Relaxation Phenomena in Ferrites

By A. M. CLOGSTON

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J. K. Galt has recently suggested a mechanism for the losses observed in ferromagnetic resonance and domain wall motion in single crystals of nickel ferrite containing small amounts of divalent iron substituted for divalent nickel. Each such substitution provides one electron able to move between the various octahedral sites in the crystal. Galt's suggestion is that the losses arise from a relaxation associated with the motion of these electrons.

This paper develops a theory of this mechanism based on a thermodynamic model. Expressions are found for the velocity of domain wall motion, for the line width in ferromagnetic resonance and for the displacement of the field for resonance.

INTRODUCTION

In a recent paper, Galt[1, 2] and Wijn and van der Heide[3] have suggested a mechanism for the losses observed during ferromagnetic resonance and domain wall motion in single crystals of nickel ferrite having a small amount of divalent iron substituted for divalent nickel in the octahedral sites. It is supposed that each substitution of a divalent iron atom for a divalent nickel atom provides one 3d electron able to move from one to another of the various iron atoms on the octahedral sites. For convenience, we shall term such electrons the "free electrons". With the crystal magnetized in a particular direction, these free 3d electrons will always so arrange themselves as to minimize the free energy of the crystal. The free energy, therefore, is going to depend upon the direction of magnetization because of this mechanism, in addition to its usual dependence upon the crystalline and shape anisotropy. It is clear that there will be an additional torque acting on the magnetization.

In what follows, we shall assume that the free electrons cannot instantly assume the equilibrium arrangements specified by the motion of the

[1] Galt, Yager and Merritt, Phys. Rev. **93,** No. 5, p. 1119, 1954.
[2] J. K. Galt, B.S.T.J., **33,** No. 5, p. 1023, 1954.
[3] H. P. J. Wijn and H. van der Heide. Rev. Mod. Phys. **80,** p. 744, 1950.

magnetization but tend toward equilibrium with a relaxation time τ. We shall show that this assumption results in energy being extracted from the motion of the magnetization and appearing as heat in the crystal.

The above mechanism has been analyzed by Galt in reference 2 in connection with the velocity of domain wall motion in nickel-iron ferrite. A similar question has been analyzed by L. Néel[4] in discussing the motion of domain walls in iron. In Néel's case the relaxation is associated with the migration of carbon atoms between various sites in the crystal. The theory used here is essentially similar to that applied by Néel. We shall try, however, to emphasize more than does Néel the connections with thermodynamics, and shall apply the theory not only to domain wall motion but also to ferromagnetic resonance. The theory is not in accord with that proposed by Galt and the points of disagreement will be mentioned below.

1. *Kinetic Model*

We shall consider the assembly of N free electrons per unit volume to constitute a thermodynamic system in heat contact with the crystal lattice. We suppose that there are m possible sites per unit volume of crystal for the free electrons to occupy; m is, of course, just the number of iron atoms per unit volume lying in octahedral sites. We shall let $\varepsilon_i(\vec{M})$ be the energy levels available to the electrons on the octahedral sites where $\vec{M}$ is the magnetization vector. We shall suppose that there are d_i levels per unit volume of energy ε_i and have therefore $\sum d_i = m$. We shall let N_i be the average number of electrons per unit volume in the level ε_i. If the magnetization is held steady in some direction, N_i will approach an equilibrium value that will be denoted by $N_{i\infty}(\vec{M})$. We shall assume a universal relaxation time τ such that N_i approaches equilibrium according to the equation

$$\frac{dN_i}{dt} = \frac{N_{i\infty} - N_i}{\tau} \tag{1-1}$$

and this equation is to hold whether or not $N_{i\infty}$ is a function of time. We observe that $\sum N_i = N$ and that therefore $\sum dN_i/dt = 0$ as must be the case.

$N_{i\infty}$ is obtained in the usual way by minimizing the free energy at constant temperature, and is given by

$$N_{i\infty} = \frac{d_i}{e^{\varepsilon_i - \varepsilon_f/kT}} \tag{1-2}$$

[4] L. Néel, Théorie du Traînage Magnétique de Diffusion, Journal de Physique et Radium, **13,** p. 249, 1952.

In what follows, we shall assume that the percentage of substituted iron is small enough so that $d_i \gg N$ for each level. We have for the total number of electrons per unit volume

$$N = \sum \frac{d_i}{e^{\varepsilon_i - \varepsilon_f/kT} + 1} \tag{1-3}$$

It is clear that $e^{\varepsilon_i - \varepsilon_f/kT}$ must be much greater than unity for all i. Consequently, we may write

$$N_{i\infty} = N \frac{d_i e^{-\varepsilon_i/kT}}{\sum d_i e^{-\varepsilon_i/kT}} \tag{1-4}$$

We now define the internal energy, the entropy, and the free energy per unit volume as

$$U = \sum_i N_i \varepsilon_i \tag{1-5}$$

$$S = k[N \ln N - \sum_i N_i \ln N_i] \tag{1-6}$$

$$F = U - TS \tag{1-7}$$

The equilibrium value of these quantities are obviously given by

$$U_\infty = \sum_i N_{i\infty} \varepsilon_i \tag{1-8}$$

$$S_\infty = k[N \ln N - \sum_i N_{i\infty} \ln N_{i\infty}] \tag{1-9}$$

$$F_\infty = U_\infty - TS_\infty \tag{1-10}$$

If we take a time derivative of equation (1-5) we obtain

$$\frac{dU}{dt} = \sum_i N_i \frac{d\varepsilon_i}{dt} + \sum_i \varepsilon_i \frac{dN_i}{dt} \tag{1-11}$$

and in accordance with the first law, identify the rate of doing work on the system as

$$\frac{dW}{dt} = \sum_i N_i \frac{d\varepsilon_i}{dt} \tag{1-12}$$

while the rate at which heat flows into the system is

$$\frac{dQ}{dt} = \sum_i \varepsilon_i \frac{dN_i}{dt} \tag{1-13}$$

2. *Domain Wall Motion*

We shall first apply the kinetic model to a discussion of the velocity of a 180° domain wall moving perpendicularly to the (110) plane. In this case the magnetization vector remains essentially in the (110) plane and its direction can be specified by the angle θ as in Fig. 1. We shall take the y-direction into the plane of the paper. Let us suppose that the wall is moving in the positive y direction, and that the magnetization turns through 180° from θ_1 to θ_2 as the wall sweeps by a given point.

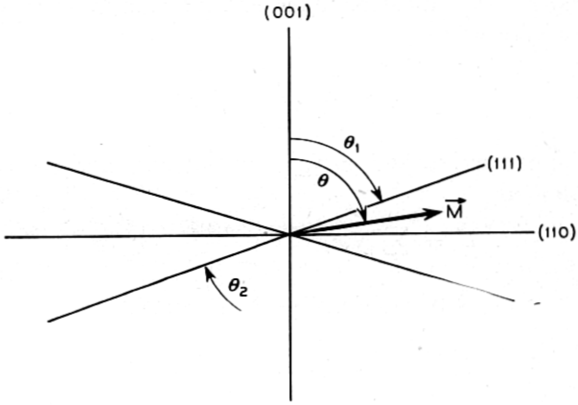

Fig. 1 — Angles used in describing motion of domain wall in (110) plane.

