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Waveguide as a Communication Medium

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The circular electric wave in round metallic tubing has an attenuation coefficient which decreases as the frequency of operation is increased. A corollary to this behavior is the fact that any preselected attenuation coefficient can in theory be obtained in any predetermined diameter of pipe through the choice of a suitably high carrier frequency. The attenuation which is characteristic of microwave radio repeater links, about 2 db/mile, is in theory attainable in a copper pipe of about 2″ diameter using a carrier frequency near 50,000 mc.

Scale-model transmission experiments, conducted at 9,000 mc, showed average transmission losses about 50 per cent above the theoretical value. These extra losses were due to (1) roughness of the copper surface and (2) transfer of power from the low-loss mode to other modes which can also propagate in the pipe.

The latter effect may have serious consequences on signal fidelity because power will transfer (at successive waveguide imperfections) from the signal mode to unused modes and, after a time delay, back to the signal mode. This effect has been studied experimentally and theoretically, and it is concluded that (1) either mode filters must be inserted periodically to absorb the power in the unused modes of propagation, or (2) the medium itself must be modified to continuously provide large attenuations for the unused modes of propagation. The latter approach is attractive in that it also provides a solution to the problem of bending this form of low-loss guide.

The general outlook, based on present knowledge, is that a waveguide system might transmit baseband widths as large as 100 to 500 mc using a rugged modulation method such as PCM. Some form of regeneration is likely to be required at each of the repeaters, which may be spaced on the order of 25 miles. A total rf bandwidth of about 40,000 mc may be available in a single guide.

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INTRODUCTION

The circular electric wave in round metallic tubing possesses a property so unique that some early research workers doubted the reality of the wave. This unique property is an attenuation coefficient which, in a given pipe, *decreases* without limit as the frequency of operation is *increased*. In parallel wire, coaxial, or ordinary waveguide lines the "skin effect" at the surface of the conductor causes the loss to increase as the frequency increases indefinitely, so the predicted circular-electric-wave loss characteristic aroused considerable interest as soon as it was discovered by S. A. Schelkunoff and G. C. Southworth in the early 1930's. Since that time considerable work has been done at Holmdel to explore the reality of the circular electric wave and to evaluate its usefulness to the Bell System. It is the purpose of this paper to report on the status of this work and to give a description of some of the basic characteristics of circular electric wave propagation.

The Bell System is interested in knowing whether waveguide can be used as a long distance communication medium in the manner in which coaxial cable or the radio relay system is now employed. Our interest in long distance waveguides is due in part to the fact that radio-wave propagation through the atmosphere becomes progressively more severely handicapped by rain, water vapor and oxygen absorptions at frequencies above 12,000 mc. Use of the spectrum above the 10,000–20,000 mc region seems to require a sheltered transmission medium.

Circular electric wave transmission may also find application in short connecting links, such as between subscribers requiring very broad band circuits, between two central offices as a multi-channel carrier link, or between a radio relay antenna site and a somewhat remote transmitter-receiver location chosen for accessibility.

In each of these cases, the broad bands available in the microwave portion of the spectrum, the complete shielding afforded by waveguides generally, combined with the low-loss properties of the circular electric wave would seem to provide an ideal transmission medium. We therefore seek knowledge of the precision required in the waveguide and some indication of general system complexity to facilitate a judgment as to whether the cost will be competitive.

ORDINARY VERSUS CIRCULAR ELECTRIC WAVES

Let us approach a discussion of circular electric waves by considering their relation to the waveguides which are now used in our radio relay systems and which found widespread use in the radars of World War II. The vast majority of waveguides in commercial use now are rectangular in cross section and have dimensions large enough so that one and only one wave-type, usually called the "dominant mode", can propagate. To simplify this discussion, such waveguides will be called ordinary waveguides. Ordinary waveguides are analogous to coaxial or parallel-wire lines in many respects. Because only one mode can propagate, departures from an absolutely straight tube of constant cross section show up as reactance effects only. A dent in the side wall of the guide or of the coaxial, an abrupt change in cross section, or a twist or bend of the line all appear as non-dissipative reflection effects which may be cancelled at one frequency (or in one band of frequencies) by the addition of another compensating reactance at a point suitably located. A great many of the components used in ordinary waveguides, including the frequency selective filters, depend on such reactance cancellation effects in order to achieve satisfactory operation.

Since the techniques for employing ordinary waveguides have been thoroughly explored, it is natural to inquire as to whether we can use them for communication purposes. We do use ordinary waveguides in lengths of the order of 100 feet and more to connect the antennas and repeaters in the 4,000 mc (TD-2) radio relay system. The attenuation is excessive, however, for long-distance applications. The particular type of brass rectangular waveguide used for TD-2 transmission lines has attenuation in excess of 50 db per mile, and use of the very best copper would only reduce the theoretical loss to about 40 db per mile. In order to reduce the loss in ordinary waveguides, just as in coaxial or parallel wire lines, one must go to lower frequencies. In particular, the theoretical loss at a carrier frequency near 1000 mc is about 2 db per mile, which is about the same as the transmitter-to-receiver attenuation in our radio relay systems. The waveguide in the 500–1000 mc region would have cross-sectional dimensions on the order of one foot, would be cumbersome to handle and would involve rather large material cost. In addition, it turns out that such a waveguide would be useful in signal bandwidths only a few mc wide as a result of delay distortion, which will be discussed further in the ensuing discussion. Thus, we have concluded that ordinary waveguide is not very attractive as a transmission medium over distances on the order of a mile or more.

It is true that the attenuation in any hollow metallic waveguide can be reduced to any desired extent at a given frequency by making the cross sectional area larger by a suitable factor. The penalty is that the transmission medium becomes capable of propagating energy in several characteristic ways, known as modes. The striking feature of a multi-mode transmission medium is that energy in one mode is entirely independent and unaltered by the presence or absence of energy in one of the other modes. This situation is sketched diagrammatically in Fig. 1. Energy can theoretically propagate between 1 and 1′, between 2 and 2′, and between 3 and 3′ at the *same time* and in the *same frequency band* without mutual interference. The separate modes represent independent transmission lines which occupy the same space. The distinguishing features of the various modes in a multi-mode waveguide are: (1) Velocity of propagation or phase constant, (2) Attenuation coefficient, and (3) Configuration of electric and magnetic field lines within the waveguide.

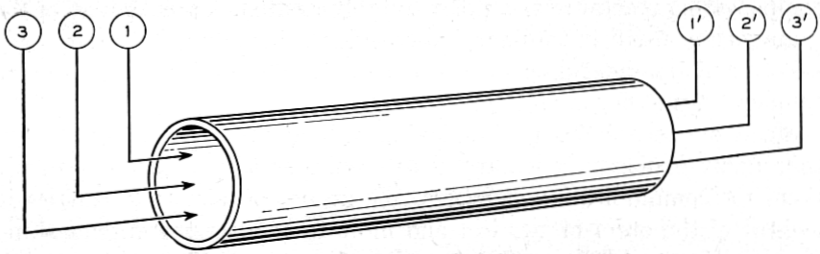

Fig. 1 — Diagram of multi-mode waveguide transmission.

The fact that it is necessary to use a waveguide whose dimensions are large enough to permit the existence of a number of modes has far-reaching influence on the research being discussed here. Practically, the independence between the various modes of propagation is limited by tolerances of various kinds. In the multimode waveguide, changes in cross section or bends or twists require design attention with regard to mode purity as well as with regard to impedance match, and it is not permissible to insert arbitrarily shaped probes or irises for impedance matching purposes as is the common practice in ordinary waveguides. This means that a complete new technique is required for the old components, such as frequency-selective filters, hybrids, and attenuators, as well as for a new series of components such as pure mode generators and mode filters.

