

# Equalization of Cables for Local Television Transmission

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*An improved equalization system has been developed for video cable. Equalizer design is based on an analysis of cable performance, which shows that an analytic expression can be obtained for the loss and phase of the cable over the video band. Fixed equalizers were designed to handle the loss characteristics of a nominal cable pair and manually-adjustable equalizers to handle expected variations. New wideband approximation methods were used to match arbitrary impedance or loss characteristics to close tolerances. With these equalizers an initial cable distortion of a hundred or more db may be reduced to less than 0.05 db.*

## INTRODUCTION

Television transmission places stringent requirements on equalization of the transmission medium. This is particularly true on short-haul intracity circuits. A large number of these short-haul links may be connected in tandem for overall long distance television service. Because the arrangement of circuits is subject to frequent change, each link must be independently equalized to close limits. In contrast with the fixed repeater spacings used in long-haul coaxial systems, the lengths of these individual short-haul circuits range from a fraction of a mile to ten or more miles. This requires an equalization system sufficiently flexible to handle all of the circuit lengths encountered in the field. Transmission over these local circuits is handled at video frequencies using special balanced cables. The video band is logarithmically very wide, extending from 30 cps to 4.5 mc — a span of over 17 octaves in frequency. This wide band adds to the complexity of the equalizers and makes necessary the use of special impedance networks for terminating the cable.

In connection with the A2A system described in a companion paper,<sup>1</sup> improved equalizers have been developed for video cable. The design of these equalizers is based on an analysis of the cable performance, which

shows that an analytic expression can be obtained for the loss and phase of the cable over the video band. Fixed equalizers were designed to handle the loss and phase characteristics of a nominal cable pair as derived from this expression, and manually-adjustable equalizers were designed to compensate for expected variations from these nominal values. The equalizers are arranged for plug-in mounting and may be installed as required to fit the desired circuit length.

#### CABLE CHARACTERISTICS

##### *Loss*

The insertion loss and phase of a section of cable between sending and receiving impedances provided by terminals or repeaters may be broken down into two terms; first the image transfer loss and phase, and second the reflection and interaction loss and phase.<sup>2</sup> The first of these two terms is an inherent property of the cable itself. The second involves the ratio of the cable image impedance to the termination. As discussed later, networks are provided in the A2A system for matching the image impedance of the cable at the two ends, thereby eliminating reflection and interaction effects. Consequently it is sufficient to consider the transfer loss and phase alone.

The image transfer constant of the cable may be expressed in the form

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (1)$$

where

$\alpha$  = attenuation

$\beta$  = phase

$\omega$  = radian frequency

$R$ ,  $L$ ,  $G$ , and  $C$  = total distributed resistance, inductance, conductance, and capacitance in the section of cable

At very low frequencies, where the conductance is negligible and the resistance is constant and large compared with  $\omega L$ , the attenuation reduces to

$$\alpha = \sqrt{\omega RC/2} = \sqrt{\pi RC} \sqrt{f} \quad (2)$$

Thus at low frequencies the attenuation is proportional to the square root of frequency.

At high frequencies the attenuation becomes

$$\alpha = \frac{R/2}{\sqrt{L/C}} + (G/2)\sqrt{L/C} \quad (3)$$

At these frequencies  $\sqrt{L/C}$  approaches a constant and the resistance varies as the square root of frequency, due to skin effects. Therefore the first term of (3) also represents a loss increasing as the square root of frequency, but with a different constant multiplier. The second term in (3) is small except at the extreme upper end of the video band and represents a component of loss which varies approximately as the first power of frequency. This component is due in part to the fact that the conductance increases linearly with frequency. Over and above this, there is an increment of loss increasing linearly with frequency, due to eddy current losses in the cable sheath.

The functional form of these losses is illustrated in Fig. 1, which is a plot of the attenuation versus frequency of a typical 1,000-ft length of 16-gauge video cable. In this figure both attenuation and frequency are plotted to a logarithmic scale. At low frequencies the attenuation  $\alpha$  is asymptotic to the straight line  $\sqrt{\pi RC} \sqrt{f}$ , and at high frequencies  $\alpha$  is asymptotic to a straight line with the same slope but with a different intercept. At the top of the band there is an additional small increment

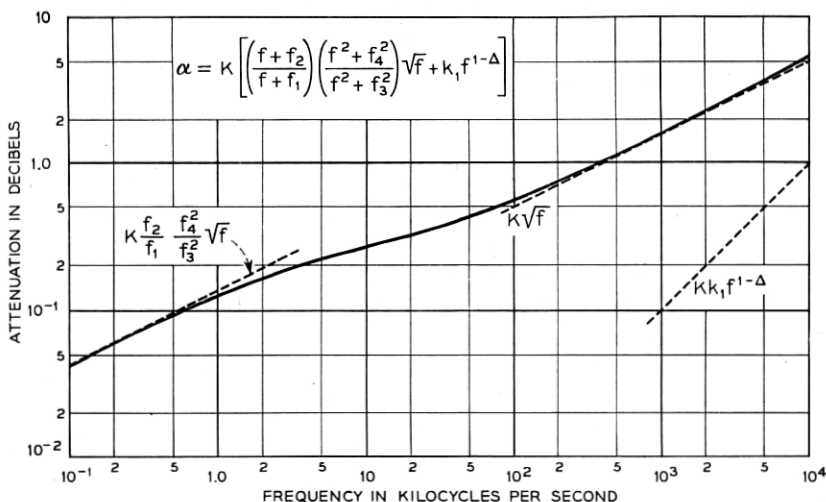


Fig. 1 — Attenuation of video pairs.

This equation provides the key for the entire equalization plan. Fixed equalizers are provided to equalize the loss of the cable as given by the equation with nominal values of the constants. Adjustable equalizers are provided at the ends of the circuits to compensate for changes in loss arising from changes in the constants from their assumed nominal values. By this means it is possible to equalize any cable likely to be encountered in the field. The fixed equalizers are provided in blocks corresponding to cables having losses of 2.5, 5, 7.5, 10, 15 or 20 db at 4.5 mc. As discussed in a companion paper,<sup>1</sup> these fixed equalizers are associated with flat-gain amplifiers to provide flexibility in application.

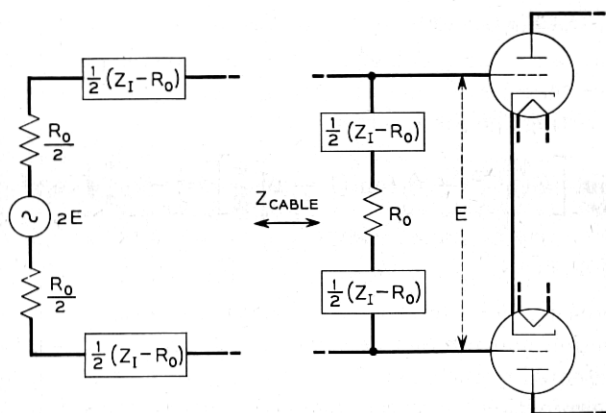


Fig. 2. — Method of terminating cable pairs.