If we follow the procedure outlined by Kittel,[5] it is easy to show that the shape of a domain wall in the (110) plane for a crystal whose easy direction of magnetization is (111), is given by

$$A\left(\frac{d\theta}{dy}\right)^2 = g(\theta) - g(\theta_1) \tag{2-1}$$

where

$$g(\theta) = K[\tfrac{1}{4}\sin^4\theta + \sin^2\theta\cos^2\theta] \tag{2-2}$$

K is the first order anisotropy constant (a negative number) and A is the usual exchange constant. We have $\sin\theta_1 = \sqrt{2/3}$, $\cos\theta_1 = \sqrt{1/3}$ so that $g(\theta_1) = \frac{1}{3}K$ and $g(\theta) - g(\theta_1) = (-K/12)(2 - 3\sin^2\theta)^2$. Equation (2-1) determines the wall except for its position along y. We have ignored in (2-2) a term depending on the magnetostriction which removes an ambiguity in the shape of the wall at the easy direction intermediate between θ_1 and θ_2. This term is unimportant in what follows.

[5] C. Kittel, Physical Theory of Ferromagnetic Domains, Rev. Mod. Physics **21,** p. 556, Oct., 1949.

We shall now assume that the wall moves along y with a constant velocity v and maintains the static shape given by (2-1). At each point in the wall we can then write,

$$\frac{d\theta}{dt} = -v\frac{d\theta}{dy} \tag{2-3}$$

The domain wall is, of course, moving as the magnetization turns to line up with an applied external field H_o. The rate at which this field does work on the wall per unit area is given by $2H_oMv$. This quantity must be set equal to the net rate at which work is done on our thermodynamic system, or, what is the same thing, the net rate at which heat flows into the crystal lattice. We may write therefore,

$$2H_oMv = \int_{-\infty}^{\infty} \frac{dW}{dt}\,dy \tag{2-4}$$

In order to compute dW/dt from (1-12) we suppose that the domain wall is moving slowly and that $N_{i\infty}$ is therefore a slowly changing function of time. Consequently we obtain from (1-1) an approximate expression for N_i

$$N_i = N_{i\infty} - \tau\frac{dN_{i\infty}}{dt} + \tau^2\frac{d^2N_{i\infty}}{dt^2} + ---. \tag{2-5}$$

We obtain immediately from (1-12) to first order in τ,

$$\frac{dW}{dt} = \sum_i N_{i\infty}\frac{d\varepsilon_i}{dt} - \tau\sum_i \frac{dN_{i\infty}}{dt}\frac{d\varepsilon_i}{dt} \tag{2-6}$$

The first term in this expression can be written as dF_∞/dt, and contributes to the integral over y a term

$$-v\int_{\theta_1}^{\theta_2} \frac{dF_\infty}{d\theta}\,d\theta$$

We assume that F_∞ has cubic symmetry, and there is therefore no net contribution from this term.

The second term may be treated as follows. Proceeding from (1-4) we find by differentiation

$$\frac{dN_{i\infty}}{dt} = -\frac{1}{kT}N_{i\infty}\frac{d\varepsilon_i}{dt} + \frac{1}{kT}N_{i\infty}\frac{1}{N}\sum_i N_{i\infty}\frac{d\varepsilon_i}{dt} \tag{2-7}$$

Then we have

$$\begin{aligned}\sum \frac{dN_{i\infty}}{dt}\frac{d\varepsilon_i}{dt} &= -\frac{1}{kT}\sum_i N_{i\infty}\left(\frac{d\varepsilon_i}{dt}\right)^2 + \frac{1}{NkT}\left(\sum_i N_{i\infty}\frac{d\varepsilon_i}{dt}\right)^2 \\ &= -\frac{N}{kT}\left[\sum_i \frac{N_{i\infty}}{N}\left(\frac{d\varepsilon_i}{dt}\right)^2 - \left(\sum_i \frac{N_{i\infty}}{N}\frac{d\varepsilon_i}{dt}\right)^2\right] \qquad (2\text{-}8) \\ &= -\frac{N}{kT}\sum_i \frac{N_{i\infty}}{N}\left[\frac{d\varepsilon_i}{dt} - \sum_i \frac{N_{i\infty}}{N}\frac{d\varepsilon_i}{dt}\right]^2\end{aligned}$$

$$\sum_i \frac{dN_{i\infty}}{dt}\frac{d\varepsilon_i}{dt} = -\frac{N}{kT}\left\langle\left[\frac{d\varepsilon_i}{d\theta} - \left\langle\frac{d\varepsilon_i}{d\theta}\right\rangle\right]^2\right\rangle\left(\frac{d\theta}{dt}\right)^2 \qquad (2\text{-}9)$$

where the angular brackets indicate a mean value over the equilibrium system. Upon substitution of (2-9) in (2-6) we find,

$$\frac{dW}{dt} = \frac{dF_\infty}{dt} + \tau\frac{N}{kT}\left\langle\left[\frac{d\varepsilon_i}{d\theta} - \left\langle\frac{d\varepsilon_i}{d\theta}\right\rangle\right]^2\right\rangle\left(\frac{d\theta}{dt}\right)^2 \qquad (2\text{-}10)$$

By using the first law, we can write for the heat flow into the system[6]

$$\frac{dQ}{dt} = \frac{dU}{dt} - \frac{dF_\infty}{dt} - \tau\frac{N}{kT}\left\langle\left[\frac{d\varepsilon_i}{d\theta} - \left\langle\frac{d\varepsilon_i}{d\theta}\right\rangle\right]^2\right\rangle\left(\frac{d\theta}{dt}\right)^2 \qquad (2\text{-}11)$$

It is clear that, in any process that takes the system between two identical thermodynamic states, the terms dU/dt and dF_∞/dt will integrate to zero. We must therefore regard the positive definite expression

$$\tau\frac{N}{kT}\left\langle\left[\frac{d\varepsilon_i}{d\theta} - \left\langle\frac{d\varepsilon_i}{d\theta}\right\rangle\right]^2\right\rangle$$

as an irreversible heat loss from the system, and it is just this heat loss that makes dW/dt greater than dF_∞/dt.

Before proceeding we wish here to make certain comparisons with the theory used by Galt in discussing domain wall motion in Reference 2.

[6] It is possible to formulate this problem in terms of a temperature T_κ such that

$$N_i = Ne^{-\varepsilon_i/kT_\kappa}/\sum e^{-\varepsilon_i/kT_\kappa}$$

One then obtains

$$\frac{dQ}{dt} = \frac{C_V}{\tau}(T_\kappa - T)$$

where

$$C_V = \frac{N}{kT^2}\left[\langle\varepsilon_i^2\rangle - \langle\varepsilon_i\rangle^2\right].$$

From (15) to (24) in Galt's paper one finds, to first order in τ,

$$\frac{dW}{dt} = \tau \frac{d^2 g_{i\infty}}{d\theta^2} \left(\frac{d\theta}{dt}\right)^2$$

which is to be compared with (2-10). This formulation seems somehow to neglect the equilibrium free energy of the free electrons, but since this term is indistinguishable from the ordinary crystalline anisotropy energy the omission is unimportant. The expression above, however must be interpreted as the irreversible heat loss and should therefore be a positive definite quantity as in (2-10). The quantity $(d^2 g_{i\infty}/d\theta^2)$, however, is not positive definite, and this would seem to be a serious objection to Galt's formulation of the theory. In Reference 2, $g_{i\infty}$ is set proportional to $g(\theta)$ defined in (2-2) so that $(d^2 g_{i\infty}/d\theta^2)$ in that case specifically has regions of positive and negative value, which should not be the case.

