Transmission Loss Due to Resonance of Loosely-Coupled Modes in a Multi-Mode System

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In a multi-mode transmission system the presence of spurious modes which resonate in a closed environment can produce an appreciable loss to the principal mode. The theory for the evaluation and control of this effect under certain conditions has been derived and checked experimentally in the particularly interesting case of a TE_{01} *transmission system, where mode conversion to* TE_{02}, TE_{03} $\cdots$ *is produced by tapered junctions between two sizes of waveguide.*

INTRODUCTION

In a transmission system, the presence of a region which supports one or more spurious modes can introduce a large change in the transmission loss of the principal mode when the region becomes resonant for one of the spurious modes. This phenomenon can occur even when the mode conversion is low and the waveguide increases in cross section smoothly to a region which supports more than one mode. In general, the conditions required to resonate the various spurious modes are not fulfilled simultaneously and, in consequence, interaction takes place between the principal mode and only one of the spurious modes for each resonating frequency. Under these conditions the resonating environment can be visualized as made of only two coupled transmission lines, one carrying the desirable mode and the other the spurious one. This simplification makes it possible to calculate the transmission loss as a function of (1) the coefficient of conversion between the two modes and (2) the attenuation of the modes in the resonating environment. The theory has shown good agreement with the measurement of transmission loss of the TE_{01} mode in a pipe wherein a portion was tapered to a larger diameter which can support the TE_{02} mode.

TRANSMISSION LOSS OF A WAVEGUIDE WITH A SPURIOUS MODE RESONATING REGION

Let us consider a single-mode waveguide connected to another of different cross-section that admits two modes. Since these two modes are orthogonal, the junctions may be considered as made of three single-mode lines connected together, provided we define the elements of the scattering matrix properly. The three modes, or lines in which they travel, are indicated by the subscripts 0, 1, and 2, as shown in Fig. 1. If a_0, a_1, a_2 and b_0, b_1, b_2 are the complex amplitudes of the electric field of the incident and reflected waves respectively, then

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = [S] \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

where

$$[S] = \begin{bmatrix} \Gamma_{00} & \Gamma_{01} & \Gamma_{02} \\ \Gamma_{01} & \Gamma_{11} & \Gamma_{12} \\ \Gamma_{02} & \Gamma_{12} & \Gamma_{22} \end{bmatrix} \tag{1}$$

is the scattering matrix.[1]

This specific type of change of cross section may be treated as a three-port junction.

Now, if a length ℓ of a two mode waveguide is terminated symmetrically at both ends with a single mode waveguide (Fig. 2), each joint is described by the same matrix (1), and the connecting two mode wave guide has the following scattering matrix:

$$\begin{bmatrix} 0 & e^{-j\theta_1} & 0 & 0 \\ e^{-j\theta_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-j\theta_2} \\ 0 & 0 & e^{-j\theta_2} & 0 \end{bmatrix} \tag{1'}$$

in which

$$j\theta_1 = \gamma_1\ell = (\alpha_1 + j\beta_1)\ell$$

$$j\theta_2 = \gamma_2\ell = (\alpha_2 + j\beta_2)\ell,$$

γ_1 and γ_2 are the propagation constants of modes 1 and 2.

[1] N. Marcuvitz, Waveguide Handbook, **10**, M.I.T., Rad. Lab. Series, McGraw-Hill, New York, 1951, pp. 107–8.

Matrices 1 and 1′ describe the system completely and from them, the transmission coefficient results,

$$\Gamma = \frac{b_0'}{a_0}$$

$$= \Gamma_{01}^2 e^{-j\theta_1} \frac{A - \Gamma_{22}^2 e^{-j2\theta_2} A^* \left(1 + \left|\frac{\Gamma_{12}}{\Gamma_{22}}\right|^2 + \frac{\Gamma_{00}}{\Gamma_{01}} \frac{\Gamma_{02}^*}{|\Gamma_{22}|^2} \Gamma_{12}\right)^2}{[1 - (\Gamma_{12}^2 - \Gamma_{11}\Gamma_{22}) e^{-j(\theta_1+\theta_2)}]^2 - (\Gamma_{11} e^{-j\theta_1} + \Gamma_{22} e^{-j\theta_2})^2} \tag{2}$$

where

$$A = 1 + \left(\frac{\Gamma_{02}}{\Gamma_{01}}\right)^2 e^{-j(\theta_2 - \theta_1)}$$