For circuits up to about 3.5 miles in length, it is possible to use flat-gain receiving and transmitting amplifiers with grid and plate directly connected to the line. For longer circuits, however, signal-to-noise considerations necessitate the use of input and output coupling networks which increase the transmission from the line to the grid and from the plate to the line at high frequencies. These networks have simple gain functions and are associated with fixed equalizers which, in combination with the networks, provide equalization for fixed lengths of line.

Since the cable and equalizers are all minimum phase structures, the phase distortion is uniquely related to the loss distortion. If the gain of the system were flat from zero to infinite frequency there would be no phase distortion. The decreasing system gain above the band, however, introduces phase distortion within the band. It is possible to associate the phase equalization directly with the loss equalizers, since (6) relates



the phase distortion of the line directly with the loss distortion. Therefore, each fixed loss equalizer, including those associated with the coupling networks, contains a fixed phase equalizer for handling the phase distortion in the related length of line. This insures proper phase equalization without the necessity of conducting phase measurements in the field. No phase equalizers need to be associated with the adjustable equalizers. Not only is the phase distortion of these equalizers small, but their out-band loss performance is such that their phase tends inherently to be complementary to the characteristics being equalized.

#### DESIGN METHODS

##### *Cable-Impedance Networks*

The design of the impedance-matching networks is based upon the fact that the image impedance of the cable is a minimum-reactance function and hence both resistance and reactance are automatically specified when the magnitude of the impedance  $|Z|$  is given.<sup>3</sup> Thus, if the impedance magnitude is matched over a sufficiently wide band of frequencies, the resistive and reactive components will also be matched.

In Fig. 3 the logarithm of the image-impedance magnitude  $|Z_I|$  for video cable is plotted against the logarithm of frequency. At low frequencies the impedance  $|Z_I|$  is inversely proportional to the square root of frequency. It appears in Fig. 3 as a curve approaching a straight-line asymptote having a slope  $k$  of  $-1/2$ . At high frequencies  $|Z_I|$  approaches a constant value.

Before proceeding with the design of a network to match this impedance, it is convenient to consider the idealized problem of approximating an impedance  $|Z|$  which is proportional to  $f^{-k}$ , or which decreases along a straight line of slope  $-k$ , as shown in Fig. 4. This impedance may be

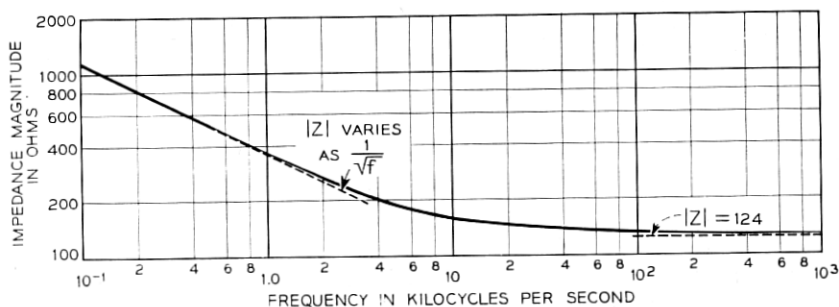


Fig. 3 — Magnitude of image impedance of video pairs.

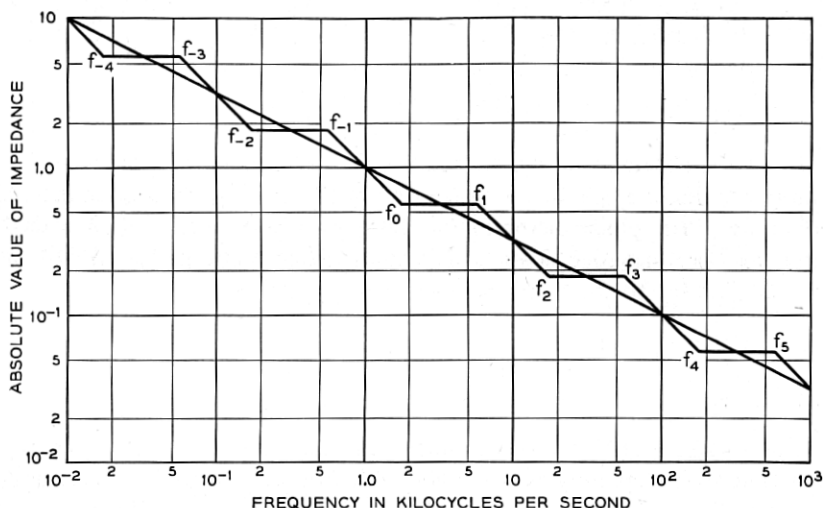


Fig. 4 — Asymptotes for network approximating infinite slope  $|Z| = f^{-1/2}$ .

approximated with a network which has an infinite number of zero-pole pairs (corresponding to an  $R$ - $C$  network). The impedance  $Z'$  of such a network is given by the infinite product

$$Z' = H \prod_{-\infty}^{\infty} \frac{p - p_{2n}}{p - p_{2n-1}} \quad (8)$$

where  $p = j2\pi f$ , and  $p_{2n}$  and  $p_{2n-1}$  are negative real numbers.

The network behavior may be represented by straight-line asymptotes with slopes of  $k = 0$  and  $k = -1$  which intersect at frequencies corresponding to  $-p_n/2\pi$ , as shown in the Fig. 4. When the zero-pole pairs are repetitive on a logarithmic frequency scale, the magnitude  $|Z'|$  computed from (8) will ripple about the desired curve with a uniform amplitude and period. The amplitude of this ripple corresponds to the percentage error in the impedance magnitude and may be computed as a function of  $k$  and of  $h$ , the number of zero-pole pairs per decade, following a method analogous to that described in Reference 4 for computing the error of an infinitely decreasing loss characteristic. Fig. 5 shows the percentage error in  $|Z'|$  when  $k$  equals  $\pm 1/2$ . For any other value of  $k$  the error varies as  $\sin \pi k$ .

