Tapered Velocity Couplers

By J. S. COOK

(Manuscript received March 15, 1955)

The advent of very broad band microwave amplifiers and oscillators necessitates microwave circuitry of comparable frequency range. A scheme is presented which makes it possible to build microwave couplers of various kinds having bandwidths of well over two octaves. A preliminary investigation of mechanical and mathematical analogs is described; and some particular devices using the new principle are mentioned.

INTRODUCTION

In waveguide and transmission line networks, it is often desired to transfer power from one line to another. To this end, various hybrid junctions and ordinary directional couplers were developed some years ago.

More recently, S. E. Miller[1] has made use of the fact that when two transmission lines with equal phase velocities are continuously coupled over some length, a signal introduced into one line will be completely transferred periodically back and forth between the lines. (Fig. 1.) This principle is employed in power-"splitting" devices where power is, for instance, equally divided between two lines. Half of the signal power is permitted to transfer from one line to the other and at this point the coupling is discontinued. (Fig. 2(a).) Another obvious application of this coupling principle is a means to effect complete transfer of a signal from one line to another. (Fig. 2(b).)

Now the trend is toward systems which are capable of handling even greater bandwidths. Miller's approach to the problem of coupled waves has led to waveguide couplers which are useful over bandwidths of 25 per cent and better. However, with the advent of traveling-wave tube amplifiers and similar devices, useful bandwidths of two-to-one and more have become desirable. The bandwidth of the Miller scheme is limited because the strength of coupling and the electrical length of the coupling

[1] S. E. Miller, Coupled Wave Theory and Waveguide Applications, B.S.T.J., **33,** pp. 677–692, May, 1954.

section vary with frequency. This directly affects the power division between the lines. In the following two paragraphs are the background and thought which led to the new broadband coupling scheme.

Miller[1] has shown that if two coupled lines have a fixed difference in phase velocities the signal power transfer also is periodic. In this case, however, only a part of the power is transferred, the amount depending on the ratio of velocity difference to coupling factor. (Fig. 3.)

Assume, now, that two coupled lines have slightly different phase velocities and a signal is introduced into one line. Let the velocity difference increase slightly at the point where maximum energy has been transferred to the adjoining line. Repeat this process until the amount of power exchange becomes vanishingly small. It is conceivable that if the velocity

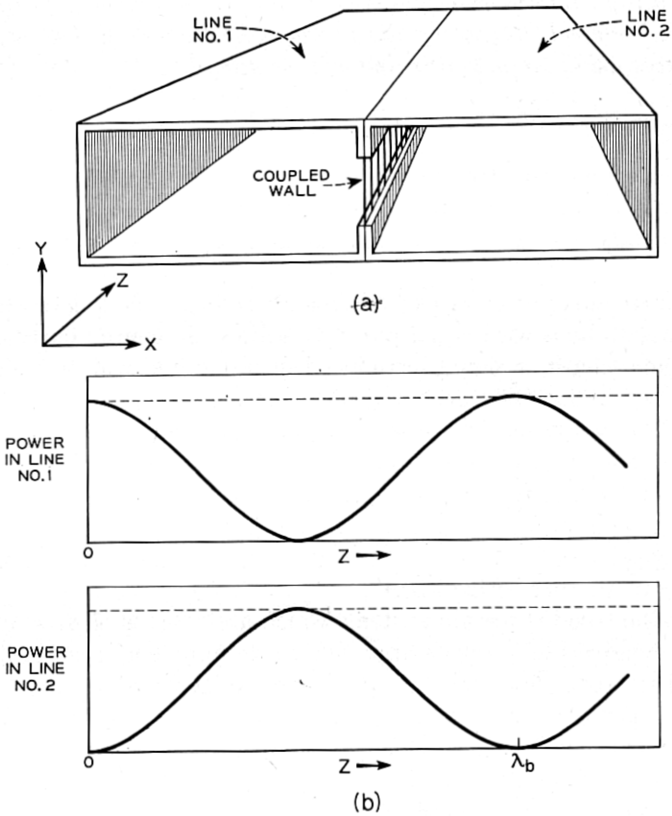

Fig. 1 — (a) Coupled waveguide transmission lines. (b) Wave power distribution along the lines.

steps are small the signal will finally be equally divided between the lines. (Fig. 4.) If this is so, it follows that the same effect must result from a continuous gradual change in the relative phase velocities of the two lines. Furthermore, if the coupling section is made long enough, this device should be frequency independent.

