Coupled Helices

By J. S. COOK, R. KOMPFNER and C. F. QUATE

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An analysis of coupled helices is presented, using the transmission line approach and also the field approach, with the objective of providing the tube designer and the microwave circuit engineer with a basis for approximate calculations. Devices based on the presence of only one mode of propagation are briefly described; and methods for establishing such a mode are given. Devices depending on the simultaneous presence of both modes, that is, depending on the beat wave phenomenon, are described; some experimental results are cited in support of the view that a novel and useful class of coupling elements has been discovered.

CONTENTS

GLOSSARY OF SYMBOLS

a	Mean radius of inner helix
b	Mean radius of outer helix
b	Capacitive coupling coefficient
$B_{10,20}$	shunt susceptance of inner and outer helices, respectively
$B_{1,2}$	Shunt susceptance plus mutual susceptance of inner and outer helices, respectively, $B_{10} + B_m$, $B_{20} + B_m$
B_m	Mutual susceptance of two coupled helices
c	Velocity of light in free space
d	Radial separation between helices, b-a
D	Directivity of helix coupler
E	Electric field intensity
F	Maximum fraction of power transferable from one coupled helix to the other
$F(\gamma a)$	Impedance parameter
$I_{1,2}$	RF current in inner and outer helix, respectively
K	Impedance in terms of longitudinal electric field on helix axis and axial power flow
L	Minimum axial distance required for maximum energy transfer from one coupled helix to the other, $\lambda_b/2$
P	Axial power flow along helix circuit
r	Radial coordinate
$\bar{r}$	Radius where longitudinal component of electric field is zero for transverse mode (about midway between a and b)
R	Return loss
s	Radial separation between helix and adjacent conducting shield
t	Time
$V_{1,2}$	RF potential of inner and outer helices, respectively
x	Inductive coupling coefficient
$X_{10,20}$	Series reactance of inner and outer helices, respectively
$X_{1,2}$	Series reactance plus mutual reactance of inner and outer helices, respectively, $X_{10} + X_m$, $X_{20} + X_m$
X_m	Mutual reactance of two coupled helices
z	Axial coordinate
$Z_{1,2}$	Impedance of inner and outer helix, respectively
$\alpha_{1,2}$	Attenuation constant of inner and outer helices, respectively
β	General circuit phase constant; or mean circuit phase constant, $\sqrt{\beta_1\beta_2}$
β_0	Free space phase constant
$\beta_{10,20}$	Axial phase constant of inner and outer helices in absence of coupling, $\sqrt{B_{10}X_{10}}$, $\sqrt{B_{20}X_{20}}$

$\beta_{1,2}$	May be considered as axial phase constant of inner and outer helices, respectively
β_b	Beat phase constant
β_c	Coupling phase constant, (identical with β_b when $\beta_1 = \beta_2$)
$\beta_{c\epsilon}$	Coupling phase constant when there is dielectric material between the helices
β_d	Difference phase constant, $\lvert \beta_1 - \beta_2 \rvert$
β_ϵ	Axial phase constant of single helix in presence of dielectric
$\beta_{t,\ell}$	Axial phase constant of transverse and longitudinal modes, respectively
γ	Radial phase constant
$\gamma_{t,\ell}$	Radial phase constant of transverse and longitudinal modes, respectively
Γ	Axial propagation constant
$\Gamma_{t,\ell}$	Axial propagation constant for transverse and longitudinal coupled-helix modes, respectively
ε	Dielectric constant
ε'	Relative dielectric constant, $\varepsilon/\varepsilon_0$
ε_0	Dielectric constant of free space
λ	General circuit wavelength; or mean circuit wavelength, $\sqrt{\lambda_1\lambda_2}$
λ_0	Free space wavelength
$\lambda_{1,2}$	Axial wavelength on inner and outer helix, respectively
λ_b	Beat wavelength
λ_c	Coupling wavelength (identical with λ_b when $\beta_1 = \beta_2$)
ψ	Helix pitch angle
$\psi_{1,2}$	Pitch angle of inner and outer helix, respectively
ω	Angular frequency

1. INTRODUCTION

Since their first appearance, traveling-wave tubes have changed only very little. In particular, if we divide the tube, somewhat arbitrarily, into circuit and beam, the most widely used circuit is still the helix, and the most widely used transition from the circuits outside the tube to the circuit inside is from waveguide to a short stub or antenna which, in turn, is attached to the helix, either directly or through a few turns of increased pitch. Feedback of signal energy along the helix is prevented by means of loss, either distributed along the whole helix or localized somewhere near the middle. The helix is most often supported along its whole length by glass or ceramic rods, which also serve to carry a conducting coating ("aquadag"), acting as the localized loss.

We therefore find the following circuit elements within the tube envelope, fixed and inaccessible once and for all after it has been sealed off:

1. The helix itself, determining the beam voltage for optimum beam-circuit interaction;

2. The helix ends and matching stubs, etc., all of which have to be positioned very precisely with relation to the waveguide circuits in order to obtain a reproducible match;

3. The loss, in the form of "aquadag" on the support rods, which greatly influences the tube performance by its position and distribution.

In spite of the enormous bandwidth over which the traveling-wave tube is potentially capable of operating — a feature new in the field of microwave amplifier tubes — it turns out that the positioning of the tube in the external circuits and the necessary matching adjustments are rather critical; moreover the overall bandwidths achieved are far short of the obtainable maximum.

Another fact, experimentally observed and well-founded in theory, rounds off the situation: The electro-magnetic field surrounding a helix, i.e., the slow wave, under normal conditions, does not radiate, and is confined to the close vicinity of the helix, falling off in intensity nearly exponentially with distance from the helix. A typical traveling-wave tube, in which the helix is supported by ceramic rods, and the whole enclosed by the glass envelope, is thus practically inaccessible as far as RF fields are concerned, with the exception of the ends of the helix, where provision is made for matching to the outside circuits. Placing objects such as conductors, dielectrics or distributed loss close to the tube is, in general, observed to have no effect whatsoever.

In the course of an experimental investigation into the propagation of space charge waves in electron beams it was desired to couple into a long helix at any point chosen along its length. Because of the feebleness of the RF fields outside the helix surrounded by the conventional supports and the envelope, this seemed a rather difficult task. Nevertheless, if accomplished, such a coupling would have other and even more important applications; and a good deal of thought was given to the problem.

Coupled concentric helices were found to provide the solution to the problem of coupling into and out of a helix at any particular point, and to a number of other problems too.

Concentric coupled helices have been considered by J. R. Pierce,[1] who has treated the problem mainly with transverse fields in mind. Such fields were thought to be useful in low-noise traveling-wave tube devices. Pierce's analysis treats the helices as transmission lines coupled uniformly over their length by means of mutual distributed capacitance and inductance. Pierce also recognized that it is necessary to wind the two helices in opposite directions in order to obtain well defined transverse and axial wave modes which are well separated in respect to their velocities of propagation.

Pierce did not then give an estimate of the velocity separation which might be attainable with practical helices, nor did anybody (as far as we are aware) then know how strong a coupling one might obtain with such helices.

It was, therefore, a considerable (and gratifying) surprise[2, 3] to find that concentric helices of practically realizable dimensions and separations are, indeed, very strongly coupled when, and these are the important points,

(*a*) They have very nearly equal velocities of propagation when uncoupled, and when

(b) They are wound in opposite senses.

It was found that virtually complete power transfer from outer to inner helix (or vice versa) could be effected over a distance of the order of *one* helix wavelength (normally between $1/10$ and $1/20$ of a free-space wavelength.

It was also found that it was possible to make a transition from a coaxial transmission line to a short (outer) helix and thence through the glass surrounding an inner helix, which was fairly good over quite a considerable bandwidth. Such a transition also acted as a directional coupler, RF power coming from the coaxial line being transferred to the inner helix predominantly in one direction.

Thus, one of the shortcomings of the "conventional" helix traveling-wave tube, namely the necessary built-in accuracy of the matching parameters, was overcome by means of the new type of coupler that might evolve around coupled helix-to-helix systems.

Other constructional and functional possibilities appeared as the work progressed, such as coupled-helix attenuators, various types of broadband couplers, and schemes for exciting pure transverse (slow) or longitudinal (fast) waves on coupled helices.

One central fact emerged from all these considerations: by placing part of the circuit outside the tube envelope with complete independence from the helix terminations inside the tube, coupled helices give back to the circuit designer a freedom comparable only with that obtained at much lower frequencies. For example, it now appears entirely possible to make one type of traveling wave tube to cover a variety of frequency bands, each band requiring merely different couplers or outside helices, the tube itself remaining unchanged.

Moreover, one tube may now be made to fulfill a number of different functions; this is made possible by the freedom with which couplers and attenuators can be placed at any chosen point along the tube.

Considerable work in this field has been done elsewhere. Reference will be made to it wherever possible. However, only that work with which the authors have been intimately connected will be fully reported here. In particular, the effect of the electron beam on the wave propagation phenomena will not be considered.

2. THEORY OF COUPLED HELICES

2.1 *Introduction*

In the past, considerable success has been attained in the understanding of traveling wave tube behavior by means of the so-called "transmission-line" approach to the theory. In particular, J. R. Pierce used it in his initial analysis and was thus able to present the solution of the so-called traveling-wave tube equations in the form of 4 waves, one of which is an exponentially growing forward traveling wave basic to the operation of the tube as an amplifier.

This transmission-line approach considers the helix — or any slow-wave circuit for that matter — as a transmission line with distributed capacitance and inductance with which an electron beam interacts. As the first approximation, the beam is assumed to be moving in an RF field of uniform intensity across the beam.

In this way very simple expressions for the coupling parameter and gain, etc., are obtained, which give one a good appreciation of the physically relevant quantities.

A number of factors, such as the effect of space charge, the non-uniform distribution of the electric field, the variation of circuit impedance with frequency, etc., can, in principle, be calculated and their effects can be superimposed, so to speak, on the relatively simple expressions deriving from the simple transmission line theory. This has, in fact, been done and is, from the design engineer's point of view, quite satisfactory.

However, physicists are bound to be unhappy over this state of affairs. In the beginning was Maxwell, and therefore the proper point to start from is Maxwell.

So-called "Field" theories of traveling-wave tubes, based on Maxwell's equation, solved with the appropriate boundary conditions, have been worked out and their main importance is that they largely confirm the results obtained by the inexact transmission line theory. It is, however, in the nature of things that field theories cannot give answers in terms of simple closed expressions of any generality. The best that can be done is in the form of curves, with step-wise increases of particular parameters. These can be of considerable value in particular cases, and when exactness is essential.

In this paper we shall proceed by giving the "transmission-line" type theory first, together with the elaborations that are necessary to arrive at an estimate of the strength of coupling possible with coaxial helices. The "field" type theory will be used whenever the other theory fails, or is inadequate. Considerable physical insight can be gotten with the use of the transmission-line theory; nevertheless recourse to field theory is necessary in a number of cases, as will be seen.

It will be noted that in all the calculations to be presented the presence of an electron beam is left out of account. This is done for two reasons: Its inclusion would enormously complicate the theory, and, as will eventually be shown, it would modify our conclusions only very slightly. Moreover, in practically all cases which we shall consider, the helices are so tightly coupled that the velocities of the two normal modes of propagation are very different, as will be shown. Thus, only when the beam velocity is very near to either one or the other wave velocity, will growing-wave interaction take place between the beam and the helices. In this case conventional traveling wave tube theory may be used.

A theory of coupled helices in the presence of an electron beam has been presented by Wade and Rynn,[4] who treated the case of weakly coupled helices and arrived at conclusions not at variance with our views.

2.2 *Transmission Line Equations*

Following Pierce we describe two lossless helices by their distributed series reactances X_{10} and X_{20} and their distributed shunt susceptances B_{10} and B_{20}. Thus their phase constants are

$$\beta_{10} = \sqrt{B_{10}X_{10}}$$

$$\beta_{20} = \sqrt{B_{20}X_{20}}$$

Let these helices be coupled by means of a mutual distributed reactance X_m and a mutual susceptance B_m, both of which are, in a way which will be described later, functions of the geometry.

Let waves in the coupled system be described by the factor

$$e^{j\omega t}e^{-\Gamma z}$$

where the Γ's are the propagation constants to be found.

The transmission line equations may be written:

$$\begin{aligned} \Gamma I_1 - jB_1V_1 + jB_mV_2 &= 0 \\ \Gamma V_1 - jX_1I_1 + jX_mI_2 &= 0 \\ \Gamma I_2 - jB_2V_2 + jB_mV_1 &= 0 \\ \Gamma V_2 - jX_2I_a + jX_mI_1 &= 0 \end{aligned} \tag{2.2.1}$$

where

$$\begin{aligned} B_1 &= B_{10} + B_m \\ X_1 &= X_{10} + X_m \\ B_2 &= B_{20} + B_m' \\ X_2 &= X_{20} + X_m \end{aligned}$$

I_1 and I_2 are eliminated from the (2.2.1) and we find

$$\frac{V_2}{V_1} = \frac{+(\Gamma^2 + X_1B_1 + X_mB_m)}{X_1B_m + B_2X_m} \tag{2.2.2}$$

$$\frac{V_1}{V_2} = \frac{+(\Gamma^2 + X_2B_2 + X_mB_m)}{X_2B_m + B_1X_m} \tag{2.2.3}$$

These two equations are then multiplied together and an expression for Γ of the 4th degree is obtained:

$$\Gamma^4 + (X_1B_1 + X_2B_2 + 2X_mB_m)\Gamma^2 + (X_1X_2 - X_m^2)(B_1B_2 - B_m^2) = 0 \tag{2.2.4}$$

We now define a number of dimensionless quantities:

$$\frac{B_m^2}{B_1B_2} = b^2 = (\text{capacitive coupling coefficient})^2$$

$$\frac{X_m^2}{X_1X_2} = x^2 = (\text{inductive coupling coefficient})^2$$

$$B_1X_1 = \beta_1^2, \qquad B_2X_2 = \beta_2^2$$

$$X_1B_1X_2B_2 = \beta^4 = (\text{mean phase constant})^4$$

With these substitutions we obtain the general equation for Γ^2

$$\Gamma^2 = \beta^2\left[-\frac{1}{2}\left(\frac{\beta^2}{\beta_2^2} + \frac{\beta^2}{\beta_1^2} + 2bx\right) \pm \sqrt{\frac{1}{4}\left(\frac{\beta^2}{\beta_2^2} + \frac{\beta^2}{\beta_1^2} + 2bx\right)^2 - (1 - x^2)(1 - b^2)}\right] \tag{2.2.5}$$

If we make the same substitutions in (2.2.2) we find

$$\frac{V_2}{V_1} = \sqrt{\frac{Z_2}{Z_1}}\left[\frac{\Gamma^2 + \beta_1^2 + \beta^2 bx}{\beta(\beta_1 b + \beta_2 x)}\right] \tag{2.2.6}$$

where the Z's are the impedances of the helices, i.e.,

$$Z_n = \sqrt{X_n/B_n}$$

2.3 *Solution for Synchronous Helices*

Let us consider the particular case where $\beta_1 = \beta_2 = \beta$. From (2.2.5) we obtain

$$\Gamma^2 = -\beta^2[1 + xb \pm (x + b)] \tag{2.3.1}$$

Each of the above values of Γ^2 characterizes a normal mode of propagation involving both helices. The two square roots of each Γ^2 represent waves going in the positive and negative directions. We shall consider only the positive roots of Γ^2, denoted Γ_ι and Γ_ℓ, which represent the forward traveling waves.

$$\Gamma_{\iota,\ell} = j\beta\sqrt{1 + xb \pm (x + b)} \tag{2.3.2}$$

If $x > 0$ and $b > 0$

$$|\Gamma_\iota| > |\beta|, \ |\Gamma_\ell| < |\beta|$$

Thus Γ_ι represents a normal mode of propagation which is slower than the propagation velocity of either helix alone and can be called the "slow" wave. Similarly Γ_ℓ represents a "fast" wave. We shall find that, in fact, x and b are numerically equal in most cases of interest to us; we therefore write the expressions for the propagation constants

$$\begin{aligned} \Gamma_\iota &= j\beta[1 + \tfrac{1}{2}(x + b)] \\ \Gamma_\ell &= j\beta[1 - \tfrac{1}{2}(x + b)] \end{aligned} \tag{2.3.3}$$

If we substitute (2.3.3) into (2.2.6) for the case where $\beta_1 = \beta_2 = \beta$ and assume, for simplicity, that the helix self-impedances are equal, we find that for $\Gamma = \Gamma_\iota$

$$\frac{V_2}{V_1} = -1$$

for $\Gamma = \Gamma_\ell$

$$\frac{V_2}{V_1} = +1$$

Thus, the slow wave is characterized by equal voltages of unlike sign on the two helices, and the fast wave by equal voltages of like sign. It follows that the electric field in the annular region between two such coupled concentric helices will be transverse for the slow wave and longitudinal for the fast. For this reason the slow and fast modes are often referred to as the transverse and longitudinal modes, respectively, as indicated by our subscripts.