Thus for any impedance magnitude and desired degree of match, the complexity of the network may be determined using Fig. 5 as a guide. The critical frequencies may be found from the asymptotic representa-

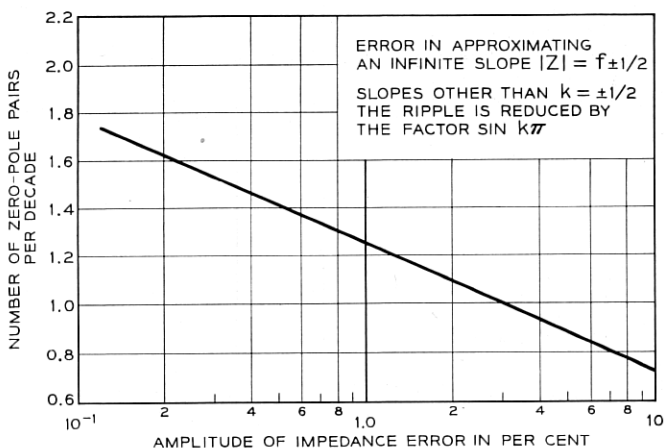


Fig. 5 — Error in approximating  $|Z| = f_{\pm 1/2}$ .

tion. This is shown in Fig. 6 for the impedance-matching networks for video cable. The figure also gives the expression for the network impedance. The resulting reflection coefficient of the network against the cable image impedance is plotted in Fig. 7. A schematic of the network appears in Fig. 8.

### Fixed Cable Equalizers

As mentioned previously, fixed equalizers are available in discrete sizes to handle cables of a variety of lengths. The smallest block equalizes a cable loss of 2.5 db and the largest a cable loss of 20 db at 4.5 mc. The design of each of these fixed equalizers is similar. Fig. 9 shows the desired loss characteristic for the 20-db equalizer. The design of this equal-

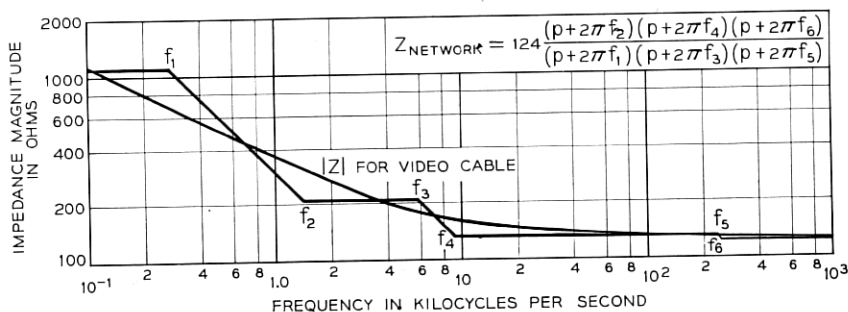


Fig. 6 — Asymptotic representation of impedance network.

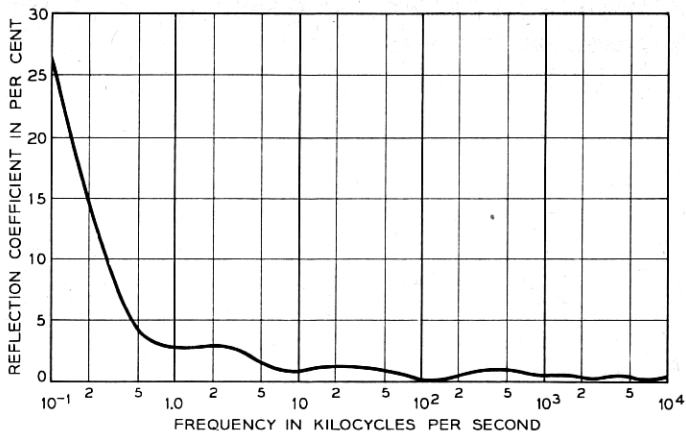


Fig. 7 — Reflection coefficient of network against cable impedance.

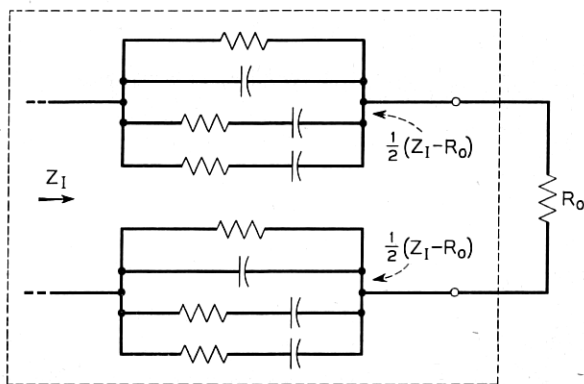


Fig. 8 — Schematic of impedance network.

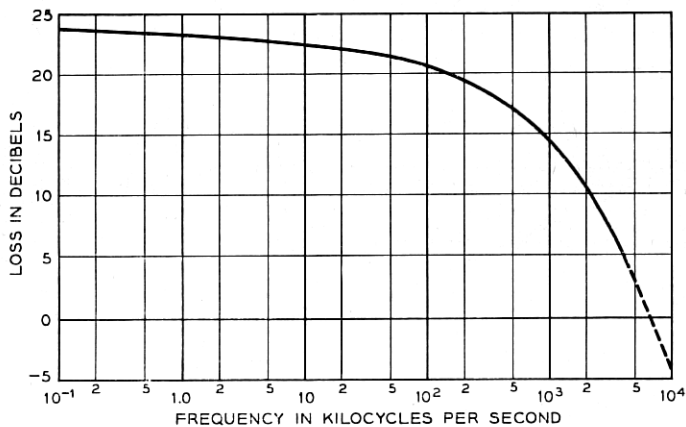


Fig. 9 — Equalizer loss objective.

izer is described in detail in Reference 4 and is analogous to the design of the impedance-matching networks as outlined above. Since the loss is monotonically decreasing over the band, it can be matched with a bridged-T equalizer whose series arm consists of a resistor shunted by resistance-capacitance branches. The complexity of the network can be determined by considering the idealized problem of matching a loss characteristic with an infinite slope of 20k db per decade by an equalizer having an infinite number of uniformly-distributed zero-pole pairs. The loss expression of such an equalizer is given by the infinite product

$$e^{\theta} = \prod_{-\infty}^{\infty} \frac{p - p_{2n}}{p - p_{2n-1}} \quad (9)$$

where  $\theta = \alpha + j\beta =$  insertion loss and phase

$$p = j2\pi f$$

$p_{2n}$  and  $p_{2n-1}$  are negative real numbers

As in the case of the impedance networks, the equalizer loss will ripple about the desired straight line characteristic. The amplitude of the ripple is a function of  $k$  and of  $h$ , the number of zero-pole pairs per decade. The amplitude of the loss error as a function of  $h$  is shown in Fig. 10 for a loss characteristic having a slope of 10 db per decade (3 db per octave) corresponding to  $k = \pm 1/2$ . For any other slope the loss error is  $\sin \pi k$  times this amplitude.