A^* is the complex conjugate of A

Γ_{01}^* is the complex conjugate of Γ_{01}

Γ_{02}^* is the complex conjugate of Γ_{02}

Furthermore, let us make the following simplifying assumptions

$$\Gamma_{00} = 0 \tag{3}$$

$$|\Gamma_{02}| \ll 1 \tag{4}$$

$$\sum_{\beta=0}^{\beta=2} \Gamma_{\beta n}\Gamma_{\beta m}^* = \begin{cases} 1, & \text{if} \quad m = n = 0, 1, 2 \\ 0, & \text{if} \quad m \neq n \end{cases} \tag{5}$$

Equation (3) indicates that if in Fig. 1, lines 1 and 2 were matched, line 0 would also be matched looking toward the junction. Equation (4) states that almost all the transmission is made from 0 to 1, or that there is small mode conversion to the spurious mode 2. Equation (5) assumes that the transition is nondissipative. The first two conditions are fulfilled when the transition is made smoothly. The last is probably the most stringent one, especially if the transition is a long tapered waveguide section, but it is always possible to imagine the transition as lossless and attribute its dissipation to the waveguides.

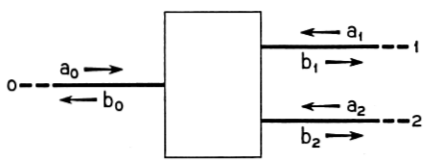

Fig. 1 — Schematic of a three-port junction.

From (2), (3), (4) and (5)

$$\Gamma = \frac{\Gamma_{01}^2 e^{-j\theta_1}}{|\Gamma_{22}|^2} \frac{1}{1 + \left|\frac{\Gamma_{11}}{\Gamma_{22}}\right| e^{-j\varphi}} \left\{1 - \frac{2 |\Gamma_{12}|^2 (1 - \cos \varphi)}{1 - [\Gamma_{22} e^{-j\theta_2} + \Gamma_{11} e^{-j\theta_1}]^2}\right\} \tag{6}$$

where

$$\Gamma_{11} = |\Gamma_{11}| e^{j\varphi_{11}}$$

$$\Gamma_{22} = |\Gamma_{22}| e^{j\varphi_{22}}$$

$$\varphi = \theta_1 - \theta_2 - \varphi_{11} + \varphi_{22}$$

In order to understand this expression physically, let us suppose first that there is no attenuation. The transmission coefficient Γ becomes 0 when the following equations are fulfilled simultaneously

$$\beta_2 \ell - \varphi_{22} = p\pi \qquad p = 0, 1, 2, 3 \cdots \tag{7}$$

and

$$\varphi = (2q + 1)\pi \qquad q = 0, 1, 2, 3 \cdots \tag{8}$$

The first of these equations states that the line carrying the feebly coupled mode must be at resonance, since this condition is satisfied when the electric length of this line is modified by a multiple of π radians. The second condition, (8), implies that both paths, in lines 1 and 2, must differ in such a way that electromagnetic waves coming through them must arrive in opposite phase at the end of the two-mode waveguide. This is quite clear if we think that, in order to get complete reflection, signals coming through lines 1 and 2 must recombine again with the same intensity and opposite phase. In order to get both modes with the same intensity, the converted mode must be built up through resonance; the opposite phase is obtained by an appropriate electric length adjustment. When attenuation is present, Γ will not be 0, and conditions (7) and (8) for minimum transmission are modified only slightly if the

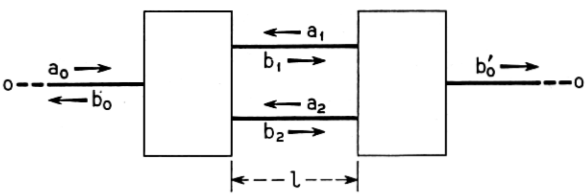

Fig. 2 — Schematic of a two-mode waveguide terminated symmetrically on each side with a single-mode waveguide.

