The Forward Characteristic of the PIN Diode

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A theory is given for the forward current-voltage characteristic of the PIN diffused junction silicon diode. The theory predicts that the device should obey a simple PN diode characteristic until the current density approaches 200 amp/cm^2. At higher currents an additional potential drop occurs across the middle region proportional to the square root of the current. A moderate amount of recombination in the middle region has little effect on the characteristic. It is shown that the middle region cannot lead to anomalous characteristics at low currents.

INTRODUCTION

In some diode applications it is desirable to have a very low ohmic resistance as well as a high reverse breakdown voltage. A device meeting these requirements, in which the resistance is low because of heavily doped P^+ and N^+ contacts and the breakdown voltage is high because of a lightly doped layer between the contacts, has been described by M. B. Prince.[1] The device is shown schematically in Figure 1a and consists of three regions, the P^+ contact, the middle P layer, and the N^+ contact. The device is called a PIN diode because the density P of uncompensated acceptors in the middle region is much less than P^+ or N^+ and in normal forward operation much less than the injected carrier density.[2]

We shall let the edge of the P^+P junction in the middle region be $x = 0$, and the edge of the PN^+ junction in the middle region be $x = w$. Thus the region $0 \leqq x \leqq w$ is space charge neutral and bounded at each end by space charge regions whose width is of the order of the Debye length

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[1] Prince, M. B., Diffused *p-n* Junction Silicon Rectifiers, B.S.T.J., page 661 of this issue.

[2] A device with similar geometry has been discussed by R. N. Hall, Proc. I.R.E., **40,** p. 1512, 1952.

$$\lambda = (K/\beta e P)^{1/2} \sim 1.5 \times 10^{-5} \text{ cm.} \tag{1}$$

where K is the dielectric constant, e is the electronic charge, and β is the constant

$$\beta = e/kT = \mu_n/D_n = \mu_P/D_P \tag{2}$$

which at room temperature is 38.7 volt^{-1}. We shall denote points in the P^+ and N^+ contacts on the edges of the space charge regions by oo and ww respectively. Thus n_{oo} is the electron density in the P^+ contact at the junction, and n_o is the electron density at the same junction in the middle region. Similarly p_{ww} is the hole density at the junction in the N^+ contact and p_w is the hole density at the junction in the middle region. We shall denote equilibrium carrier densities in the three regions by n_{P^+}, n_P, p_p, p_{N^+}. Typical values for the parameters characterizing

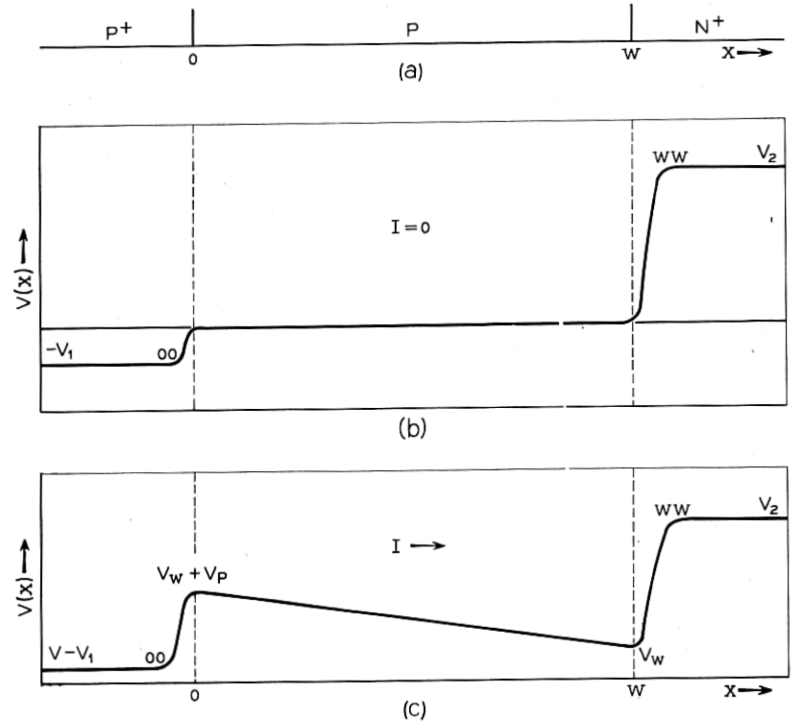

Fig. 1 — Schematic representation of the PIN diode with the P+ and N+ contacts regarded as extending to infinity. (b) shows the electrostatic potential in equilibrium and (c) shows the potential when a forward current flows.

the device are

$$\begin{aligned} w &\sim 2 \times 10^{-3} \text{ cm} \\ P &\sim 10^{15} \text{ cm}^{-3} \\ N^+, P^+ &\sim 10^{18} \text{ cm}^{-3} \\ L_n, L_p &\sim 10^{-4} \text{ cm} \end{aligned} \tag{3}$$

where L_n, L_p are minority carrier diffusion lengths in the contacts.

The present treatment makes three distinct approximations. The first is to neglect the voltage drop in the contacts. The highest currents ordinarily used are of the order of 500 amp/cm^2 which should produce an ohmic drop in the contacts of about 1 volt/cm. Since the entire diode has a length of about 0.01 cm we are neglecting only about 0.01 volts in this approximation.

The second approximation is to regard the Debye length as small compared to w and the diffusion lengths L_n, L_p. If L_n, L_p are as small as the typical values given in (3) the error made in this approximation is not completely negligible. Nevertheless, we use the approximation because it enables us to regard the device as three relatively large neutral regions and two relatively narrow space charge regions. The behavior of the device can then be determined by solving for the diffusion and drift of carriers in the neutral regions subject to boundary conditions connecting the carrier densities across the space charge layers.

The third approximation is to neglect any increase in majority carrier density in the contacts due to injection of minority carriers. This approximation is valid until the current density approaches 5×10^4 amp/cm^2, which is well above anticipated operating currents. It is conceivable that in some junctions all the current may flow through small active spots at which the current density is very high, perhaps exceeding the above figure. In such cases the current flow is two or three dimensional and the present analysis would not apply.