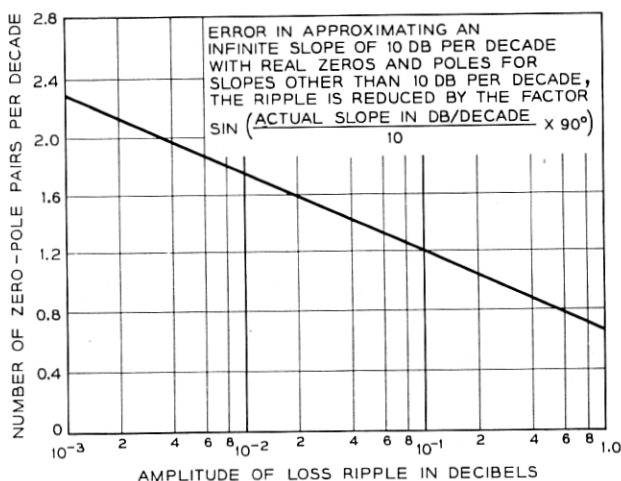


Fig. 10 — Error in approximating infinite loss slope.

This curve was used to determine the complexity necessary for each fixed equalizer. In order to meet tolerances for the requisite number of equalizers in tandem, a design ripple of  $\pm .005$  db was selected. The resulting equalizer as shown in Fig. 11 contains nine resistance-capacitance branches in the series arm. A resonant-frequency branch has been added to bring the loss to zero above the band in order to conserve system gain by reducing the total equalizer loss. The configuration of the inverse shunt branch has been modified to allow the inductor dissipation to be absorbed. As shown in the figure, one all-pass section is included to equalize the phase distortion. Fig. 12 shows the sum of the equalizer and cable losses.

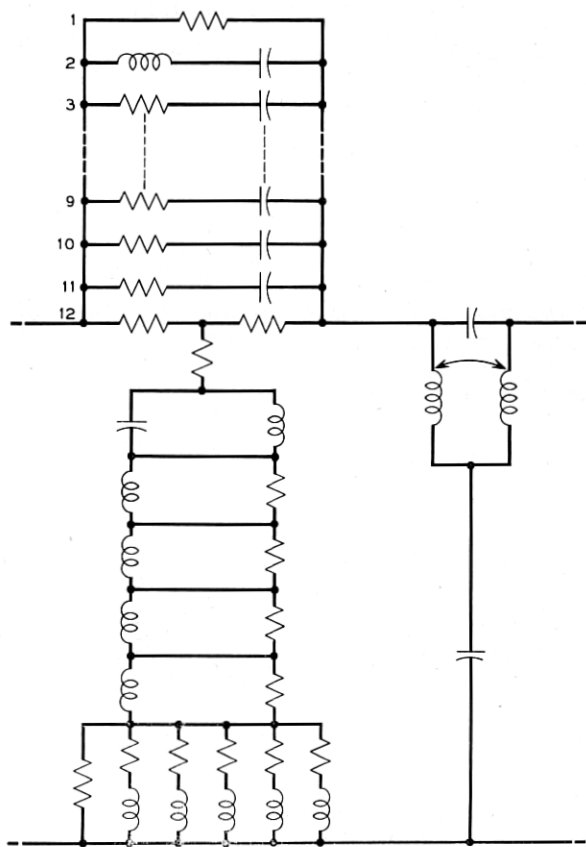


Fig. 11 — Schematic of fixed cable equalizer.

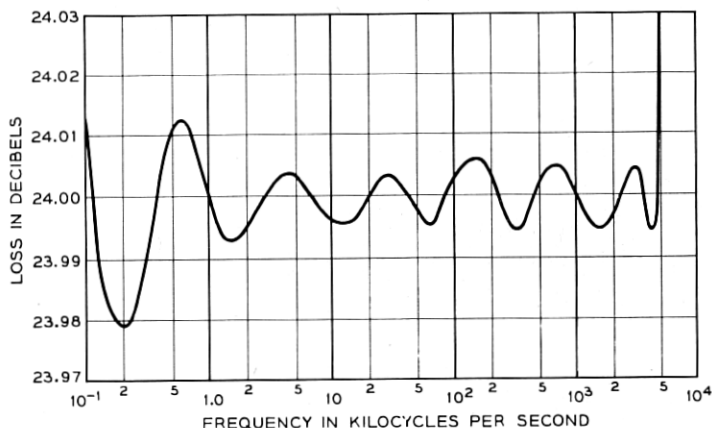


Fig. 12 — Sum of equalizer loss and cable loss.

### Input and Output Networks

With flat-gain amplifiers and the fixed equalizers described above, the maximum length of a single A2A link is about 3.5 miles or 65 db.<sup>1</sup> This length can be increased to 4.5 miles through the use of coupling networks which step up the gain at high frequencies. Each network improves the signal-to-noise ratio by approximately 10 db. These networks are balanced-to-ground and use a transformer with a 6 to 1 turns ratio, corresponding to an asymptotic gain of 15.6 db at high frequencies.

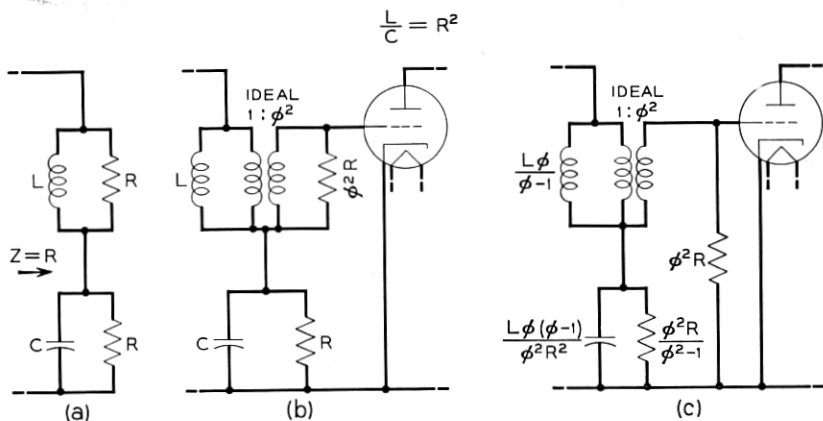


Fig. 13 — Ideal coupling network.

The design of the A2A input and output networks is similar in principle to the design used by E. W. Holman in 1939 for the A2 video system. The basic circuit was derived from the resistive network shown in Fig. 13(a). If the inductor is stepped-up with an ideal transformer of turns ratio  $1:\phi$ , as shown in Fig. 13(b), the input impedance remains resistive but the ratio of output voltage to input voltage is given by

$$\frac{\text{output voltage}}{\text{input voltage}} = e^{\theta} = \phi \frac{p + R/L\phi}{p + R/L} \quad (10)$$

where

$$\theta = \alpha + j\beta = \text{gain and phase}$$

$$p = j2\pi f$$

The gain characteristic is given in Fig. 14. At low frequencies the gain is zero, and at high frequencies it is asymptotic to  $20 \log \phi$ . The location of the characteristic in the frequency spectrum is controlled by the ratio of  $R$  to  $L$ .