THEORETICAL CHARACTERISTICS OF THE CIRCULAR ELECTRIC WAVE

Since it has been found necessary to use a waveguide in the multimode region in order to get the desired losses in a reasonable size waveguide, we may inquire as to which of the modes is best suited to our problem. At a given frequency the loss for any one of the modes may be reduced as much as is desired by making the cross sectional area of the guide large enough, but there is a mode for which the loss decreases with increasing guide size much more rapidly than for any other mode. This is the circular electric (TE_{01}) mode in straight round pipe. It turns out that no current flows in the direction of propagation in the metallic walls of a straight round pipe carrying the circular electric mode. It is the absence of current in the direction of propagation which permits the circular-electric-wave attenuation to decrease indefinitely as the frequency increases, and this difference between ordinary transmission lines and the circular-electric wave is further illustrated in Fig. 2. In the familiar parallel-wire line the electric field extends directly from one conductor to the other, resulting in charge accumulations at half-wave intervals along the axis of propagation and associated conduction currents in the copper wires. These conduction currents in the direction of propagation do not diminish as the frequency of operation increases, since they are associated with the energy transmitted to the end of the transmission line. With the circular electric wave the electric field lines close upon themselves, are always tangential to the conducting wall, and do not result in a charge accumulation on the walls due to the main energy flow. The wall currents which do flow are merely sufficient to prevent the propagating energy from spreading out as it would if the metallic walls were not present, but these currents decrease rapidly with increasing frequency in a given waveguide size. Only in a straight circular pipe can all of the electric field lines close upon themselves, and only in the straight circular pipe does the attenuation approach zero at infinite frequency.*

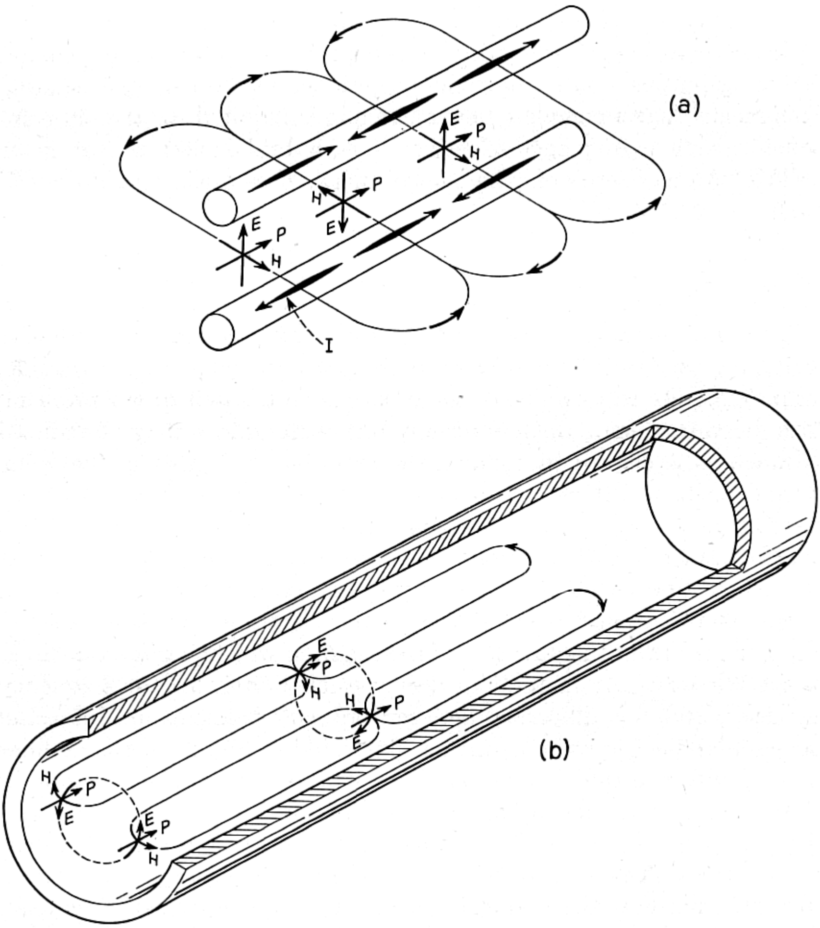

Fig. 2 — Sketch of the magnetic intensity (*H*), electric intensity (*E*) and Poynting Vector (*P*) for parallel-wire and circular-electric waveguides. Because the main energy flow (*P*) in the circular electric waveguide is associated with electric field lines that close on themselves and do not produce accumulations of charge on the metal walls, the wall currents and associated losses are very small.

* For further discussion, see G. C. Southworth, *Principles and Applications of Waveguide Transmission*, D. Van Nostrand, Inc. pp. 175–178.

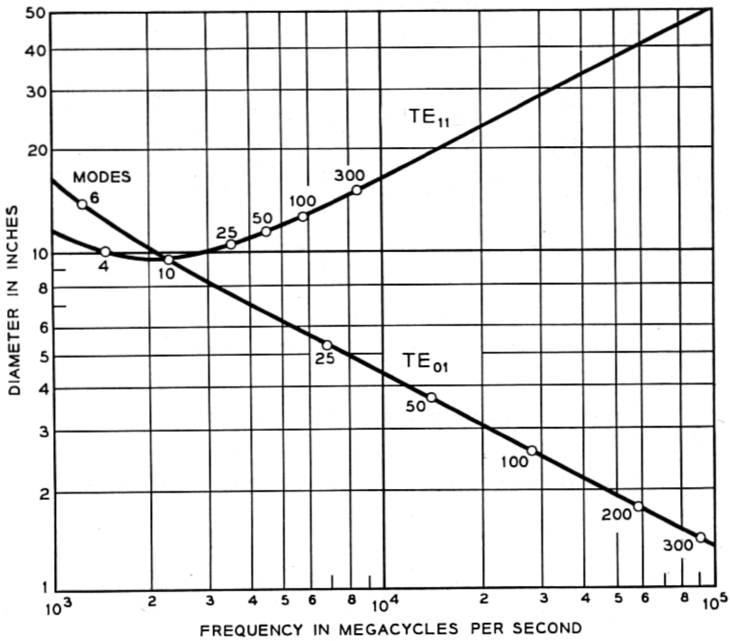

Fig. 3 — Round guide diameter versus frequency for attenuation of 2 db/mile.

As a consequence of the unusual loss versus frequency characteristic of the circular electric wave, the diameter required in order to achieve a given loss decreases as the carrier frequency increases. This is illustrated by the curves labeled TE_{01} in Figs. 3 and 4. All other waveguide modes (except higher-order circular-electric waves, TE_{0m}) have a characteristic of the general form sketched for the dominant wave (TE_{11}) also shown in Figs. 3 and 4. The longitudinal wall currents contribute a loss component which rises at increasing frequencies due to skin effect; this accounts for the positive slope of the TE_{11} curve at the right-hand side of Figs. 3 and 4. The negative slope of the TE_{01} curves and of the left-hand portion of the TE_{11} curves in Figs. 3 and 4 is a consequence of losses associated with the wall currents which prevent the wave from spreading as it would in an unbounded medium; these currents and the losses associated with them decrease as the operating frequency becomes farther removed from cut-off.