It is also necessary to assume some law for carrier recombination. We shall assume that recombination in the contacts is linear in the injected minority carrier density

$$\frac{dI_n}{dx} \sim \frac{n - n_{P^+}}{\tau} \tag{4}$$

Modification of the theory to suit other recombination laws is simple in principle, although considerable analytical complications might be encountered. It seems most likely that in silicon PN junctions the recombination actually is nonlinear. It can be shown that if the recombination follows some power ν of the injected density

$$\frac{dI_n}{dx} \sim n^{\nu} \tag{5}$$

the forward characteristic of a simple PN junction is of the form

$$\exp\left[\tfrac{1}{2}\beta(\nu + 1)V\right] \tag{6}$$

Thus nonlinear recombination can account for the observation that in silicon diodes the slope of V versus log I is usually much less than β. Our purpose here is not to study this interesting effect, but to study those effects which are due to the presence of the middle region. Therefore, we assume linear recombination for the sake of simplicity. In the last section we give a brief consideration of what to expect in the case of nonlinear recombination in the contacts. Recombination in the middle region will also be assumed to be linear in the injected carrier density, but this assumption is not critical, since it turns out that a moderate amount of recombination in the middle region does not change the qualitative behavior of the device.

BASIC EQUATIONS

Fig. 1(b)[3] shows the electrostatic potential $V(x)$ for the equilibrium case $I = 0$. The potential is constant except in the space charge layers. If we call the potential of the middle region zero, the P^+ and N^+ contacts are at the potentials $-V_1$ and V_2 respectively, where

$$\begin{aligned} \beta V_1 &= \ell n\,(P^+/p_P) \\ \beta V_2 &= \ell n\,(N^+/n_P) \end{aligned} \tag{7}$$

Figure 1c shows the potential when a forward current I flows and a forward bias V is produced across the device. We shall define the potential so that the N^+ contact remains at V_2, which puts the P^+ contact at potential $V - V_1$. The potential at a point x is then given by

$$V(x) = V_2 - \int_{ww}^{x} E(x)\,dx \tag{8}$$

where $E(x)$ is the electric field assumed zero in the contact regions $x >$ ww and $x < oo$. The applied bias V consists of three terms

$$V = V_0 + V_P + V_w \tag{9}$$

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[3] This potential distribution has been discussed by A. Herlet and E. Spenke, Zeits. f. Ang. Phys., **B7, H3,** p. 149, 1955.

where V_0 is the forward bias across the junction at $x = 0$, V_P is the potential drop in the middle region, and V_w is the forward bias across the junction at $x = w$. In this notation $V(0) = V_w + V_P$ and $V(w) = V_w$.

The total current density is constant

$$I_n(x) + I_p(x) = I \tag{10}$$

We shall denote electric current densities by eI_n, eI_p, so that I_n, I_p, I have the dimensions of (particles/cm^2-sec). At $x = 0$ and $x = w$ the minority carrier currents must flow into the contacts by diffusion, which gives the boundary conditions

$$\begin{aligned} I_p(w) &= I_{ps}\left\{\frac{p_{ww}}{p_{N^+}} - 1\right\} \\ I_n(0) &= I_{ns}\left\{\frac{n_{oo}}{n_{P^+}} - 1\right\} \end{aligned} \tag{11}$$

where I_{ps}, I_{ns} are saturation current densities

$$I_{ps} = \frac{p_N{}^+ D_p}{L_P}, \qquad I_{ns} = \frac{n_P{}^+ D_n}{L_n} \tag{12}$$

The order of magnitude of the saturation current density is given by

$$e(I_{ns} + I_{ps}) \sim 3 \times 10^{-10} \text{ amp/cm}^2 \text{ in } Si$$

based on the typical values of (3). Equations (11) contain the assumptions of linear recombination and small injection into the contacts as discussed in the introduction.

In the middle region the current densities satisfy

$$\begin{aligned} I_p(x) &= D_p\left\{-\frac{dp}{dx} + \beta pE\right\} \\ I_n(x) &= D_n\left\{\frac{dn}{dx} + \beta nE\right\} \end{aligned} \tag{13}$$

Let us assume these equations remain valid in the space charge regions.[4] Since these space charge regions are narrow I_n and I_p can be considered constant and the solution of (13) in the space charge regions is

$$\begin{aligned} p(x) &= e^{-\beta V(x)}\left\{p_{ww}e^{\beta V_2} - \frac{I_p(w)}{D_P}\int_{ww}^{x} e^{\beta V(x)}\,dx\right\} \\ n(x) &= e^{\beta V(x)}\left\{n_{00}e^{-\beta(V-V_1)} + \frac{I_n(0)}{D_n}\int_{00}^{x} e^{-\beta V(x)}\,dx\right\} \end{aligned} \tag{14}$$

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[4] Shockley, W., B.S.T.J., **28,** p. 435, 1949.

Since $\lambda/L_p \ll 1$ we can write for the junction at $x = w$

$$\begin{aligned} p(w) &= e^{-\beta V_w} \left\{ p_{ww} e^{\beta V_2} - \frac{(p_{ww} - p_N{}^+)}{L_p} \int_{ww}^{w} e^{\beta V} dx \right. \\ &= p_{ww} e^{\beta(V_2 - V_w)} \left\{ 1 - 0 \left(\frac{\lambda}{L_p} \right) \right\} \end{aligned} \tag{15}$$

where $0(\lambda/L_p)$ means a term of order λ/L_p. Thus we see that if we may neglect λ/L_p and λ/L_n we have the following simple boundary conditions at the junctions

$$\begin{aligned} n_{oo} &= n_o(n_P{}^+/n_P)e^{\beta V_0} \\ p_o &= p_P \, e^{\beta V_0} \\ n_w &= n_P \, e^{\beta V_w} \\ p_{ww} &= p_w(p_N{}^+/p_P)e^{\beta V_w} \end{aligned} \tag{16}$$

It is clear that in order to divide the device into three neutral regions we must also be able to neglect λ/w.

Finally, we have the condition of space charge neutrality

$$p - n = P \tag{17}$$

It can be shown that the term $K^{-1} \, dE/dx$ is of order $(\lambda/L)^2$ or $(\lambda/w)^2$ and therefore negligible in our approximation. Therefore (17) is the Poisson equation for the middle region in our approximation. When we use (17) we are not saying that $E(x)$ is constant but only that $K^{-1} \, dE/dx$ is negligible compared to $p(x)$ and $n(x)$. The basic equations then are (10), (11), (13), (16), (17).