In an actual network this ideal gain characteristic is limited by tube and transformer capacitance and by transformer leakage reactance. The parasitic elements may be handled more readily by using the equivalent circuit of Fig. 13(c), which was originally suggested by S. Darlington. In Fig. 13 the voltage from cathode to grid is the sum of the voltages across the  $R$ - $L$  and  $R$ - $C$  branches. These voltages add in amplitude and phase to produce the simple characteristic of Fig. 14. Any low-pass filter structure used to absorb parasitic capacitances should be designed to shift the phase of both components of the voltage equally in order to avoid ripples in the characteristic. This can be done if the resistor termi-

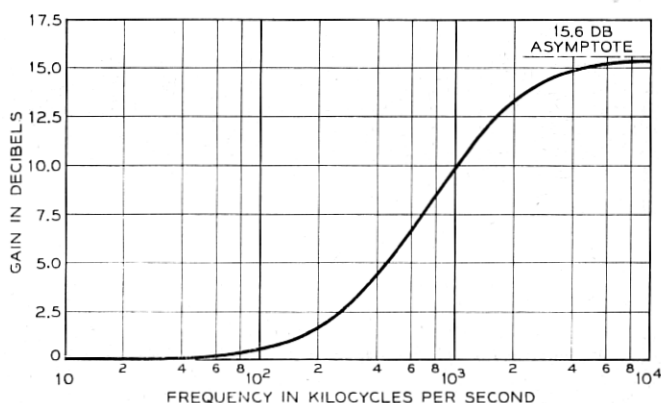


Fig. 14 — Gain characteristic of ideal coupling network.



nating the low-pass filter is connected from grid to ground. Another merit of the configuration of Fig. 13(e) is that, in the case of the output network, the plate resistance can be absorbed by modifying the terminating resistor.

The actual networks are built as balanced-to-ground structures as shown in Figs. 15 and 16. The high-frequency parasites are absorbed into a seven-element low-pass filter having Tchebycheff pass-band characteristics. In the input network the grid is connected to an intermediate point in the filter, as shown. Since the impedance at this point rises with frequency the gain characteristic, shown in Fig. 17, exhibits a rise above the nominal 15.6 db of Fig. 14. In the output network the plate is also connected to an intermediate point in the filter. This filter was designed to have the proper response with a shunt conductance across the capacitor to which the plate is connected in order to allow the plate resistance of the tube to be absorbed. The transmission characteristic is similar to that of the input network. As noted in Figs. 15 and 16, cable impedance networks are added as a shunt element in the input network and as a series element in the output network.

Although the gain characteristic shown in Fig. 17 rises with frequency

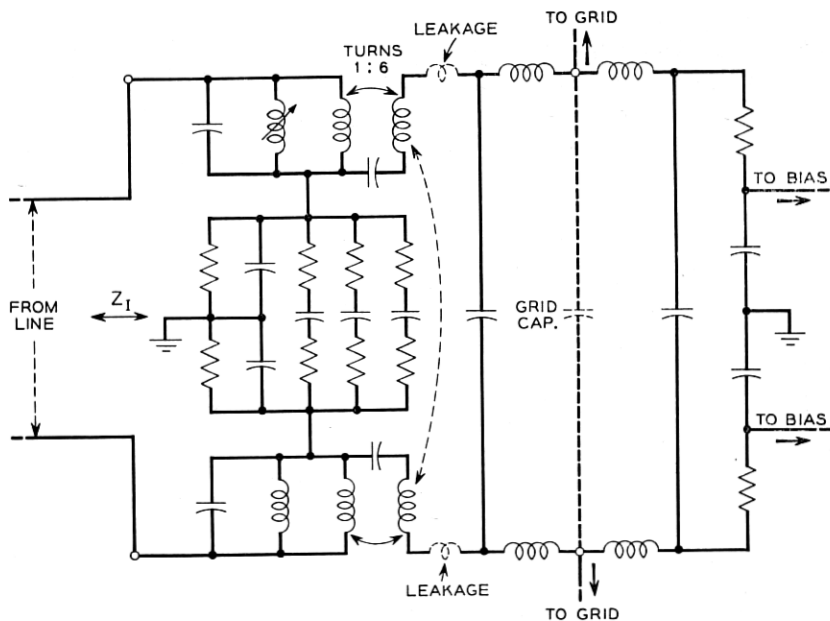


Fig. 15 — Schematic of input network.

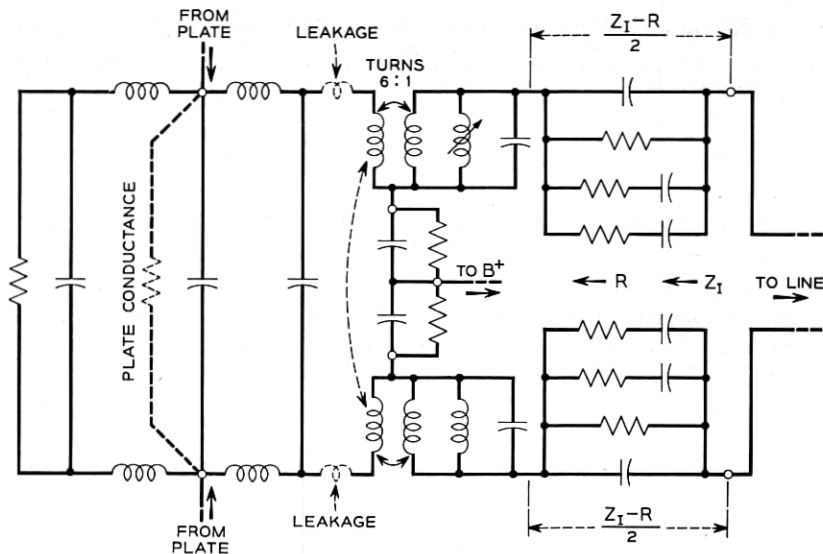


Fig. 16 — Schematic of output network.

the shape is not complementary to the loss of the cable. Fixed equalizers are associated with each of the networks in such a manner that the combination of network and equalizer compensates for 32 db of cable loss at 4.5 mc. The design of these equalizers is similar to that of the fixed cable equalizers. Loss characteristics are shown in Fig. 18 and a schematic in Fig. 19.

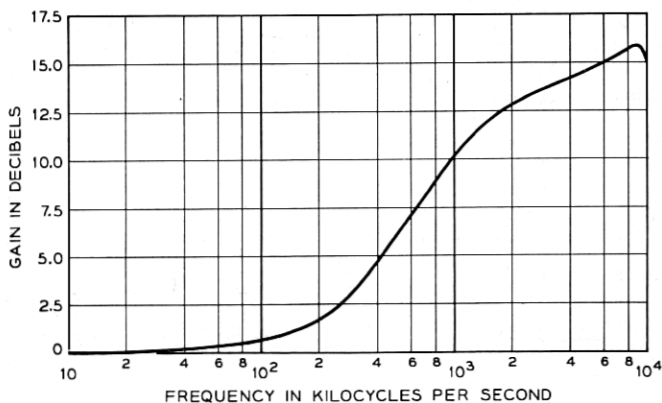


Fig. 17 — Gain characteristic of input network.