For a loss of 2 db per mile Fig. 3 shows that a waveguide 7″ in diameter is required in the frequency band near 4,000 mc where the TD-2 system operates. Whereas this may not be prohibitive in a connecting

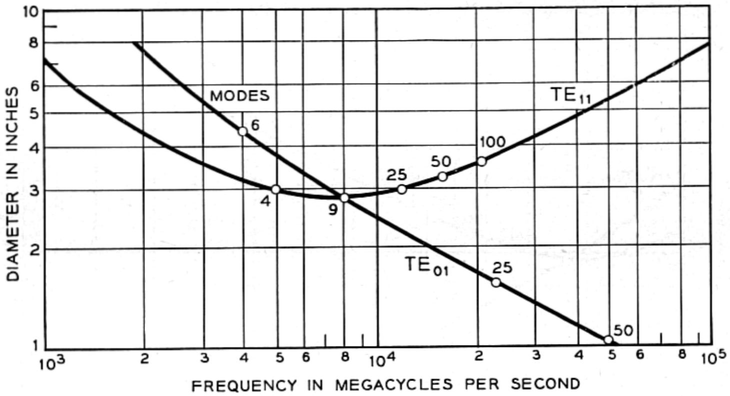

Fig. 4 — Round guide diameter versus frequency for attenuation of 13.2 db/mile (0.25 db/100 ft.).

link application, the waveguide size is definitely too large for long-distance application. In the vicinity of 50,000 mc, however, Fig. 3 shows that the required waveguide size is on the order of 2″, and this is comparable to the size of the present standard 8-pipe coaxial cable. From these simple calculations, it is evident that carrier frequencies in the vicinity of 50,000 mc or more are very desirable for long-distance waveguide applications in order to minimize the size of the waveguide.

Other reasons for wanting a high carrier frequency arise from a consideration of bandwidth. Any hollow conductor waveguide has a cutoff characteristic of the form sketched in Fig. 5. Above cutoff the group velocity approaches asymptotically to the velocity in an unbounded medium composed of the dielectric used in the waveguide. Because the group velocity varies across the frequency band, a signal being transmitted in a waveguide will experience delay distortion; the components transmitted at $f_0 \pm \Delta f$ (Fig. 5) would be delayed compared with their relation at the input to the line. When this delay is 180° a baseband signal of frequency Δf would be severely distorted regardless of the modulation method, and this condition may be regarded as an upper limit to the usable bandwidths in the waveguide unless correction for delay distortion is employed. This particular type of phase distortion has been analyzed in unpublished papers, first by D. H. Ring and later by S. Darlington. The work of Darlington leads to the following relation between the parameters of the waveguide and the baseband width f_B associated with the 180° phase difference noted above:

$$f_B = \frac{304 f^{1/2}(1 - \upsilon^2)^{3/4}}{\upsilon L^{1/2}} \text{ (cps)}$$

where f is the carrier frequency (cps), v is the ratio of the waveguide cutoff frequency to the carrier frequency, and L is the line length in miles.

From the above relation, the maximum baseband width available has been calculated for one mile of line as a function of carrier frequency with the loss held constant at 2 db per mile and 13.2 db per mile, and the results are plotted in Fig. 6. The conditions for these curves are directly comparable to those for which Figs. 3 and 4 were calculated. At a carrier frequency near 50,000 mc, the circular electric wave in 2-inch diameter pipe makes available a baseband width on the order of 500 mc for one mile of line, or 100 mc for 25 miles of line. At lower frequencies, with the waveguide enlarged to hold the loss constant, there is less bandwidth available.

The 13.2 db per mile condition (in a smaller diameter of waveguide) permits the use of approximately one-half the bandwidth available at the 2 db per mile condition.

The above design considerations are the basis for concluding that the most attractive communication possibility is the use of carrier frequencies on the order of 50,000 megacycles and associated waveguide

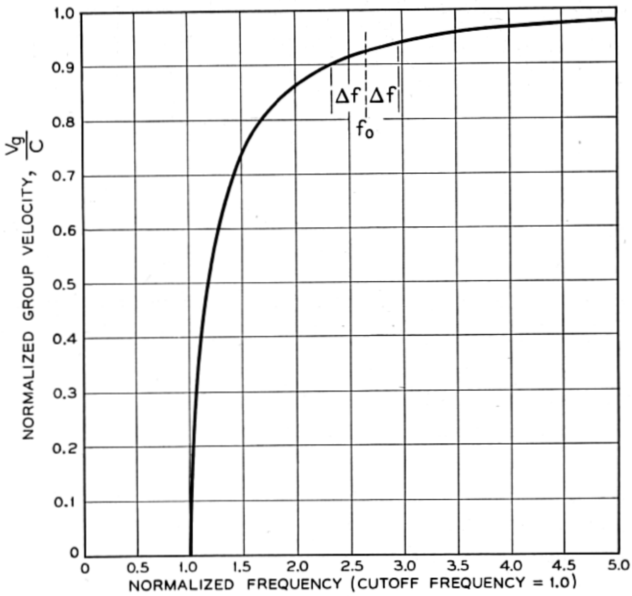

Fig. 5 — Normalized group velocity versus normalized frequency for hollow metallic waveguides.

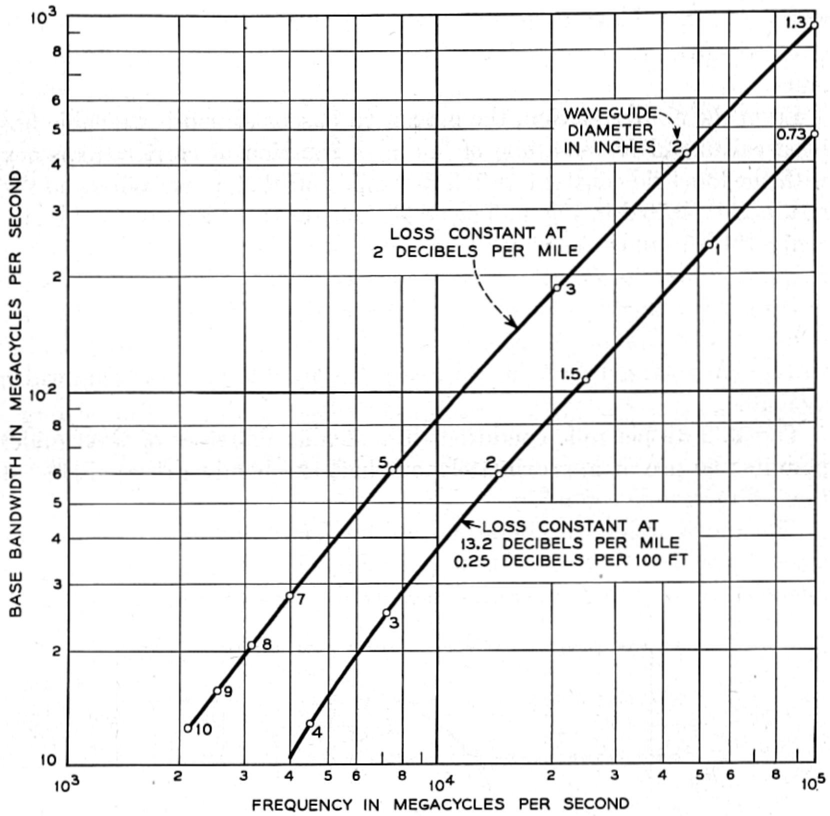

Fig. 6 — Base band width per channel versus frequency (for one-mile of waveguide) using the circular electric mode, loss held constant by varying pipe diameter.

diameters on the order of 1 to 2 inches. Having selected the general region which appears attractive, it is important to know the communication characteristics of a given waveguide. Figs. 7 and 8 show the attenuation and bandwidth characteristics respectively for a 2-inch round copper tube. The theoretical loss (Fig. 7) for the TE_{01} wave varies from 4 db per mile to 0.64 db per mile over the frequency range from 30,000 to 100,000 mc. The higher order circular electric waves have somewhat more attenuation but may be found useful as auxiliary transmission channels for short distances.