It should be noted here that we arbitrarily chose b and x positive. A different choice of signs cannot alter the fact that the transverse mode is the slower and the longitudinal mode is the faster of the two.

Apart from the interest in the separate existence of the fast and slow waves as such, another object of interest is the phenomenon of the simultaneous existence of both waves and the interference, or spatial beating, between them.

Let V_2 denote the voltage on the outer helix; and let V_1, the voltage on the inner halix, be zero at $z = 0$. Then we have, omitting the common factor $e^{j\omega t}$,

$$\begin{aligned} V_1 &= V_{t1}e^{-\Gamma_t z} + V_{\ell 1}e^{-\Gamma_\ell z} \\ V_2 &= V_{t2}e^{-\Gamma_t z} + V_{\ell 2}e^{-\Gamma_\ell z} \end{aligned} \qquad (2.3.4)$$

Since at $z = 0$, $V_1 = 0$, $V_{t1} = -V_{\ell_1}$. For the case we have considered we have found $V_{t1} = -V_{t2}$ and $V_{\ell 1} = V_{\ell 2}$. We can write (2.3.4) as

$$\begin{aligned} V_1 &= \frac{V}{2}\,(e^{-\Gamma_\ell z} - e^{-\Gamma_t z}) \\ V_2 &= \frac{V}{2}\,(e^{-\Gamma_\ell z} + e^{-\Gamma_t z}) \end{aligned} \qquad (2.3.5)$$

V_2 can be written

$$\begin{aligned} V_2 &= \frac{V}{2}\,e^{-1/2(\Gamma_t+\Gamma_\ell)z}[e^{+1/2(\Gamma_t-\Gamma_\ell)z} + e^{-1/2(\Gamma_t-\Gamma_\ell)z}] \\ &= Ve^{-1/2(\Gamma_t+\Gamma_\ell)z}\cos\,[-j\tfrac{1}{2}(\Gamma_t - \Gamma_\ell)z] \end{aligned}$$

In the case when $x = b$, and $\beta_1 = \beta_2 = \beta$

$$V_2 = Ve^{-j\beta z}\cos\,[\tfrac{1}{2}(x + b)\beta z] \qquad (2.3.6)$$

Correspondingly, it can be shown that the voltage on the inner helix is

$$V_1 = jVe^{-j\beta z}\sin\,[\tfrac{1}{2}(x + b)\beta z] \qquad (2.3.7)$$

The last two equations exhibit clearly what we have called the spatial beat phenomenon, a wave-like transfer of power from one helix to the other and back. We started, arbitrarily, with all the voltage on the outer helix at $z = 0$, and none on the inner; after a distance, z', which makes the argument of the cosine $\pi/2$, there is no voltage on the outer helix and all is on the inner.

To conform with published material let us define what we shall call the "coupling phase-constant" as

$$\beta_c = \beta(b + x) \tag{2.3.8}$$

From (2.3.3) we find that for $\beta_1 = \beta_2 = \beta$, and $x = b$,

$$\Gamma_t - \Gamma_\ell = j\beta_c$$

2.4 *Non-Synchronous Helix Solutions*

Let us now go back to the more general case where the propagation velocities of the (uncoupled) helices are not equal. Equation (2.2.5) can be written:

$$\Gamma^2 = -\beta^2 [1 + (1/2)\Delta + xb \pm \sqrt{(1 + xb)\Delta + (1/4)\Delta^2 + (b + x)^2}] \tag{2.4.1}$$

where

$$\Delta \equiv \left[\frac{\beta_2 - \beta_1}{\beta}\right]^2$$

In the case where $x = b$, (2.4.1) has an exact root.

$$\Gamma_{t,\ell} = j\beta [\sqrt{1 + \Delta/4} \pm 1/2 \sqrt{\Delta + (x + b)^2}] \tag{2.4.2}$$

We shall be interested in the difference between Γ_t and Γ_ℓ,

$$\Gamma_t - \Gamma_\ell = j\beta \sqrt{\Delta + (x + b)^2} \tag{2.4.3}$$

Now we substitute for Δ and find

$$\Gamma_t - \Gamma_\ell = j \sqrt{(\beta_1 - \beta_2)^2 + \beta^2 (b + x)^2} \tag{2.4.4}$$

Let us define the "beat phase-constant" as:

$$\beta_b = \sqrt{(\beta_1 - \beta_2)^2 + \beta^2(b + x)^2}$$

so that

$$\Gamma_t - \Gamma_\ell = j\beta_b \tag{2.4.5}$$

Further, let us define

$$\beta_d = | \beta_1 - \beta_2 |$$

and call this the "difference phase-constant," i.e., the hase constant corresponding to two uncoupled waves of the same frequency but differing phase velocities. We can thus state the relation between these phase constants:

$$\beta_b^2 = \beta_d^2 + \beta_c^2 \tag{2.4.6}$$

This relation is identical (except for notation) with expression (33) in S. E. Miller's paper.[5] In this paper Miller also gives expressions for the voltage amplitudes in two coupled transmission systems in the case of unequal phase velocities. It turns out that in such a case the power transfer from one system to the other is necessarily incomplete. This is of particular interest to us, in connection with a number of practical schemes. In our notation it is relatively simple, and we can state it by saying that the maximum fraction of power transferred is

$$F = \left(\frac{\beta_c}{\beta_b}\right)^2 \tag{2.4.7}$$

or, in more detail,

$$F = \frac{\beta_c^2}{\beta_d^2 + \beta_c^2} = \frac{\beta^2(b + x)^2}{(\beta_1 - \beta_2)^2 + \beta^2(b + x)^2}$$

This relationship can be shown to be a good approximation from (2.2.6), (2.3.4), (2.4.2), on the assumption that $b \approx x$ and $Z_1 \approx Z_2$, and the further assumption that the system is lossless; that is,

$$|V_2|^2 + |V_1|^2 = \text{constant} \tag{2.4.8}$$

We note that the phase velocity difference gives rise to two phenomena: It reduces the coupling wavelength and it reduces the amount of power that can be transferred from one helix to the other.

Something should be said about the case where the two helix impedances are not equal, since this, indeed, is usually the case with coupled concentric helices. Equation (2.4.8) becomes:

$$\frac{|V_2|^2}{Z_2} + \frac{|V_1|^2}{Z_1} = \text{constant} \tag{2.4.9}$$

Using this relation it is found from (2.3.4) that

$$\frac{V_2}{V_1}\sqrt{\frac{Z_1}{Z_2}} = \pm\sqrt{\frac{1}{F}}\,(1 \pm \sqrt{1 - F}) \tag{2.4.10}$$

When this is combined with (2.2.6) it is found that the impedances drop out with the voltages, and that "F" is a function of the β's only. In other words, complete power transfer occurs when $\beta_1 = \beta_2$ regardless of the relative impedances of the helices.

The reader will remember that β_{10} and β_{20}, not β_1 and β_2, were defined as the phase constants of the helices in the absence of each other. If the assumption that $b \approx x$ is maintained, it will be found that all of the derived relationships hold true when β_{n0} is substituted for β_n. In other words, throughout the paper, β_1 and β_2 may be treated as the phase constants of the inner and outer helices, respectively. In particular it should be noted that if these quantities are to be measured experimentally each helix must be kept in the same environment as if the helices were coupled; only the other helix may be removed. That is, if there is dielectric in the annular region between the coupled helices, β_1 and β_2 must each be measured in the presence of that dielectric.

Miller also has treated the case of lossy coupled transmission systems. The expressions are lengthy and complicated and we believe that no substantial error is made in simply applying his conclusions to our case.

If the attenuation constants α_1 and α_2 of the two transmission systems (helices) are equal, no change is required in our expressions; when they are unequal the total available power (in both helices) is most effectively reduced when

$$\frac{\alpha_1 - \alpha_2}{\beta_c} \approx 1 \tag{2.4.11}$$

This fact may be made use of in designing coupled helix attenuators.

2.5 *A Look at the Fields*

It may be advantageous to consider sketches of typical field distributions in coupled helices, as in Fig. 2.1, before we go on to derive a quantitative estimate of the coupling factors actually obtainable in practice.

Fig. 2.1(a) shows, diagrammatically, electric field lines when the coupled helices are excited in the fast or "longitudinal" mode. To set up this mode only, one has to supply voltages of like sign and equal amplitudes to both helices. For this reason, this mode is also sometimes called the "(++) mode."

Fig. 2.1(b) shows the electric field lines when the helices are excited in the slow or "transverse" mode. This is the kind of field required in the transverse interaction type of traveling wave tube. In order to excite this mode it is necessary to supply voltages of equal amplitude and opposite signs to the helices and for this reason it is sometimes called the "(+−) mode." One way of exciting this mode consists in connecting one helix to one of the two conductors of a balanced transmission line ("Lecher"-line) and the other helix to the other.

Fig. 2.1(c) shows the electric field configuration when fast and slow modes are both present and equally strongly excited. We can imagine the two helices being excited by a voltage source connected to the outer

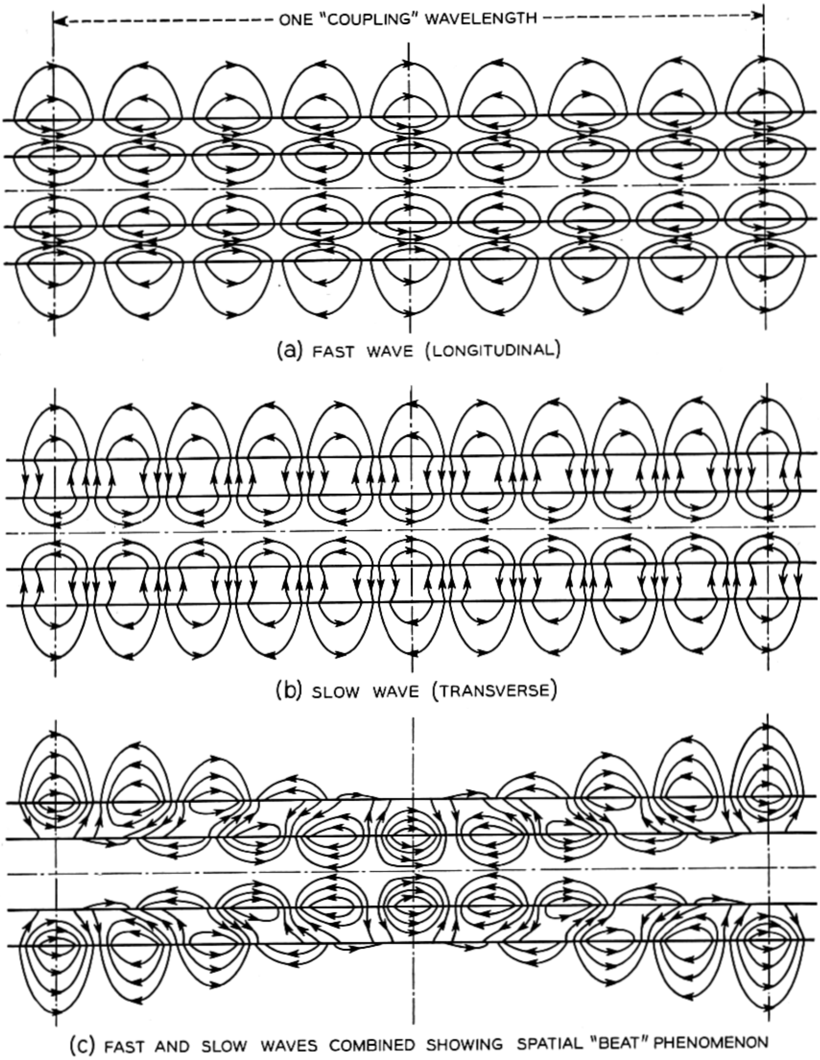

Fig. 2.1 — Typical electric field distributions in coupled coaxial helices when they are excited in: (a) the in-phase or longitudinal mode, (b) the out-of-phase or transverse mode, and (c) both modes equally.

helix only at the far left side of the sketch. One, perfectly legitimate, view of the situation is that the RF power, initially all on the outer helix, leaks into the inner helix because of the coupling between them, and then leaks back to the outer helix, and so forth.

Apart from noting the appearance of the stationary spatial beat (or interference) phenomenon these additional facts are of interest:

1) It is a simple matter to excite such a beat-wave, for instance, by connecting a lead to either one or the other of the helices, and

2) It should be possible to discontinue either one of the helices, at points where there is no current (voltage) on it, without causing reflections.

2.6 *A Simple Estimate of b and x*

How strong a coupling can one expect from concentric helices in practice? Quantitatively, this is expressed by the values of the coupling factors x and b, which we shall now proceed to estimate.