Using equation (2-10) we find

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{dW}{dt}\, dy &= \tau \frac{N}{kT} \int_{-\infty}^{\infty} \left\langle \left[\frac{d\varepsilon_i}{d\theta} - \left\langle \frac{d\varepsilon_i}{d\theta} \right\rangle\right]^2 \right\rangle \left(\frac{d\theta}{dt}\right)^2 dy \\ &= v^2 \tau \frac{N}{kT} \int_{\theta_1}^{\theta_2} \left\langle \left[\frac{d\varepsilon_i}{d\theta} - \left\langle \frac{d\varepsilon_i}{d\theta} \right\rangle\right]^2 \right\rangle \left(\frac{d\theta}{dy}\right) d\theta \end{aligned} \tag{2-12}$$

where $(d\theta/dy)$ is taken as positive by convention. This integral is the net rate at which work is done on the wall and is positive definite. The corresponding expression developed by Galt (30) is

$$\int_{-\infty}^{\infty} \frac{dW}{dt}\, dy = v^2 \tau \int_{\theta_1}^{\theta_2} \frac{d^2 g_{1\infty}}{d\theta^2} \left(\frac{d\theta}{dy}\right) d\theta$$

We can only regard a positive value for this integral as accidental and to depend upon the shape of the domain wall.

Combining (2-4), (2-12) and (2-1), we find for the velocity of domain wall motion

$$v = \frac{2H_0 M}{\tau \left(\frac{N}{kT}\right) \sqrt{\frac{-K}{12A}} \int_{\theta_1}^{\theta_2} \left\langle \left[\frac{d\varepsilon_i}{d\theta} - \left\langle \frac{d\varepsilon_i}{d\theta} \right\rangle\right]^2 \right\rangle \left| 2 - 3 \sin^2 \theta \right| d\theta} \tag{2-13}$$

This expression exhibits the characteristic dependence of v on $1/\tau$ discussed by Galt in reference 2 and shown there to account for the very marked dependence of v on temperature found in his experiments. It also contains the obvious result that v depends inversely on the number of free electrons.

It would be very desirable now to proceed to calculate the coefficient of $1/\tau$. This can only be done by introducing some physical ideas to arrive at an explicit dependence of ε_i on θ. This may be done according to two simple schemes that we shall consider later. First, however, we shall discuss the phenomenon of ferromagnetic resonance. One point that will soon become clear is that the present theory allows no direct comparison between domain wall motion experiments and ferromagnetic resonance experiments unless a particular model is adopted.

3. *Ferromagnetic Resonance*

We shall now apply the kinetic model to a calculation of the line width for ferromagnetic resonance and the corresponding displacement of the field for resonance. We suppose that the applied field and equilibrium direction of magnetization lie in the (110) plane at angles η and θ_0 as shown in Fig. 2. The direction z lies along $\vec{M}_0$. The direction x lies in the 110 plane perpendicular to z, while the y direction is perpendicular to the (110) plane in such a sense that x, y and z form a right-handed system.

We begin by writing down an expression for the total free energy of the system.

$$F_T = -MH \cos(\theta - \eta) \cos \Phi + G(\theta, \Phi) + \sum_i N_i \varepsilon_i - kT(N \ln N - \sum_i N_i \ln N_i) \quad (3\text{-}1)$$

where θ and Φ are the instantaneous directions of the magnetization.

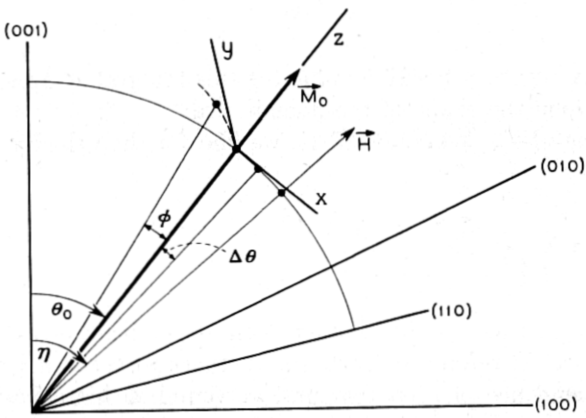

Fig. 2 — Coordinate systems used in discussing ferromagnetic resonance in the (110) plane.

It will be observed that θ and Φ are not the usual polar coördinates but are angles especially suitable to describing events occurring in the (110) plane. The first term in this expression is the energy of the magnetization in the external field. The second term includes the crystalline anisotropy energy, the shape anisotropy of the specimen and any energy due to magnetostriction. The third term is the free energy of the free electrons. We now wish to obtain the x and y components of the torque acting on the magnetization. These torques will be given by

$$T_y = -\frac{\partial F}{\partial \theta} \qquad T_x = \frac{\partial F}{\partial \Phi} \tag{3-2}$$

where the derivatives are taken with N_i held constant. This procedure is consistent with (1-12) for dW/dt.

We obtain for T_y, setting $\Phi = 0$

$$T_y = -MH \sin(\theta - \eta) - \frac{\partial G}{\partial \theta} - \sum_i N_i \frac{\partial \varepsilon_i}{\partial \theta} \tag{3-3}$$

Returning to (1-1), we can write generally for N_i, ignoring a transient term,

$$N_i = N_{i\infty} - e^{-t/\tau} \int^t e^{t/\tau} \frac{dN_{i\infty}}{dt} \, dt \tag{3-4}$$

If we insert (3-4) into (3-3) we obtain,

$$\begin{aligned} T_y = {} & -MH \sin(\theta - \eta) - \frac{\partial G}{\partial \theta} - \sum_i N_{i\infty} \frac{\partial \varepsilon_i}{\partial \theta} \\ & + \sum_i \frac{\partial \varepsilon_i}{\partial \theta} e^{-t/\tau} \int^t e^{t/\tau} \left(\frac{\partial N_{i\infty}}{\partial \theta} \right) \frac{d\theta}{dt} \, dt \end{aligned} \tag{3-5}$$