It is interesting and important, as will be pointed out later, to note that several of the higher order transverse electric waves have attenuation constants which may be within a factor of 3 to 6 times the attenuation for the circular electric wave. These waves, TE_{12} and TE_{13} , have very little wall current flowing in the direction of propagation and therefore approximate the low-loss properties of the circular electric wave family. Under certain conditions of mode coupling, which will be described at a later point, it is undesirable for the medium to be able to propagate modes with attenuation coefficients comparable to that of the mode which is used for communication purposes.

SOME RESULTS OF TRANSMISSION EXPERIMENTS

Transmission experiments have been conducted on the 500-ft waveguide line shown in Fig. 9.* Supports for the line were set in concrete

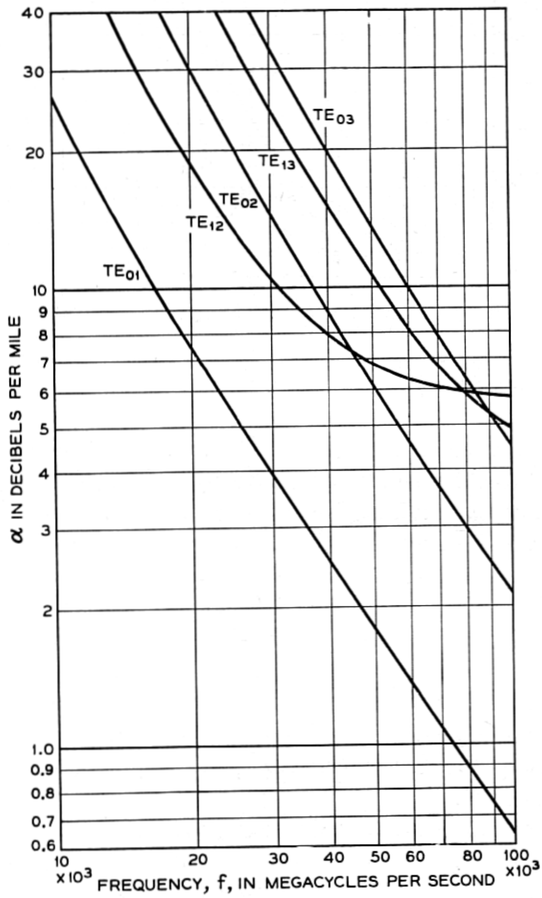

Fig. 7 — Attenuation versus frequency for a 2″ diameter round waveguide.

* This is the same line used for the work reported in Reference 1. Some of the experiments described in Reference 1 are also described here in order to furnish background for the new material.

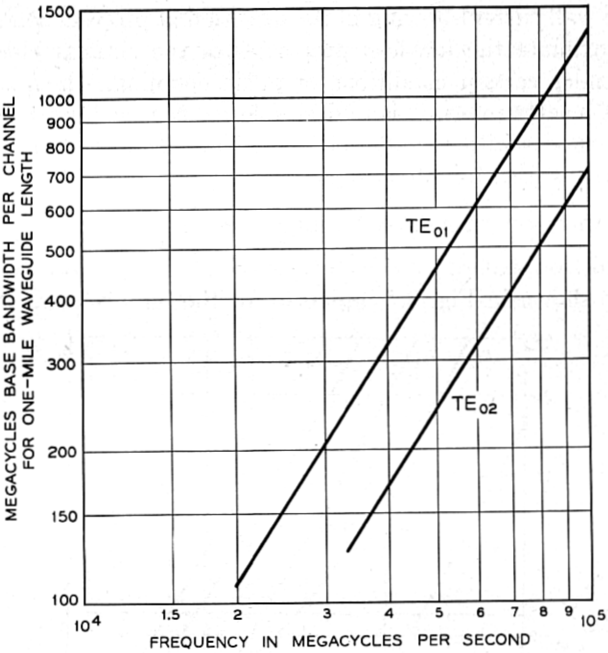

Fig. 8 — Base band width per channel versus frequency for a 2″ diameter pipe (one-mile waveguide length).

and optically aligned so as to provide a waveguide straight within about ⅛″ over its entire length. The philosophy behind this installation was the familiar one of providing for experimental purposes, as close to the ideal line as possible so that deviations could be created in a controlled manner. The inside diameter is about 4.73″, chosen to obtain the desired theoretical loss of about 2 db per mile at 9,000 mc, where measuring equipment was readily available. The difference between the major and minor inside diameters of the pipe was in the range 0.005″ to 0.008″ at the ends of the sections which averaged 20 ft in length. At the time this work was initiated, in 1946, generators of higher frequencies which would permit the use of smaller waveguides had not yet become available for use in this research.

Tests were conducted on this line using a technique due to A. C. Beck,[1] and involving the layout of equipment shown in Fig. 10. Short bursts of *RF* energy approximately 1/10 microsecond in duration, were injected into the line at intervals of about three hundred microseconds. Except for two small holes through which to couple to the transmitter and receiver, the waveguide line was short-circuited at both ends. The injected $\frac{1}{10}$ microsecond pulse occupied at any instant a space interval of 100 feet and therefore this pulse, while travelling from one end to the other between the short circuits, produced at the receiver coupling hole spurts of energy corresponding to the time when the pulse passed the sending end. Each such pulse passing through the receiver coupling hole was amplified, detected, and placed as a vertical deflection on the oscilloscope. The horizontal deflection on the oscilloscope was a linear time

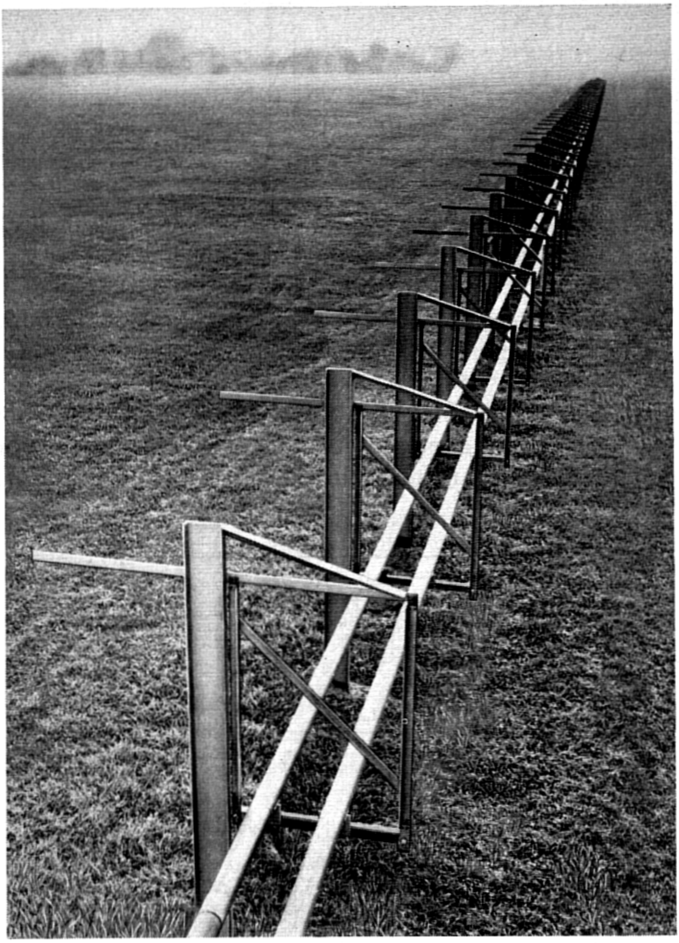

Fig. 9 — Experimental waveguide installation, 5″ diameter Holmdel line.