Large Injection, No Recombination

In this section we consider current densities of the order of magnitude of those that flow in normal operation of the diode as a power rectifier. These currents inject large densities of electrons and holes into the middle region greatly increasing its conductivity. The result is that the voltage drop V_P is small even though the normal resistivity of the middle region is high. For this reason the device has been called a conductivity modulated rectifier. Also in this section we shall neglect recombination in the middle region, which makes $I_n(x)$ and $I_p(x)$ constant and greatly simplifies the analysis. The effect of recombination is to remove carriers and increase the drop across the middle region. Therefore, it is desirable to keep recombination in the middle region as low as possible.

Under conditions of large injection we can say

$$\begin{aligned} n &\gg P, & p &\gg P \\ n_{oo} &\gg n_P{}^+ & p_{ww} &\gg p_N{}^+ \end{aligned} \tag{18}$$

so that (11) becomes

$$\begin{aligned} I_n &= I_{ns}(n_{oo}/n_P{}^+) \\ I_p &= I_{ps}(p_{ww}/p_N{}^+) \end{aligned} \tag{19}$$

and (17) becomes

$$n(x) = p(x) \qquad 0 \leqq x \leqq w \tag{20}$$

Equation (16) becomes

$$\begin{aligned} n_{oo} &= n_o(n_P{}^+/n_P)e^{\beta V_0} \\ n_o &= p_P e^{\beta V_0} \\ n_w &= n_P e^{\beta V_w} \\ p_{ww} &= n_w(p_N{}^+/p_P)e^{\beta V_w} \end{aligned} \tag{21}$$

Equations (13) can be written

$$\begin{aligned} \beta E &= \frac{I_n + bI_p}{2D_n n} \\ \frac{dn}{dx} &= \frac{I_n - bI_p}{2D_n} \end{aligned} \tag{22}$$

where $b = D_n/D_p$. Combining (19) and (21) gives the equations

$$\begin{aligned} n_o &= n_i(I_n/I_{ns})^{1/2} \\ n_w &= n_i(I_p/I_{ps})^{1/2} \end{aligned} \tag{23}$$

where $n_i^2 = n_P p_P$ is a constant, and also

$$\begin{aligned} \beta V_0 &= \tfrac{1}{2} \ln \frac{n_P}{p_P} \frac{I_n}{I_{ns}} \\ \beta V_w &= \tfrac{1}{2} \ln \frac{p_P}{n_P} \frac{I_p}{I_{ps}} \end{aligned} \tag{24}$$

From the first equation (22) we have

$$\beta V_P = \frac{I_n + bI_p}{2D_n} \int_0^w \frac{dx}{n(x)} \tag{25}$$

Upon invoking the second equation of (22) we get

$$\beta V_P = \frac{I_n + bI_p}{I_n - bI_p} \ell n \frac{n_w}{n_o} \tag{26}$$

and

$$n_w = n_o + \frac{I_n - bI_p}{2D_n} w. \tag{27}$$

We see that V_p is always positive in sign whatever the sign of $I_n - bI_p$.

We now define a parameter

$$\gamma \equiv n_o/n_w \tag{28}$$

and a device constant

$$R \equiv I_{ns}/I_{ps} \tag{29}$$

Then from (23) and (10)

$$\begin{gathered} I_n/I_p = R\gamma^2 \\ I_n = \frac{R\gamma^2}{1 + R\gamma^2} I \qquad I_p = \frac{1}{1 + R\gamma^2} I \end{gathered} \tag{30}$$

Combining (23), (27) and (30) gives the equation for γ as a function of total current

$$\begin{aligned} \gamma &= 1 - \frac{I_n - bI_p}{2D_n} \frac{w}{n_w} \\ &= 1 - \sqrt{\frac{I}{I_0}} \frac{(\gamma/\gamma_\infty)^2 - 1}{\sqrt{1 + b(\gamma/\gamma_\infty)^2}} \end{aligned} \tag{31}$$

where

$$\gamma_\infty^2 \equiv b/R \tag{32}$$

and I_0 is a unit of (particle) current density characteristic of the device

$$I_0 = \frac{4D_p^2 n_i^2}{w^2 I_{ps}} = 4\left(\frac{L_p}{w}\right)^2 \frac{N^+ D_p}{L_p} \tag{33}$$

A typical value for $e\, I_0$ in a silicon diode is

$$e\, I_0 \sim 200 \text{ amp/cm}^2 \tag{34}$$

based on (3).

From (26) the potential drop in the middle region can be written

$$\beta V_P = -\frac{\gamma^2 + \gamma_\infty^2}{\gamma^2 - \gamma_\infty^2} \ln \gamma \tag{35}$$

From (24) and (30)

$$\beta(V_0 + V_w) = \ln \frac{I}{I_0} + \ln \frac{\gamma}{1 + b(\gamma/\gamma_\infty)^2} + \ln \frac{I_0}{I_{ps}} \tag{36}$$

Thus the total applied bias V as a function of total current density I is given by

$$\beta V = \ln \frac{I}{I_0} - \frac{\gamma^2 + \gamma_\infty^2}{\gamma^2 - \gamma_\infty^2} \ln \gamma + \ln \frac{\gamma}{1 + b(\gamma/\gamma_\infty)^2} + \ln \frac{I_0}{I_{ps}} \tag{37}$$

where $\gamma(I)$ is the (positive) solution of (31).

