller. Therefore, delay distortion requirements may dictate antenna sizes that are not justified by power considerations alone.

SEASONAL DEPENDENCE

Both the effective earth radius, R_e, and the magnitude of the discontinuities in gradient, K_1, are related to the season of the year. During the summer when the water vapor content of the air is high, the effective radius and the discontinuities in gradient are larger than in winter. Substituting a/R_e for θ and assigning summer and winter values for R_e and K_1, (12) may be used to calculate the ratio of the power received in summer and in winter.

$$\frac{P_R\text{ (Summer)}}{P_R\text{ (Winter)}} = \left(\frac{K_{1S}}{K_{1W}}\right)^2 \left(\frac{R_{eS}}{R_{eW}}\right)^5 \frac{2 + \dfrac{\alpha R_{eW}}{a}}{2 + \dfrac{\alpha R_{eS}}{a}} \; \frac{f\left(\dfrac{\alpha R_{eS}}{a}\right)}{f\left(\dfrac{\alpha R_{eW}}{a}\right)} \tag{21}$$

$$\approx \left(\frac{K_{1S}}{K_{1W}}\right)^2 \left(\frac{R_{eS}}{R_{eW}}\right)^6$$

For example, if we assume $K_{1S} = 2K_{1W}$ and $R_{eS} = 1.2\,R_{eW}$, then $P_{RS}/P_{RW} = 11.9$ (10.75 db). A seasonal variation has been observed.[8, 10]

DEPENDENCE OF RECEIVED POWER ON ANTENNA ORIENTATION

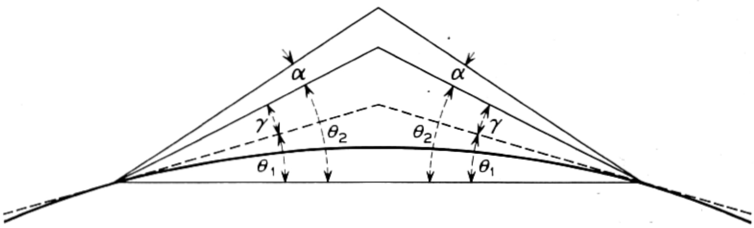
The variation of received power with orientation of the antennas at the terminals of a beyond-the-horizon circuit differs considerably from that observed under line-of-sight conditions. Consider, for example, Fig. 10 which shows the beams of the transmitting and receiving antennas elevated simultaneously. The variation of received power can be calculated from (13). As an example, consider the 188-mile circuit between

Crawford Hill, N. J., and Round Hill, Mass., for which experimental data is published.[10] For this circuit $\alpha = 0.65°$ (3 db points) and $\theta_1 = 1°$ (4/3 earth radius). The table below gives the calculated variation of received power as angle θ_2 is varied.

$\theta_2 =$	1°	1.1°	1.2°	1.4°	1.6°	1.8°	2°	2.2°
$10 \log_{10} (P_{R1}/P_{R2}) =$	0	2.3	4.5	8.5	12	15	17.9	20.5

The received power versus elevation angle, $\gamma = \theta_2 - \theta_1$, is plotted in Fig. 10. The calculated and experimental curves are in good agreement.

If the beams of the antennas are steered simultaneously in the horizontal plane, Fig. 11, the calculation of the variation of received power is



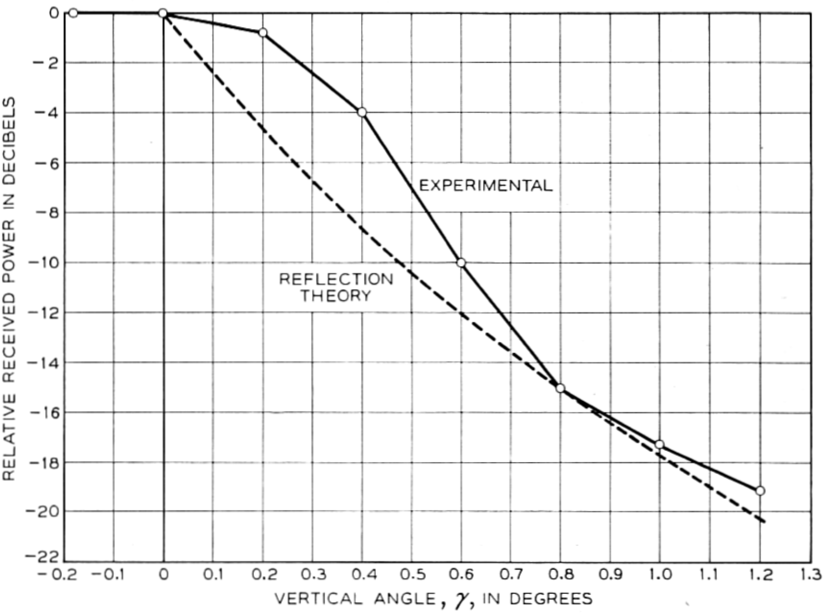


Fig. 10 — Relation between received power and vertical angle γ.

comparatively simple. In horizontal steering, the intersection of the axes of the antenna beams moves along line AB in the figure labelled "Cross section at 0." If the intersection of the beams moved along the circle A-C, the received power would not change. The decrease in power caused by moving the beams from position A to B is given by (13). The calcu-