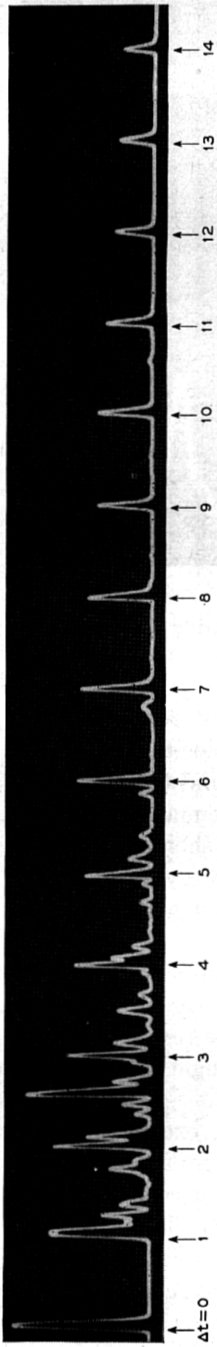

Fig. 12 — Same as Fig. 11 except that a longer time interval is shown.

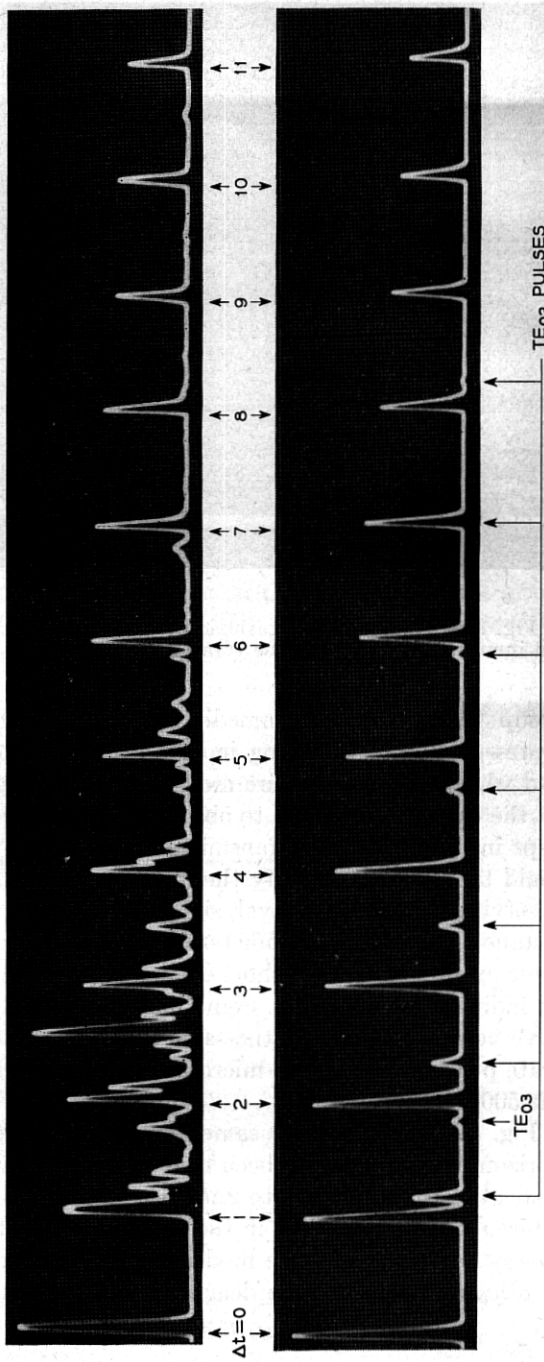

Fig. 14 — The effect of a mode filter in the 500-foot line. The unlabeled pulses in the lower part are in the TE_{01} mode.

appear at regular intervals and with smoothly decaying amplitude. This behavior indicates that the major portion of the energy in the line was travelling in a single mode, and we deduced that this mode was TE_{01} as follows: We observed that the velocity of propagation was near that for the TE_{01} mode by measuring the absolute time between pulses, averaged over many round trips. This excluded all but about 6 modes whose cut-off frequencies are near that of TE_{01}. Measurement of transmission loss was made by observing the rate of decay of the received pulses averaged over 10 or more round trips. The measured loss was found to be approximately 3 db per mile compared to a theoretical value of 1.9 db per mile for TE_{01} propagation. It follows that propagation must have

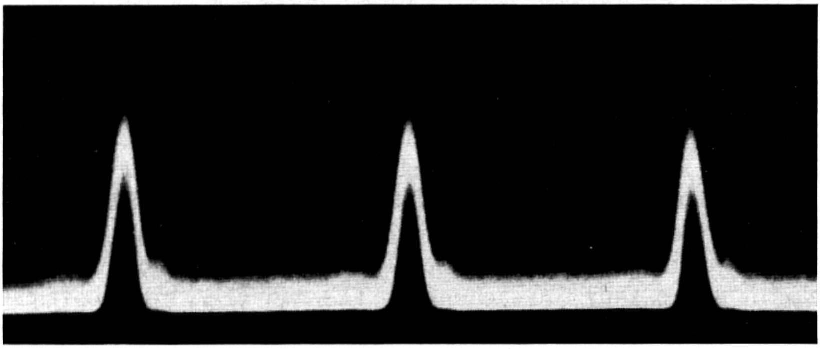

Fig. 13 — Record of pulses after 40 miles of repeated traversal over the 500-foot line.

been taking place in the TE_{01} mode, for all other modes near TE_{01} in velocity have theoretical losses well in excess of the observed value.

To summarize the effects shown in Fig. 12, a great many modes including TE_{01} were launched by exciting the waveguide through a small aperture in the end plate. All these modes propagated back and forth in the line for a while, but due to the fact that TE_{01} has appreciably less loss than the other modes, the energy remaining in the line after a suitable time delay was substantially all in the TE_{01} mode. This permitted measuring TE_{01} loss over a distance of many miles by allowing the energy to traverse the 500-foot line many times.

Fig. 13 records three successive trips of a pulse which had travelled up and down the 500-foot waveguide for a total distance of 40 miles. The pulse shape was still essentially the same as that of the transmitted pulse, although background noise had become clearly visible. We certainly can conclude from this that circular electric wave transmission over great distances is possible.

The long waveguide line also provided a very convenient way of demonstrating additional multi-mode transmission effects. For example, in a multimode medium we may use mode filters. One such filter may have a very low loss for the circular electric waves but very high loss to other waves. Such mode filters have been built, and Fig. 14 shows the transmission changes which result when introducing one. The upper half of Fig. 14 shows the photograph of the oscilloscope in the time interval 0 to $11\Delta t$ with no mode filter in the line. (This is the same as Fig. 12.) On introducing the mode filter into the waveguide, the received signal is altered as shown in the lower half of Fig. 14. We observed that energy propagating in the undesired modes has largely disappeared; it was absorbed by the mode filter.