MECHANICAL ANALOG

It has long been recognized that coupled pendulums are analogous to coupled transmission lines. This analogy is demonstrated in the Appendix. To test the above conclusions about lines with changing phase velocities two pendulums were coupled together in such a way that their relative lengths could be adjusted while in motion. (Fig. 5.) It was ob-

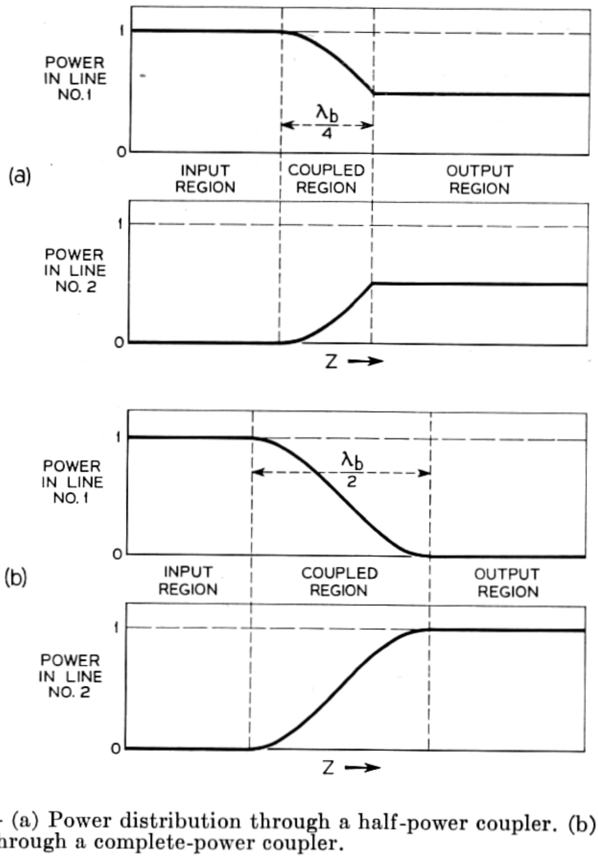

Fig. 2 — (a) Power distribution through a half-power coupler. (b) Power distribution through a complete-power coupler.

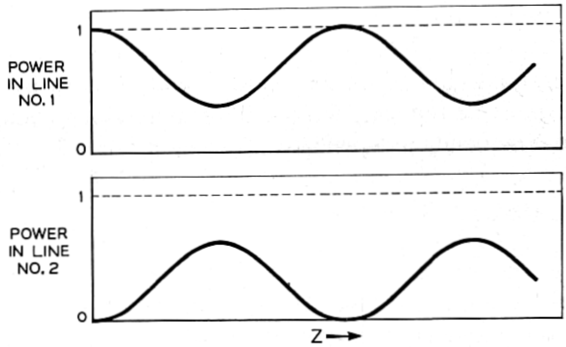

Fig. 3 — Incomplete power transfer resulting from coupling two lines having different phase velocities

served that when the pendulum lengths were fixed and equal, and one pendulum was set swinging, it would swing less and less until it stopped altogether, while the other pendulum would swing higher and higher until it reached a maximum. Then the process reversed until the first pendulum reached a maximum and the second pendulum came to rest once more. This cycle repeated until friction finally brought both pen-

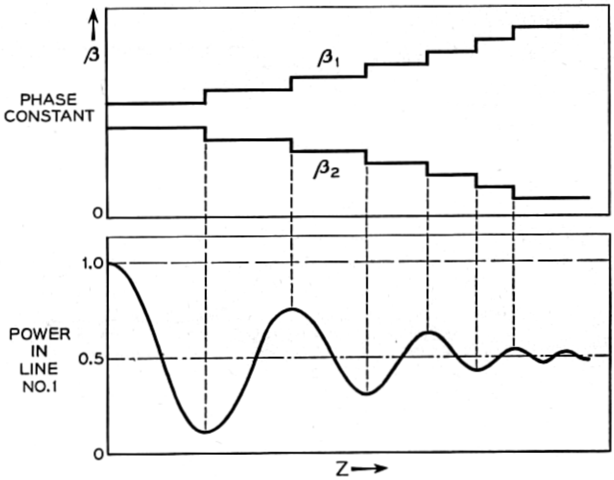

Fig. 4 — Power distribution in the driving line where the difference in phase velocity between two coupled lines is increased each half beat wavelength.

β_1 = phase constant of line 1

β_2 = phase constant of line 2

dulums to rest. This is a classical experiment performed in most early physics courses.

Now one pendulum was made quite long and the other relatively short. The longer pendulum was set swinging. It was then gradually shortened and the other gradually lengthened until they were finally of equal length. At that point they were swinging nearly together and with about the same amplitude. It was found, also, that if the short pendulum instead of the long was set swinging, and then the pendulum lengths were gradually equalized, the pendulums ended up swinging opposite to each other and again, of course, with about equal amplitudes. The initial and final conditions of the pendulums in these two experiments are indicated in Fig. 6. Although these experiments are the inverse of the original proposition reciprocity must hold, and the proposition, is thus valid.