## ADJUSTABLE EQUALIZERS

The smallest fixed equalizer, which compensates for a cable loss of 2.5 db, allows the circuit to be equalized to the nearest  $\pm 1.25$  db at 4.5 mc. Equalization for incremental lengths finer than this is handled by manually-adjustable equalizers located at the receiving terminal. These equalizers also allow correction to be made for changes resulting from: (1) manufacturing variations in the cables and changes in cable types; (2) reflection and interaction losses of the resistance-terminated office cables at the ends of the circuits; and (3) variations in amplifiers and equalizers which produce high-frequency loss changes approximately proportional to  $f^3$ . The resulting characteristics are shown in Fig. 20. The shapes in the first category were obtained by differentiating (4) with respect to each of the constants. The reflection and interaction shape is that computed for short lengths of coaxial cable terminated in 75 ohms. Provision of the flexibility inherent in these eight shapes would appear to allow any loss variations expected in the system to be successfully equalized.

The optimum choice of loss shapes for a system of adjustable equalizers is dependent on the type of measuring facility available for system line-up. In the A2A system a test set is used which compares the gain at a single test frequency to the gain at 300 kc. The characteristics shown in Fig. 20 overlap in the frequency spectrum. Consequently they are not

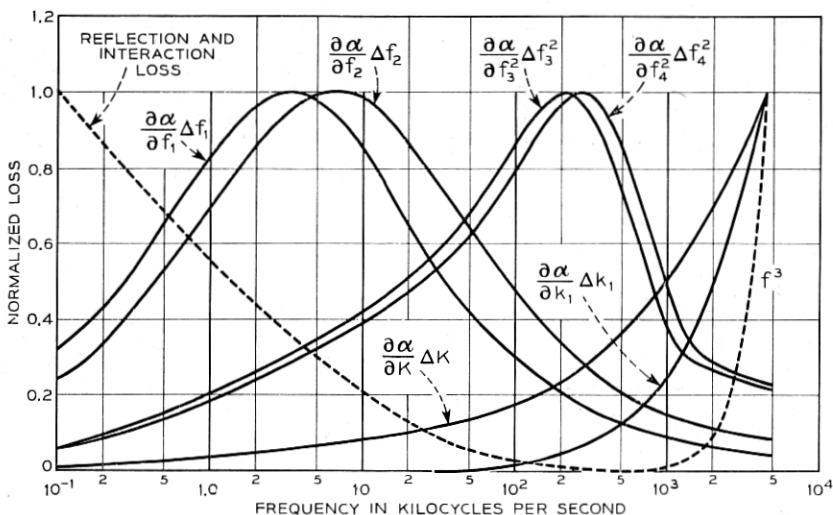


Fig. 20 — Types of loss changes to be equalized.

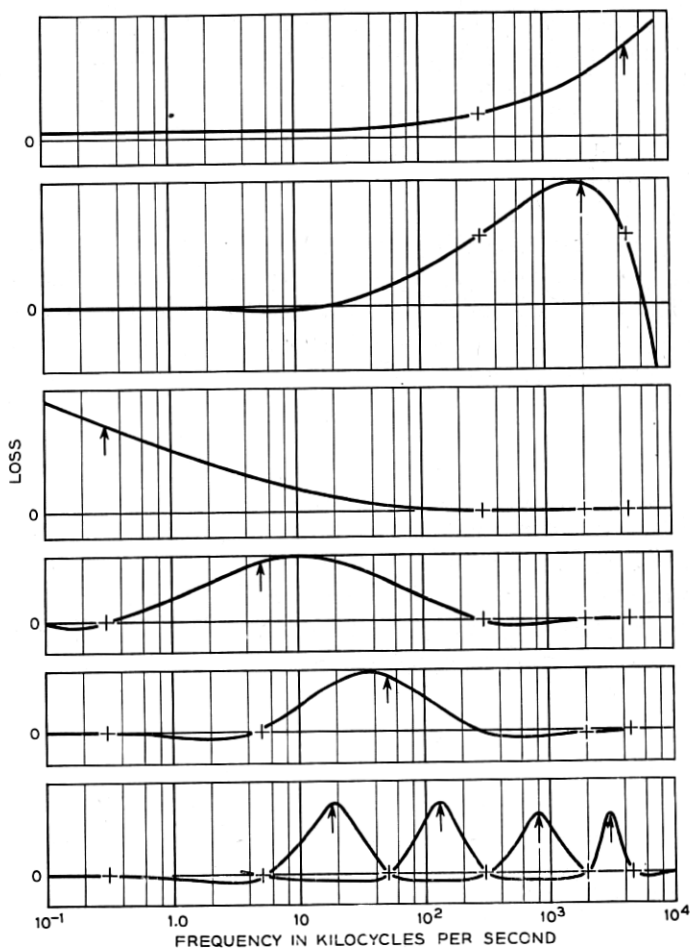


Fig. 21 — Loss characteristics of adjustable equalizers.

amenable to straightforward line-up using this test facility. However, it is possible to resolve this difficulty by taking linear combinations of these characteristics such that each successive shape remains invariant at any of the preceding adjustment frequencies. A ninth shape was added when it became apparent that not all of the changes in the constants of (4) could be treated as differentially small changes. The resulting shapes are illustrated in Fig. 21 with the adjustment frequencies indicated by arrows. Each equalizer is adjusted so that at its adjustment frequency the system gain is made equal to the gain at 300 kc.

The first five shapes, which are the most important, have been packaged in one container known as the "A" Equalizer. The next four shapes comprise the "B" Equalizer, which is normally used only on circuits over 3.5 miles in length.