There are still a few small pulses in the lower half of Fig. 14 which cannot be in the TE_{01} mode because of their time position. Starting at time $1.15\Delta t$, there is a series of regularly spaced pulses in Fig. 14 labeled TE_{02}, and a single small pulse labeled TE_{03}. The geometric placement of resistive material in the mode filter leads us to anticipate low filter losses for the entire circular electric (TE_{0n}) family of modes, and therefore the extra pulses were suspected of being in higher-order circular electric modes. Only two such modes, TE_{02} and TE_{03}, were above cut-off. The TE_{03} pulse was tentatively identified by noting that its group velocity was 55 to 60 per cent of that of the TE_{01} pulses. High attenuation in the TE_{03} mode prevented additional TE_{03} pulses from being observed.

In the case of the TE_{02} series of pulses, it was possible to get a fairly accurate measure of relative group velocity, confirming the identification as TE_{02}. Note that the seventh TE_{01} pulse coincides with the sixth TE_{02} pulse, and that the pulse at $7\Delta t$ shows on the TE_{01} train as being too large in amplitude.

The smooth decay of the TE_{01} train in the lower half of Fig. 14 in the interval Δt to $6\Delta t$ is in marked contrast to the corresponding pulse train in the upper half of the figure, and is graphic illustration of the important effects that mode filtering can produce.

Another very important transmission observation appeared during Mr. Beck's experiments with the 5″ diameter line. He observed that the attenuation, as measured by the amplitude decay of the shuttling pulse, was a function of the position of the piston at the far end of the line. Translations of the far end piston on the order of 10 to 40 centimeters changed the overall transmission from a condition in which the original pulse shape was preserved for as many as 40 miles of travel (Fig. 13), to a condition wherein the shape of the pulse was badly distorted after only 3 or 4 miles of travel. This general behavior is illustrated by the photographs shown in Fig. 15. We will concentrate for the moment on the top two rows of photographs which record the pulse transmission in the bare waveguide as a function of distance of pulse travel for both favorable and unfavorable piston settings. However, all of the rows of the photographs were taken under such conditions as to permit direct comparison.

The photographs at the extreme left end represent the outgoing pulse and the first echo pulses which travelled one round trip in the line, approximately 340 yards. All the other photographs show two principal pulses which record two successive trips of the pulse as it passed the transmitting end (Fig. 10). The second photograph from the left represents the pulse as it passed the sending end after 10 and 11 round trips. The third picture from the left records the 20th and 21st trip, the fourth picture the 30th and 31st trip, etc. The numbers placed directly beneath the individual photographs represent the relative sensitivity of the receiver for that particular photograph. Reference receiver sensitivity was taken as the condition under which the 10th and 11th trip in the bare waveguide were recorded with a favorable piston setting (0 db beneath the photograph), and the designation −6 db under the adjacent photograph indicates that 6 db more receiver sensitivity was used in the latter case. The relation between display amplitude and actual pulse amplitude was approximately square law. The distance of pulse travel associated with each of the pictures is given at the bottom of the figure.

Comparing the top two rows representing favorable and unfavorable piston positions, we note that the attenuation was appreciably different — the values being 2.6 db per mile and 3.1 db per mile respectively. Serious distortion of the transmitted pulse also occurred for the unfavorable piston setting. Since the receiver in the experiment was sensitive to very many modes, one might suspect that the spurious pulses which appeared at more than 7,000 yards with the unfavorable piston setting might represent energy present in some of the other modes. Actually, all of the pulses and wiggles shown in the photographs at ranges greater than 3,500 yards were in the circular electric mode. This was deduced by first noting that every two successive pulses were not appreciably different from each other (see Fig. 15). If some of the distortion effects shown by the received pulse were due to energy being received in modes other than the signal mode, then successive pulses would be different in shape because the various modes have different phase constants. The very gradual change in pulse shape which did

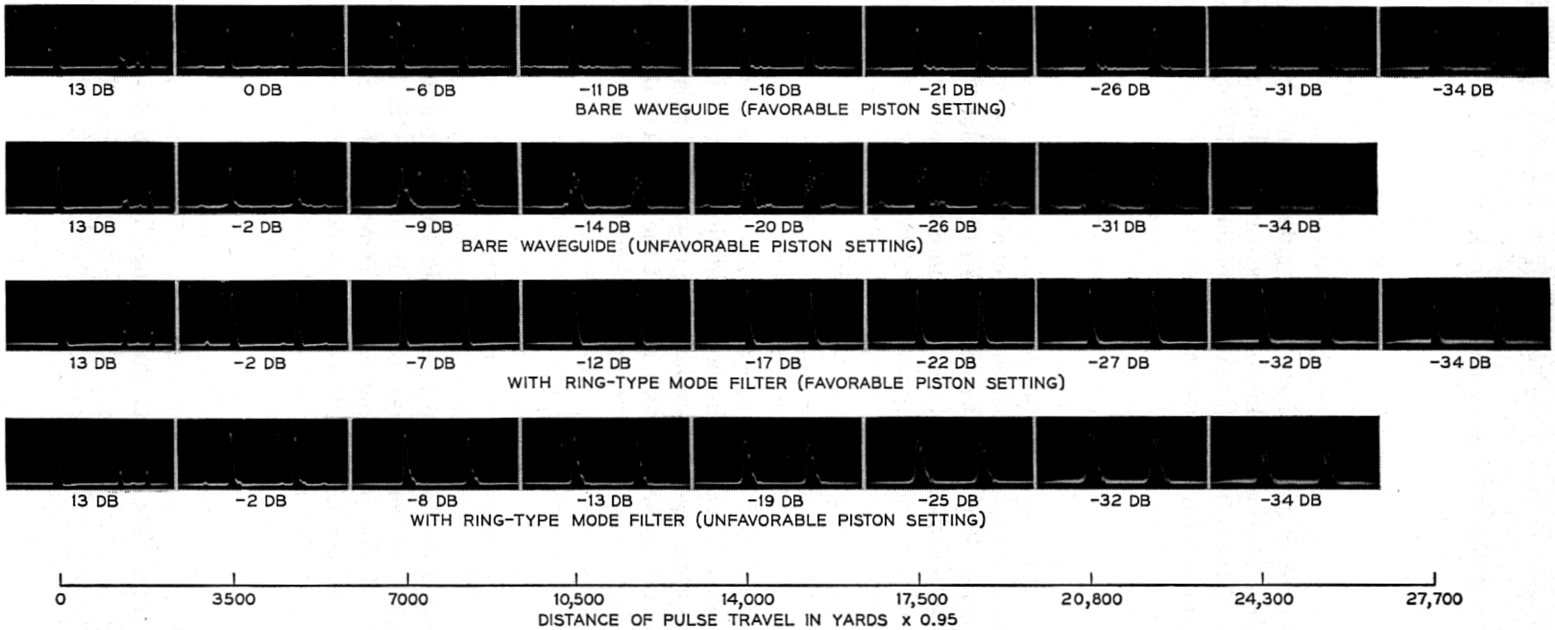

Fig. 15 — Photographs of pulse transmission with both favorable and unfavorable piston settings at the end of the 500-ft. waveguide.

occur as the pulse travelled up and down the line had the general form of an amplitude component which led or lagged the signal pulse by a constant interval but which gradually increased in amplitude with increasing distance that the signal pulse had travelled. Note, for example, the growth of spurious peaks before and after the signal pulse in row 2 of Fig. 15 at 3,500, 7,000, 10,500 and 14,000 yards of travel.