It is known that a fixed system of two coupled pendulums will support two independent or "normal" modes of oscillation. These two normal modes are characterized by the pendulums swinging exactly "in phase"

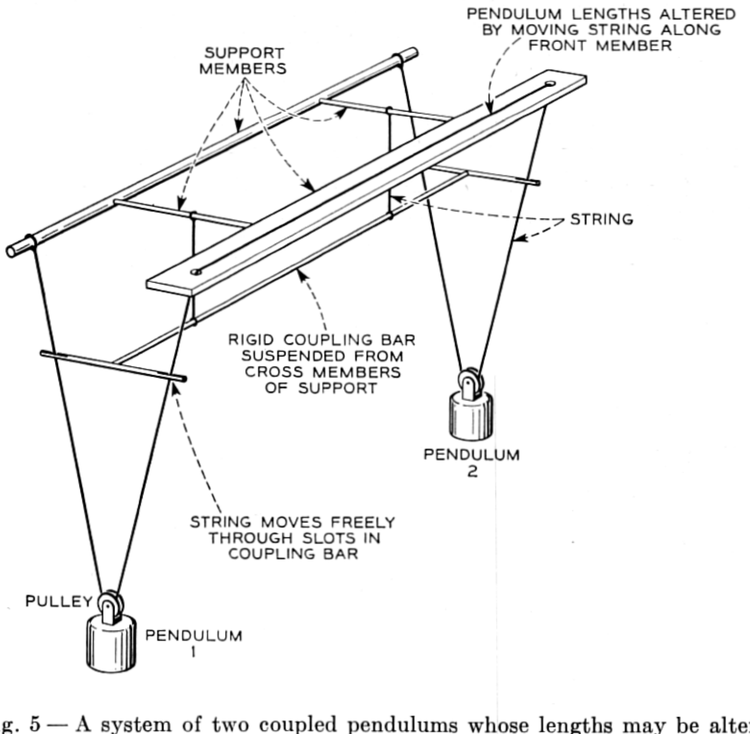

Fig. 5 — A system of two coupled pendulums whose lengths may be altered while in motion.

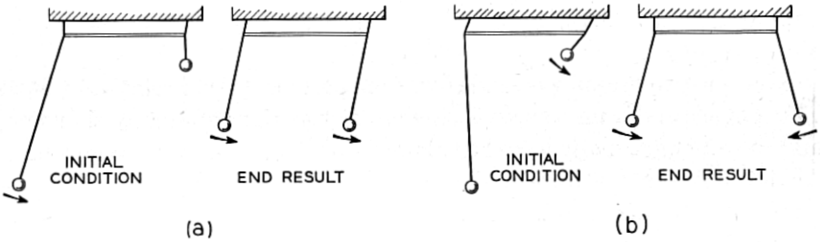

Fig. 6 — Initial and final states of oscillation of a system of two coupled pendulums where, (a), only the longer one is initially excited, and (b), only the shorter one is initially excited. In each case the lengths have been slowly equalized to produce the end results.

or exactly "out of phase" with each other. The end results of the two excitations in the experiment, then, were the two normal modes.

MATHEMATICAL ANALOG COMPUTER

As is shown in the Appendix, the pendulum analogy could not provide true quantitative solutions in terms of the transmission line constants. For this reason, and because of the mathematical complexity encountered in trying to solve the transmission line equations directly, we resorted to an analog computer. The coupled transmission line equations as proposed by Miller and given in the Appendix were appropriately rationalized and adapted to permit the phase constants of the lines to vary linearly with distance. Symbolically:

$$\beta_1 = \beta + \mu z, \qquad \beta_2 = \beta - \mu z \tag{1}$$

where β_1 = phase constant of line 1

β_2 = phase constant of line 2

β = initial phase constant of both lines

z = distance along coupled system

and μ = constant which determines rate of phase change.

In the process of rationalizing it was found convenient to measure z in terms of coupling wavelengths, λ_{b0}. The latter quantity is defined as the hypothetical distance required to transfer all the power from one line to the other and back again where $\beta_1 = \beta_2 = \beta$ = constant. If we let

$$\zeta = 2\pi z/\lambda_{b0}$$

we can write (1) as

$$\beta_1 = \beta\left(1 + \frac{\zeta}{2\pi n}\right), \qquad \beta_2 = \beta\left(1 - \frac{\zeta}{2\pi n}\right) \tag{2}$$

where n = number of coupling wavelengths the problem is allowed to run, i.e., until $\beta_2 = 0$, or $0 \leqq \zeta \leqq 2\pi n$.

The quantity E_1^2 was plotted as a function of ζ where E_1 was the wave amplitude in line 1 and $E_1(0) = 1$, $E_2(0) = 0$.

These plots are shown in Fig. 7 for $n = 2$ and $n = 6$. The dotted lines roughly indicate the centers of oscillation of these curves.

Inspection of the curves shows that:

1. The accuracy with which the power was ultimately divided between the two guides seemed to be determined within the first two or three exchange cycles.

2. The ultimate phase constant difference here chosen was not great enough since there was too much power still being exchanged at the end of the plot.

3. The exchange amplitude was much too constant after the first few cycles. This indicates that a linear phase constant change is inefficient if minimum coupler length is to be realized.