The structure used to obtain these shapes is a constant-resistance variable equalizer which can readily be adjusted to obtain proportional characteristics varying in either direction from a reference flat condition. This equalizer is a bridged-T structure whose series arm contains a four-terminal constant-resistance network terminated in an adjustable resistor. The overall change in loss and phase from the flat condition may be expressed in terms of the transfer constant of the component four-terminal network. Thus

$$\theta - \alpha_0 = 2 \tanh^{-1} k\rho e^{-2\phi} \approx k\rho e^{-2\phi} \quad (11)$$

where

$\theta - \alpha_0$  = over all change in loss and phase

$k\rho$  is a function of the maximum equalizer range and the setting of the adjustable resistor

$\phi$  = transfer loss and phase of the four-terminal network

From the approximate relation the transfer constant  $\phi$  may be determined from the desired equalizer loss and phase,  $\theta - \alpha_0$ . Although Fig. 21 shows only the loss, the phase component of  $\theta - \alpha_0$  may be computed

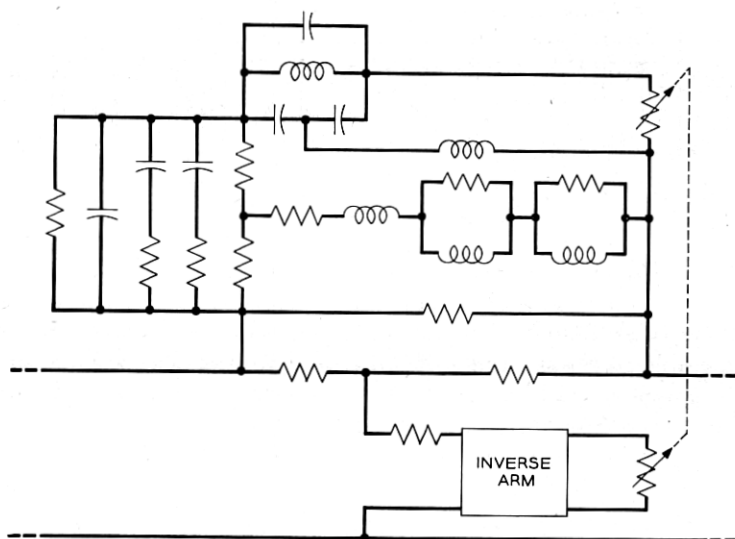


Fig. 22 — Adjustable equalizer with one control.

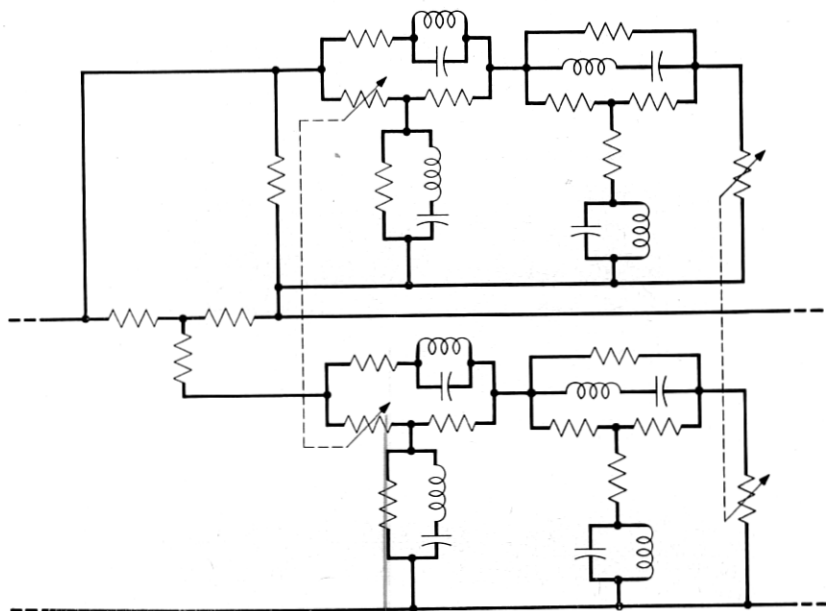


Fig. 23 — Adjustable equalizer with two controls.

using a minimum phase matrix developed by B. A. Kingsbury. This matrix is similar to the one described by W. R. Lundry<sup>5</sup> but is arranged for logarithmic frequency intervals.

With the transfer constant  $\phi$  available, the four-terminal networks can be synthesized by standard methods. Typical schematics of the overall equalizers are shown in Figs. 22 and 23. The configuration of Fig. 23 conserves gain by combining the functions of two controls into one structure.<sup>6</sup>

#### CONSTRUCTION

The quality of performance of the manufactured product depends on the types of components used, the choice of configuration and the mechanical arrangement of parts. The video band is logarithmically very wide and requires large element values for operation at the low end of the range. The inductors and capacitors should be precise elements at low frequencies. Throughout the rest of the band the inductors should maintain high impedance and the capacitors low impedance. However, low frequency inductors and capacitors characteristically exhibit parasitic secondary resonances and antiresonances within the video band.

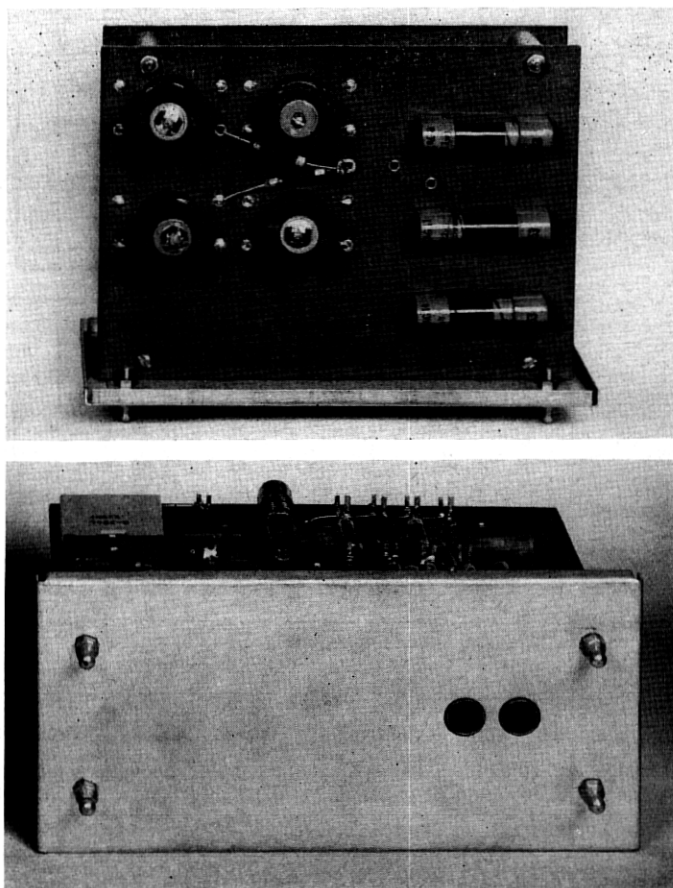


Fig. 24 — Photograph of fixed cable equalizer.

Accordingly it was necessary to develop new types of components which would be free of these spurious characteristics. In the case of the inductors this was done with a ferrite core structure. The magnetic properties of ferrite are superior to those of other magnetic materials over the video range and it is feasible to manufacture ferrite parts in shapes that allow simple uniform windings.<sup>7</sup> The secondary antiresonances in the capacitors were avoided by reducing the lead lengths of the individual units to a minimum and by pairing two units not too disparate in size to achieve the desired precision of adjustment. At high frequencies the



element values are low, and standard molded-mica capacitors and air-core inductors are satisfactory.