Thus far we have referred the problem of the $V - I$ characteristic to the problem of calculating $\gamma(I)$ from (31). We see that in the limits of high and low current γ approaches the limits

$$\begin{aligned} \gamma &\rightarrow 1 & I &\ll I_0 \\ \gamma &\rightarrow \gamma_\infty & I &\gg I_0 \end{aligned} \tag{38}$$

and in general lies between these limits. A good approximate solution is readily obtained by replacing (31) with the quadratic equation

$$\begin{aligned} \gamma &= 1 - z[(\gamma/\gamma_\infty)^2 - 1] \\ z &= (I/I_0)^{1/2} (1 + b)^{-1/2} \end{aligned} \tag{39}$$

which has the solution

$$\gamma = \frac{\sqrt{\gamma_\infty^4 + 4(1 + z)z\gamma_\infty^2} - \gamma_\infty^2}{2z} \tag{40}$$

A plot of this solution is shown in Fig. 2 as a function of z for $\gamma_\infty = \frac{1}{2}$, $\gamma_\infty = 2$. Since $\gamma(I)$ is bounded by unity and γ_∞, which usually will be of order unity, we can reject some of the dependence of V upon γ and retain only its essential dependence upon I. This appears in the first and second terms of (37). By means of (31) this second term can be written

$$\beta V_p = \left[\frac{\ln \gamma}{\gamma - 1} \frac{(\gamma/\gamma_\infty)^2 + 1}{\sqrt{1 + b(\gamma/\gamma_\infty)^2}}\right] \sqrt{\frac{I}{I_0}} \tag{41}$$

Retaining only the essential dependence on I we write this equation

$$\beta V_p = C(I/I_o)^{1/2} \tag{42}$$

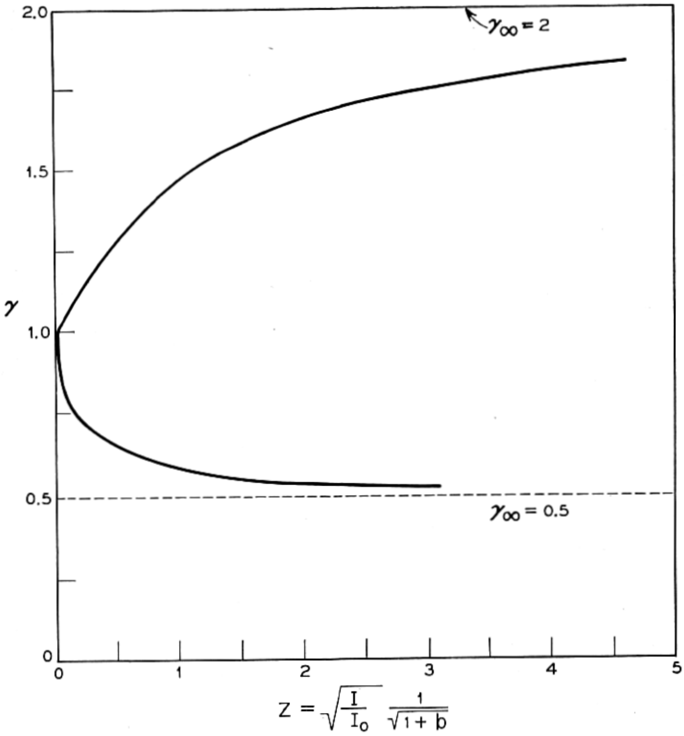

Fig. 2 — The function $\gamma(z)$ given by equation (40) for two choices of γ_∞.

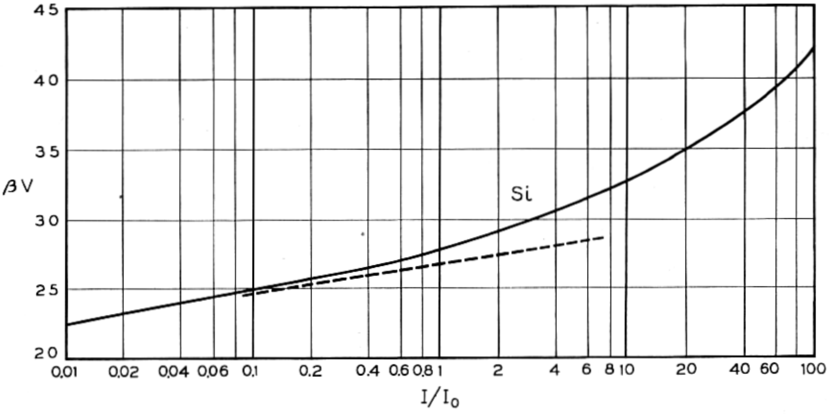

Fig. 3 — The voltage-current characteristic of the PIN diode according to equation (44). The dashed line represents an ideal *PN* diode and $eI_0 \sim 200$ amp/cm^2 in silicon.

where C is a constant representing the slowly varying coefficient of $(I/I_o)^{1/2}$ in (41). We choose C such that (42) becomes exact at high current density when βV_p is large

$$C = \frac{\ell n\, \gamma_\infty}{\gamma_\infty - 1} \frac{2}{\sqrt{b+1}} \tag{43}$$

When we regard the third and fourth terms of (37) together as a constant βV_c we obtain the simplified voltage-current characteristic

$$\beta V = \ell n \frac{I}{I_0} + C \sqrt{\frac{I}{I_0}} + \beta V_c \tag{44}$$

In this approximation it is unnecessary to evaluate $\gamma(I)$ from (31).

Fig. 3 shows plots of βV versus I/I_o calculated from (44). For plotting the curves the value $C = 1.1$ was used. To choose a value for βV_c we put $\gamma = 1$, which gives

$$\ell n \frac{\gamma}{1 + b(\gamma/\gamma_\infty)^2} \rightarrow \ell n \frac{1}{1+R} \qquad \gamma \rightarrow 1 \tag{45}$$

so that

$$\beta V_c \rightarrow \ell n[I_o/(I_{ns} + I_{ps})] \tag{46}$$

which has the value 27 in silicon according to the values in (3). The dotted line is the asymptote approached by the curve at low current densities

$$\beta V \rightarrow \ell n \frac{I}{I_{ns} + I_{ps}} \qquad I \ll I_0 \tag{47}$$

This is the characteristic of a simple PN junction when

$$I \gg I_{ns} + I_{ps}\,.$$

We return now to the question of when the large injection conditions (18) are satisfied. Let us suppose I is much less than I_0 so that $\gamma \sim 1$, $I_n/I_p \approx R$. It follows from (30) and (23) that

$$n_o \approx n_w \approx n_i[I/(I_{ns} + I_{ps})]^{1/2} \tag{48}$$

Now let us set $n_o \gg P$ which gives a condition on the current density

$$I \gg (P/n_i)^2\,(I_{ns} + I_{ps}). \tag{49}$$

Setting $n_{oo} \gg n_p{}^+$, $p_{ww} \gg p_N{}^+$ gives

$$I \gg I_{ns} + I_{ps}\,. \tag{50}$$

Usually $P \gg n_i$ so that (49) includes (50). When numbers are put in from (3) we get the condition for large injection

$$eI \gg 0.07 \text{ amp/cm}^2 \text{ in } Si \tag{51}$$

Since this current in (51) is much less than eI_o, we may quite properly speak of large injection $n \gg P$ and small currents $I \ll I_o$ at the same time.