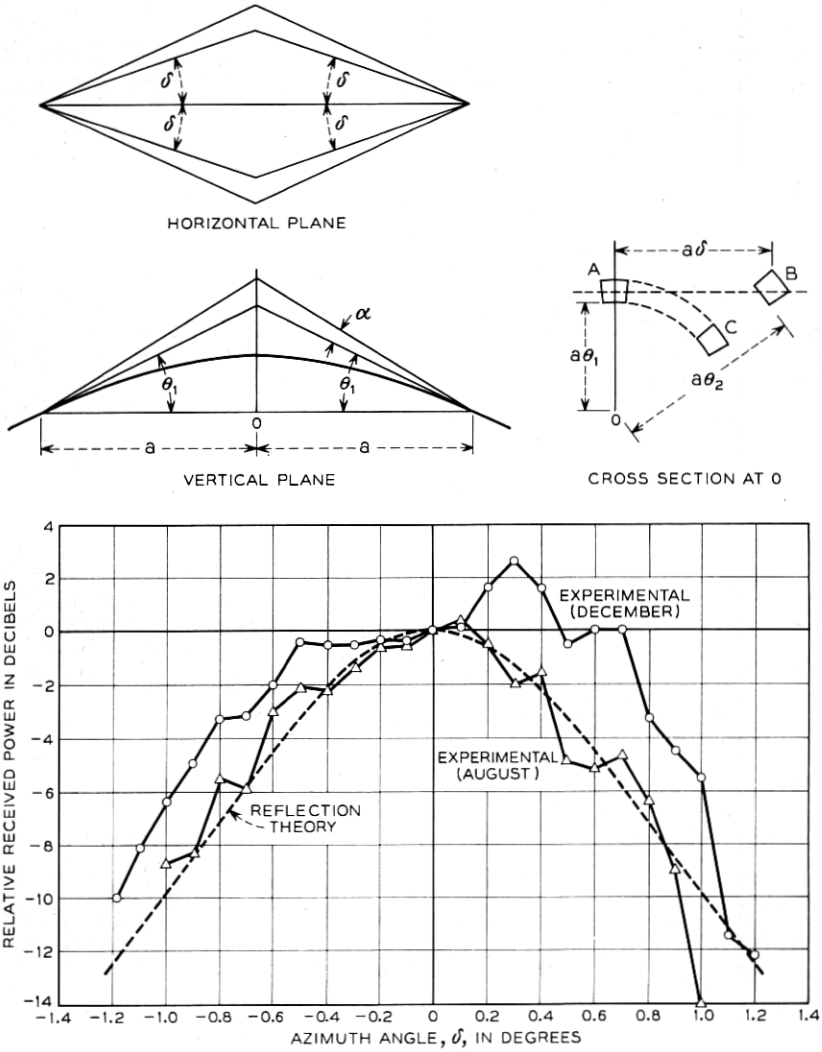


Fig. 11 — Relation between received power and azimuth angle δ.

lations are identical with the calculations for elevation steering; the elevation angle θ_2 is related to the azimuth angle δ and the beamwidth angle α by

$$\delta^2 = (\theta_2 - \theta_1)(\theta_2 + \theta_1 + \alpha)$$

A calculated curve of received power versus azimuth angle is shown in Fig. 11 for the Crawford Hill-Round Hill circuit together with the reported experimental data.[10] The agreement is considered good.

THE VALUE OF FACTOR M IN EQUATION (12)

An average value for the factor M can be obtained from propagation data. Using equation (12), the ratio of received powers corresponding to free space and beyond-horizon transmission is

$$\frac{P_R\text{ (free space)}}{P_R\text{ (beyond-horizon)}} = \frac{0.75\,\theta^5\left(2 + \dfrac{\alpha}{\theta}\right)}{M\lambda\alpha f\left(\dfrac{\alpha}{\theta}\right)} \tag{22}$$

This ratio was found experimentally to be 5×10^6 (67 db) for the circuit between St. Anthony and Gander in Newfoundland.[8] For this circuit, $\alpha = 0.081$, $\theta = 0.0164$, $\lambda = 0.6$. Substituting these values in (22) we obtain,

$$M = 3 \times 10^{-14} \tag{23}$$

Substituting this value for M in (12) leads to the following equation for a beyond-horizon tropospheric circuit,

$$P_R = P_T \times 10^{-14}\,\frac{\lambda^3}{\alpha^3}\,\frac{1}{\theta^5 a^2}\,\frac{1}{2 + \dfrac{\alpha}{\theta}}\,f\left(\frac{\alpha}{\theta}\right) \tag{24}$$

Equations (6) and (23) give

$$b^2 K_1^2 N = 1.5 \times 10^{-17} \tag{25}$$

Although values of the layer dimension, b, the change in gradient, K_1, and the number of layers per unit volume, N, are not known, it is interesting to calculate N from (25) assuming reasonable values for b and K_1.

Assuming $K_1 = 4 \times 10^{-8}$, which is half the value of K', the average gradient of the dielectric constant in the troposphere, and $b = 1{,}000$ (1 km) we find $N = 10^{-8}$ or 10 layers per cubic kilometer.

CONCLUDING REMARKS

The interpretation of propagation beyond the horizon in terms of reflection from layers of limited size formed by variations in the gradient of the dielectric constant of the atmosphere leads to relatively simple results which are in good agreement with reported experimental data. The received power depends on the wavelength, the distance, and the size of the antennas used for the circuit and on the strength and size of the reflecting layers.

As mentioned earlier, the structure of the atmosphere may change markedly from time to time so that large, small and intermediate size layers play their parts at different times. Furthermore, the effective size of a given layer may be different for widely separated wavelengths, depending on the roughness of the layer in terms of the wavelength. All that can be expected of a study such as the present one is that it serve as a guide for estimating the roles of the various parameters involved in beyond-the-horizon propagation.

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