A first crude estimate is based on the fact that slow-wave fields are known to fall off in intensity somewhat as $e^{-\beta r}$ where β is the phase constant of the wave and r the distance from the surface guiding the slow wave. Thus a unit charge placed, say, on the inner helix, will induce a charge of opposite sign and of magnitude

$$e^{-\beta(b-a)}$$

on the outer helix. Here b = mean radius of the outer helix and a = mean radius of the inner. We note that the shunt mutual admittance coupling factor is negative, irrespective of the directions in which the helices are wound. Because of the similarity of the magnetic and electric field distributions a current flowing on the inner helix will induce a similarly attenuated current, of amplitude

$$e^{-\beta(b-a)}$$

on the outer helix. The direction of the induced current will depend on whether the helices are wound in the same sense or not, and it turns out (as one can verify by reference to the low-frequency case of coaxial coupled coils) that the series mutual impedance coupling factor is negative when the helices are oppositely wound.

In order to obtain the greatest possible coupling between concentric helices, both coupling factors should have the same sign. This then requires that the helices should be wound in opposite directions, as has been pointed out by Pierce.

When the distance between the two helices goes to zero, that is to say, if they lie in the same surface, it is clear that both coupling factors b and x will go to unity.

As pointed out earlier in Section 2.3, the choice of sign for b is arbitrary. However, once a sign for b has been chosen, the sign of x is necessarily the opposite when the helices are wound in the same direction, and vice versa. We shall choose, therefore,

$$\begin{aligned} b &= +e^{-\beta(b-a)} \\ x &= \mp e^{-\beta(b-a)} \end{aligned} \tag{2.6.1}$$

the sign of the latter depending on whether the helices are wound in the same direction or not.

In the case of unequal velocities, β, the propagation constant, would be given by

$$\beta = \sqrt{\beta_1\beta_2} \tag{2.6.2}$$

2.7 *Strength of Coupling versus Frequency*

The exponential variation of coupling factors with respect to frequency (since $\beta = \omega/v$) has an important consequence. Consider the expression for the coupling phase constant

$$\beta_c = \beta(b + x) \tag{2.3.8}$$

or

$$| \beta_c | = 2\beta e^{-\beta(b-a)} \tag{2.7.1}$$

The coupling wavelength, which is defined as

$$\lambda_c = \left| \frac{2\pi}{\beta_c} \right| \tag{2.7.2}$$

is, therefore,

$$\lambda_c = \frac{\pi}{\beta} e^{\beta(b-a)}$$

or

$$\lambda_c = \frac{\lambda}{2} e^{(2\pi/\lambda)(b-a)} \tag{2.7.3}$$

where λ is the (slowed-down) RF wavelength on either helix. It is convenient to multiply both sides of (2.7.1) with a, the inner helix radius, in order to obtain a dimensionless relation between β_c and β:

$$\beta_c a = 2\beta a e^{-\beta a((b/a)-1)} \tag{2.7.4}$$

This relation is plotted on Fig. 2.2 for several values of b/a.

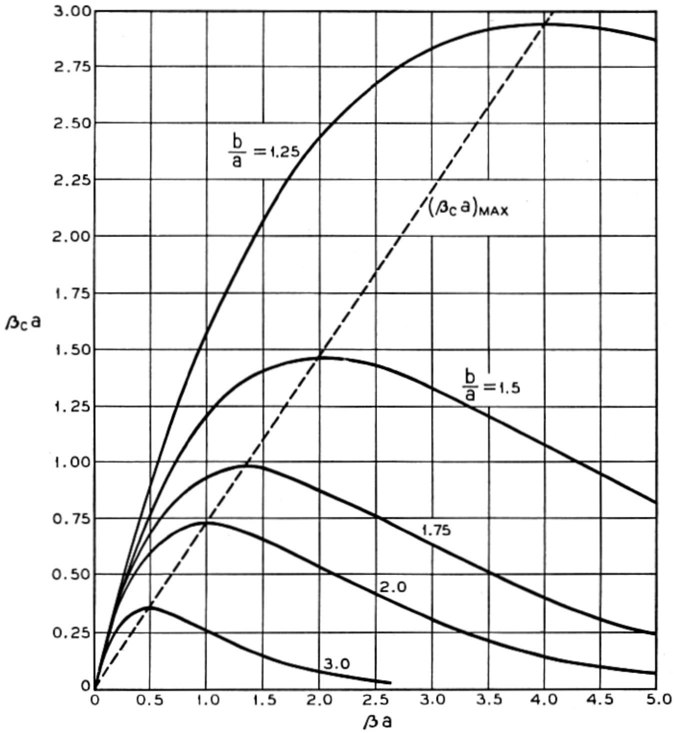

Fig. 2.2 — Coupling phase-constant plotted as a function of the single helix phase-constant for synchronous helices for several values of b/a. These curves are based on simple estimates made in Section 2.7.

There are two opposing tendencies determining the actual physical length of a coupling beat-wavelength:

1) It tends to grow with the RF wavelength, being proportional to it in the first instance;

2) Because of the tighter coupling possible as the RF wavelength increases in relation to the helix-to-helix distance, the coupling beat-wavelength tends to shrink.

Therefore, there is a region where these tendencies cancel each other, and where one would expect to find little change of the coupling beat-wavelength for a considerable change of RF frequency. In other words, the "bandwidth" over which the beat-wavelength stays nearly constant can be large.

This is a situation naturally very desirable and favorable for any device in which we rely on power transfer from one helix to the other by means of a length of overlap between them an integral number of half beat-wavelengths long. Obviously, one will design the helices in such a way as to take advantage of this situation.

Optimum conditions are easily obtained by differentiating β_c with respect to β and setting $\partial\beta_c/\partial\beta$ equal to zero. This gives for the optimum conditions

$$\beta_{\text{opt}} = \frac{1}{b - a} \tag{2.7.5}$$

or

$$\beta_{c\,\text{opt}} = \frac{2e^{-1}}{b - a} = 2e^{-1}\beta_{\text{opt}} \tag{2.7.6}$$

Equation (2.7.5), then, determines the ratio of the helix radii if it is required that deviations from a chosen operating frequency shall have least effect.

2.8 *Field Solutions*

In treating the problem of coaxial coupled helices from the transmission line point of view one important fact has not been considered, namely, the dispersive character of the phase constants of the separate helices, β_1 and β_2. By dispersion we mean change of phase velocity with frequency. If the dispersion of the inner and outer helices were the same it would be of little consequence. It is well known, however, that the dispersion of a helical transmission line is a function of the ratio of helix radius to wavelength, and thus becomes a parameter to be considered. When the theory of wave propagation on a helix was solved by means of Maxwell's equations subject to the boundary condition of a helically conducting cylindrical sheath, the phenomenon of dispersion first made its appearance. It is clear, therefore, that a more complete theory of

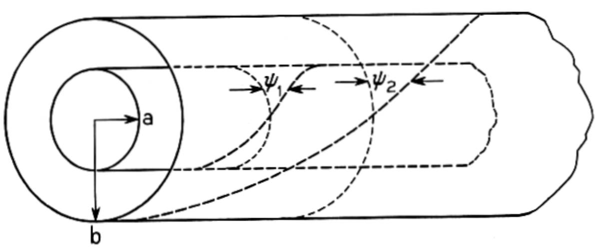

Fig. 2.3 — Sheath helix arrangement on which the field equations are based.

coupled helices will require similar treatment, namely, Maxwell's equations solved now with the boundary conditions of two cylindrical helically conducting sheaths. As shown on Fig. 2.3, the inner helix is specified by its radius a and the angle ψ_1 made by the direction of conductivity with a plane perpendicular to the axis; and the outer helix by its radius b (not to be confused with the mutual coupling coefficient b) and its corresponding pitch angle ψ_2. We note here that oppositely wound helices require opposite signs for the angles ψ_1 and ψ_2; and, further, that helices with equal phase velocities will have pitch angles of about the same absolute magnitude.

The method of solving Maxwell's equations subject to the above mentioned boundary conditions is given in Appendix I. We restrict ourselves here to giving some of the results in graphical form.

The most universally used parameter in traveling-wave tube design is a combination of parameters:

$$\beta_0 a \cot \psi_1$$

where $\beta_0 = 2\pi/\lambda_0$, λ_0 being the free-space wavelength, a the radius of the inner helix, and ψ_1 the pitch angle of the inner helix. The inner helix is chosen here in preference to the outer helix because, in practice, it will be part of a traveling-wave tube, that is to say, inside the tube envelope. Thus, it is not only less accessible and changeable, but determines the important aspects of a traveling-wave tube, such as gain, power output, and efficiency.

The theory gives solutions in terms of radial propagation constants which we shall denote γ_t and γ_ℓ (by analogy with the transverse and longitudinal modes of the transmission line theory). These propagation constants are related to the axial propagation constants β_t and β_ℓ by

$$\gamma_n = \sqrt{\beta_n^2 - \beta_0^2}$$

Of course, in transmission line theory there is no such thing as a radial propagation constant. The propagation constant derived there and denoted Γ corresponds here to the axial propagation constant $j\beta$. By analogy with (2.4.5) the beat phase constant should be written

$$\beta_b = \beta_t - \beta_\ell$$

However, in practice β_0 is usually much smaller than β and we can therefore write with little error

$$\beta_b = \gamma_t - \gamma_\ell$$

for the beat phase constant. For practical purposes it is convenient to

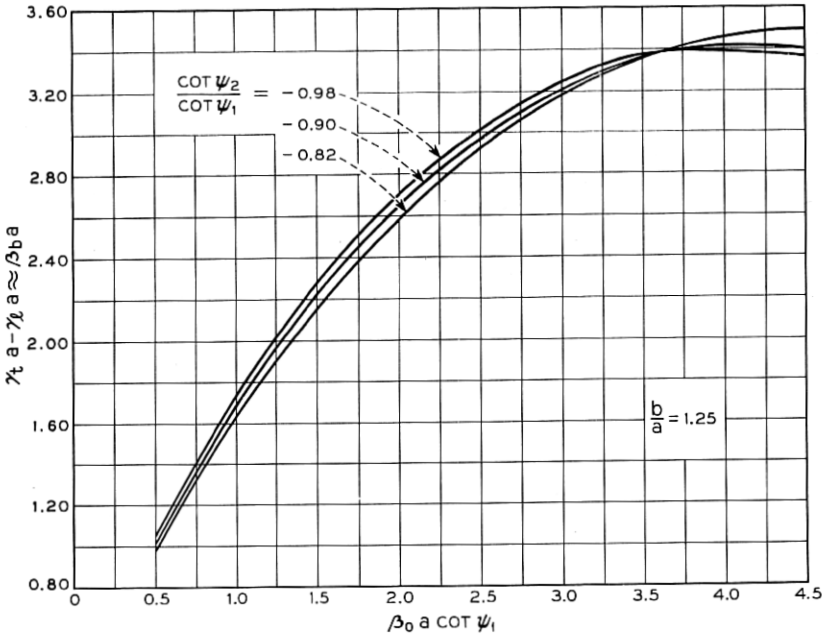

Fig. 2.4.1 — Beat phase-constant plotted as a function of $\beta_0 a \cot \psi_1$. These curves result from the solution of the field equations given in the appendix. For $b/a = 1.25$.

normalize in terms of the inner helix radius, a:

$$\beta_b a = \gamma_t a - \gamma_\ell a$$

This has been plotted as a function of $\beta_0 \, a \cot \psi_1$ in Fig. 2.4, which should be compared with Fig. 2.2. It will be seen that there is considerable agreement between the results of the two methods.

2.9 *Bifilar Helix*

The failure of the transmission line theory to take into account dispersion is well illustrated in the case of the bifilar helix. Here we have two identical helices wound in the same sense, and at the same radius. If the two wires are fed in phase we have the normal mode characterized by the sheath helix model whose propagation constant is the familiar Curve A of Fig. 2.5. If the two wires of the helix are fed out of phase we have the bifilar mode; and, since that is a two wire transmission system, we shall have a TEM mode which, in the absence of dielectric, propagates along the wire with the velocity of light. Hence, the propagation constant for this mode is simply $\beta_0 a \cot \psi$ and gives rise to the horizontal

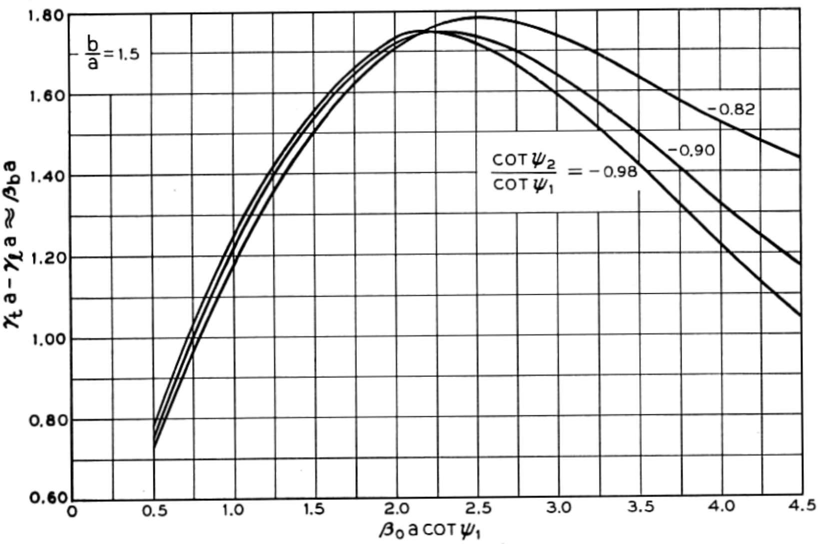

Fig. 2.4.2 — Beat phase-constant plotted as a function of $\beta_0 a \cot \psi_1$. These curves result from the solution of the field equations given in the appendix. For $b/a = 1.5$.

line of Curve B in Fig. 2.5. Again the coupling phase constant β_c is given by the difference of the individual phase constants:

$$\beta_c a = \beta_0 a \cot \psi - \gamma a \tag{2.9.1}$$

which is plotted in Fig. 2.6. Now note that when $\beta_0 \ll \gamma$ this equation is accurate, for it represents a solution of the field equations for the helix.

From the simple unsophisticated transmission line point of view no coupling between the two helices would, of course, have been expected, since the two helices are identical in every way and their mutual capacity and inductance should then be equal and opposite.