Let us denote by

$$I_{CM} = (P/n_i)^2 \, (I_{ns} + I_{ps}) \tag{52}$$

the current density at which conductivity modulation starts to be important. Then we may distinguish three ranges of current: (a) very small current $I < I_{CM}$ for which large injection analysis does not apply; (b) low current $I_{CM} < I < I_o$ for which large injection analysis applies, but the voltage drop V_p in the middle region is negligible; (c) large current $I > I_o$ for which V_P is sizable. The treatment of this section has covered ranges (b) and (c). Range (c) (as treated here) does not extend to infinity but only up to current densities of the order

$$\frac{eN^+D_p}{L_p} \sim 8 \times 10^4 \text{ amp/cm}^2$$

so that the diffusion currents in the contacts may be treated as a small injection.

Small Injection, No Recombination

In this section, we shall cover ranges (a) and (b) in current density. We must go back to the basic equations, but we shall make use of two facts that have come out of the large injection analysis: (a) βV_p is negligible when $I \ll I_o$; (b) $\gamma = n_o/n_w \approx 1$ which means $n(x)$ and $p(x)$ are essentially constant in the middle region $0 \leqq x \leqq w$ when $I \ll I_o$. When we set

$$n_o = n_w, \qquad p_o = p_w \tag{53}$$

equations (16) give us

$$\begin{aligned} n_{oo} &= n_P{}^+ e^{\beta(V_0+V_w)} \\ p_{ww} &= p_N{}^+ e^{\beta(V_0+V_w)} \end{aligned} \tag{54}$$

Then (11) gives

$$I = I_n + I_p = (I_{ns} + I_{ps}) \, [e^{\beta(V_0+V_w)} - 1] \tag{55}$$

Now $V_o + V_w$ is the total applied bias when V_P can be neglected; therefore we obtain the characteristic

$$\beta V = \ell n \left(\frac{I}{I_{ns} + I_{ps}} + 1 \right) \tag{56}$$

which is valid until I approaches I_o. Of course we would not have obtained this ideal characteristic of a simple *PN* junction had we taken recombination into account; our result depends upon the constancy of $n(x)$ and $p(x)$ in the middle region. For the case of no recombination in the middle region (56) and (44) cover ranges (a), (b) and (c). Instead of (44) the more exact expression (37) could be used requiring the evaluation of $\gamma(I)$ from (31). It seems that the extra refinement is of no help in understanding the device and unnecessary in treating experimental data. Therefore, we shall adopt (44) and the approximations leading to it as a model for treating the more complicated recombination case. That is, we shall seek a generalization of (44) which takes recombination into account in a sufficiently good approximation.

Large Injection with Recombination

We are interested in determining the effect of recombination in the middle region upon the operating characteristics of the device. Therefore we go immediately to the large injection case $n = p$. Equation (16) become

$$\begin{aligned} n_w &= n_P e^{\beta V_w} & \qquad p_{ww} &= n_w (p_N{}^+/p_P) e^{\beta V_w} \\ n_o &= p_P e^{\beta V_0} & \qquad n_{oo} &= n_o (n_P{}^+/n_P) e^{\beta V_0} \end{aligned} \tag{57}$$

which gives

$$\beta(V_o + V_w) = \ell n (n_w n_o / n_i^2) \tag{58}$$

We shall assume that recombination is linear in the injected carrier density to simplify the calculation. It will be possible, later to approximate bimolecular recombination by using an appropriate value for the lifetime τ corresponding to the injected carrier density. Therefore we write

$$\frac{dI_n}{dx} = -\frac{dI_p}{dx} = \frac{n}{\tau} \tag{59}$$

Eliminating $I_n(x)$ by use of (13) gives the equation for $n(_x)$

$$\frac{d^2n}{dx^2} = \frac{n}{L^2} \tag{60}$$

where L is the effective diffusion length in the middle region

$$L = [2D_n \tau/(b + 1)]^{1/2} \tag{61}$$

The solution of (60) may be written

$$n(z) = \frac{n_0 \sinh (w - z) + n_w \sinh z}{\sinh \omega} \tag{62}$$

where $z = x/L$ is the position variable and $\omega = w/L$ is the length of the middle region in units of L. Fig. 4 shows several of these solutions for the case $n_o = n_w$.

In equation (60) and the solution (62) we have neglected the equilibrium carrier densities n_p, p_P. The criterion for the validity of this approximation is

$$\sinh \tfrac{1}{2}\omega \ll (n_o/P), (n_w/P) \tag{63}$$

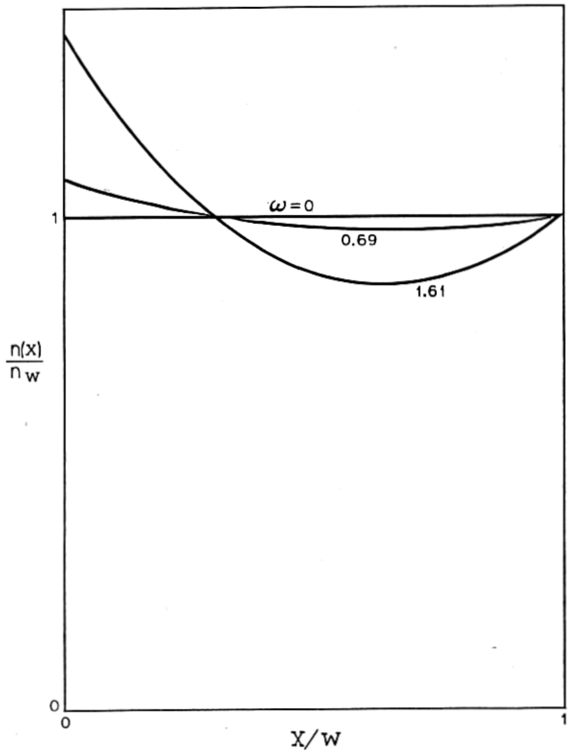

Fig. 4 — The carrier density according to equation (62) for the case $n_0 = n_w$ and several values of ω.

arrived at by considering the minima in the solutions for $\omega \gg 1$. This is really a criterion for conductivity modulation, so we shall assume henceforth that it is satisfied.