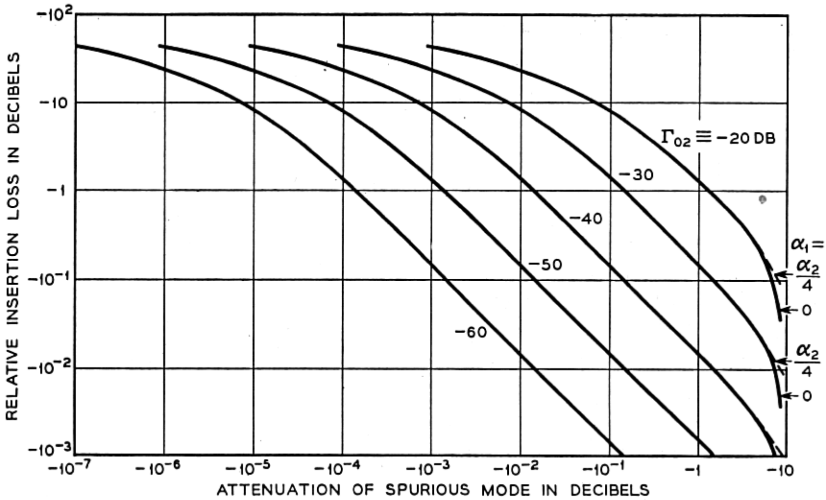

Fig. 3 — Relative insertion loss as a function of the spurious mode attenuation and mode conversion level.

attenuation is low, but the general interpretation of the phenomenon is still the one given above.

From (6) we can calculate the extreme values of $|\Gamma|$ differentiating with respect to ℓ, and we define the relative insertion loss I in db, as the ratio between the minimum and maximum transmitted power expressed in db.

$$I = 20 \log_{10} \left| \frac{\Gamma_{\min}}{\Gamma_{\max}} \right| = 20 \log_{10} \frac{B}{C} \frac{1 - \dfrac{2 \mid \Gamma_{12} \mid^2 (1 + \cosh \alpha\ell)}{1 - C^2 \mid \Gamma_{22} \mid^2 e^{-2\alpha_2 \ell}}}{1 - \dfrac{2 \mid \Gamma_{12} \mid^2 (1 - \cosh \alpha\ell)}{1 + B^2 \mid \Gamma_{22} \mid^2 e^{-2\alpha_2 \ell}}} \tag{9}$$

where

$$B = 1 + \left| \frac{\Gamma_{12}}{\Gamma_{22}} \right|^2 e^{-\alpha\ell} \qquad C = 1 - \left| \frac{\Gamma_{12}}{\Gamma_{22}} \right|^2 e^{-\alpha\ell} \qquad \alpha = \alpha_1 - \alpha_2$$

For the most important practical case, that is, when the maximum value attainable by $\cosh \alpha\ell$ is of the order of 1, and knowing from (3), (4) and (5) that

$$\mid \Gamma_{12} \mid^2 = \mid \Gamma_{02} \mid^2 (1 - \mid \Gamma_{02} \mid^2)$$

$$\mid \Gamma_{22} \mid^2 \cong 1 - 2 \mid \Gamma_{02} \mid^2$$

$$I \cong 20 \log_{10} (1 + 2 |\Gamma_{02}|^2 e^{-\alpha \ell}) \left\{ 1 - \frac{2 |\Gamma_{02}|^2 (1 + \cosh \alpha \ell)}{1 - e^{-2\alpha_2 \ell} + 2 |\Gamma_{02}|^2 (1 + e^{-\alpha \ell}) e^{-2\alpha_2 \ell}} \right\} \tag{10}$$

From this expression we deduce

(a), I is strongly reduced when $\alpha_2 \ell \gg \Gamma_{02}$.