Experiments confirm the essential correctness of (2.9.1). In one experiment, which was performed to measure the coupling wavelength for the bifilar helices, we used helices with a $\cot \psi = 3.49$ and a radius of 0.036 cm which gave a value, at 3,000 mc, of $\beta_0 a \cot \psi = 0.51$. In these experiments the coupling length, L, defined by

$$(\beta_0 a \cot \psi - \gamma a) \frac{L}{a} = \pi$$

was measured to be $15.7a$ as compared to a value of $13.5a$ from Fig. 2.6. At 4,000 mc the measured coupling length was $14.6a$ as compared to

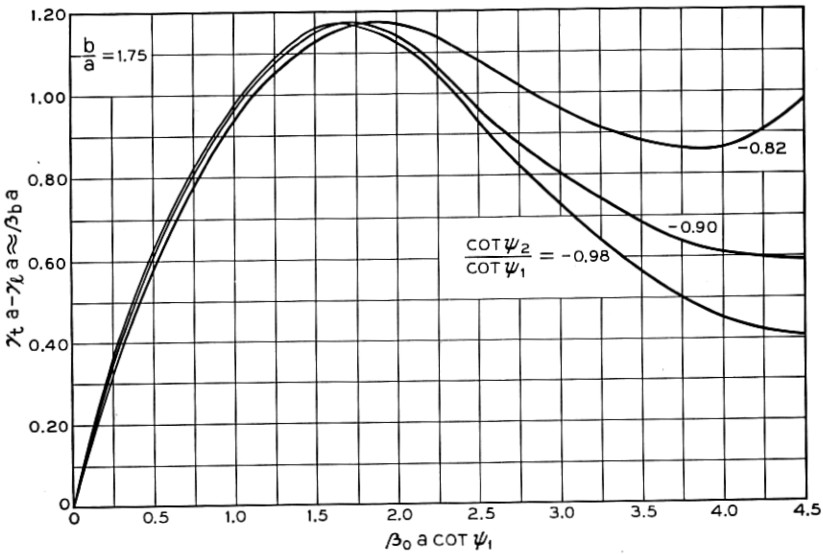

Fig. 2.4.3 — Beat phase-constant plotted as a function of $\beta_0 a \cot \psi_1$. These curves result from the solution of the field equations given in the appendix. For $b/a = 1.75$.

12.6a computed from Fig. 2.6, thus confirming the theoretical prediction rather well. The slight increase in coupling length is attributable to the dielectric loading of the helices which were supported in quartz tubing. The dielectric tends to decrease the dispersion and hence reduce β_c. This is discussed further in the next section.

2.10 *Effect of Dielectric Material between Helices*

In many cases which are of interest in practice there is dielectric material between the helices. In particular when coupled helices are used with traveling-wave tubes, the tube envelope, which may be of glass, quartz, or ceramic, all but fills the space between the two helices.

It is therefore of interest to know whether such dielectric makes any difference to the estimates at which we arrived earlier. We should not be surprised to find the coupling strengthened by the presence of the dielectric, because it is known that dielectrics tend to rob RF fields from the surrounding space, leading to an increase in the energy flow through the dielectric. On the other hand, the dielectric tends to bind the fields closer to the conducting medium. To find a qualitative answer to this question we have calculated the relative coupling phase constants for two sheath helices of infinite radius separated by a distance "d" for 1)

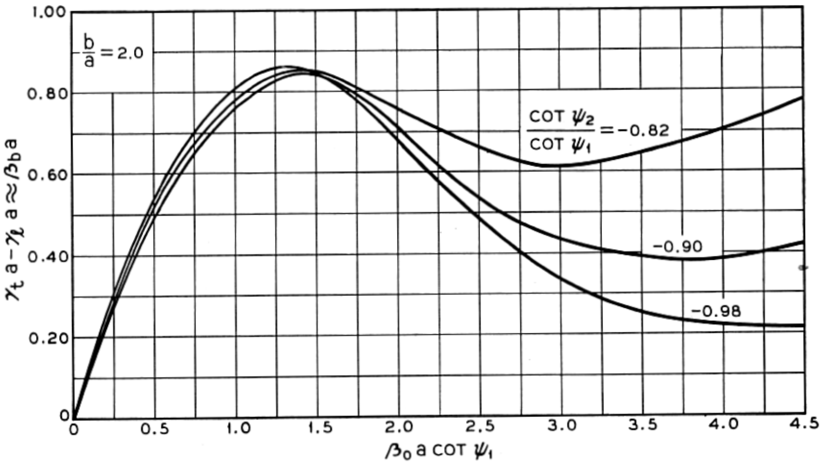

Fig. 2.4.4 — Beat phase-constant plotted as a function of $\beta_0 a$ cot ψ_1. These curves result from the solution of the field equations given in the appendix. For $b/a = 2.0$.

the case with dielectric between them having a relative dielectric constant $\epsilon' = 4$, and 2) the case of no dielectric. The pitch angles of the two helices were ψ and $-\psi$, respectively; i.e., the helices were assumed to be synchronous, and wound in the opposite sense.

Fig. 2.7 shows a plot of the ratio of $\beta_{c\epsilon}/\beta_\epsilon$ to β_c/β versus β_0 $(d/2)$ cot ψ,

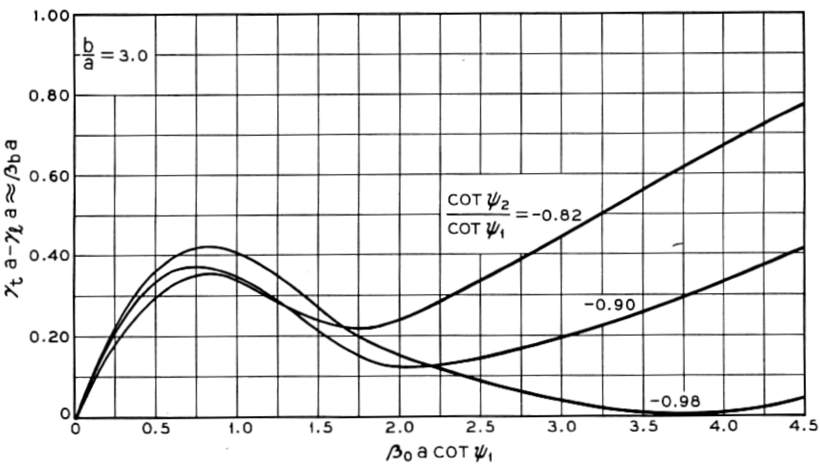

Fig. 2.4.5 — Beat phase-constant plotted as a function of $\beta_0 a$ cot ψ_1. These curves result from the solution of the field equations given in the appendix. For $b/a = 3.0$.

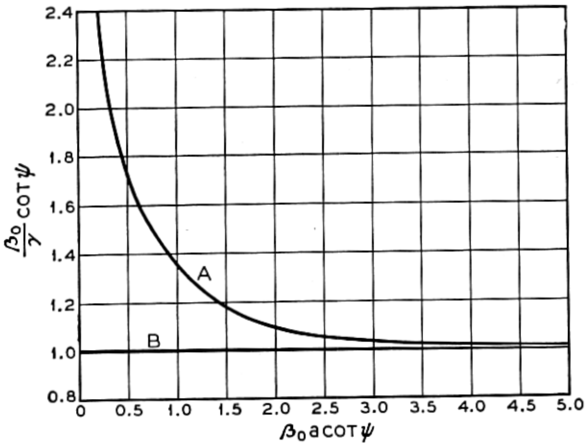

Fig. 2.5 — Propagation constants for a bifilar helix plotted as a function of $\beta_0 a \cot \psi_1$. The curves illustrate, (A) the dispersive character of the in-phase mode and, (B) the non-dispersive character of the out-of-phase mode.

where $\beta_{c\epsilon}$ is the coupling phase-constant in the presence of dielectric, β_ϵ is the phase-constant of each helix alone in the presence of the same dielectric, β_c is the coupling phase-constant with no dielectric, and β is the phase constant of each helix in free space. In many cases of interest $\beta_0(d/2) \cot \psi$ is greater than 1.2. Then

$$\frac{\beta_{c\varepsilon}/\beta_\epsilon}{\beta_c/\beta} = \left[\frac{3\varepsilon' + 1}{2\varepsilon' + 2}\right] e^{-(\sqrt{2\epsilon'+2}-2)\beta_0 \, (d/2) \cot \psi} \qquad (2.10.1)$$

Appearing in the same figure is a similar plot for the case when there is a conducting shield inside the inner helix and outside the outer, and separated a distance, "s," from the helices. Note that

$$d \equiv b - a.$$

It appears from these calculations that the effect of the presence of dielectric between the helices depends largely on the parameter $\beta_0 \, (d/2) \cot \psi$. For values of this parameter larger than 0.3 the coupling wavelength tends to increase in terms of circuit wavelength. For values smaller than 0.3 the opposite tends to happen. Note that the curve representing (2.10.1) is a fair approximation down to $\beta_0(d/2) \cot \psi = 0.6$ to the curve representing the exact solution of the field equations. J. W. Sullivan, in unpublished work, has drawn similar conclusions.

2.11 *The Conditions for Maximum Power Transfer*

The transmission line theory has led us to expect that the most efficient power transfer will take place if the phase velocities on the two helices, prior to coupling, are the same. Again, this would be true were it not for the dispersion of the helices. To evaluate this effect we have used the field equation to determine the parameter of the coupled helices which gives maximum power transfer. To do this we searched for combinations of parameters which give an equal current flow in the helix sheath for either the longitudinal mode or the transverse mode. This was suggested by L. Stark, who reasoned that if the currents were equal for the individual modes the beat phenomenon would give points of zero RF current on the helix.

The values of $\cot \psi_2/\cot \psi_1$ which are required to produce this condition are plotted in Fig. 2.8 for various values of b/a. Also there are shown values of $\cot \psi_2/\cot \psi_1$ required to give equal axial velocities for the helices before they are coupled. It can be seen that the uncoupled velocity of the inner helix must be slightly slower than that of the outer.

A word of caution is necessary for these curves have been plotted without considering the effects of dielectric loading, and this can have a rather marked effect on the parameters which we have been discussing. The significant point brought out by this calculation is that the optimum

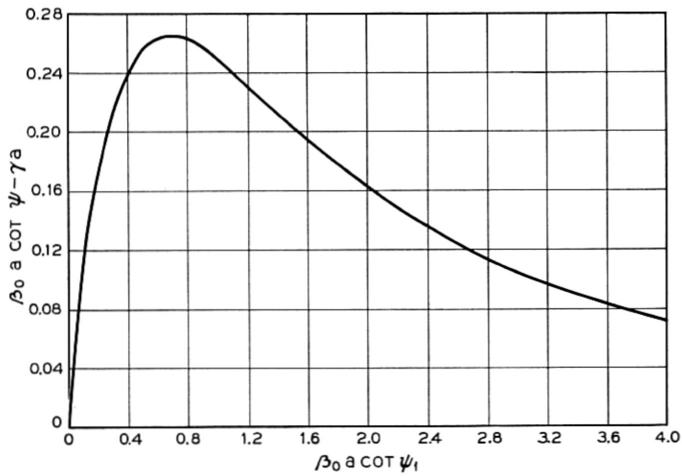

Fig. 2.6 — The coupling phase-constant which results from the two possible modes of propagation on a bifilar helix shown as a function of $\beta_0 a \cot \psi_1$.

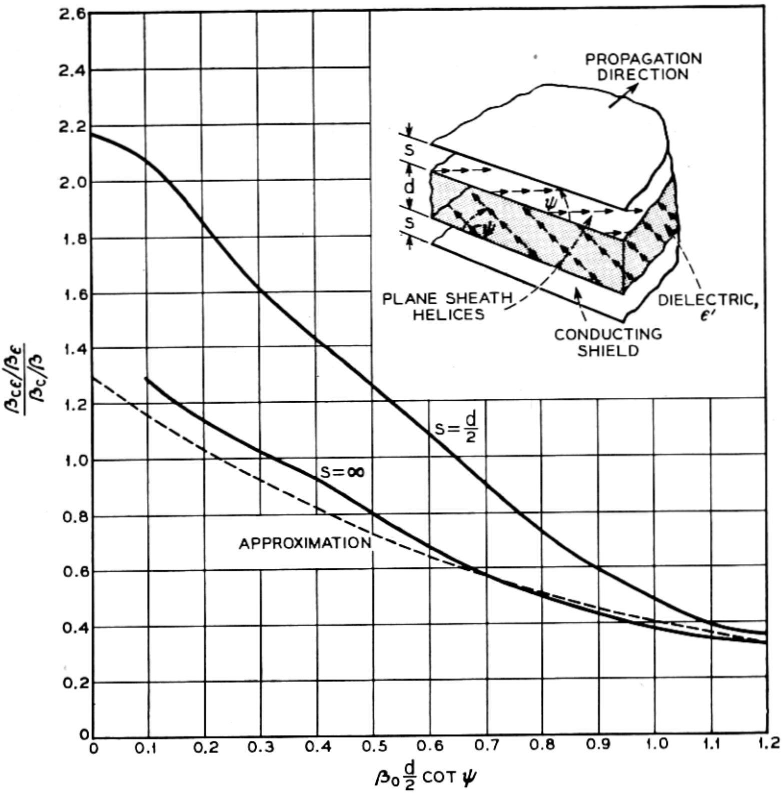

FIG. 2.7 — The effect of dielectric material between coupled infinite radius sheath helices on their relative coupling phase-constant shown as a function of $\beta_0 d/2 \cot \psi_1$. The effect of shielding on this relation is also indicated.

condition for coupling is not necessarily associated with equal velocities on the uncoupled helices.

2.12 *Mode Impedance*

Before leaving the general theory of coupled helices something should be said regarding the impedance their modes present to an electron beam traveling either along their axis or through the annular space between them. The field solutions for cross wound, coaxially coupled helices, which are given in Appendix I, have been used to compute the impedances of the transverse and longitudinal modes. The impedance, K, is defined, as usual, in terms of the longitudinal field on the axis and the power flow along the system.

$$K = \frac{E_z^2}{\beta^2 P} = \left(\frac{\beta}{\beta_0}\right)^{1/3} \left(\frac{\gamma}{\beta}\right)^{4/3} F(\gamma a)$$

In Fig. 2.9, $F(\gamma a)$, for various ratios of inner to outer radius, is plotted for both the transverse and longitudinal modes together with the value of $F(\gamma a)$ for the single helix ($b/a = \infty$). We see that the longitudinal mode has a higher impedance with cross wound coupled helices than does a single helix. We call attention here to the fact that this is the same phenomenon which is encountered in the contrawound helix[6], where the structure consists of two oppositely wound helices of the same radius.

As defined here, the transverse mode has a lower impedance than the single helix. This, however, is not the most significant comparison; for it is the transverse field midway between helices which is of interest in the transverse mode. The factor relating the impedance in terms of the transverse field between helices to the longitudinal field on the axis is $E_r^2(\bar{r})/E_z^2(0)$, where $\bar{r}$ is the radius at which the longitudinal component of the electric field E_z, is zero for the transverse mode. This factor, plotted in Fig. 2.10 as a function of $\beta_0 a \cot \psi_r$, shows that the impedance in terms of the transverse field at $\bar{r}$ is interestingly high.