4. Although, initially, a slow rate of change of phase velocity is necessary, subsequently, a progressively faster rate should reduce the final exchange amplitude.

From the dotted lines indicating the centers of oscillation of the curves it may be seen that the initial rate of phase change for $n = 2$ is too great while that for $n = 6$ is slow enough to keep the oscillation fairly well centered.

It can be shown from Miller's equations for coupled lines of different fixed phase velocities[1] that the peak-to-peak amplitude of the power exchange between the lines, P_x, when the distance-averaged powers in the lines are equal is given by the following equation.

$$P_x = \frac{P}{\sqrt{\left[\frac{\Delta\beta}{2c}\right]^2 + 1}} \tag{3}$$

where P = total power in the coupled system,

$$\Delta\beta = \beta_1 - \beta_2$$

and c = coefficient of coupling between the lines.

In spite of the fact that this calculation assumes a constant velocity

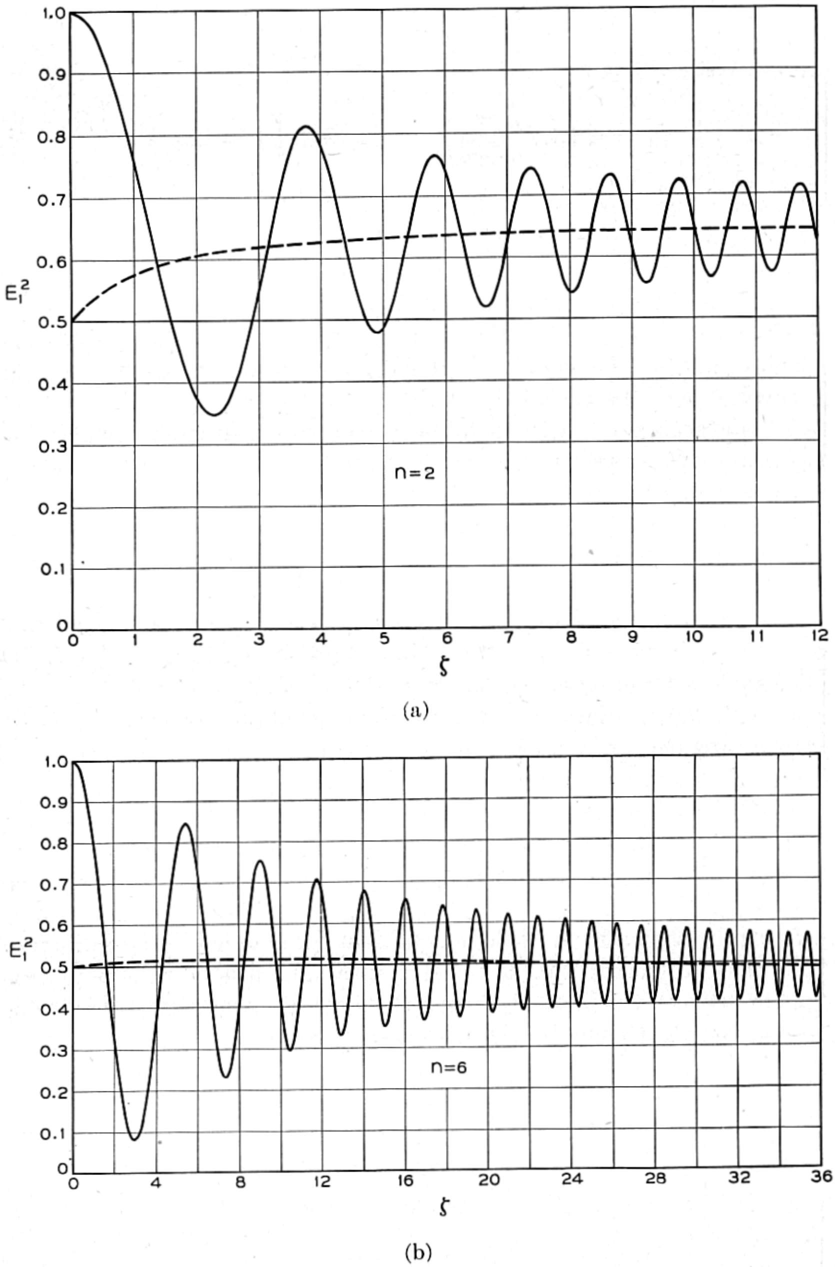

Fig. 7 — Power distribution in the driving line where the difference in phase constant between the two lines is increased linearly. Curve (a) shows the result of increasing too rapidly, (b) the result of an acceptable rate of increase. These curves are reproductions of those recorded by a mathematical analogue computer. Curve (b) discloses that there was a small amount of computer "drift."