(b), Attenuation in line 1 is not an important factor until $\alpha_1 \ell$ and $| \alpha_1 - \alpha_2 | \ell$ are of the order of 1. In other words, for low attenuation in both lines, $\alpha_2 \ell$ assumes a major importance in the determination of I because it influences the conditions of resonance. That the effect of $\alpha_1 \ell$ is small is shown in Fig. 3 (dotted line for the particular case $\alpha_1 = \alpha_2/4$).

In order to handle the general problem, (10) has been plotted in Fig. 3. We can enter with any two and obtain the third following quantities: I_1, relative insertion loss in db; $10 \log_{10} e^{-2\alpha_2 \ell}$, attenuation in db of the spurious mode in the resonating environment; and $20 \log_{10} \Gamma_{02}$ conversion level at the junction, in db, of power in the spurious mode relative to that in the first line.

APPLICATION OF THESE RESULTS TO A TE_{01} TRANSMITTING SYSTEM

The results of the preceding section have been checked experimentally by measuring the relative insertion loss of different lengths of ⅞″ diameter round waveguide tapered at both ends to round waveguides of 7⁄16″ diameter. This waveguide is shown in Fig. 4 with a schematic diagram of the measuring set. In the round transmission line A-B, section A will propagate only TE_{01}. Section B, which has been expanded by means of the conical taper T_1, can support TE_{02} and TE_{03} in addition to the principal TE_{01} mode. This section is a closed region to the spurious modes (TE_{02}, TE_{03}) whose length can be adjusted to resonate each one of these modes. A sliding piston provides a means for varying the length, ℓ, of section B.

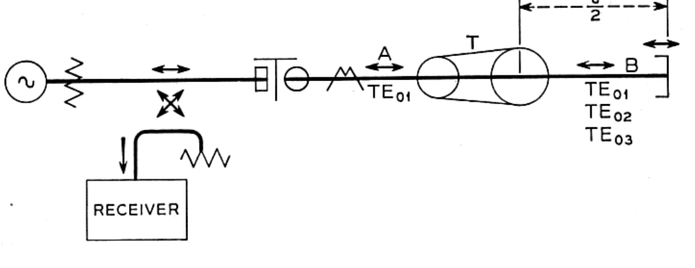

Fig. 4 — Circuit used to measure TE_{01} insertion loss due to resonance of the TE_{02} and TE_{03} modes.