Of the many methods available for network synthesis, some allow the network configuration to be prescribed in advance, others place only general restrictions on the functional forms such that the network meets the requirements of physical realizability without specifying in advance what the configuration will be. The advantages of prescribing the configurations in advance are twofold. In the first place, a configuration may be selected in which the effects of element variations are minimized. This is done by arranging the circuit so that the share contributed by each element to the overall characteristic is small and adds to, rather than partially cancels, the portion contributed by other elements. In the second place, the configuration can be chosen in a manner such that the parasites can be absorbed in the structure, or at least such that their effects are minimized. These were the basic considerations in the selection of the configurations for the A2A networks and equalizers.

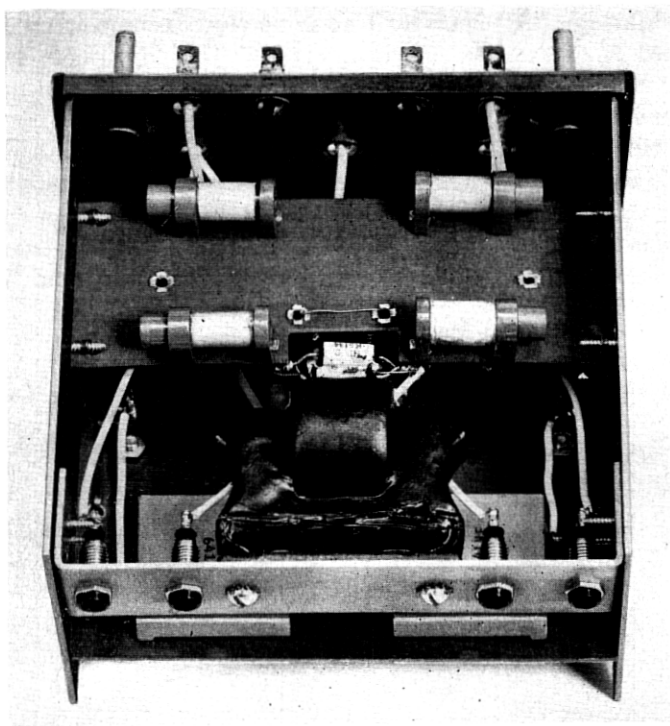


Fig. 25 — Photograph of output network.

The construction of one of the fixed equalizers is illustrated in Fig. 24. A mechanical assembly has been developed to accommodate the diverse types of components used. To permit equipment flexibility the equalizers were designed for plug-in mounting, as shown in the illustration. The locating pins fit into spring catches on the equalizer mounting plate. In the assembly of the equalizer the high-frequency resistance-capacitance branches were placed close to the input-output jacks to keep lead inductance as low as possible. With these precautions the insertion loss of the manufactured product matches the computed performance to high precision. At the same time the maximum reflection coefficient at input and output terminals is held to 2 per cent.

Fig. 25 shows a view of the output network. The ferrite-core transformer appears at the bottom of the figure. The pin jacks adjacent to the transformer are used to short out the cable impedance networks in testing the amplifier. Since this is a balanced network, care was taken to maintain constructional symmetry in order to avoid introduction of longitudinal unbalances.

The adjustable "A" equalizer is made up of five component equalizers assembled in one can as shown in Fig. 26. The adjustment sequence calls for the setting of the knobs from left to right. A typical component equalizer is illustrated in Fig. 27. Control of the characteristic is obtained with a step-type dual rheostat wired with deposited carbon resistors as shown in the figure. Where necessary, the phase shift of the rheostat and its connecting leads was absorbed in the design of the four-terminal network which the rheostat terminates. Each equalizer maintains a maximum reflection of 2 per cent for any setting of the control knob.



Fig. 26 — Adjustable "A" Equalizer.

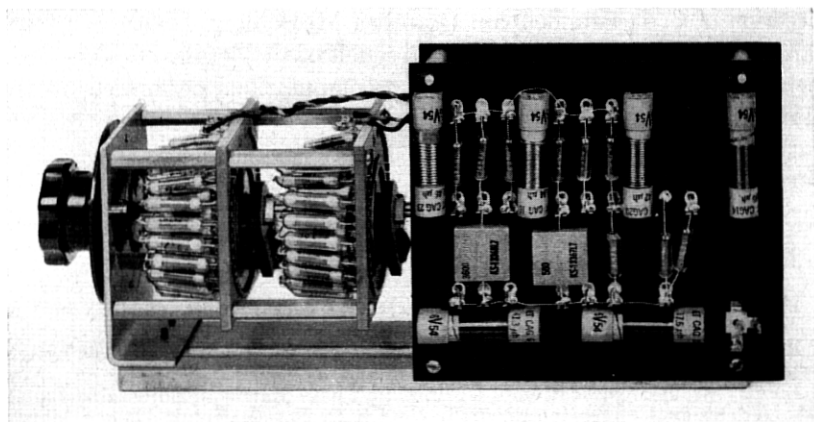


Fig. 27 — Component of "A" Equalizer.

The adjustable "B" equalizer is assembled in a container similar to that used for the fixed equalizer, and hence may be plugged in as required on long circuits. Controls on this equalizer utilize small dual composition-type rheostats.

#### CONCLUSION

Experience with these equalizers in commercial service indicates that the wide variety of circuit lengths and cable types encountered in the field can be successfully equalized. Fixed blocks of equalization are installed only as required for the circuit length involved. Use of a new design technique allows sufficient precision in the fixed equalizers so that no mop-up shapes are required for their design deviations; the only adjustable equalizers used are those which compensate for manufacturing variations in the cable and changes in cable types, as predicted from the analytic expression for cable losses. These factors aid in reducing costs on the preponderant number of short circuits which exist in the video plant. Circuit alignment is straightforward and rapid, using simple portable test equipment. With this equalization system an initial cable distortion of a hundred or more db may be reduced to less than 0.05 db.

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## REFERENCES

1. Doba, S. and Kolding, A. R., A New Local Video Transmission System, page 677 of this issue.
2. Johnson, K. S., Transmission Circuits for Telephonic Communication, D. Van Nostrand Co.
3. Bode, H. W., Network Analysis and Feedback Amplifier Design, D. Van Nostrand Co.
4. Rounds, P. W., Equalization of Video Cables, Convention Record of the I.R.E., Part 2, Circuit Theory, March, 1954.
5. Lundry, W. R., Application of a Minimum Phase Matrix to Adjustable Equalizer Design, Convention Record of the I.R.E., Part 2, Circuit Theory, March, 1954.
6. Lundry, W. R., Attenuation and Delay Equalizers for Coaxial Lines, A.I.E.E. Trans., **68**, Part 2, pp. 1174-1179, 1949.
7. Stone, H. A., Jr., Ferrite Core Inductors, B.S.T.J., **32**, 265, March, 1953.
8. Graham, R. S., Relay Computer for Network Analysis, Bell Lab. Record., **31**, p. 152, April, 1953.