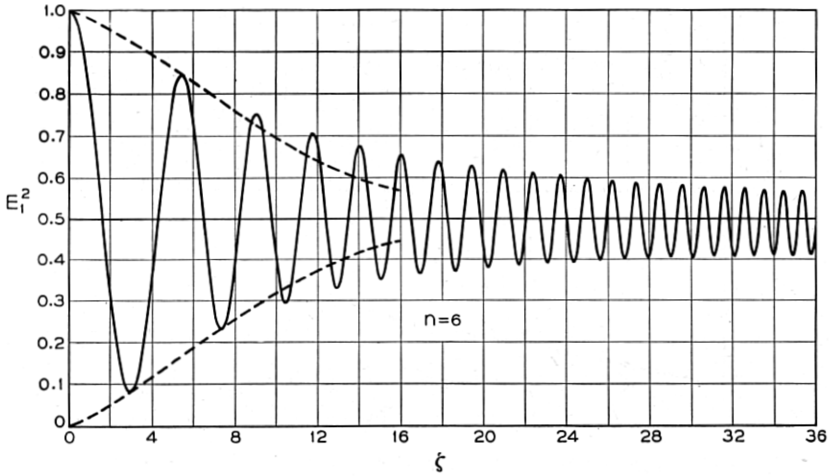

Fig. 8 — A reproduction of the curve of Fig. 7(b) showing the proposed envelope which might result from a more efficient coupler.

difference it can be shown from Fig. 7(b) that it provides a good approximation for the power exchange at any point along a coupled system where the phase velocities are varying slowly. Equation (3) indicates that the final power exchange amplitude may be reduced either by increasing the final velocity difference or by decreasing the coupling.

The next question is how to choose a variation of phase velocities in such a way that the power will be well divided in the shortest possible distance. Assume for the present that the velocity difference between the lines is constant throughout each power exchange shown in Fig. 7(b). An envelope of oscillation amplitude was arbitrarily superimposed on the $n = 6$ curve as shown in Fig. 8. It was proposed to find a law of velocity change which would produce such an envelope. From the equations for fixed velocity difference the value of $\Delta\beta/2c$ was calculated which would produce the power amplitude predicted by this envelope after each power exchange. The value thus calculated was considered as the value half way through the exchange distance, this distance also being determined by the particular value of $\Delta\beta/2c$. These points were then plotted as a function of ζ as shown in Fig. 9. The solid line represents the linear variation actually used for the computed curve. Note that the first three points lie very close to this line proving the validity of the approximation.

W. H. Louisell has made a thorough mathematical examination of this sort of coupler.[2] It is interesting to note that Louisell's analysis

[2] B.S.T.J., page 853 of this issue.

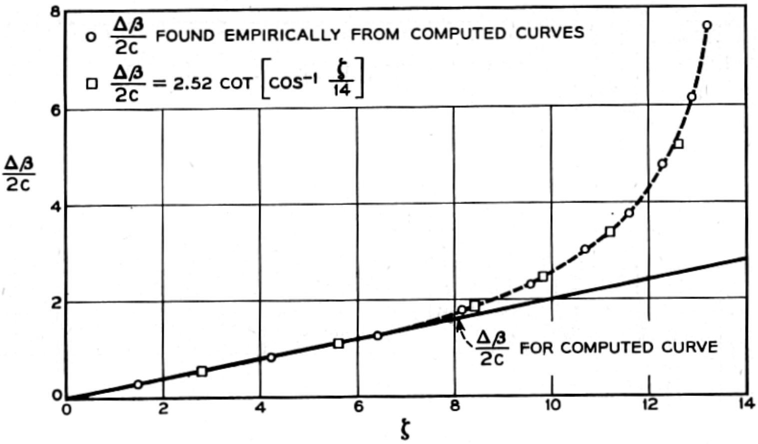

Fig. 9 — The solid line shows the linear variation of $\Delta\beta/2c$ used on the analogue computer to produce the curve of Fig. 7(b). The broken line is the variation of $\Delta\beta/2c$ calculated to produce the envelope superimposed on that curve in Fig. 8.

indicates that when the coupling is held constant the phase velocity difference, $\Delta\beta$, should vary as follows for greatest efficiency:

$$\Delta\beta \sim \cot[\cos^{-1} \zeta/2\pi n]$$

where n = the length of the coupler measured in coupling wave lengths, λ_{b0}. A suitable multiplier and a value for n were chosen for the above formula to make it coincide with the empirical curve of Fig. 9 as nearly as possible. Points found from the resulting equation appear as boxes on that figure.

WAVEGUIDE COUPLER

Using the function of $\Delta\beta/2c$ thus calculated, an experimental coupler was made. Two rectangular waveguides (1.145″ x 2.290″) were placed side by side so that they had a common narrow wall. They were electrically coupled through a slot like that shown in Fig. 1. One guide was loaded with a gradually increasing amount of polystyrene such as to taper the phase velocity to match the curve of Fig. 9. Since the phase constant difference $\Delta\beta$ could not be made great enough by this method, the coupling near the end was decreased by tapering the slot width linearly to zero through the section where $\Delta\beta$ was maximum, i.e., where the loaded guide was completely filled with polystyrene. To reduce reflections the other end of the slot was also tapered to zero, though over a much shorter distance. Finally, the polystyrene at the fully loaded end was tapered off outside the coupled section slowly enough to provide a low reflection transition to unloaded guide. The effective coupling length was about 88″.