Let us expand this expression around the equilibrium angle θ_0 to obtain

$$\begin{aligned} T_y = {} & -MH \sin(\theta_0 - \eta) - \left(\frac{\partial G}{\partial \theta} \right)_{\theta_0} - \sum_i N_{i\infty} \left(\frac{\partial \varepsilon_i}{\partial \theta} \right)_{\theta_0} \\ & - \left(MH \cos(\theta_0 - \eta) + \left(\frac{\partial^2 G}{\partial \theta^2} \right)_{\theta_0} + \sum_i \left[\frac{\partial}{\partial \theta} \left(N_{i\infty} \frac{\partial \varepsilon_i}{\partial \theta} \right) \right]_{\theta_0} \right) \Delta\theta \\ & + \sum_i \left(\frac{\partial \varepsilon_i}{\partial \theta} \right)_{\theta_0} \left(\frac{\partial N_{i\infty}}{\partial \theta} \right)_{\theta_0} e^{-t/\tau} \int^t e^{t/\tau} \left(\frac{d\theta}{dt} \right) dt \end{aligned} \tag{3-6}$$

which can be expressed in terms of the equilibrium free energy as

$$
\begin{aligned}
T_y = {} & -MH \sin (\theta_0 - \eta) - \left(\frac{\partial G}{\partial \theta}\right)_{\theta_0} - \left(\frac{\partial F_\infty}{\partial \theta}\right)_{\theta_0} \\
& - \left(MH \cos (\theta_0 - \eta) + \left(\frac{\partial^2 G}{\partial \theta^2}\right)_{\theta_0} + \left(\frac{\partial^2 F_\infty}{\partial \theta^2}\right)_{\theta_0}\right) \Delta\theta \qquad (3\text{-}7) \\
& + \sum_i \left(\frac{\partial \varepsilon_i}{\partial \theta}\right)_{\theta_0} \left(\frac{\partial N_{i\infty}}{\partial \theta_0}\right) e^{-t/\tau} \int^t e^{t/\tau} \left(\frac{d\theta}{dt}\right) dt
\end{aligned}
$$

We choose θ_0 so that $MH \sin(\theta_0 - \eta) + (\partial G/\partial \theta)_{\theta_0} + (\partial F_\infty/\partial \theta)_{\theta_0} = 0$ and suppose that H is large enough so that θ_0 is substantially equal to η. We shall now drop subscripts and suppose that the field and equilibrium magnetization are in the direction θ. We further recognize that $\Delta\theta = M_x/M$. The expression for T_y then becomes,

$$
\begin{aligned}
T_y = {} & -\left(MH + \frac{\partial^2 G}{\partial \theta^2} + \frac{\partial^2 F_\infty}{\partial \theta^2}\right) \frac{M_x}{M} \\
& + \sum_i \left(\frac{\partial \varepsilon_i}{\partial \theta}\right) \left(\frac{\partial N_{i\infty}}{\partial \theta}\right) e^{-t/\tau} \int^t e^{t/\tau} \frac{1}{M} \frac{dM_x}{dt} dt
\end{aligned} \qquad (3\text{-}8)
$$

An exactly similar expression is obtained for T_x

$$
\begin{aligned}
T_x = {} & \left(MH + \frac{\partial^2 G}{\partial \Phi^2} + \frac{\partial^2 F_\infty}{\partial \Phi^2}\right) \frac{M_y}{M} \\
& - \sum_i \left(\frac{\partial \varepsilon_i}{\partial \Phi}\right) \left(\frac{\partial N_{i\infty}}{\partial \Phi}\right) e^{-t/\tau} \int^t e^{t/\tau} \frac{1}{M} \frac{dM_y}{dt} dt
\end{aligned} \qquad (3\text{-}9)
$$

The torques T_x and T_y may now be substituted into the equation of motion of the magnetization

$$
\frac{d\vec{M}}{dt} = -\gamma \vec{T} - \gamma \vec{M} \times \vec{h} \qquad (3\text{-}10)
$$

where $\gamma = ge/2mc$ and $\vec{h}$ is the driving field. It will be observed that M_x and M_y will vary sinusoidally with frequency ω of the driving field for small amplitude motion. In that case, we can write

$$
e^{-t/\tau} \int^t e^{t/\tau} \frac{dM_x}{dt} dt = \frac{\tau}{1 + (\omega\tau)^2} \frac{dM_x}{dt} + \frac{(\omega\tau)^2}{1 + (\omega\tau)^2} M_x \qquad (3\text{-}11)
$$

and similarly for M_y. It now becomes an elementary matter to compute the resonance line width and the field shift brought about by the relaxation process.

The total line width is given by,

$$\Delta H = -\frac{1}{M}\sum_i \left(\left(\frac{\partial \varepsilon_i}{\partial \theta}\right)\left(\frac{\partial N_{i\infty}}{\partial \theta}\right) + \left(\frac{\partial \varepsilon_i}{\partial \Phi}\right)\left(\frac{\partial N_{i\infty}}{\partial \Phi}\right)\right)\frac{\omega\tau}{1+(\omega\tau)^2} \quad (3\text{-}12)$$

while the field required for resonance is increased by,

$$\delta H = -\frac{1}{2M}\left(\frac{\partial^2 F_\infty}{\partial \theta^2} + \frac{\partial^2 F_\infty}{\partial \Phi^2}\right) + \frac{1}{2M}\sum_i \left(\left(\frac{\partial \varepsilon_i}{\partial \theta}\right)\left(\frac{\partial N_{i\infty}}{\partial \theta}\right) + \left(\frac{\partial \varepsilon_i}{\partial \Phi}\right)\left(\frac{\partial N_{i\infty}}{\partial \Phi}\right)\right)\frac{(\omega\tau)^2}{1+(\omega\tau)^2} \quad (3\text{-}13)$$

The expression

$$\sum_i \left(\left(\frac{\partial \varepsilon_i}{\partial \theta}\right)\left(\frac{\partial N_{i\infty}}{\partial \theta}\right) + \left(\frac{\partial \varepsilon_i}{\partial \Phi}\right)\left(\frac{\partial N_{i\infty}}{\partial \Phi}\right)\right)$$

is very similar to an expression that arose in discussing domain wall motion. We can treat this term exactly as was done in arriving at (2-9). We shall find that

$$\sum_i \left(\left(\frac{\partial \varepsilon_i}{\partial \theta}\right)\left(\frac{\partial N_{i\infty}}{\partial \theta}\right) + \left(\frac{\partial \varepsilon_i}{\partial \Phi}\right)\left(\frac{\partial N_{i\infty}}{\partial \Phi}\right)\right) = -\frac{N}{kT}\left\langle \left[\frac{d\varepsilon_i}{d\theta} - \left\langle \frac{d\varepsilon_i}{d\theta}\right\rangle\right]^2 + \left[\frac{d\varepsilon_i}{d\Phi} - \left\langle \frac{d\varepsilon_i}{d\Phi}\right\rangle\right]^2\right\rangle \quad (3\text{-}14)$$

Let us first notice that ΔH and δH depend upon the quantity

$$\left\langle \left[\frac{d\varepsilon_i}{\partial \Phi} - \left\langle \frac{d\varepsilon_i}{d\Phi}\right\rangle\right]^2\right\rangle$$

which does not appear at all in (2-13) for domain wall velocity. Without making more explicit assumptions about ε_i, therefore, we cannot make any connection between the two experiments. The essential point here of course is that $\vec{M}$ is not constrained in the resonance experiment to move in the (110) plane as it is in the domain wall experiment.