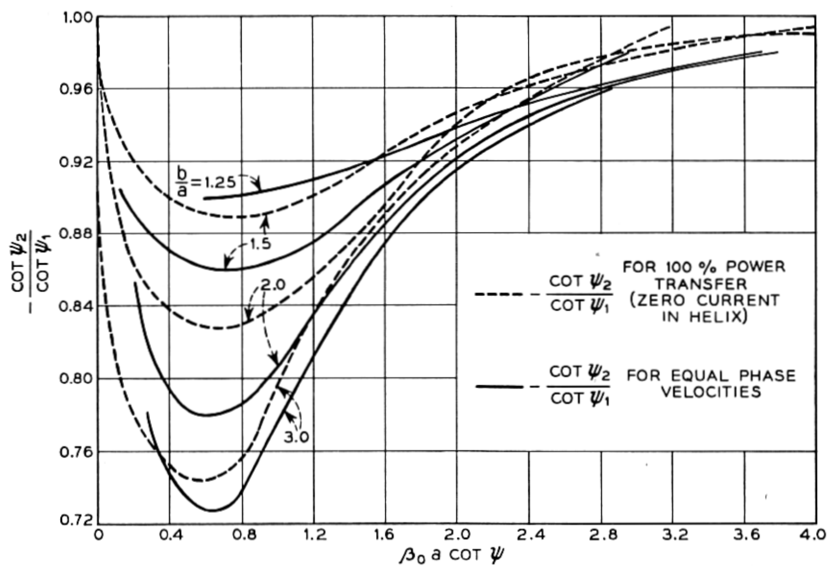

Fig. 2.8 — The values of cot ψ_2/cot ψ_1 required for complete power transfer plotted as a function of $\beta_0 a$ cot ψ_1 for several values of b/a. For comparison, the value of cot ψ_2/cot ψ_1 required for equal velocities on inner and outer helices is also shown.

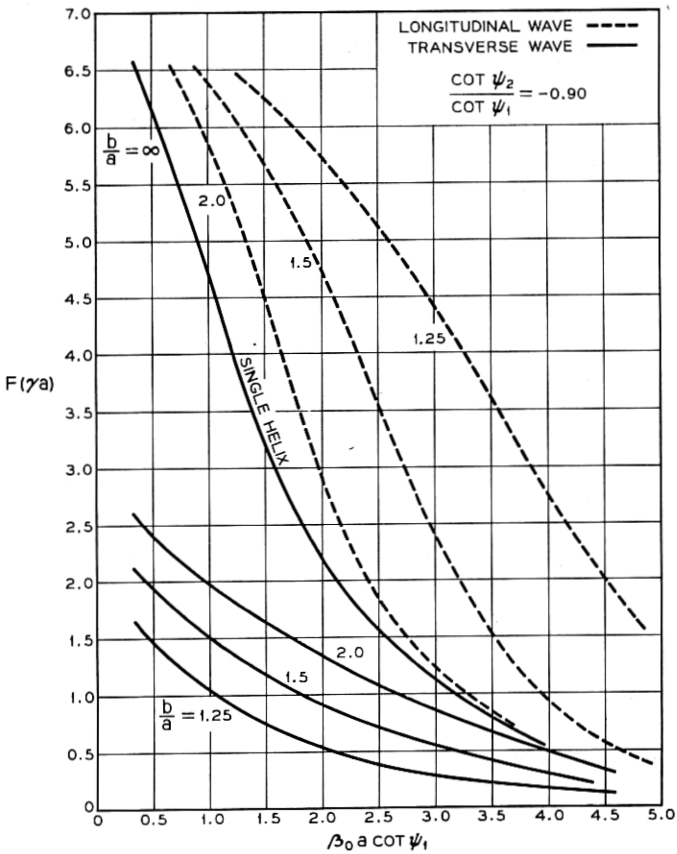

Fig. 2.9 — Impedance parameter, $F(\gamma a)$, associated with both transverse and longitudinal modes shown for several values of b/a. Also shown is $F(\gamma a)$ for a single helix.

It is also of interest to consider the impedance of the longitudinal mode in terms of the longitudinal field between the two helices. The factor, $E_z^2(\bar{r})/E_z^2(0)$, relating this to the axial impedance is plotted in Fig. 2.11. We see that rather high impedances can also be obtained with the longitudinal field midway between helices. This, in conjunction with a hollow electron beam, should provide efficient amplification.[7]

3. APPLICATION OF COUPLED HELICES

When we come to describe devices which make use of coupled helices we find that they fall, quite naturally, into two separate classes. One class contains those devices which depend on the presence of only one of the two normal modes of propagation. The other class of devices depends on the simultaneous presence, in roughly equal amounts, of *both* normal modes of propagation, and is, in general, characterized by the words "spatial beating." Since spatial beating implies energy surging to and fro between inner and outer helix, there is no special problem in exciting both modes simultaneously. Power fed exclusively to one or the other

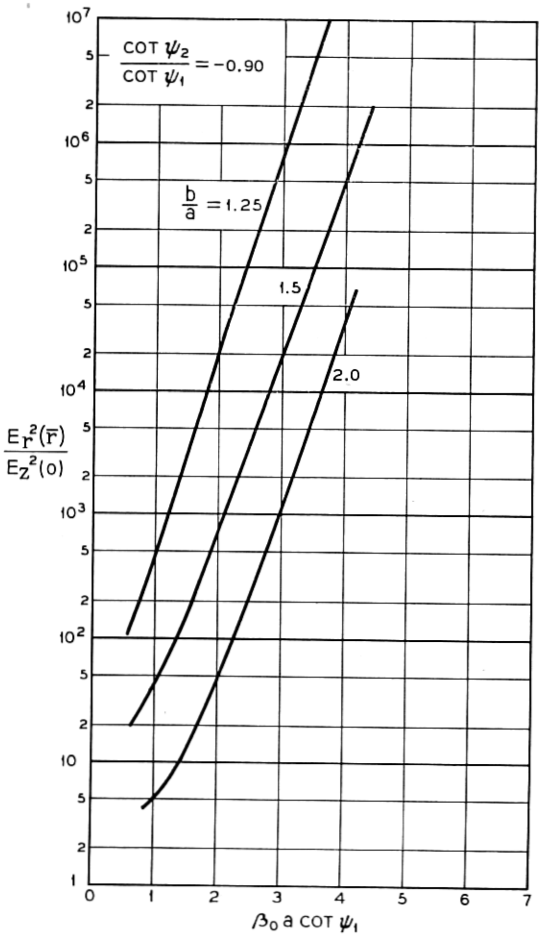

Fig. 2.10 — The relation between the impedance in terms of the transverse field between coupled helices excited in the out-of-phase mode, and the impedance in terms of the longitudinal field on the axis shown as a function of $\beta_0 a \cot \psi_1$.

helix will inevitably excite both modes equally. When it is desired to excite one mode exclusively a more difficult problem has to be solved. Therefore, in section 3.1 we shall first discuss methods of exciting one mode only before going on to discuss in sections 3.2 and 3.3 devices using one mode only.

In section 3.4 we shall discuss devices depending on the simultaneous presence of both modes.

3.1 *Excitation of Pure Modes*

3.1.1 *Direct Excitation*

In order to set up one or the other normal mode on coupled helices, voltages with specific phase and amplitudes (or corresponding currents)

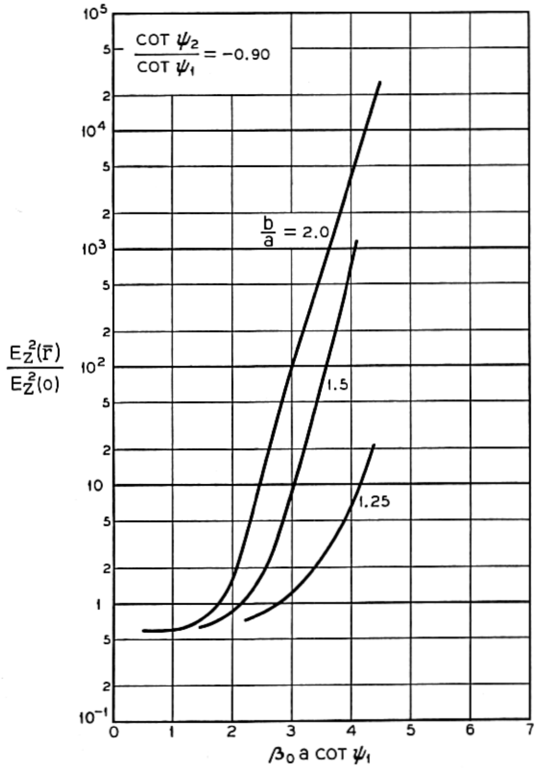

Fig. 2.11 — The relation between the impedance in terms of the longitudinal field between coupled helices excited in the in-phase mode, and the impedance in terms of the longitudinal field on the axis shown as a function of $\beta_0 a \cot \psi_1$.

have to be supplied to each helix at the input end. A natural way of doing this might be by means of a two-conductor balanced transmission line (Lecher-line), one conductor being connected to the inner helix, the other to the outer helix. Such an arrangement would cause something like the transverse (+ −) mode to be set up on the helices. If the two conductors and the balanced line can be shielded from each other starting some distance from the helices then it is, in principle, possible to introduce arbitrary amounts of extra delay into one of the conductors. A delay of one half period would then cause the longitudinal (+ +) mode to be set up in the helices. Clearly such a coupling scheme would not be broad-band since a frequency-independent delay of one half period is not realizable.

Other objections to both of these schemes are: Balanced lines are not generally used at microwave frequencies; it is difficult to bring leads through the envelope of a TWT without causing reflection of RF energy and without unduly encumbering the mechanical design of the tube plus circuits; both schemes are necessarily inexact because helices having different radii will, in general, require different voltages at either input in order to be excited in a pure mode.

Thus the practicability, and success, of any general scheme based on the existence of a pure transverse or a pure longitudinal mode on coupled helices will depend to a large extent on whether elegant coupling means are available. Such means are indeed in existence as will be shown in the next sections.

3.1.2 *Tapered Coupler*

A less direct but more elegant means of coupling an external circuit to either normal mode of a double helix arrangement is by the use of the so-called "tapered" coupler.[8, 9, 10] By appropriately tapering the relative propagation velocities of the inner and outer helices, outside the interaction region, one can excite either normal mode by coupling to one helix only.

The principle of this coupler is based on the fact that any two coupled transmission lines support two, and only two, normal modes, regardless of their relative phase velocities. These normal modes are characterized by unequal wave amplitudes on the two lines if the phase velocities are not equal. Indeed the greater the phase velocity difference and/or the smaller the coupling coefficient between the lines, the more their wave amplitudes diverge. Furthermore, the wave amplitude on the line with the slower phase velocity is greater for the out-of-phase or transverse normal mode, and the wave amplitude on the faster line is greater for the longitudinal normal mode. As the ratio of phase constant to coupling constant approaches infinity, the ratio of the wave amplitudes on the two lines does also. Finally, if the phase velocities of, or coupling between, two coupled helices are changed gradually along their length the normal modes existing on the pair roughly maintain their identity even though they change their character. Thus, by properly tapering the phase velocities and coupling strength of any two coupled helices one can cause the two normal modes to become two separate waves, one existing on each helix.

For instance, if one desires to extract a signal propagating in the in-phase, or longitudinal, normal mode from two concentric helices of equal phase velocity, one might gradually increase the pitch of the outer helix and decrease that of the inner, and at the same time increase the diameter of the outer helix to decrease the coupling, until the longitudinal mode exists as a wave on the outer helix only. At such a point the outer helix may be connected to a coaxial line and the signal brought out.

This kind of coupler has the advantage of being frequency insensitive; and, perhaps, operable over bandwidths upwards of two octaves. It has the disadvantage of being electrically, and sometimes physically, quite long.

3.1.3 *Stepped Coupler*

There is yet a third way to excite only one normal mode on a double helix. This scheme consists of a short length at each end of the outer helix, for instance, which has a pitch slightly different from the rest. This has been called a "stepped" coupler.

The principle of the stepped coupler is this: If two coupled transmission lines have unlike phase velocities then a wave initiated in one line can never be completely transferred to the other, as has been shown in Section 2.4. The greater the velocity difference the less will be the maximum transfer. One can choose a velocity difference such that the maximum power transfer is just one half the initial power. It is a characteristic of incomplete power transfer that at the point where the maximum transfer occurs the waves on the two lines are exactly either in-phase or out-of-phase, depending on which helix was initially excited. Thus, the conditions for a normal mode on two equal-velocity helices can be produced at the maximum transfer point of two unlike velocity helices by initiating a wave on only one of them. If at that point the helix pitches are changed to give equal phase velocities in both helices, with equal current or voltage amplitude on both helices, either one or the other of the two normal modes will be propagated on the two helices from there on. Although the pitch and length of such a stepped coupler are rather critical, the requirements are indicated in the equations in Section 2.4.

The useful bandwidth of the stepped coupler is not as great as that of the tapered variety, but may be as much as an octave. It has however the advantage of being very much shorter and simpler than the tapered coupler.

3.2 *Low-Noise Transverse-Field Amplifier*

One application of coupled helices which has been suggested from the very beginning is for a transverse field amplifier with low noise factor. In such an amplifier the RF structure is required to produce a field which is purely transverse at the position of the beam. For the transverse mode there is always such a cylindrical surface where the longitudinal field is zero and this can be obtained from the field equation of Appendix II. In Fig. 3.1 we have plotted the value of the radius $\bar{r}$ at which the longitudinal field is zero for various parameters. The significant feature of this plot is that the radius which specifies zero longitudinal field is not constant with frequency. At frequencies away from the design frequency the electron beam will be in a position where interaction with longitudinal components might become important and thus shotnoise power will be introduced into the circuit. Thus the bandwidth of the amplifier over which it has a good noise factor would tend to be limited. However, this effect can be reduced by using the smallest practicable value of b/a.

Section 2.12 indicates that the impedance of the transverse mode is very high, and thus this structure should be well suited for transverse field amplifiers.

3.3 *Dispersive Traveling-Wave Tube*

Large bandwidth is not always essential in microwave amplifiers. In particular, the enormous bandwidth over which the traveling-wave tube is potentially capable of amplifying has so far found little application, while relatively narrow bandwidths (although quite wide by previous standards) are of immediate interest. Such a relatively narrow band, if it is an inherent electronic property of the tube, makes matching the tube to the external circuits easier. It may permit, for instance, the use of non-reciprocal attenuation by means of ferrites in the ferromagnetic resonance region. It obviates filters designed to deliberately reduce the band in certain applications. Last, but not least, it offers the possibility of trading bandwidth for gain and efficiency.