When a signal was fed into either guide at one end of the coupler the signal output at the other end was equally divided between the two guides within 0.2 db over a frequency range of 2,700 to 3,100 mc/s. Since guide cutoff is about 2,600 mc/s this frequency range represented a large range of λ_{b0}. The coupling wavelength, indeed, varied from about 18″ at 2,700 mc/s to 40″ at 3,100 mc/s. The coupler accuracy degenerated rapidly for frequencies above 3,100 mc/s both because the coupling wavelength got too large compared with the length of the coupler and because of large reflections due, possibly, to the TE_{20} mode which can exist above about 3,200 mc/s in the fully loaded guide.

APPLICATIONS

One rather serious shortcoming besets this kind of coupler. It is inherently many wavelengths long. This presents a physical size problem for fast-wave structures at frequencies of 5000 mc/s and lower, and a loss problem at all frequencies. Thus, some of the most practical applications are found in connection with traveling-wave tubes where the associated helices provide an ideal medium for varying phase and coupling constants on a slow wave structure. If two helices are wound in opposite sense and mounted coaxially, strong coupling occurs between them.[3] If the axial phase constants of the two helices are equal, and an out-of-phase normal mode is launched on the system, the electric field in the region between the helices will be oriented transverse to the axis. It has long been proposed that an axially directed electron beam in such a transverse field region would provide low noise amplification.[4] C. F. Quate has suggested tapering the relative pitch of the two helices (Fig. 10) at the beginning such as to launch the transverse normal mode over a wide frequency range by introducing the signal to the outer helix only. By reversing the relative pitches at the input in Fig. 10, an in-phase or or longitudinal, normal mode could be excited.

Coupling power into and out from a traveling-wave tube over a wide frequency range has long been a problem. R. Kompfner[5, 6] has sugested using a short section of helix after the principle Miller used, i.e., an outer

[3] J. R. Pierce, Travelling Wave Tubes, pp. 46–47, D. Van Nostrand, 1950.
[4] J. R. Pierce, Travelling Wave Tubes, p. 156, D. Van Nostrand, 1950.
[5] Kompfner, Experiments on Coupled Helices, A.E.R.E. Report No. G/M98, Sept., 1951.
[6] R. Kompfner, Coupled Helices, Paper presented at I.R.E. Electron Tube Conference, 1953, Stanford, Cal.

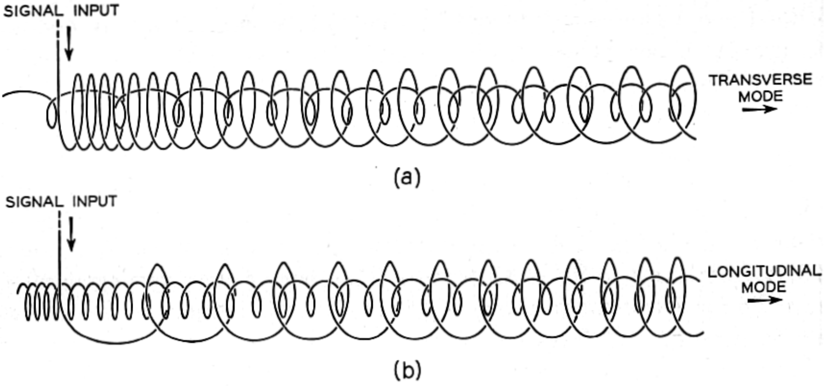

Fig. 10 — Coupled helices arranged so that essentially pure, (a), transverse or, (b), longitudinal normal modes may be excited over a wide frequency range by introducing the signal to the outer helix only.

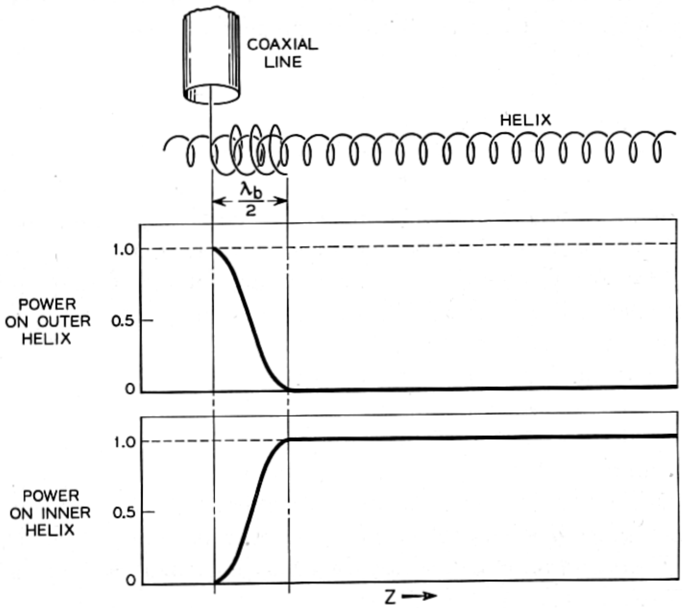

Fig. 11 — Helix coupler proposed by R. Kompfner and currently in common use for coupling to traveling-wave tubes.