We now modify (13) by setting $n = p$ and eliminating $E(x)$ by use of (22)

$$\begin{aligned} I_n(x) &= \frac{bI + 2D_n n'(x)}{b+1} \\ I_p(x) &= \frac{I - 2D_n n'(x)}{b+1} \end{aligned} \tag{64}$$

where $n'(x) = dn/dx$. Inserting these currents into (22) gives $E(x)$ and integrating gives the potential drop V_P in the middle region

$$\beta V_p = \frac{bI}{(b+1)D_n} \int_0^w \frac{dx}{n} - \frac{b-1}{b+1} \ln \frac{n_w}{n_0} \tag{65}$$

This is the generalization of (26) for linear recombination.

The direct evaluation of (58) and (65) in terms of the total current I leads to a very complicated expression for the applied voltage. It will be shown in the next section that this result reduces in its simplest approximate form retaining only the essential dependence on ω to the formula

$$\beta V \approx \ln \frac{I}{I_{ns} + I_{ps}} + C \sqrt{\frac{I}{I_0(\omega)}} \tag{66}$$

which is identical with (44) except that the characteristic current density is a function of ω

$$\begin{aligned} I(\omega) &= I_0 g(\omega) \\ g(\omega) &= \frac{(\omega/2)^2}{\left[\cosh \frac{\omega}{2} \tan^{-1}\left(\sinh \frac{\omega}{2}\right)\right]^2} \\ &= 1 - \frac{\omega^2}{6} + \frac{\omega^4}{48} - \cdots \end{aligned} \tag{67}$$

Fig. 5 shows a plot of $g(\omega)$. These results show that if $\omega < 1$ as we might expect in a good diode recombination has no significant effect on the forward voltage-current characteristic in the conductivity modulation range of operation.

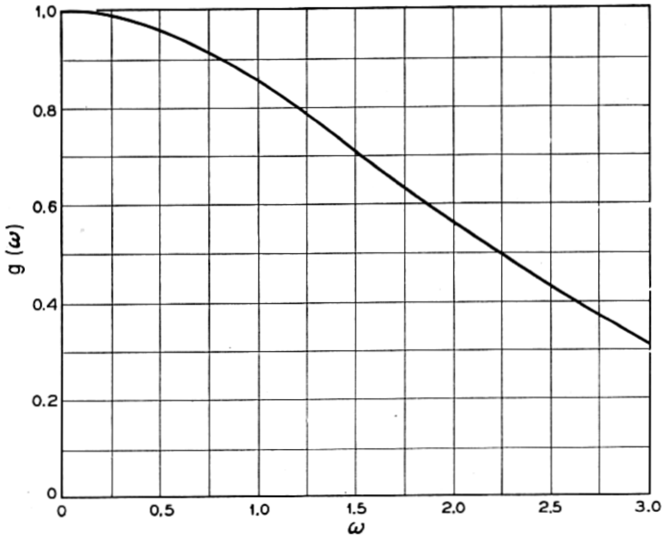

Fig. 5 — The function $g(\omega)$ of equation (67).

Analysis

We denote

$$\xi = \frac{n_w}{n_i}, \qquad \zeta = \frac{n_0}{n_i} \tag{68}$$

From (11) and (67)

$$I_p(\omega) = I_{ps}\xi^2, \qquad I_n(0) = I_{ns}\zeta^2 \tag{69}$$

By means of (62) and (64) we eliminate I_n and I_p and obtain the equations

$$\begin{aligned} (b + 1)I_{ps}\xi^2 &= I - I_r(\xi \cosh \omega - \zeta) \\ (b + 1)RI_{ps}\zeta^2 &= bI + I_r(\xi - \zeta \cosh \omega) \end{aligned} \tag{70}$$

where I_r is a (particle) current density

$$I_r = \frac{2D_n n_i}{L \sinh \omega} \tag{71}$$

In principle we could solve (70) for ξ and ζ as functions of I with R and ω as parameters; this would determine βV through (58) and (65) and complete the problem. First we shall rewrite these equations in terms of γ as in the analysis of the second section.

If we eliminate I from equations (24) we get

$$\begin{aligned} bI_{ps} + \frac{I_r}{\xi}\,\frac{b\cosh\omega + 1}{b + 1} &= RI_{ps}\gamma^2 \\ &\quad + \gamma\,\frac{I_r}{\xi}\,\frac{\cosh\omega + b}{b + 1} \end{aligned} \tag{72}$$

which can be solved for ξ

$$\xi = \frac{I_r}{I_{ps}}\,\frac{\cosh\omega + b}{b + 1}\,\frac{\gamma_0 - \gamma}{R\gamma^2 - b} \tag{73}$$

where

$$\gamma_0 = \frac{b\cosh\omega + 1}{\cosh\omega + b} \tag{74}$$

Substituting (73) into (70) gives the equation satisfied by γ

$$\left(\gamma - \frac{R\gamma^2\cosh\omega + 1}{R\gamma^2 + \cosh\omega}\right)(\gamma - \gamma_0) = \frac{I}{I_{00}}\,\frac{[(\gamma/\gamma_\infty)^2 - 1]^2}{R\gamma^2 + \cosh\omega} \tag{75}$$

where I_{00} is a characteristic (particle) current density

$$I_{oo} = I_o\left[\frac{\omega}{\sinh\omega}\right]^2\frac{\cosh\omega + b}{b + 1} \tag{76}$$

Now the solution of (75) has two branches which as $I \rightarrow 0$ approach values given by

$$\begin{aligned} &\text{a)} && \gamma \rightarrow \gamma_0 \\ &\text{b)} && \gamma \rightarrow \frac{R\gamma^2\cosh\omega + 1}{R\gamma^2 + \cosh\omega} \end{aligned} \tag{77}$$

As I increases the first branch remains positive and approaches γ_∞ as $I \rightarrow \infty$. The second branch becomes negative and approaches $-\gamma_\infty$. Therefore, we choose that branch which satisfies

$$\begin{aligned} \gamma(0) &= \gamma_0 = \frac{b\cosh\omega + 1}{b + 1} \\ \gamma(\infty) &= \gamma_\infty = (b/R)^{1/2} \\ \gamma &> 0 \end{aligned} \tag{78}$$

On this branch γ always lies between γ_0 and γ_∞, and γ never approaches the quantity in (77b). Therefore we replace $R\gamma^2$ by b (as if $\gamma = \gamma_\infty$) in the first factor on the left of (75), and obtain the simpler form

$$\gamma - \gamma_0 = -\sqrt{\frac{I}{I_{00}}}\,\frac{(\gamma/\gamma_\infty)^2 - 1}{\sqrt{R\gamma^2 + \cosh \omega}} \tag{79}$$

which is the generalization of (31).