Let us now make the reasonable assumption that the quantity

$$\left\langle \left[\frac{d\varepsilon_i}{d\theta} - \left\langle \frac{d\varepsilon_i}{d\theta}\right\rangle\right]^2 + \left[\frac{d\varepsilon_i}{d\Phi} - \left\langle \frac{d\varepsilon_i}{d\Phi}\right\rangle\right]^2\right\rangle$$

varies in the (110) plane with cubic symmetry. We write therefore that

$$\left\langle \left[\frac{d\varepsilon_i}{d\theta} - \left\langle \frac{d\varepsilon_i}{d\theta}\right\rangle\right]^2 + \left[\frac{d\varepsilon_i}{d\Phi} - \left\langle \frac{d\varepsilon_i}{d\Phi}\right\rangle\right]^2\right\rangle = p + q[\tfrac{1}{4}\sin^4\theta + \sin^2\theta\cos^2\theta] \quad (3\text{-}15)$$

When we come to discuss specific models, we shall give particular expressions for p and q in terms of one unknown constant. We see that $p > 0$ and $p + q/3 > 0$ to maintain the right hand side of (3-15) positive. We observe that it is not possible to claim cubic symmetry for the terms

$$\left\langle \left[\frac{d\varepsilon_i}{d\theta} - \left\langle \frac{d\varepsilon_i}{d\theta} \right\rangle \right]^2 \right\rangle \quad \text{and} \quad \left\langle \left[\frac{d\varepsilon_i}{d\Phi} - \left\langle \frac{d\varepsilon_i}{d\Phi} \right\rangle \right]^2 \right\rangle$$

separately.

Using (3-15), the expressions for line width and line shift become,

$$\Delta H = \frac{1}{M} \frac{N}{kT} [p + q(\tfrac{1}{4} \sin^4 \theta + \sin^2 \theta \cos^2 \theta)] \frac{\omega\tau}{1 + (\omega\tau)^2} \tag{3-16}$$

$$\begin{aligned} \delta H = & -\frac{1}{2M} \left(\frac{\partial^2 F_\infty}{\partial \theta^2} + \frac{\partial^2 F_\infty}{\partial \Phi^2} \right) \\ & - \frac{1}{2M} \frac{N}{kT} [p + q(\tfrac{1}{4} \sin^4 \theta + \sin^2 \theta \cos^2 \theta)] \frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \end{aligned} \tag{3-17}$$

Equation (3-16) for the line width shows a dependence on $(\omega\tau)$ that is characteristic of relaxation processes. If a measurement is made of ΔH versus temperature at a given frequency and in a given crystallographic direction, a peak will be observed in the line width at very nearly the temperature where $\tau = 1/\omega$. The peak will not fall accurately at $\tau = 1/\omega$ because of the temperature dependence of the coefficient of $\omega\tau/1 + (\omega\tau)^2$ but comes very close to $1/\omega$ because of the exponential dependence of τ on temperature. Observations of this kind have been made by Galt, Yager and Merritt on nickel-iron ferrite and reported by them in Reference 1.

If we may now extend the measurements over a range of frequencies, and observe how the line shape varies as a function of frequency at constant temperature, ΔH will have its maximum accurately at $\omega = 1/\tau$ In principle, then, τ can be determined as a function of temperature.

If τ is known at a particular temperature, a measurement of ΔH versus θ at constant temperature and frequency will serve to determine $1/M\ N/kT\ p$ and $1/M\ N/kT\ q$ at the temperature in question.

Let us now consider the field shift δH given by (3-17). In the first place, this shift is not the ordinary displacement that accompanies the introduction of loss into any resonant system since such a field shift would be proportional to $(\Delta H)^2$. To the contrary, the field shift is of the same order of magnitude as the line width and we must consider seriously what effect this will have on the measurement of g-value. We have assumed previously that F_∞ has cubic symmetry, and in that case, the term in (3-17) depending on F_∞ will be indistinguishable from the usual crystal anisotropy and need not be considered further. The second term we must consider more closely.

The following expression is an identity.

$$p + q[\tfrac{1}{4} \sin^4 \theta + \sin^2 \theta \cos^2 \theta] = \left(p + \frac{q}{5}\right) - \frac{q}{10}\,(2 - \tfrac{5}{2} \sin^2 \theta - \tfrac{15}{8} \sin^2 2\theta) \tag{3-18}$$

Now, the second term on the right has the angular dependence in terms of which one usually expresses the first order anisotropy constant K_1. We may expect then that the measured anisotropy constant will have the form

$$K_1 = K_0 - \frac{N}{kT}\left(\frac{q}{20}\right)\frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \tag{3-19}$$

where K_0 is the static first order anisotropy constant.

We may look, therefore, for a dependence of K_1 on frequency and temperature like that given in (3-19) and make comparisons with the behavior of the line width.

If the total anisotropy field has been thus accounted for, we will be left with a line shift

$$-\frac{1}{2M}\frac{N}{kT}\left(p + \frac{q}{5}\right)\frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \tag{3-20}$$

which will appear as a change in the g-value. If a measurement of the field H for resonance is made for a given ω, T and θ, and if this field is corrected for the anisotropy field to give an effective field H_e, we will have

$$H_e = \frac{\omega}{\gamma} - \frac{1}{2M}\frac{N}{kT}\left(p + \frac{q}{5}\right)\frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \tag{3-21}$$

We shall measure an effective γ, however, given by

$$H_e = \frac{\omega}{\gamma_e} \tag{3-22}$$

Equating (3-21) and (3-22), we obtain

$$\frac{\gamma_e - \gamma}{\gamma} = \frac{1}{H_e}\frac{1}{2M}\frac{N}{kT}\left(p + \frac{q}{5}\right)\frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \tag{3-23}$$

or, if $\gamma_e = g_e e/2mc$ and $\Delta g = g_e - g$,

$$\frac{\Delta g}{g} = \frac{1}{2H_e M}\frac{N}{kT}\left(p + \frac{q}{5}\right)\frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \tag{3-24}$$

We, therefore, see that the g-factor will also have a dispersion in frequency and temperature, and the measurement of g-factor may indeed be the most sensitive way of observing the relaxation phenomenon.

4. *First Model*

There are two simple models we may adopt to obtain an explicit dependence of ε_i on θ and Φ. In the first case, let us suppose that there are four distinguishable energy levels in the crystal ε_1, ε_2, ε_3 and ε_4 and that each is associated with one of the body diagonals of the cubic crystal according to the relation

$$\varepsilon_i = w \cos^2 \theta_i \tag{4-1}$$

where θ_i is the angle between the direction of magnetization and the i^{th} body diagonal and w is some constant depending on temperature.