helix concentric with the traveling-wave tube helix (Fig. 11) and just long enough for all the power launched on it from a matched coaxial line to be transferred to the inner helix. Under certain conditions of coupling and geometry this kind of coupler may have a useful frequency range of two to one. There are times, however, when even wider bandwidths are desirable. The new coupling scheme again may be applied. Let the inner helix turns be wound very close at point "A" (Fig. 12) and then the pitch increased gradually, until it is a very loose or "fast" helix at point "B". By constrast let the outer helix be very fast at A and slow at B. At some point between A and B the helix phase velocities will be equal. Now if a wave is launched on the outer helix at A it will appear as the in-phase normal mode at the point where the helix velocities are equal. As it proceeds along the coupler the wave energy will again be

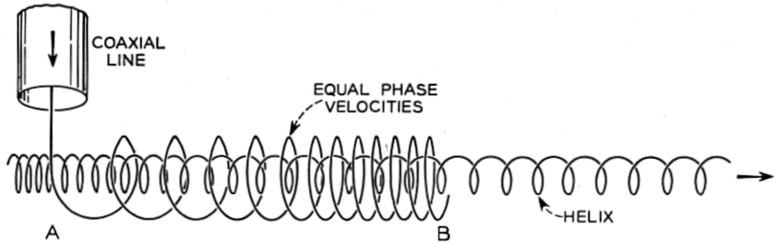

Fig. 12 — A tapered-velocity coupler to a traveling-wave tube helix. Though the inner helix is here shown tapered, it might, in practice, be made uniform if the outer helix is sufficiently tapered.

transferred to the faster helix which now is the inner one. At point B, then, the wave energy will be entirely on the inner helix. In other words, if the coupling is not too great and the taper sufficiently long this device is a 100 per cent, or complete, coupler. It turns out that such a coupler on a 4,000 mc/s traveling-wave tube may be less than 2″ long and operate effectively over a bandwidth of three or more octaves.

Since a helix may be matched to a coaxial transmission line over a considerable bandwidth, and since a 3-db coupler may be made with concentric helices to cover a similar frequency range, it should be possible to build a compact hybrid junction with coaxial connectors which will operate usefully throughout a bandwidth of three octaves.

CONCLUSION

Using distributed coupling between transmission lines with tapered phase velocities has made possible extremely broad band directional couplers. This principle can be used to give an equal and accurate division of power delivered to two conjugate lines (a hybrid coupler) or to give essentially complete power transfer from one line to the other.

Probably the most useful applications are to traveling-wave tubes. A very broad band coupling can be made through the envelope to a standard traveling-wave tube helix. The principle also provides a means to couple into and out from either normal mode of a double, concentric traveling-wave tube helix.

Finally, it should be possible to make a broad band (3 or more octaves) hybrid junction by coupling to tapered helices from coaxial lines.

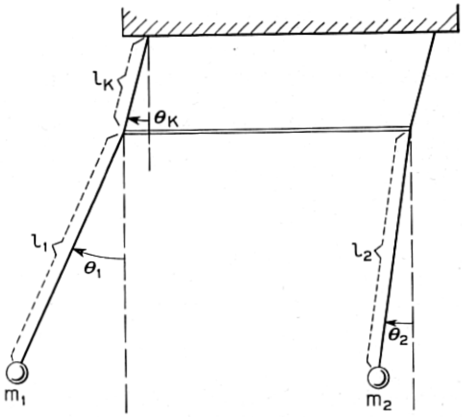

Fig. 13 — A system of two coupled pendulums with the symbols used in the appendix.

APPENDIX

Fig. 13 shows the coupled pendulums and the symbols used in the following analysis. From the energy equations for the system the Lagrangian and Hamiltonian equations were found. All third order and greater terms were dropped and since no second order terms appeared the following linear equations resulted.

$$m_1\ell_1\ddot{\theta}_1 + m_1\ell_k\ddot{\theta}_k = -m_1g\theta_1 \tag{A1}$$

$$m_2\ell_2\ddot{\theta}_2 + m_2\ell_k\ddot{\theta}_k = -m_2g\theta_2 \tag{A2}$$

$$(m_1 + m_2)\ell_k\ddot{\theta}_k + m_1\ell_1\ddot{\theta}_1 + m_2\ell_2\ddot{\theta}_2 = -(m_1 + m_2)g\theta_k \tag{A3}$$