A very simple method of making a traveling-wave tube narrow-band

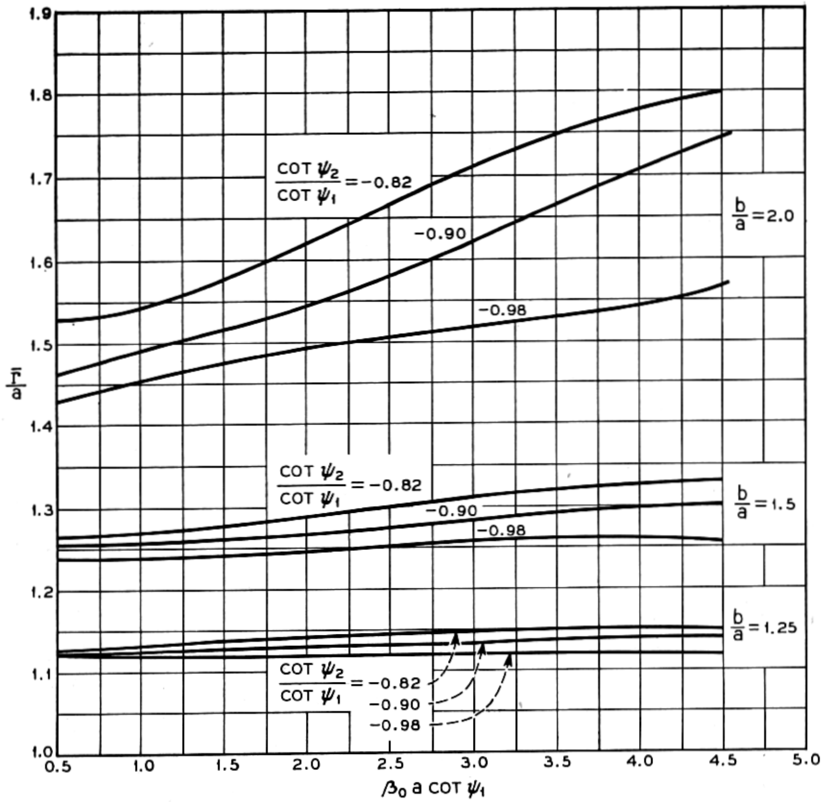

Fig. 3.1 — The radius $\bar{r}$ at which the longitudinal field is zero for transversely excited coupled coaxial helices.

is by using a dispersive circuit, (i.e. one in which the phase velocity varies significantly with frequency). Thus, we obtain an amplifier that can be *tuned* by varying the beam voltage; being dispersive we should also expect a low group velocity and therefore higher circuit impedance.

Calculations of the phase velocities of the normal modes of coupled concentric helices presented in the appendix show that the fast, longitudinal or (++) mode is highly dispersive. Given the geometry of two such coupled helices and the relevant data on an electron beam, namely current, voltage and beam radius, it is possible to arrive at an estimate of the dependence of gain on frequency.

Experiments with such a tube showed a bandwidth 3.8 times larger than the simple estimate would show. This we ascribe to the presence of the dielectric between the helices in the actual tube, and to the neglect of power propagated in the form of spatial harmonics.

Nevertheless, the tube operated satisfactorily with distributed nonreciprocal ferrite attenuation along the whole helix and gave, at the center frequency of 4,500 mc/s more than 40 db stable gain.

The gain fell to zero at 3,950 mc/s at one end of the band and at 4,980 mc/s at the other. The forward loss was 12 db. The backward loss was of the order of 50 db at the maximum gain frequency.

3.4 *Devices Using Both Modes*

In this section we shall discuss applications of the coupled-helix principle which depend for their function on the simultaneous presence of both the transverse and the longitudinal modes. When present in substantially equal magnitude a spatial beat-phenomenon takes place, that is, RF power transfers back and forth between inner and outer helix.

Thus, there are points, periodic with distance along each helix, where there is substantially no current or voltage; at these points a helix can be terminated, cut-off, or connected to external circuits without detriment.

The main object, then, of all devices discussed in this section is power *transfer* from one helix to the other; and, as will be seen, this can be accomplished in a remarkably efficient, elegant, and broad-band manner.

3.4.1 *Coupled-Helix Transducer*

It is, by now, a well known fact that a good match can be obtained between a coaxial line and a helix of proportions such as used in TWT's. A wire helix in free space has an effective impedance of the order of 100 ohms. A conducting shield near the helix, however, tends to reduce the helix impedance, and a value of 70 or even 50 ohms is easily attained. Provided that the transition region between the coaxial line and the helix does not present too abrupt a change in geometry or impedance, relatively good transitions, operable over bandwidths of several octaves, can be made, and are used in practice to feed into and out of tubes employing helices such as TWT's and backward-wave oscillators.

One particularly awkward point remains, namely, the necessity to lead the coaxial line through the tube envelope. This is a complication in manufacture and requires careful positioning and dimensioning of the helix and other tube parts.

Coupled helices offer an opportunity to overcome this difficulty in the form of the so-called coupled-helix transducer, a sketch of which is shown in Fig. 3.2. As has been shown in Section 2.3, with helices having the same velocity an overlap of one half of a beat wavelength will result in a 100 per cent power transfer from one helix to the other. A signal introduced into the outer helix at point A by means of the coaxial line will be all on the inner helix at point B, nothing remaining on the outer helix. At that point the outer helix can be discontinued, or cut off; since there is no power there, the seemingly violent discontinuity represented by the 'open" end of the helix will cause no reflection of power. In practice, unfortunately, there are always imperfections to consider, and there will often be some power left at the end of the coupler helix. Thus, it is desirable to terminate the outer helix at this point non-reflectively, as, for instance, by a resistive element of the right value, or by connecting to it another matched coaxial line which in turn is then non-reflectively terminated.

It will be seen, therefore, that the coupled-helix transducer can, in principle, be made into an efficient device for coupling RF energy from a coaxial line to a helix contained in a dielectric envelope such as a glass tube. The inner helix will be energized predominantly in one direction, namely, the one away from the input connection. Conversely, energy traveling initially in the inner helix will be transferred to the outer, and made available as output in the respective coaxial line. Such a coupled-helix transducer can be moved along the tube, if required. As long as the outer helix completely overlaps the inner, operation as described above should be assured. By this means a new flexibility in design, operation and adjustment of traveling-wave tubes is obtained which could not be achieved by any other known form of traveling-wave tube transducer.

Naturally, the applications of the coupled-helix transducer are not restricted to TWT's only, nor to 100 per cent power transfer. To obtain

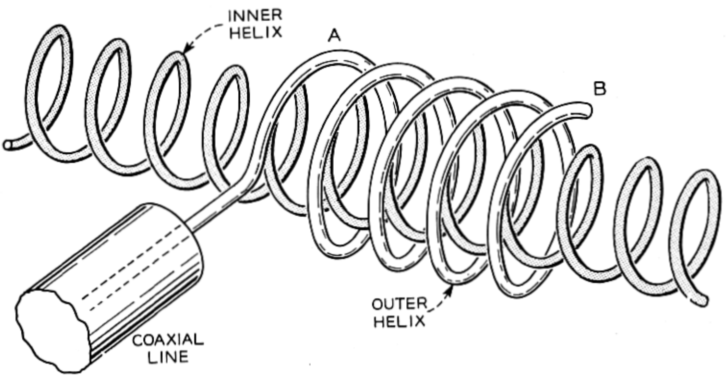

Fig. 3.2 — A simple coupled helix transducer.

power transfer of proportions other than 100 per cent two possibilities are open: either one can reduce the length of the synchronous coupling helix appropriately, or one can deliberately make the helices non-synchronous. In the latter case, a considerable measure of broad-banding can be obtained by making the length of overlap again equal to one half of a beat-wavelength, while the fraction of power transferred is determined by the difference of the helix velocities according to 2.4.7. An application of the principle of the coupled-helix transducer to a variable delay line has been described by L. Stark[12] in an unpublished memorandum.

Turning again to the complete power transfer case, we may ask: How broad is such a coupler?

In Section 2.7 we have discussed how the radial falling-off of the RF energy near a helix can be used to broad-band coupled-helix devices which depend on relative constancy of beat-wavelength as frequency is varied. On the assumption that there exists a perfect broad-band match between a coaxial line and a helix, one can calculate the performance of a coupled-helix transducer of the type shown in Fig. 3.2.

Let us define a center frequency ω, at which the outer helix is exactly one half beat-wavelength, λ_b, long. If ω is the frequency of minimum beat wavelength then at frequencies ω_1 and ω_2, larger and smaller, respectively, than ω, the outer helix will be a fraction δ shorter than $\frac{1}{2}\lambda_b$, (Section 2.7). Let a voltage amplitude, V_2, exist at the point where the outer helix is joined to the coaxial line. Then the magnitude of the voltage at the other end of the outer helix will be $|V_2 \cdot \sin(\pi\delta/2)|$ which means that the power has not been completely transferred to the inner helix. Let us assume complete reflection at this end of the outer helix. Then all but a fraction of the reflected power will be transferred to the inner helix in a *reverse* direction. Thus, we have a first estimate for the "directivity" defined as the ratio of forward to backward power (in db) introduced into the inner helix:

$$D = \left| 10 \log \sin^2 \left(\frac{\pi\delta}{2} \right) \right| \tag{3.4.1.1}$$

We have assumed a perfect match between coaxial line and outer helix; thus the power reflected back into the coaxial line is proportional to $\sin^4(\pi\delta/2)$. Thus the reflectivity defined as the ratio of reflected to incident power is given in db by

$$R = 10 \log \sin^4 \left(\frac{\pi\delta}{2} \right) \tag{3.4.1.2}$$

For the sake of definiteness, let us choose actual figures: let $\beta a = 2.0$. and $b/a = 1.5$. And let us, arbitrarily, demand that R always be less than -20 db.

This gives $\sin(\pi\delta/2) < 0.316$ and $\pi\delta/2 < 18.42°$ or 0.294 radians, $\delta < 0.205$. With the optimum value of $\beta_c a = 1.47$, this gives the minimum permissible value of $\beta_c a$ of $1.47/(1 + 0.205) = 1.22$. From the graph on Fig. 2.2 this corresponds to values of βa of 1.00 and 3.50. Therefore, the reflected power is down 20 db over a frequency range of $\omega_2/\omega_1 = 3.5$ to one. Over the same range, the directivity is better than 10 to one. Suppose a directivity of better than 20 db were required. This requires $\sin(\pi\delta/2) = 0.10$, $\delta = 0.0638$ and is obtained over a frequency range of approximately two to one. Over the same range, the reflected power would be down by 40 db.

In the above example the full bandwidth possibilities have not been used since the coupler has been assumed to have optimum length when $\beta_c a$ is maximum. If the coupler is made longer so that when $\beta_c a$ is maximum it is electrically short of optimum to the extent permissible by the quality requirements, then the minimum allowable $\beta_c a$ becomes even smaller. Thus, for $b/a = 1.5$ and directivity 20 db or greater the realizable bandwidth is nearly three to one.

When the coupling helix is non-reflectively terminated at both ends, either by means of two coaxial lines or a coaxial line at one end and a resistive element at the other, the directivity is, ideally, infinite, irrespective of frequency; and, similarly, there will be no reflections. The power transfer to the inner helix is simply proportional to $\cos^2(\pi\delta/2)$. Thus, under the conditions chosen for the example given above, the coupled-helix transducer can approach the ideal transducer over a considerable range of frequencies.

So far, we have inspected the performance and bandwith of the coupled-helix transducer from the most optimistic theoretical point of view. Although a more realistic approach does not change the essence of our conclusions, it does modify them. For instance, we have neglected dispersion on the helices. Dispersion tends to reduce the maximum attainable bandwidth as can be seen if Fig. 2.4.2 rather than Fig. 2.2 is used in the example cited above. The dielectric that exists in the annular region between coupled concentric helices in most practical couplers may also affect the bandwidth.

In practice, the performance of coupled-helix transducers has been short of the ideal. In the first place, the match from a coaxial line to a helix is not perfect. Secondly, a not inappreciable fraction of the RF power on a real wire helix is propagated in the form of spatial harmonic

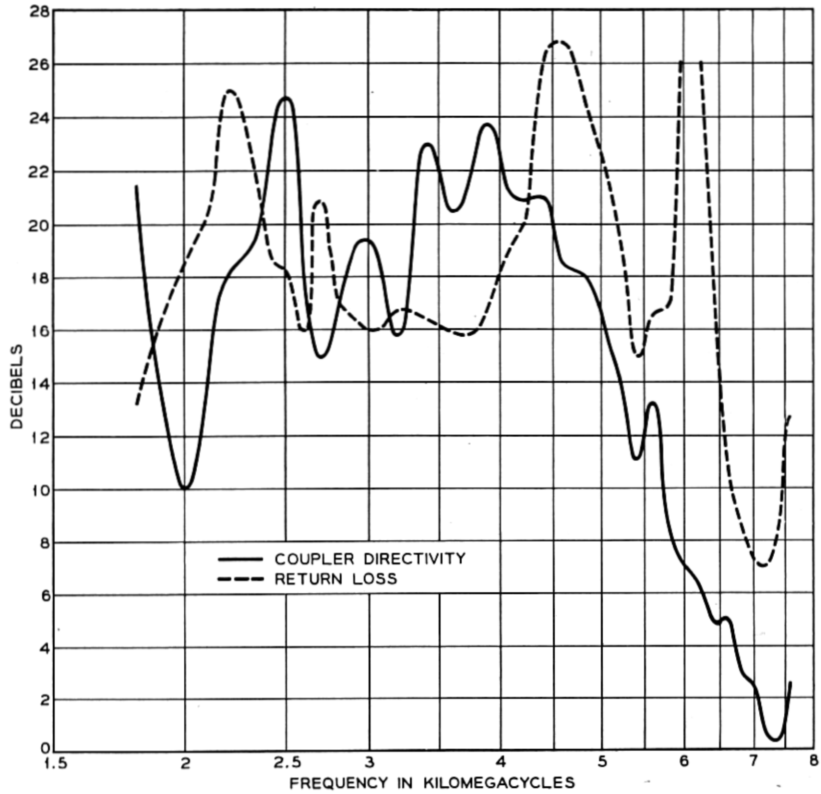

Fig. 3.3 — The return loss and directivity of an experimental 100 per cent coupled-helix transducer.

wave components which have variations with angle around the helix-axis, and coupling between such components on two helices wound in opposite directions must be small. Finally, there are the inevitable mechanical inaccuracies and misalignments.

Fig. 3.3 shows the results of measurements on a coupled-helix transducer with no termination at the far end.

3.4.2 *Coupled-Helix Attenuator*

In most TWT's the need arises for a region of heavy attenuation somewhere between input and output; this serves to isolate input and output, and prevents oscillations due to feedback along the circuit. Because of the large bandwidth over which most TWT's are inherently capable of amplifying, substantial attenuation, say at least 60 db, is required over a bandwidth of maybe 2 octaves, or even more. Furthermore, such attenuation should present a very good match to a wave on the helix, particularly to a wave traveling backwards from the output of the tube since such a wave will be amplified by the output section of the tube.

Another requirement is that the attenuator should be physically as short as possible so as not to increase the length of the tube unnecessarily.

Finally, such attenuation might, with advantage, be made movable during the operation of the tube in order to obtain optimum performance, perhaps in respect of power output, or linearity, or some other aspect.

Coupled-helix attenuators promise to perform these functions satisfactorily.

A length of outer helix (synchronous with the inner helix) one half of a beat wavelength long, terminated at either end non-reflectively, forms a very simple, short, and elegant solution of the coupled-helix attenuator problem. A notable weakness of this form of attenuator is its relatively narrow bandwidth. Proceeding, as before, on the assumption that the attenuator is a fraction δ larger or smaller than half a beat wavelength at frequencies ω_1 and ω_2 on either side of the center frequency ω, we find that the fraction of power transferred from the inner helix to the attenuator is then given by $(1 - \sin^2(\pi\delta/2))$. The attenuation is thus simply

$$A = \sin^2\left(\frac{\pi\delta}{2}\right)$$

For helices of the same proportions as used before in Section 3.4.1, we find that this will give an attenuation of at least 20 db over a frequency band of two to one. At the center frequency, ω_0, the attenuation is infinite; — in theory.