This model is particularly tempting because there are four non-equivalent octahedral sites in the ferrite crystal, each associated with a given body diagonal. Expressed in terms of the direction cosines of the magnetization $(\alpha_1\alpha_2\alpha_3)$, we find

$$\begin{aligned}
\varepsilon_1 &= \frac{w}{3}[1 + 2(\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1)] \\
\varepsilon_2 &= \frac{w}{3}[1 + 2(\alpha_1\alpha_2 - \alpha_2\alpha_3 - \alpha_3\alpha_1)] \\
\varepsilon_3 &= \frac{w}{3}[1 + 2(-\alpha_1\alpha_2 + \alpha_2\alpha_3 - \alpha_3\alpha_1)] \\
\varepsilon_4 &= \frac{w}{3}[1 + 2(-\alpha_1\alpha_2 - \alpha_2\alpha_3 + \alpha_3\alpha_1)]
\end{aligned} \tag{4-2}$$

while expressed in terms of the angles θ and Φ, the energies become,

$$\begin{aligned}
\varepsilon_1 &= \frac{w}{3}\left\{1 + [\sin^2\theta\cos^2\Phi - \sin^2\Phi + 2\sqrt{2}\sin\theta\cos\theta\cos^2\Phi]\right\} \\
\varepsilon_2 &= \frac{w}{3}\left\{1 + [\sin^2\theta\cos^2\Phi - \sin^2\Phi - 2\sqrt{2}\sin\theta\cos\theta\cos^2\Phi]\right\} \\
\varepsilon_3 &= \frac{w}{3}\left\{1 + [-\sin^2\theta\cos^2\Phi + \sin^2\Phi + 2\sqrt{2}\cos\theta\sin\Phi\cos\Phi]\right\} \\
\varepsilon_4 &= \frac{w}{3}\left\{1 + [-\sin^2\theta\cos^2\Phi + \sin^2\Phi - 2\sqrt{2}\cos\theta\sin\Phi\cos\Phi]\right\}
\end{aligned} \tag{4-3}$$

Let us consider now how we shall calculate the various mean values that will be required. If we consider a set of values g_i we have

$$\langle g_i \rangle = \frac{\sum_i g_i\, d_i e^{-\epsilon_i/kT}}{\sum_i d_i e^{-\epsilon_i/kT}} = \frac{\sum_i g_i\, d_i e^{-(\epsilon_i - \langle \epsilon_i \rangle)/kT}}{\sum_i d_i e^{-(\epsilon_i - \langle \epsilon_i \rangle)/kT}} \tag{4-4}$$

We shall suppose that T is large enough so that $(\varepsilon_i - \langle \varepsilon_i \rangle) \ll kT$, and assume that d_i is the same for each level. In that case, approximately

$$\langle g_i \rangle = \frac{1}{k} \sum_{n=1}^{k} g_n \tag{4-5}$$

if there are k levels. We shall show shortly that this assumption is consistent with the experimental data.

Using (4-5) we can calculate for our first model the following mean values,

$$\left\langle \frac{d\varepsilon_i}{d\theta} \right\rangle = \left(\frac{w}{3}\right) \tag{4-6}$$

$$\left\langle \left[\frac{d\varepsilon_i}{d\theta} - \left\langle \frac{d\varepsilon_i}{d\theta} \right\rangle\right]^2 \right\rangle = \left(\frac{w}{3}\right)^2 [1 + 3\cos^2 2\theta] \tag{4-7}$$

$$\left\langle \left[\frac{d\varepsilon_i}{d\Phi} - \left\langle \frac{d\varepsilon_i}{d\Phi} \right\rangle\right]^2 \right\rangle = \left(\frac{w}{3}\right)^2 4\cos^2 \theta \tag{4-8}$$

$$\left\langle \left[\frac{d\varepsilon_i}{d\theta} - \left\langle \frac{d\varepsilon_i}{d\theta} \right\rangle\right]^2 + \left[\frac{d\varepsilon_i}{d\Phi} - \left\langle \frac{d\varepsilon_i}{d\Phi} \right\rangle\right]^2 \right\rangle = \left(\frac{w}{3}\right)^2 [1 + 3\cos^2 2\theta + 4\cos^2 \theta] \tag{4-9}$$

$$= \tfrac{8}{9} w^2 [1 - 2(\tfrac{1}{4}\sin^4 \theta + \sin^2 \theta \cos^2 \theta)] \tag{4-10}$$

Referring back to (2-13), the velocity of domain wall motion becomes

$$v = \frac{2H_0 M}{\tau \left(\frac{N}{kT}\right) \sqrt{\frac{-K}{12A}} \int_{\theta_1}^{\theta_2} \left(\frac{w}{3}\right)^2 [1 + 3\cos^2 2\theta]\, |\, 2 - 3\sin^2 \theta \,|\, d\theta} = \frac{2H_0 M}{\tau \sqrt{\frac{-K}{12A}} \left(\frac{N}{kT}\right) w^2 (1.02)} \tag{4-11}$$

Comparing with (3-15) we find $p = 8/9\, w^2$ and $q = -16/9\, w^2$. The line width for resonance becomes

$$\Delta H = \frac{1}{M}\frac{8}{9}\left(\frac{N}{kT}\right) w^2 \left[1 - 2\left(\frac{1}{4}\sin^4\theta + \sin^2\theta\cos^2\theta\right)\right]\frac{\omega\tau}{1 + (\omega\tau)^2} \tag{4-12}$$

The anisotropy constant is given by

$$K_1 = K_0 + \frac{4}{45}\left(\frac{N}{kT}\right) w^2 \frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \tag{4-13}$$

and the shift in g-factor by

$$\frac{\Delta g}{g} = \frac{1}{H_e M}\frac{4}{15}\left(\frac{N}{kT}\right) w^2 \frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \tag{4-14}$$

We thus have four different measurements which we may try to fit with the same value of $N/kT\ w^2$. It should be noted that (4-12) predicts a maximum line width in the (100) direction and a minimum in the (111) direction.

5. *Second Model*

For the second model, we shall assume that there are three distinguishable energy levels ε_1 ε_2 and ε_3 each associated with one of the cube edges of the crystal according to the relation

$$\varepsilon_i = w \cos^2\theta_i \tag{5-1}$$

where θ_i is the angle between the direction of magnetization and the ith cube edge. Expressed in terms of the direction cosines of $\vec{M}$ we have

$$\begin{aligned} \varepsilon_1 &= w\,\alpha_1^2 \\ \varepsilon_2 &= w\,\alpha_2^2 \\ \varepsilon_3 &= w\,\alpha_3^2 \end{aligned} \tag{5-2}$$

while expressed in terms of the angles θ and Φ the energies become,

$$\begin{aligned} \varepsilon_1 &= \frac{w}{2}(\sin\theta\cos\Phi - \sin\Phi)^2 \\ \varepsilon_2 &= \frac{w}{2}(\sin\theta\cos\Phi + \sin\Phi)^2 \\ \varepsilon_3 &= w(\cos\theta\cos\Phi)^2 \end{aligned} \tag{5-3}$$

This model we now consider is the same as the one used by Néel in Reference 3 to discuss the relaxation of interstitial atoms in iron. Proceeding as before we find,

$$\left\langle \frac{d\varepsilon_i}{d\Phi} \right\rangle = \left(\frac{w}{3}\right) \tag{5-4}$$

$$\left\langle \left[\frac{d\varepsilon_i}{d\theta} - \left\langle \frac{d\varepsilon_i}{d\theta} \right\rangle\right]^2 \right\rangle = 2w^2 \sin^2\theta \cos^2\theta \tag{5-5}$$

$$\left\langle \left[\frac{d\varepsilon_i}{d\Phi} - \left\langle \frac{d\varepsilon_i}{d\Phi} \right\rangle\right]^2 \right\rangle = \frac{2}{3} w^2 \sin^2\theta \tag{5-6}$$

$$\left\langle \left[\frac{d\varepsilon_i}{d\theta} - \left\langle \frac{d\varepsilon_i}{d\theta} \right\rangle\right]^2 \right\rangle + \left\langle \left[\frac{d\varepsilon_i}{d\Phi} - \left\langle \frac{d\varepsilon_i}{d\Phi} \right\rangle\right]^2 \right\rangle = \frac{8}{3} w^2 \left(\frac{1}{4}\sin^4\theta + \sin^2\theta\cos^2\theta\right) \tag{5-7}$$