The drop βV_P in the middle region given by (65) can be written

$$\beta V_P = \frac{b-1}{b+1}\,\ell n\,\gamma + \frac{2}{b+1}\sqrt{\frac{I}{I_{00}}}\,\sqrt{R\gamma^2 + \cosh \omega}\;F_\omega(\gamma) \tag{80}$$

where $F_\omega(\gamma)$ comes from $\int dx/n$ and is defined

$$\begin{aligned} F_\omega(\gamma) &= \int_0^1 \frac{\omega du}{\gamma \sinh\,[\omega(1-u)] + \sinh\,[\omega u]} \\ &= \frac{\ell n \left| \dfrac{1+Q}{1+Q} \right| - \ell n \left| \dfrac{1+e^\omega Q}{1-e^\omega Q} \right|}{\sqrt{1 - 2\gamma \cosh \omega + \gamma^2}} \end{aligned} \tag{81}$$

or

$$2\,\frac{\tan^{-1} e^\omega Q - \tan^{-1} Q}{\sqrt{2\gamma \cosh \omega - 1 - \gamma^2}}$$

The first form applies when $\gamma > e^\omega$, or $\gamma < e^{-\omega}$, and the second applies when $e^{-\omega} < \gamma < e^\omega$, and Q is the quantity

$$Q = \frac{1 - \gamma e^{-\omega}}{\sqrt{|\,1 - 2\gamma \cosh \omega + \gamma^2\,|}} \tag{82}$$

It can readily be shown that when $\omega \to 0$

$$F_0(\gamma) = \frac{\ln \gamma}{\gamma - 1} \tag{83}$$

Thus when $\omega = 0$ (80) reduces to

$$\begin{aligned} \beta V_P &\rightarrow \frac{\ell n\,\gamma}{\gamma - 1}\,\frac{b-1}{b+2}\,(\gamma - 1) + \frac{2}{b+1}\sqrt{\frac{I}{I_0}}\,\sqrt{R\gamma^2 + 1} \\ &= \left[\frac{\ell n\,\gamma}{\gamma - 1}\,\frac{(\gamma/\gamma_\infty)^2 - 1}{R\gamma^2 + 1}\right]\sqrt{\frac{I}{I_0}} \end{aligned} \tag{84}$$

which is identical with (41). It is also clear that (79) reduces to (31) as the recombination goes to zero. Finally we write from (58)

$$\beta(V_0 + V_w) = \ell n\,\gamma\xi^2 = \ell n\,\frac{I}{I_{ps}} + \ell n\,\frac{\gamma}{R\gamma^2 + \cosh \omega} \tag{85}$$

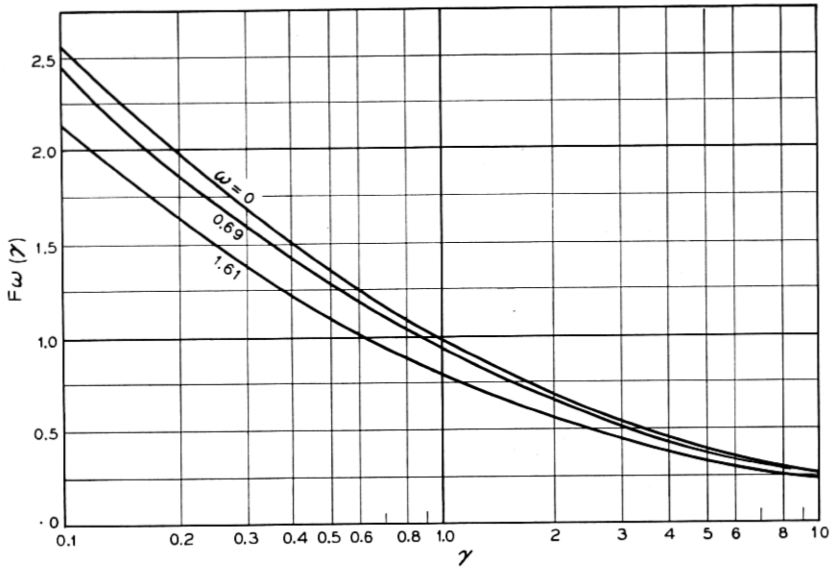

Fig. 6 — The function $F_\omega(\gamma)$ of equation (81) for several values of ω.

which reduces to (36) when $\omega = 0$. Thus the whole theory reduces correctly in the case $\omega = 0$.

The function $F_\omega(\gamma)$ is plotted in Fig. 6 for several values of ω including $\omega = 0$. The expansion of $F_\omega(\gamma)$ to order ω^2 is

$$F_\omega(\gamma) = \frac{\ln \gamma}{\gamma - 1} - \frac{\omega^2}{4} f(\gamma)$$

$$f(\gamma) = \frac{(\gamma + 1) - 2\gamma \dfrac{\ln \gamma}{\gamma - 1}}{(\gamma - 1)^2} \tag{86}$$

$$= 1 - 2\gamma \ln \frac{1}{\gamma} + \cdots$$

Our next step is to eliminate from (80) and (85) unimportant dependencies on I which would be difficult or impossible to detect experimentally. If in (85) we let $\gamma = 1$, $\cosh \omega = 1$ we get

$$\beta(V_0 + V_w) \approx \ln \frac{I}{I_{ns} + I_{ps}} \tag{87}$$

In (80) we drop the first term (as if $\gamma = 1$) and in the second term we put $R\gamma^2 = b$ (as if $\gamma = \gamma_\infty$) and $F_\omega(\gamma) = F_\omega(1)$,

$$\beta V_p \approx \frac{2}{b+1}\sqrt{\frac{I}{I_{00}}}\sqrt{b + \cosh\omega}\,F_\omega(1) \tag{88}$$