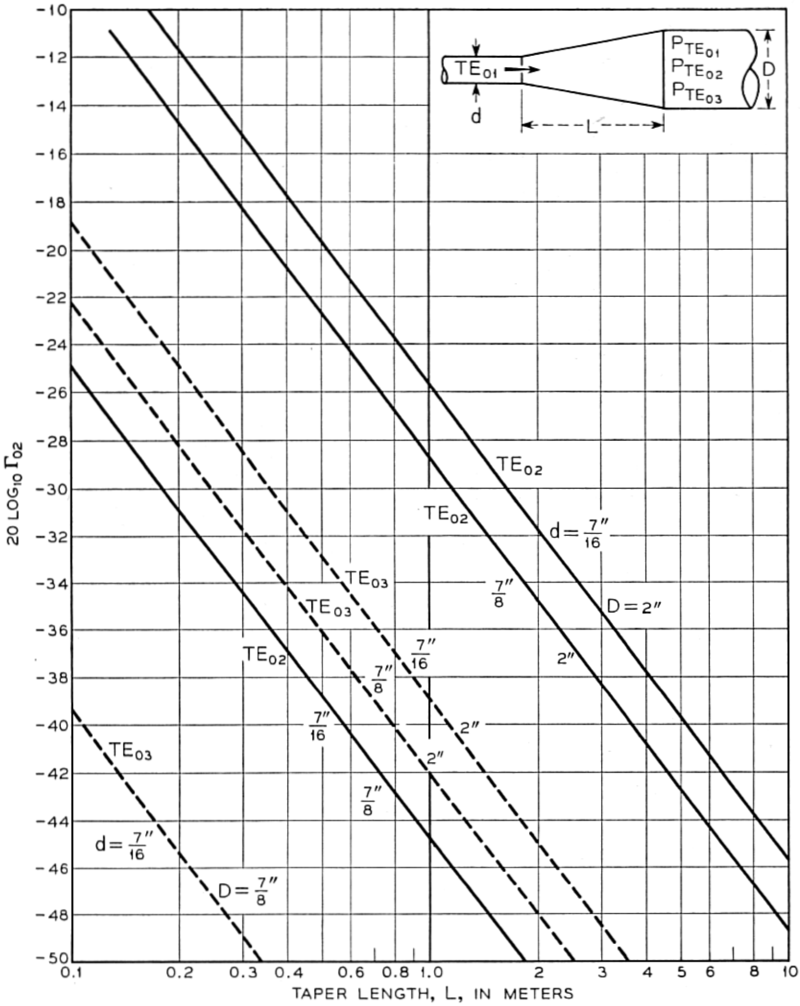

Fig. 5 — Mode conversion of TE_{02} and TE_{03} relative to TE_{01} generated by a conical taper.

The relative levels of TE_{02} and TE_{03} conversions, which have been calculated from unpublished work of S. P. Morgan are shown plotted in Fig. 5 for the waveguide sizes employed in the millimeter wavelength band. The conversions, $20 \log_{10} \Gamma_{02}$, are plotted in terms of the TE_{02} and TE_{03} powers relative to the TE_{01} mode power and are expressed in db as a function of the taper length L, in meters.

Fig. 6 shows the theoretical and experimental values obtained for TE_{01} relative insertion loss. Since the minimum length of pipe tested is

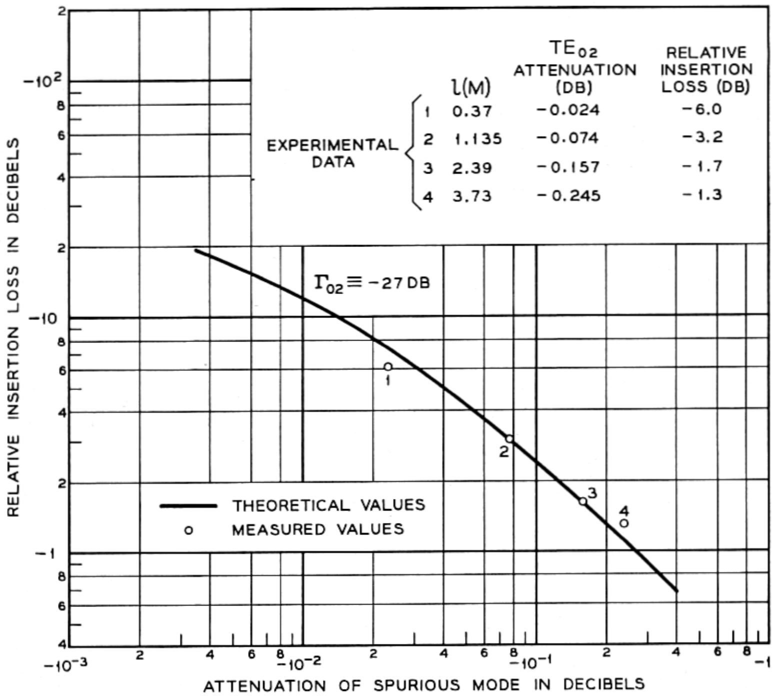

Fig. 6 Theoretical and measured relative insertion loss in the TE_{01} transmission system of Fig. 4.

several times the length of the tapers, the losses in the transitions are fairly small compared to the losses in the multimode guide and this justifies assumption (5). The resonance due to the other modes is too small to be appreciable. This is understandable since, according to (10), the value of the mode conversion for the TE_{03} (Fig. 5) and the attenuation for the shortest length of pipe tested, the calculated relative insertion loss is less than -0.1 db.

CONCLUSIONS

The resonance of spurious modes in a closed environment can produce a large insertion loss of the transmitting mode. In a fairly narrow band device it is possible to avoid this problem by selecting a proper waveguide size for the closed environment. In a broad-band system the losses can be minimized by providing a high attenuation and a low mode conversion for the spurious mode. For example, it may be noted, by referring to Fig. 3, that mode conversion as high as -20 db with a spurious mode loss of -8 db results in only an -0.1 db insertion loss for the transmitting mode.