In this case we obtain,

$$v = \frac{2H_0M}{\tau\left(\frac{N}{kT}\right)\sqrt{\frac{-K}{12A}}\int_{\theta_1}^{\theta_2} 2w^2\sin^2\theta\cos^2\theta\,|\,2 - 3\sin^2\theta\,|\,d\theta} = \frac{2H_0M}{\tau\sqrt{\frac{-K}{12A}}\left(\frac{N}{kT}\right)w^2\,(.583)} \tag{5-8}$$

and by comparison with (3-15) $p = 0$ and $q = 8/3\, w^2$. The line width for resonance is

$$\Delta H = \frac{1}{M}\frac{8}{3}\left(\frac{N}{kT}\right) w^2 \left(\frac{1}{4}\sin^4\theta + \sin^2\theta\cos^2\theta\right)\frac{\omega\tau}{1 + (\omega\tau)^2} \tag{5-9}$$

The anisotropy constant is

$$K_1 = K_0 - \frac{2}{15}\left(\frac{N}{kT}\right) w^2 \frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \tag{5-10}$$

The effective field at resonance is

$$H_e = \frac{\omega}{\gamma} - \frac{4}{15}\left(\frac{NW^2}{MkT}\right)\frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \tag{5-11}$$

and the g-factor shift is

$$\frac{\Delta g}{g} = \frac{1}{H_eM}\left(\frac{4}{15}\right)\left(\frac{N}{kT}\right) w^2 \frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \tag{5-12}$$

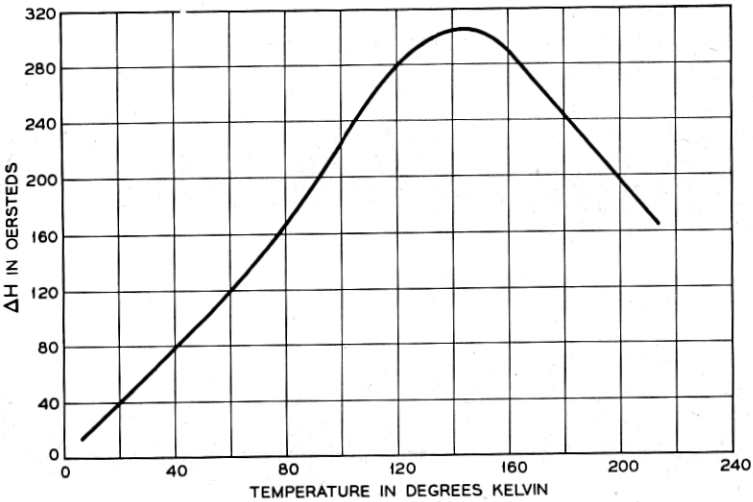

Fig. 3 — Corrected line width in the (111) direction versus temperature for $Ni_{0.75}Fe_{2.25}O_4$ after Yager, Galt and Merritt.

In this case the line width due to relaxation is zero in the (100) direction and a maximum in the (111) direction.

6. *Comparison with Experiment*

We wish now to make some comparisons between the theory developed in the preceding sections and measurements upon nickel-iron ferrite of composition $Ni_{0.75}Fe_{2.25}O_4$ made by Yager, Galt and Merritt.[7] The data we shall consider were made upon a .009″ sphere and are presented in Fig. 4 and Table II of Reference 7. These data include a measurement of line width versus temperature for the (111) direction and (100) direction in the crystal; a measurement of the first order anisotropy constant (as K_1/M) versus temperature; and a measurement of the effective field at resonance, H_e, versus temperature. It is observed from these data that the line width is maximum in the (111) direction and we shall therefore try to fit the data with the second model as discussed in Section 5. This model requires the line width to be zero in the (100) direction. In fitting the data, we shall assume that the line width in the (100) direction represents non-angle-dependent contributions from other mechanisms and shall subtract this width at each temperature from the width in the (111) direction. The resulting curve is shown in

[7] Yager, Galt and Merritt, Phys. Rev., to be published.

Fig. 3. The data for K_1/M and H_e are presented in Table I. The effective fields H_e have been corrected in each case to a common frequency 24,388 mc/sec.

The equations we need are contained in Section 5 and are repeated here for convenience,

$$\Delta H = \frac{8}{9}\left(\frac{Nw^2}{MkT}\right)\frac{\omega\tau}{1 + (\omega\tau)^2} \tag{6-1}$$

$$H_e = \frac{\omega}{\gamma} - \frac{4}{15}\left(\frac{Nw^2}{MkT}\right)\frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \tag{6-2}$$

$$\frac{-K_1}{M} = \left(\frac{-K_0}{M}\right) + \frac{2}{15}\left(\frac{Nw^2}{MkT}\right)\frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \tag{6-3}$$

To proceed, we must make some assumptions about how w depends upon temperature, and shall therefore suppose that (Nw^2/MkT) is a constant independent of temperature. If we assumed instead, for instance, that w was a constant, the curves to be calculated would differ considerably at low temperatures but not enough to affect the conclusions to be drawn.

Proceeding on this assumption, we conclude from Fig. 3 that $\omega\tau = 1$ at $T = 145°$ and calculate that $(Nw^2/MkT) = 690$. We may now use Fig. 3 and (6-1) to calculate $\omega\tau$ at each temperature. With this we may calculate H_e at each temperature, and the resulting curve is shown in Fig. 4. Since we have no independent value for γ, the curve has been fitted to the data at 160°K. It is seen that the predicted changes in H_e are in the correct direction and are of approximately the right magnitude. The experimental points are not fitted in detail, however. The fit above 85°K could be somewhat improved by assuming a different dependence of w on T. The theory as presently developed cannot explain the in-

TABLE I — ANISOTROPY FIELD $-K_1/M$ AND EFFECTIVE FIELD H_e FOR VARIOUS VALUES OF ABSOLUTE TEMPERATURE AFTER YAGER, GALT AND MERRITT.