The explanation of this behavior involves transfer of energy from the circular electric mode to one or more of the unused modes of propagation and reconversion of the same energy back to the circular electric mode. This process is one of the characteristic features of multimode waveguide systems and is discussed at greater length in the following sections.

MODE CONVERSION AND RECONVERSION AS A SIGNAL-LOSS EFFECT

In beginning discussion of the mode conversion-reconversion phenomena, we take an idealized case of a dissipationless waveguide containing two deformities, Fig. 16. We assume a *c-w* signal entirely in the circular electric mode entering this waveguide. After passing the first deformity there will be energy present in some other mode, and this is designated as TX_1. When the combination of $TE_{01} + TX_1$ strikes the second deformity, another conversion takes place and the output will be a large TE_{01} component, two smaller components in the unused mode, TX_1 and TX_2, and a still smaller circular electric wave component, TE_{01}', which is due to reconversion of energy from TX_1 to the circular electric wave in traversing the second deformity. It can be shown[2] that for the proper distance between two identical symmetrical deformities, the wave emerging from the line may be purely circular electric; the two components TX_1 and TX_2 cancel each other under this condition. Another separation between the deformities results in a maximum energy transfer from circular electric to other modes. Therefore, it follows that any mechanism which varies the effective spacing between conversion points will produce conversion loss variations. This accounts for the change in the attenuation of the circular electric wave pulses in Fig. 15 as a function of the far end piston setting.

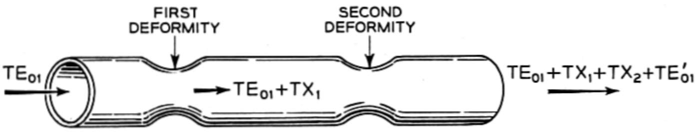

Fig. 16 — A distorted waveguide and the associated mode-conversion signal-loss effects.

MODE CONVERSION AND RECONVERSION AS AN INTERFERENCE EFFECT

The mode conversion-reconversion phenomena can also produce a signal distortion or interference effect. Fig. 17 shows another idealized waveguide containing two deformities, but in this case we assume a *short pulse* in the circular electric wave as the input signal. The amplitude-time plots below the waveguide show the energy present in the circular electric and unused waves respectively at various physical points along the line. The key item in this diagram is the time displacement between the converted energy TX and the circular electric wave energy at the input to the second deformity. This time difference appears as a result of propagation over an identical line length at two different group velocities which, in general, the circular electric and unused modes will possess. Since the signal and unused mode pulses strike the second deformity at different times the second conversion process results in energy appearing back in the circular electric wave at a time separated from the signal pulse itself. When the distance between deformities is too short for the pulses to be resolved at the second deformation, the result will be a distortion of the signal pulse rather than the appearance of a separate pulse.

The above very much simplified picture of the mode conversion and reconversion effects allows one to visualize several general properties of this phenomenon:

1. In general, there will be a large number of unused modes which will be coupled to the signal mode through the various imperfections in the transmission line. Since these unused modes have unequal phase constants, and since the imperfections will be randomly spaced along the line, the reconverted signal pulses in a time-division system will be spread

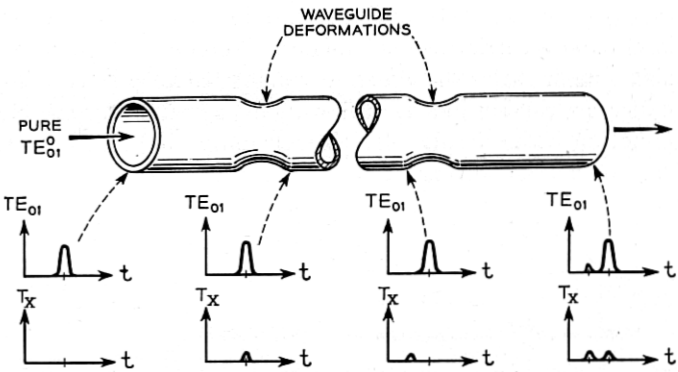

Fig. 17 — Signal interference effects due to mode conversion and reconversion.

out on the time scale rather than appear as a single well-defined distortion pulse.

2. Because some of the unused modes of propagation have group velocities greater than the group velocity of the circular electric wave, the reconverted energy pulses may reach the receiving end of the transmission system before the signal pulse itself appears.

3. The reconverted energy will, in general, be out of phase with the signal from which it was derived. When the differential time of travel in the unused mode is short, the principal effect of the conversion-reconversion process will be to distort the signal wave. When the differential time of travel in the unused mode becomes as large as the reciprocal of the modulation frequencies involved, the reconverted energy will appear more like an echo. In a time-division system such echo pulses would interfere with the pulses representing other signal components. Because of the large number of conversions contributing at random time delays, this "echo-interference" may be unintelligible. In this sense the interference may be thought of as a noise effect, just as multi-channel cross-talk due to amplitude non-linearities in a single sideband AM system may be thought of as noise.

4. The general case of a signal pulse, both preceded and succeeded by a series of reconverted energy pulses, is sketched in Fig. 18. It is quite apparent that if the reconverted signal pulses are allowed to become of the same order as the signal pulse itself, even a pulse code modulation system will be rendered inoperative. Other types of modulation will experience difficulty at appreciably smaller magnitudes of reconverted energy.

5. The level of the reconverted energy relative to the signal is determined by the transmission medium. It is not possible to avoid this interference by using more power at the sending end of the transmission link, for the interference rises with the signal. The need for low-noise receivers is just as acute as in other transmission systems, because better noise figure means that correspondingly less power is required from the transmitter.

6. If the loss to the mode TX is very large in the region between successive waveguide imperfections, the TX pulse can be attenuated to a negligibly small value before reaching the second deformation, thus preventing any significant reconversion back to the signal mode.

7. Limits can be placed on the time delay between the signal energy and the reconverted energy returned to the signal mode. The lower limit on this time delay is obviously zero, corresponding to a series of imperfections very close together. The upper limit can be taken as the differ

ence between the times it takes the signal mode and the unused mode to travel a distance in which there is a large difference between the ohmic losses in the unused-mode and signal mode. This concept may be expanded as follows: Energy is transferred at an imperfection located at coordinate z_1 on the axis of propagation, producing an amplitude in unused mode x. At z_2 on the axis of propagation the amplitude in mode x will be attenuated relative to the signal-mode amplitude at the same point by the factor

$$\epsilon^{-(\alpha_x - \alpha_1)(z_2 - z_1)}$$

where α_x and α_1 are the normal heat loss coefficients in mode x and the signal mode respectively. When the exponential factor is small enough (i.e., z_2 large enough), reconversion will no longer be important compared to reconversion near z_1. For order of magnitude we might assume that 10 db more attenuation for the x-mode amplitude than for the signal-mode amplitude would render further reconversion unimportant.