Thus to get higher attenuation, it would be necessary to arrange for a sufficient number of such attenuators in tandem along the TWT. Moreover, by properly staggering their lengths within certain ranges a wider attenuation band may be achieved. The success of such a scheme largely depends on the ability to terminate the helix ends non-reflectively. Considerable work has been done in this direction, but complete success is not yet in sight.

Another basically different scheme for a coupled-helix attenuator rests on the use of distributed attenuation along the coupling helix. The difficulty with any such scheme lies in the fact that *unequal* attenuation in the two coupled helices reduces the coupling between them and the more they differ in respect to attenuation, the less the coupling. Naturally, one would wish to have as little attenuation as practicable associated with the inner helix (inside the TWT). This requires the attenuating element to be associated with the outer helix. Miller[5] has shown that the maximum total power reduction in coupled transmission systems is obtained when

$$\left| \frac{\alpha_1 - \alpha_2}{\beta_b} \right| \approx 1$$

where α_1 and α_2 are the attenuation constants in the respective systems, and β_b the beat phase constant. If the inner helix is assumed to be lossless, the attenuation constant of the outer helix has to be effectively equal to the beat wave phase constant. It turns out that 60 db of attenuation requires about 3 beat wavelengths (in practice 10 to 20 helix wavelengths). The total length of a typical TWT is only 3 or 4 times that, and it will be seen, therefore, that this scheme may not be practical as the only means of providing loss.

Experiments carried out with outer helices of various resistivities and thicknesses by K. M. Poole (then at the Clarendon Laboratory, Oxford, England) tend to confirm this conclusion. P. D. Lacy[11] has described a coupled helix attenuator which uses a multifilar helix of resistance material together with a resistive sheath between the helices.

Experiments were performed at Bell Telephone Laboratories with a TWT using a resistive sheath (graphite on paper) placed between the outer helix and the quartz tube enclosing the inner helix. The attenuations were found to be somewhat less than estimated theoretically. The attenuator helix was movable in the axial direction and it was instructive to observe the influence of attenuator position on the power output from the tube, particularly at the highest attainable power level. As one might expect, as the power level is raised, the attenuator has to be moved nearer to the input end of the tube in order to obtain maximum gain and power output. In the limit, the attenuator helix has to be placed right close to the input end, a position which does not coincide with that for maximum low-level signal gain. Thus, the potential usefulness of the feature of mobility of coupled-helix elements has been demonstrated.

4. CONCLUSION

In this paper we have made an attempt to develop and collect together a considerable body of information, partly in the form of equations, partly in the form of graphs, which should be of some help to workers in the field of microwave tubes and devices. Because of the crudity of the assumptions, precise agreement between theory and experiment has not been attained nor can it be expected. Nevertheless, the kind of physical phenomena occurring with coupled helices are, at least, qualitatively described here and should permit one to develop and construct various types of devices with fair chance of success.

ACKNOWLEDGEMENTS

As a final note the authors wish to express their appreciation for the patient work of Mrs. C. A. Lambert in computing the curves, and to G. E. Korb for taking the experimental data.

Appendix I

I. SOLUTION OF FIELD EQUATIONS

In this section there is presented the field equations for a transmission system consisting of two helices aligned with a common axis. The propagation properties and impedance of such a transmission system are discussed for various ratios of the outer helix radius to the inner helix radius. This system is capable of propagating two modes and as previously pointed out one mode is characterized by a longitudinal field midway between the two helices and the other is characterized by a transverse field midway between the two helices.

The model which is to be treated and shown in Fig. 2.3 consists of an inner helix of radius a and pitch angle ψ_1 which is coaxial with the outer helix of radius b and pitch angle ψ_2. The sheath helix model will be treated, wherein it is assumed that helices consist of infinitely thin sheaths which allow for current flow only in the direction of the pitch angle ψ.

The components of the field in the region inside the inner helix, between the two helices and outside the outer helix can be written as follows — inside the inner helix

$$H_{z_1} = B_1 I_0(\gamma r) \tag{1}$$

$$E_{z_2} = B_2 I_0(\gamma r) \tag{2}$$

$$H_{\varphi_2} = j \frac{\omega \varepsilon}{\gamma} B_2 I_1(\gamma r) \tag{3}$$

$$H_{r_1} = \frac{j\beta}{\gamma} B_1 I_1(\gamma r) \tag{4}$$

$$E_{\varphi_1} = -j \frac{\omega \mu}{\gamma} B_1 I_1(\gamma r) \tag{5}$$

$$E_{r_2} = \frac{j\beta}{\gamma} B_2 I_1(\gamma r) \tag{6}$$

and between the two helices

$$H_{z_3} = B_3 I_0(\gamma r) + B_4 K_0(\gamma r) \tag{7}$$

$$E_{z_5} = B_5 I_0(\gamma r) + B_6 K_0(\gamma r) \tag{8}$$

$$H_{\varphi_5} = \frac{j\omega\varepsilon}{\gamma} [B_5 I_1(\gamma r) - B_6 K_1(\gamma r)] \tag{9}$$

$$H_{r_3} = \frac{j\beta}{\gamma} [B_3 I_1(\gamma r) - B_4 K_1(\gamma r)] \tag{10}$$

$$E_{\varphi_3} = -j \frac{\omega\mu}{\gamma} [B_3 I_1(\gamma r) - B_4 K_1(\gamma r)] \tag{11}$$

$$E_{r_5} = \frac{j\beta}{\gamma} [B_5 I_1(\gamma r) - B_6 K_1(\gamma r)] \tag{12}$$

and outside the outer helix

$$H_{z_7} = B_7 K_0(\gamma r) \tag{13}$$

$$E_{z_8} = B_8 K_0(\gamma r) \tag{14}$$

$$H_{\varphi_8} = -j \frac{\omega\varepsilon}{\gamma} B_8 K_1(\gamma r) \tag{15}$$

$$H_{r_7} = \frac{-j\beta}{\gamma} B_7 K_1(\gamma r) \tag{16}$$

$$E_{\varphi_7} = j \frac{\omega\mu}{\gamma} B_7 K_1(\gamma r) \tag{17}$$

$$E_{r_8} = \frac{-j\beta}{\gamma} B_8 K_1(\gamma r) \tag{18}$$

With the sheath helix model of current flow only in the direction of wires we can specify the usual boundary conditions that at the inner and outer helix radius the tangential electric field must be continuous and perpendicular to the wires, whereas the tangential component of magnetic field parallel to the current flow must be continuous. These can be written as

$$E_z \sin\psi + E_\varphi \cos\psi = 0 \tag{19}$$

E_z, E_φ and $(H_z \sin\psi + H_\varphi \cos\psi)$ be equal on either side of the helix.

By applying these conditions to the two helices the following equations are obtained for the various coefficients.

First, we will define a more simple set of parameters. We will denote

$$I_0(\gamma a) \text{ by } I_{01} \qquad \text{and} \qquad I_0(\gamma b) \text{ by } I_{02}\text{, etc.}$$

Further let us use the notation introduced by Humphrey, Kite and James[11] in his treatment of coaxial helices.

$$\begin{aligned} P_{01} &\equiv I_{01}K_{01} \qquad & P_{02} &\equiv I_{02}K_{02} \qquad & R_0 &\equiv I_{01}K_{02} \\ P_{11} &\equiv I_{11}K_{11} \qquad & P_{12} &\equiv I_{12}K_{12} \qquad & R_1 &\equiv I_{11}K_{12} \end{aligned} \tag{20}$$

and define a common factor (C.F.) by the equation

$$\text{C.F.} = -\left[\frac{(\beta_0 a \cot \psi_2)^2}{(\gamma a)^2} P_{01}P_{02} - \frac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2} \frac{\cot \psi_2}{\cot \psi_1} R_1 R_0 + R_0^2 - P_{01}P_{01}\right] \tag{21}$$

With all of this we can now write for the coefficients of equations 1 through 18:

$$\frac{B_1}{B_2} = -j\sqrt{\frac{\varepsilon}{\mu}} \frac{\gamma a}{\beta_0 a \cot \psi_1} \frac{I_{01}}{I_{02}} \tag{22}$$

$$\frac{B_3}{B_2} = -i\sqrt{\frac{\varepsilon}{\mu}} \frac{\beta_0 a \cot \psi_1}{\gamma a} \frac{I_{01}K_{12}}{\text{C.F.}} \left[\frac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2} R_1 - \frac{\cot \psi_2}{\cot \psi_1} R_0\right] \tag{23}$$

$$\frac{B_4}{B_2} = -j\sqrt{\frac{\varepsilon}{\mu}} \frac{\beta_0 a \cot \psi_1}{\gamma a} \frac{I_{01}I_{11}}{\text{C.F.}} \left[\frac{(\beta_0 a \cot \psi_2)^2}{(\gamma a)^2} P_{12} - P_{02}\right] \tag{24}$$

$$\frac{B_5}{B_2} = -\frac{R_0}{\text{C.F.}} \left[R_0 - \frac{(\beta_0 a \cot \psi_1)^2}{(\gamma a^2)} \frac{\cot \psi_2}{\cot \psi_1} R_1\right] \tag{25}$$

$$\frac{B_6}{B_2} = -\frac{I_{01}^2}{\text{C.F.}} \left[\frac{(\beta_0 a \cot \psi_2)^2}{(\gamma a)^2} P_{12} - P_{02}\right] \tag{26}$$

$$\frac{B_7}{B_2} = j\sqrt{\frac{\varepsilon}{\mu}} \frac{\beta_0 a \cot \psi_1}{\gamma a} \frac{1}{\text{C.F.}} \frac{I_{01}}{K_{12}} \left[P_{02}R_1 - \frac{\cot \psi_2}{\cot \psi_1} P_{12}R_0\right] \tag{27}$$

$$\frac{B_8}{B_2} = \frac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2} \frac{\cot \psi_2}{\cot \psi_1} \frac{I_{01}^2}{\text{C.F.} R_0} \left[P_{02}R_1 - \frac{\cot \psi_2}{\cot \psi_1} P_{12}R_0\right] \tag{28}$$

The last equation necessary for the solution of our field problem is the transcendental equation for the propagation constant, γ, which can be written

$$\left[R_0 - \frac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2} \frac{\cot \psi_2}{\cot \psi_1} R_1\right]^2 = \left[P_{02} - \frac{(\beta_0 a \cot \psi_2)^2}{(\gamma a)^2} P_{12}\right]\left[P_{01} - \frac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2} P_{11}\right] \quad (29)$$

The solutions of this equation are plotted in Fig. 4.1.

There it is seen that there are two values of γ, one, γ_t, denoting the slow mode with transverse fields between helices and the other, γ_l, denoting the fast mode with longitudinal fields midway between the two helices.

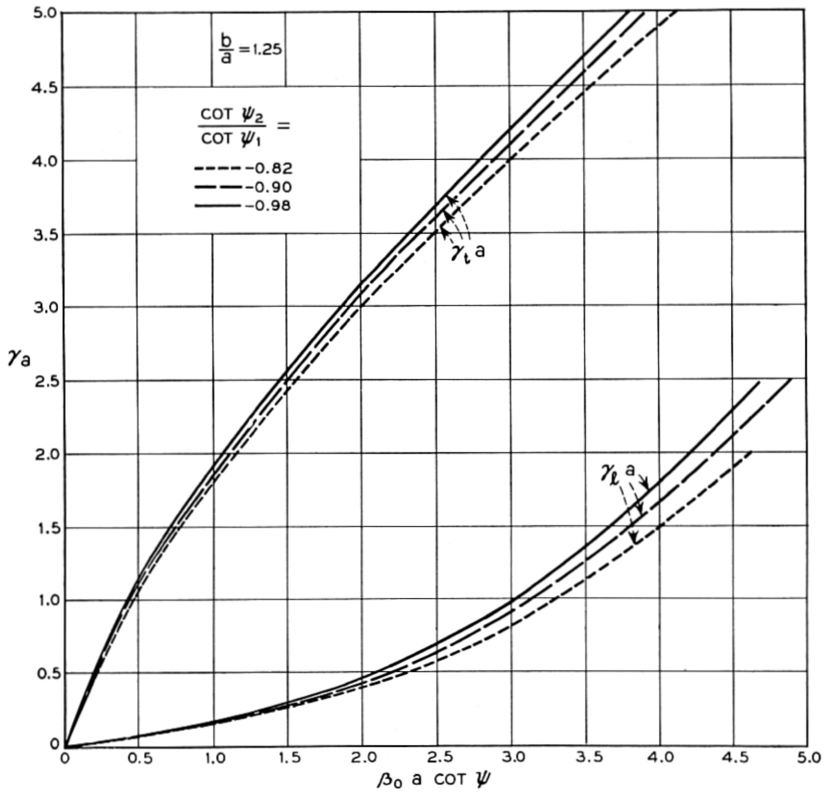

Fig. 4.1.1 — The radial propagation constants associated with the transverse and longitudinal modes on coupled coaxial sheath helices given as a function of $\beta_0 a \cot \psi_1$ for several values of $b/a = 1.25$.