In this way we retain the correct form of dependence on ω, but throw out the dependence on I that comes from $\gamma(I)$. It can be shown from (81) that

$$\begin{aligned} F_\omega(1) &= \frac{\tan^{-1}\left(\sinh\frac{\omega}{2}\right)}{\sinh\frac{\omega}{2}} \\ &= 1 - \frac{\omega^2}{12} + \frac{\omega^4}{180} + \cdots \end{aligned} \tag{89}$$

Thus we define the characteristic (particle) current density of the device

$$\begin{aligned} I_0(\omega) &= \frac{(b+1)I_{00}}{(b+\cosh\omega)F_\omega(1)^2} \\ &= I_0\left[\frac{\omega}{F_\omega(1)\sinh\omega}\right]^2 = I_0 g(\omega) \end{aligned} \tag{90}$$

and (88) can be written

$$\beta V_p \approx \frac{2}{\sqrt{b+1}}\sqrt{\frac{I}{I_0(\omega)}} \tag{91}$$

This formula corresponds to (42) with $C = 2/\sqrt{b+1}$. In the spirit of the present theory the exact value of this constant is not important, so we may replace $2/\sqrt{b+1}$ in (91) by C. Then the sum of (87) and (91) gives the total applied bias (66).

Non Linear Recombination

In this section we shall consider the forward characteristic of a *PIN* diode in which the current densities at the contacts obey the law

$$\begin{aligned} I_n &= I_{ns}\left(\frac{n_{00}}{n_p{}^+}\right)^a \\ I_p &= I_{ps}\left(\frac{p_{ww}}{p_N{}^+}\right)^a \end{aligned} \tag{92}$$

where I_{ns} and I_{ps} are characteristic of the device and a is a number between 0 and 1. We see that (30) must be replaced by

$$\begin{gathered} I_n/I_p = R\gamma^{2a} \\ I_p = \frac{I}{I + R\gamma^{2a}} \qquad I_n = \frac{R\gamma^{2a} I}{1 + R\gamma^{2a}} \end{gathered} \tag{93}$$

and (23) must be replaced by

$$\begin{aligned} n_0 &= n_i(I_n/I_{ns})^{1/2a} \\ n_w &= n_i(I_p/I_{ps})^{1/2a} \end{aligned} \tag{94}$$

The equation for γ is now

$$\gamma = 1 - \left(\frac{I}{I_1}\right)^{1-(1/2a)} \frac{(\gamma/\gamma_\infty')^{2a} - 1}{[1 + b(\gamma/\gamma_\infty')^{2a}]^{1-(1/2a)}} \tag{95}$$

where $\gamma_\infty' = (b/R)^{1/2a}$ and

$$I_1 = I_0(I_{ps}/I_0)^{(a-1/2a-1)} \tag{96}$$

is a characteristic (particle) current density of the device. We now obtain βV_p from (26)

$$\beta V_P \approx C'(I/I_1)^{1-(1/2a)} \tag{97}$$

where C' is a slowly varying function

$$C' = \frac{(\gamma/\gamma_\infty')^{2a} + 1}{[1 + b(\gamma/\gamma_\infty')^{2a}]^{1-(1/2a)}} \frac{\ell n\, \gamma}{\gamma - 1} \tag{98}$$

similar to the coefficient in brackets in (41). From (21) and (94) we get

$$\beta(V_0 + V_w) = \frac{1}{2a}\, \ell n\, \frac{I_n I_p}{I_{ns} I_{ps}} \tag{99}$$

If now $\gamma \sim 1$ we get

$$\frac{I}{I_{ns} + I_{ps}} = e^{\alpha\beta(V_0+V_w)} \tag{100}$$

This shows how we must choose a to agree with the low current characteristic. On the basis of experience with silicon diodes we would choose $a \sim 0.6$, which would give

$$\beta V_P \sim C'(I/I_1)^{0\cdot17} \tag{101}$$

The characteristic current density would be

$$eI_1 \sim 200 \times (I_{ps}/I_0)^{-2} \text{amp/cm}^2 \text{ in } Si \tag{102}$$

The value to use for I_{ps} is very uncertain, but it certainly is much less than I_0, so $I_1 \gg I_0$. Thus we would not expect to observe βV_P, and the characteristic should have the form

$$I \sim I_s e^{a\beta} \tag{103}$$

up to the highest attainable currents.

We have shown in this section how the law of recombination in the contacts affects the dependence of V_P upon I. In particular if $a = \frac{1}{2}$ there is no dependence of V_P upon I, which means that the conductivity due to injection increases just as rapidly as the current. We may conclude from (97) that the smaller the value of a the more effective is conductivity modulation in keeping down the drop V_P in the middle region.

Discussion

We have considered the *PIN* structure of Fig. 1 having typical parameters given in (3). We find that the presence of the middle region causes no significant deviation in the voltage-current characteristic from that of a simple *PN* diode until very high current densities are reached, of the order of 200 amp/cm^2 in silicon. In particular the middle region is not responsible for an anomalous slope in the plot of V versus log I. We find that recombination in the middle region can be accounted for by replacing the characteristic current density eI_0 of the device with $eI_0 g(w/L)$ where $g(w/L) < 1$ is shown in Fig. 5. Thus qualitatively there is no change in the form of the voltage-current characteristic due to recombination in the middle region, although the effect of $g(w/L)$ is to make the voltage drop somewhat higher than if recombination were absent.

We have suggested that the anomalous slope of V versus log I usually observed in silicon diodes might be due to non-linear recombination. If the recombination obeys a power law chosen to give a typical (anomalous) $V - I$ characteristic for a *PN* diode, we have shown that the *PIN* diode should manifest the same characteristic up to extremely large current densities many times eI_0. Thus the drop across the middle region should be even more negligible with non-linear than with linear recombination.

I am pleased to acknowledge my great benefit from discussions with M. B. Prince and I. M. Ross.