Substituting equations (A1) and (A2) in (A3) we find

$$m_1g\theta_1 + m_2g\theta_2 = (m_1 + m_2)g\theta_k \tag{A4}$$

In our experimental pendulums the masses were equal so

$$m_1 = m_2$$

and equations (A1), (A2), and (A4) become

$$\ell_1\ddot{\theta}_1 + \ell_k\ddot{\theta}_k = -g\theta_1 \tag{A5}$$

$$\ell_2\ddot{\theta}_2 + \ell_k\ddot{\theta}_k = -g\theta_2 \tag{A6}$$

$$\theta_1 + \theta_2 = 2\theta_k \tag{A7}$$

Let us choose S_1 and S_2 as a measure of pendulum displacement from rest; and define it as

$$\begin{aligned} S_1 &= \ell_1\theta_1 + \ell_k\theta_k\,, \\ S_2 &= \ell_2\theta_2 + \ell_k\theta_k \end{aligned} \tag{A8}$$

Then

$$\begin{aligned} \ddot{S}_1 &= \ell_1\ddot{\theta}_1 + \ell_k\ddot{\theta}_k \\ \ddot{S}_2 &= \ell_2\ddot{\theta}_2 + \ell_k\ddot{\theta}_k \end{aligned} \tag{A9}$$

Substitute (A9) in (A5) and (A6)

$$\begin{aligned} \ddot{S}_1 &= -g\theta_1 \\ \ddot{S}_2 &= -g\theta_2 \end{aligned} \tag{A10}$$

From (A7) and (A8) find

$$\begin{aligned} \theta_1 &= \frac{\ell_k S_1 + 2\ell_2 S_1 - \ell_k S_2}{\ell_k(\ell_1 + \ell_2) + 2\ell_1\ell_2} \\ \theta_2 &= \frac{\ell_k S_2 + 2\ell_1 S_2 - \ell_k S_1}{\ell_k(\ell_1 + \ell_2) + 2\ell_1\ell_2} \end{aligned} \tag{A11}$$

Substitute (A11) in (A10)

$$\begin{aligned} \ddot{S}_1 &= -g\,\frac{[\ell_k S_1 + 2\ell_2 S_1 - \ell_k S_2]}{\ell_k(\ell_1 + \ell_2) + 2\ell_1\ell_2} \\ \ddot{S}_2 &= -g\,\frac{[\ell_k S_2 + 2\ell_1 S_2 - \ell_k S_1]}{\ell_k(\ell_1 + \ell_2) + 2\ell_1\ell_2} \end{aligned} \tag{A12}$$

Miller has expressed the general wave equations for two coupled lines as

$$\begin{aligned} \frac{dE_1}{dz} &= -j(c + \beta_1)E_1 + jcE_2 \\ \frac{dE_2}{dz} &= -j(c + \beta_2)E_2 + jcE_1 \end{aligned} \tag{A13}$$

Differentiate (A13) and make the proper substitutions to find

$$\frac{d^2E_1}{dz^2} = -c(2c + \beta_1 + \beta_2)E_1 - [c(\beta_1 - \beta_2) + \beta_1^2]E_1 + c(2c + \beta_1 + \beta_2)E_2$$

$$\frac{d^2E_2}{dz^2} = -c(2c + \beta_1 + \beta_2)E_2 - [c(\beta_2 - \beta_1) + \beta_2^2]E_2 + c(2c + \beta_1 + \beta_2)E_1$$

where E_1 = wave amplitude in line 1

E_2 = wave amplitude in line 2

β_1 = wave phase constant in line 1 before coupling

β_2 = wave phase constant in line 2 before coupling

c = coupling constant

Now equations (A12) and (A14) are of exactly the same form, namely:

$$\begin{aligned} \ddot{S}_1 &= -AS_1 - BS_1 + AS_2 \\ \ddot{S}_2 &= -AS_2 - BS_2 + AS_1 \end{aligned} \tag{A15}$$

Thus the solutions for the two sets of equations differ only in their constants. The constants have the following correspondence:

$$\frac{g\ell_k}{\ell_k(\ell_1 + \ell_2) + 2\ell_1\ell_2} \doteq c(2c + \beta_1 + \beta_2) \tag{A16}$$

$$\frac{2g\ell_2}{\ell_k(\ell_1 + \ell_2) + 2\ell_1\ell_2} \doteq c(\beta_1 - \beta_2) + \beta_1^2 \tag{A17}$$

The complexity of the constants seems to defy separating them out. Suffice it to say that ℓ_k roughly corresponds to c and ℓ_n to reciprocal β_n. Note, however, that if $\beta_1 = \beta_2 = \beta$ and $\ell_1 = \ell_2 = \ell$, (A17) becomes

$$\beta^2 \doteq \frac{g}{\ell + \ell_k} \tag{A18}$$

It may be recognized from the equations for a simple pendulum that $\sqrt{g/l + l_k}$ is, indeed, the phase constant of the uncoupled pendulums, just as β is, by definition, the phase constant for the uncoupled transmission lines.