These equations can now be used to compute the power flow as defined by

$$P = \tfrac{1}{2}\, Re \int E \times H^* \, dA \tag{30}$$

which can be written in the form

$$\left[\frac{E_z^2(0)}{\beta^2 P}\right]^{\frac{1}{3}} = \frac{\beta}{\beta_0}\left(\frac{\gamma}{\beta}\right)^4 F(\gamma a, \gamma b) \tag{31}$$

where

$$\begin{aligned}
[F(\gamma a, \gamma b)]^{-3} = \frac{(\gamma a)^2}{240}\,\frac{I_{01}^2}{(\text{C.F.})^2}\Bigg\{ & \frac{\left(I_{01}^2 + \dfrac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2} I_{11}^2\right)(I_{11}^2 - I_{01}I_{21})(\text{C.F.})^2}{I_{01}^2 I_{11}^2 \left(\dfrac{\beta_0 a \cot \psi_1}{\gamma a}\right)^2} \\
& - \left(K_{02}^2 + \frac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2} K_{12}^2\right)\left(R_0 - \frac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2}\,\frac{\cot \psi_2}{\cot \psi_1} R_1\right)^2 \\
& \quad \left[\left(\frac{b}{a}\right)^2 (I_{02}I_{22} - I_{12}^2) + (I_{11}^2 - I_{01}I_{21})\right] \\
& + \left(R_0 - \frac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2}\,\frac{\cot \psi_2}{\cot \psi_1} R_1\right)^2 \left(P_{02} - \frac{(\beta_0 a \cot \psi_2)^2}{(\gamma a)^2} P_{12}\right) \\
& \left[\left(\frac{b}{a}\right)^2 (2I_{12}K_{12} + I_{02}K_{22} + I_{22}K_{02}) - (2I_{11}K_{11} + I_{01}K_{21} + I_{21}K_{01})\right] \\
& - \left[I_{01}^2 + \frac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2} I_{11}^2\right]\left[P_{02} - \frac{(\beta_0 a \cot \psi_2)^2}{(\gamma a)^2} P_{12}\right]^2 \\
& \quad \left[\left(\frac{b}{a}\right)^2 (K_{02}K_{22} - K_{12}^2) - (K_{01}K_{21} - K_{11}^2)\right] \\
& + \frac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2 K_{12}^2 R_0^2}\left(\frac{b}{a}\right)^2 \left[R_0^2 + \frac{(\beta_0 a \cot \psi_2)^2}{(\gamma a)^2} I_{01}^2 K_{12}^2\right] \\
& \quad \left[P_{02}R_1 - \frac{\cot \psi_2}{\cot \psi_1} P_{12}R_0\right]^2 [K_{02}K_{22} - K_{12}^2]\Bigg\}
\end{aligned} \tag{32}$$

In (32) we find the power in the transverse mode by using values of

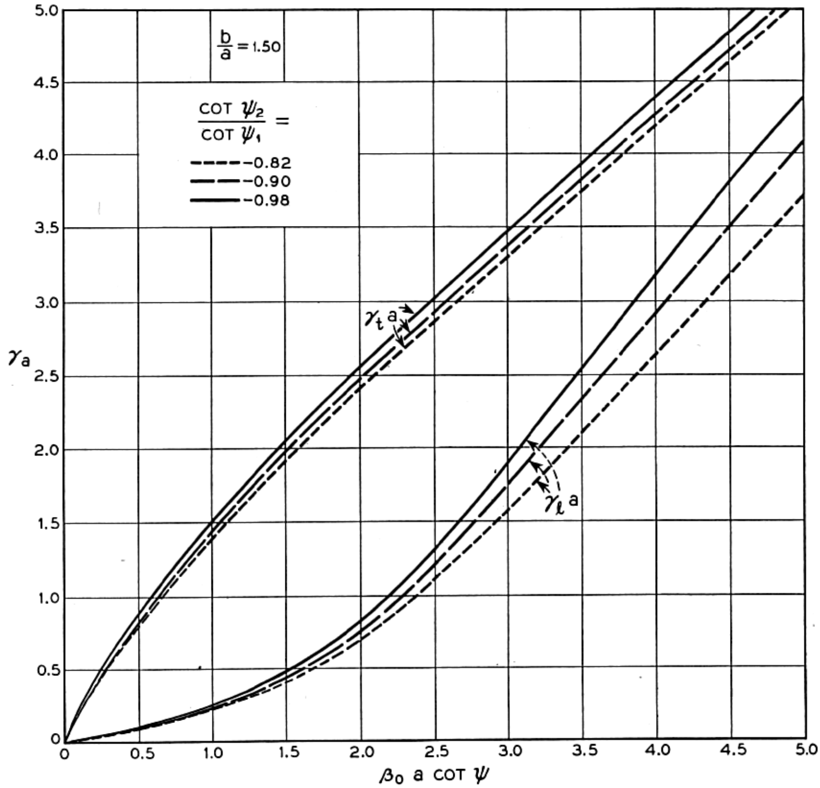

Fig. 4.1.2 — The radial propagation constants associated with the transverse and longitudinal modes on coupled coaxial sheath helices given as a function of $\beta_0 a \cot \psi_1$ when $b/a = 1.50$.

γ_t obtained from (29) and similarly the power in the longitudinal mode is found by using values of γ_ℓ.

II. FINDING $\bar{r}$

When coaxial helices are used in a transverse field amplifier, only the transverse field mode is of interest and it is important that the helix parameters be adjusted such that there is no longitudinal field at some radius, $\bar{r}$, where the cylindrical electron beam will be located. This condition can be expressed by equating E_z to zero at $r = \bar{r}$ and from (8)

$$B_5 I_0(\gamma \bar{r}) + B_6 K_0(\gamma \bar{r}) = 0 \tag{33}$$

which can be written with (25) and (26) as

$$K_{02}\left[R_0 - \frac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2} \frac{\cot \psi_2}{\cot \psi_1} R_1\right] I_0(\gamma \bar{r}) = I_{01}\left[P_{02} - \frac{(\beta_0 a \cot \psi_2)^2}{(\gamma a)^2} P_{12}\right] K_0(\gamma \bar{r}) \tag{34}$$

This equation together with (29) enables one to evaluate $\bar{r}/a$ versus $\beta_0 a \cot \psi_1$ for various ratios of b/a and $\cot \psi_2/\cot \psi_1$. The results of these calculations are shown in Fig. 3.1.

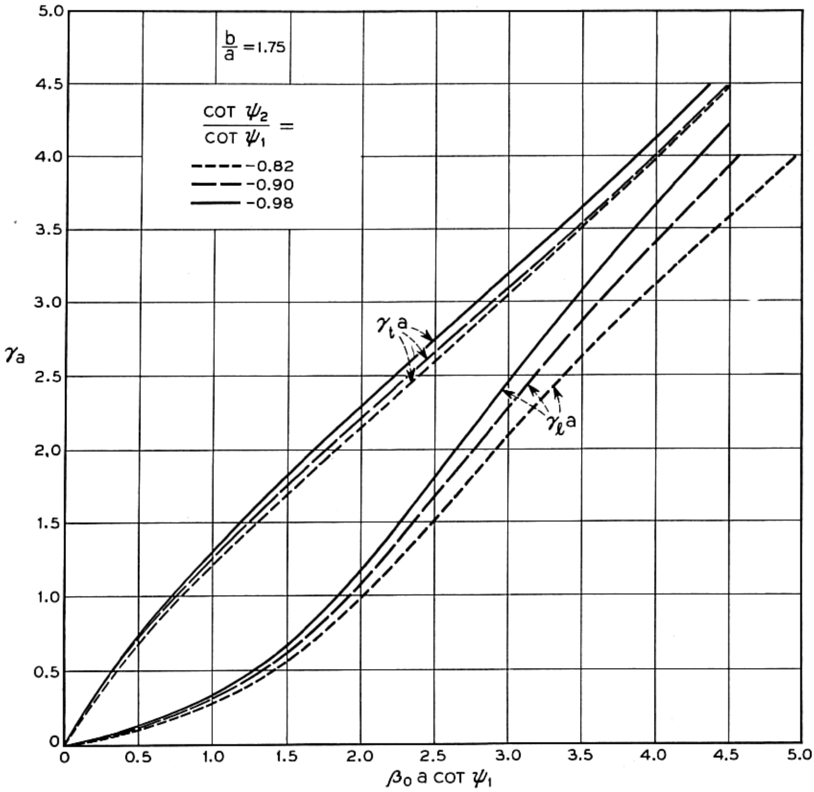

Fig. 4.1.3 — The radial propagation constants associated with the transverse and longitudinal modes on coupled coaxial sheath helices given as a function of $\beta_0 a \cot \psi_1$ when $b/a = 1.75$.

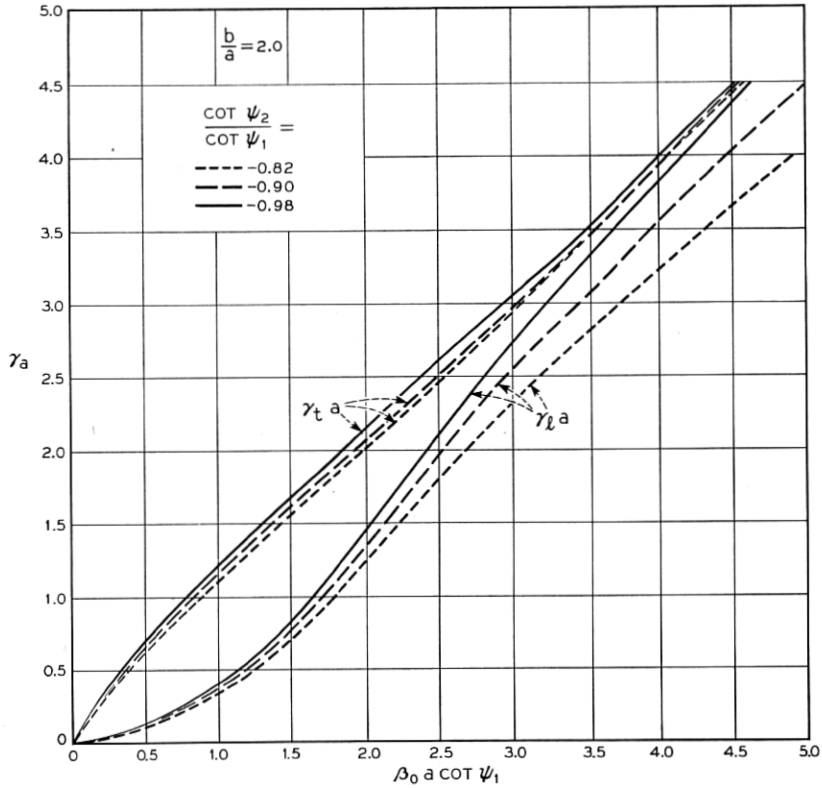

Fig. 4.1.4 — The radial propagation constants associated with the transverse and longitudinal modes on coupled coaxial sheath helices given as a function of $\beta_0 a \cot \psi_1$ when $b/a = 2.0$.

III. COMPLETE POWER TRANSFER

For coupled helix applications we require the coupled helix parameters to be adjusted so that RF power fed into one helix alone will set up the transverse and longitudinal modes equal in amplitude. For this condition the power from the outer helix will transfer completely to the inner helix. The total current density can be written as the sum of the current in the longitudinal mode and the transverse mode. Thus for the inner helix we have

$$J_a = J_{a\ell} e^{-j\beta_\ell z} + J_{at} e^{-j\beta_t z} \tag{35}$$

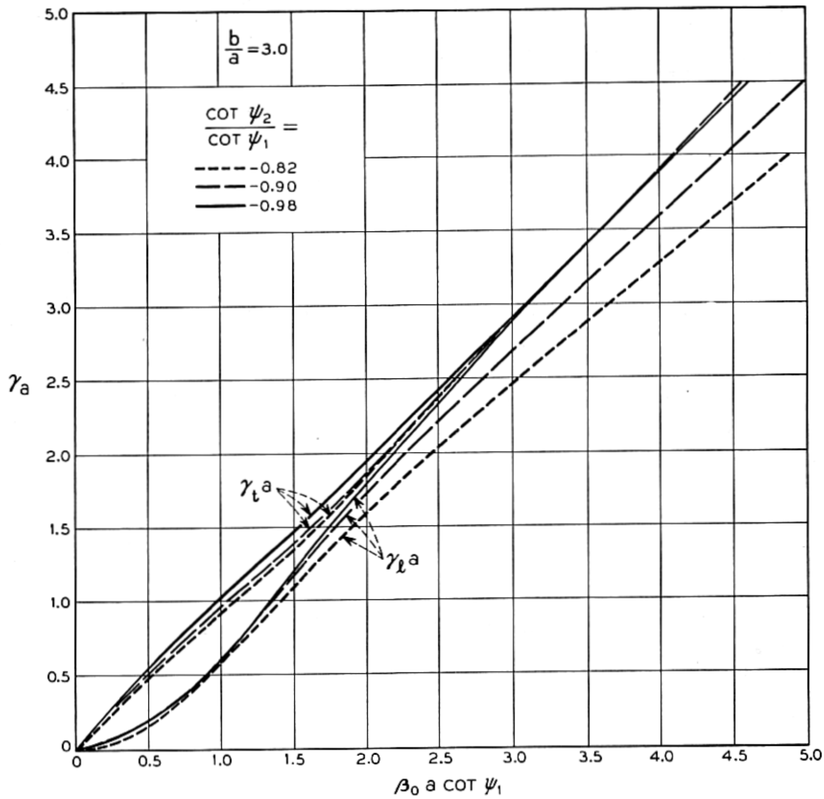

Fig. 4.1.5 — The radial propagation constants associated with the transverse and longitudinal modes on coupled coaxial sheath helices given as a function of $\beta_0 a \cot \psi_1$ when $b/a = 3.0$.

and for the outer helix

$$J_b = J_{b\ell} e^{-j\beta_\ell z} + J_{bt} e^{-j\beta_t z} \tag{36}$$

For complete power transfer we ask that

$$J_{b\ell} = J_{bt}$$

when J_a is zero at the input ($z = 0$)

$$J_{a\ell} = -J_{at}$$

or

$$\frac{J_{b\ell}}{J_{a\ell}} = -\frac{J_{bt}}{J_{at}} \tag{37}$$

Now $J_{a\ell}$ is equal to the discontinuity in the tangential component of magnetic field which can be written at $r = a$

$$J_{a\ell} = (H_{z3} \cos \psi_1 - H_{\varphi 5} \sin \psi_1) - (H_{z1} \cos \psi_1 - H_{\varphi 2} \sin \psi_1)$$

which can be written as

$$J_{a\ell} = -(H_{z1} - H_{z3})_{a\ell}(\cot \psi_1 + \tan \psi_1) \sin \psi_1 \tag{38}$$

and similarily at $r = b$

$$J_{b\ell} = -(H_{z7} - H_{z3})_{b\ell}(\cot \psi_2 + \tan \psi_2) \sin \psi_2 \tag{39}$$

Equations (38) and (39) can be combined with (37) to give as the condition for complete power transfer

$$A_\ell = -A_t \tag{40}$$

where

$$A = \frac{(I_{12}K_{02} + I_{02}K_{12})\left(P_{01} - \dfrac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2} P_{11}\right)}{(I_{01}K_{11} + I_{11}K_{01})\left(R_0 - \dfrac{(\beta_0 a \cot \psi_1)^2}{(\gamma a)^2} \dfrac{\cot \psi_2}{\cot \psi_1} R_1\right)} \tag{41}$$

In (40) A_ℓ is obtained by substituting γ_ℓ into (41) and A_t is obtained by substituting γ_t into (41).

The value of $\cot \psi_2/\cot \psi_1$ necessary to satisfy (40) is plotted in Fig. 2.8.

In addition to $\cot \psi_2/\cot \psi_1$ it is necessary to determine the interference wavelength on the helices and this can be readily evaluated by considering (36) which can now be written

$$J_b = J_{b\ell}(e^{-j\beta_\ell z} + e^{-j\beta_t z})$$

or

$$J_b = J_{b\ell} e^{-j((\beta_\ell + \beta_t)z/2)} \cos \frac{(\beta_t - \beta_\ell)}{2} z \tag{48}$$

and

$$J_b = J_{b\ell} e^{-j((\beta_\ell + \beta_t)z/2)} \cos \tfrac{1}{2}\beta_b z \tag{49}$$

where we have defined

$$\beta_b a = (\gamma_t a - \gamma_\ell a) \tag{50}$$

This value of β_b is plotted versus $\beta_0 a \cot \psi_1$ in Fig. 2.4.

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