T°K	$-K_1/M$ (oe)	H_e (oe)
300	152	8195
159	218	8070
135	238	
85	259	7953
4.2	421	8047

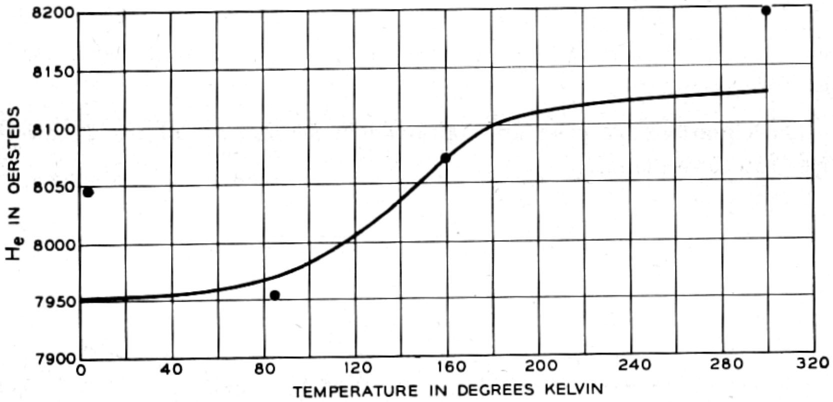

Fig. 4 — Measured and calculated values of effective field H_e as a function of absolute temperature.

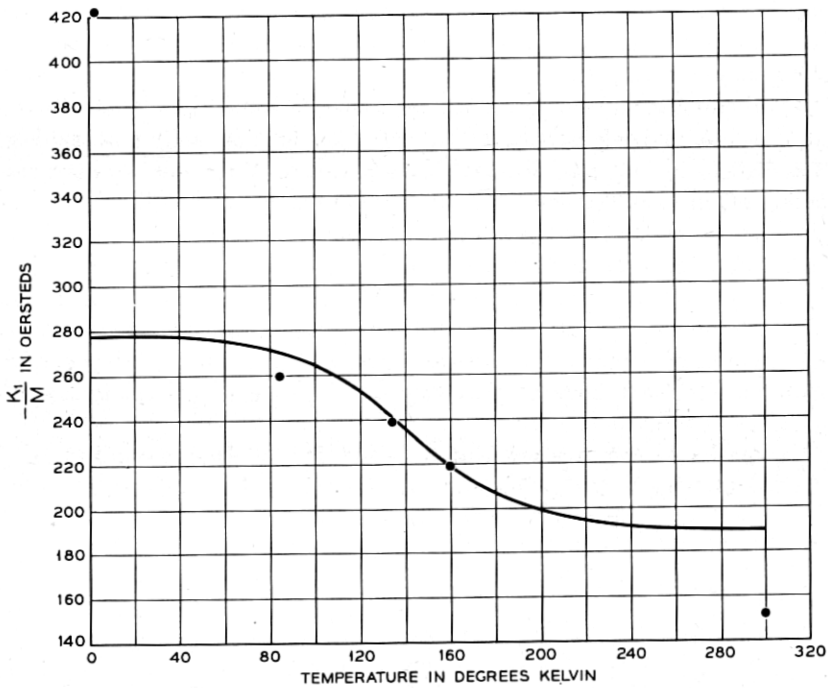

Fig. 5 — Measured and calculated values of anisotropy field $-K_1/M$ as a function of absolute temperature.

crease of H_e at 4.2°K. Yager, Galt, and Merritt believe that the changes observed between 85°K and 4.2°K must be ascribed to some other mechanism.

In arriving at (6-1) to (6-3) we have assumed as in Section 4 that $w/kT \ll 1$. At a temperature of 150°K, kT equal 2.07×10^{-14} ergs. The molecular weight M of nickel ferrite is 234.39 and the density ρ is 5.37 gms/cc. There are then $\rho L/M = 1.38 \times 10^{22}$ molecules per cubic cm, where $L = 6.025 \times 10^{23}$ is Avogadro's number. In nickel ferrite of composition $Ni_{0.75}Fe_{2.25}O_4$ one divalent iron is introduced for each four molecules, and the number of free electrons N is therefore 3.45×10^{21} per cc. The magnetization of this ferrite is about 340 cgs units. We have determined above that $(Nw^2/MkT) = 690$ and can therefore calculate that $w/kT = 0.057$ at $T = 150$°K. We have thus proceeded consistently in assuming $w \ll kT$. If w is constant, our calculation would not however be correct below about 50°K or if w^2/kT is constant, below about 10°K. It would be entirely possible to make accurate calculations for the entire temperature range and, by assuming special variations of w with temperature, perhaps account for some of the rise in H_e at 4.2°K.

We must also recognize that the ratio m/N of levels to number of free electrons in this composition is 5. Since we have assumed the levels

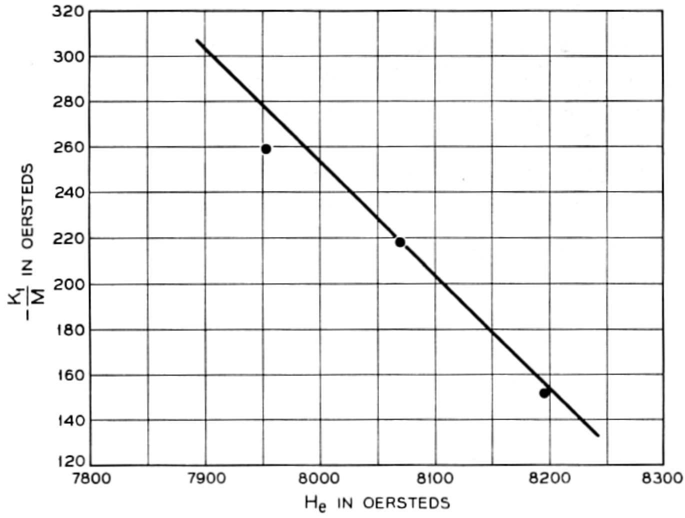

Fig. 6 — The theoretical relation between anisotropy field $-K_1/M$ and effective field H_e compared with experimental points.

divided into three equal groups, $d_i = \frac{5}{3}N$ so that we are hardly justified in considering the electrons to be non-degenerate. We should then really use (1-2) instead of (1-4) in computing our averages. Furthermore, when degeneracy effects are important, the relaxation equation (1-1) should be replaced by a more sophisticated expression. Since the general effect of degeneracy is to reduce the number of electrons taking part in the relaxation process, particularly at low temperatures, a calculation considering degeneracy has a good chance of accounting for the rise in H_e at liquid helium temperatures.

In a ferrite of composition similar to that considered here, Bozorth, Cetlin, Galt, Merritt and Yager[8] report measurements of the static anisotropy which show K_1/M to be nearly constant between 300°K and 196°K. If we make the assumption that the static anisotropy varies very little down to 85°K, we may use (6-3) to draw the curve in Fig. 5. It will be observed that the predicted changes in K_1/M are again in the right direction and of approximately the right size although the detailed fit is poor.

We may make one further comparison with the data. By eliminating between (6-2) and (6-3) one obtains

$$\left(-\frac{K_1}{M}\right) = \left(-\frac{K_0}{M}\right) + \frac{1}{2}\left(\frac{\omega}{\gamma} - H_e\right) \qquad (6\text{-}4)$$

With the same assumption about K_0, a plot of $(-K_1/M)$ versus H_e should be a straight line with slope $-\frac{1}{2}$. The data has been plotted in Fig. 6 and a straight line of the appropriate slope drawn through the point at 160°K. Although only three experimental points are available, the fit is very satisfactory.

The author would like to acknowledge many helpful discussions with J. K. Galt, P. W. Anderson, H. Suhl and L. R. Walker.

[8] Bozorth, Cetlin, Galt, Merritt and Yager, Physical Review, to be published.