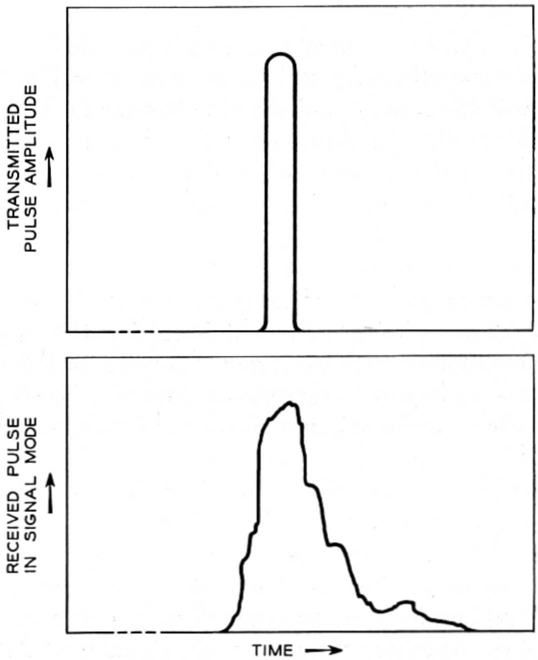

Fig. 18 — Schematic of signal distortion due to conversion and reconversion effects in a line with randomly placed conversion points.

Then we know the distance $(z_2 - z_1)$ from

$$(z_2 - z_1) = \frac{1.15}{(\alpha_x - \alpha_1)}$$

The corresponding "upper limit" on time delay between the signal and the reconverted energy in the signal mode is

$$t = (z_2 - z_1)\left(\frac{1}{v_x} - \frac{1}{v_s}\right) \tag{1}$$

where v_x and v_s are the group velocities in the mode x and the signal mode.

It is well known that the unused modes of a circular electric waveguide have attenuation coefficients which are appreciably larger than attenuation coefficients for the circular electric wave itself. In the light of this attenuation to the unused modes plus the fact that the reconverted energy has undergone two mode conversion losses before it reaches the signal mode again, one might wonder whether the mode conversion-reconversion phenomenon would really be an important effect. The first indication that the magnitudes of the reconversion amplitudes are significant came during experimental work on the 5″ diameter 9,000-mc line, described above in connection with Fig. 15. A more quantitative theoretical discussion which follows shows that the reconversion phenomena will continue to be important even when mode filters are introduced into the line.

The effects of the mode conversion-reconversion process are very similar to the effects of multipath transmission through the atmosphere. In microwave radio there are under unusual fading conditions 2 or 3 subsidiary signals, and these are representative of propagation over different path lengths in space but at the same velocity of propagation. In the waveguide, there will in general be a large number of subsidiary transmission paths, each of which corresponds to the identical distance of propagation but at velocities of propagation which are different for the various modes. The radio multipath phenomenon exists only occasionally, whereas the waveguide multipath phenomenon is a steady characteristic present 100 per cent of the time. Long-distance radio transmission in the 6–20 mc region by way of the ionosphere encounters multi-path effects more like those expected in the waveguide, with the exception that waveguide multi-path effects are expected to have far greater short-time stability.

Quantitative relations describing the conversion-reconversion process may be derived by considering an infinitely long waveguide composed of a series of identical sections each containing a single mode-conversion discontinuity at the midpoint. (The actual 500-foot line contains many conversion points, as will be discussed later, and is effectively repeated over and over as the pulse traverses the identical line many times.) A very short signal-pulse of unit amplitude is assumed as an input to the idealized line containing identical conversion points. After the first conversion point the amplitude in the unused mode is k and the amplitude in the signal mode is $(1 - k^2)^{1/2}$, where k is a measure of the size of the conversion irregularity. There is no reconverted wave at this point since there is no input to the first conversion point in the unused mode. After the second conversion point, however, there is a reconverted-wave amplitude $k^2\epsilon^{\theta_x}$, where θ_x is defined below. After the n^{th} conversion point, it may be shown that the amplitude in the signal mode is

$$\epsilon^{(n-1)\theta_1}(1 - k^2)^{n/2} \tag{2}$$

the amplitude in the unused mode is

$$\begin{aligned} &k(1 - k^2)^{\frac{(n-1)}{2}}\epsilon^{(n-1)\theta_x} \\ &+ k(1 - k^2)^{\frac{(n-1)}{2}}\epsilon^{(n-2)\theta_x+\theta_1} \\ &+ k(1 - k^2)^{\frac{(n-1)}{2}}\epsilon^{(n-3)\theta_x+2\theta_1} \\ &+ \text{etc. to} + k(1 - k^2)^{\frac{(n-1)}{2}}\epsilon^{(n-1)\theta_1} \end{aligned} \tag{3}$$

and the reconverted wave amplitude is

$$\begin{aligned} &(n - 1)k^2(1 - k^2)^{\frac{(n-2)}{2}}\epsilon^{\theta_x+(n-2)\theta_1} \\ &+ (n - 2)k^2(1 - k^2)^{\frac{(n-2)}{2}}\epsilon^{2\theta_x+(n-3)\theta_1} \\ &+ (n - 3)k^2(1 - k^2)^{\frac{(n-2)}{2}}\epsilon^{3\theta_x+(n-4)\theta_1} \\ &+ \text{etc. to } k^2(1 - k^2)^{\frac{(n-2)}{2}}\epsilon^{(n-1)\theta_x} \end{aligned} \tag{4}$$

in which

z = distance between adjacent conversion points

α_1 and α_x are the heat loss coefficients (applying to wave amplitudes) for the signal and unused modes respectively.

β_1 and β_x are the phase constants for the signal and unused modes respectively.

$\theta_1 = -(\alpha_1 + j\beta_1)z$

$\theta_x = -(\alpha_x + j\beta_x)z$

k = amplitude of the converted wave for a single conversion point and for unit wave incident on the conversion point.

In deriving the series represented by (2), (3) and (4), it is assumed that all of the converted power travels in the forward direction, and that reflection effects are negligible. These conditions are typical of imperfections in practical multi-mode waveguides.

When the input pulse for the idealized line is sufficiently short, the various terms of (3) and (4) (representing successive conversions and reconversions) are non-overlapping pulses. It is instructive to write down the ratio of the signal-wave amplitude to the reconverted wave amplitude which is separated from the amplitude of the signal wave at the same point by the time difference $z(1/v_x - 1/v_1)$, in which v_x and v_1 are group velocities. This ratio is [ratio of (2) to the first term of (4)]

$$\frac{(1 - k^2)}{(n - 1)k^2\epsilon^{-(\alpha_x - \alpha_1)z}} \tag{5}$$

It is clear from this ratio that the signal wave may be smaller than the reconverted wave if n is sufficiently large. Physically, what happens is that the reconverted amplitude created at each successive conversion point adds in phase with the reconverted wave amplitude present at that point due to previous conversions and reconversions. This happens of course because the line contains identically spaced conversion points. With random location of conversion points, a less severe build-up of reconverted wave energy would certainly occur.

The first and second reconverted pulses [i.e., the first and second terms in (4)] are separated by the time difference $z(1/v_x - 1/v_1)$ and the ratio of the amplitude of the second to the first reconverted pulse is

$$\frac{(n - 2)}{(n - 1)}\epsilon^{-(\alpha_x - \alpha_1)z} \tag{6}$$

The largest amplitude existing in the unused mode at a point immediately following the n^{th} conversion is the component converted at the n^{th} conversion point and the ratio of this component to the amplitude of the signal pulse after the n^{th} conversion is:

$$\frac{k}{(1 - k^2)^{1/2}} \tag{7}$$

Note that this ratio is independent of n. The unused-mode amplitude converted at the first conversion point is, after n trips and relative to the signal pulse at the same point,

$$\frac{k}{(1 - k^2)^{1/2}} \epsilon^{-(n-1)(\alpha_x - \alpha_1)z} \tag{8}$$

These expressions show that the largest unused mode amplitude at any point in the line will be the one most recently converted from the signal wave and will be nearest to the signal pulse in time position.

The sketch shown in Fig. 19 represents schematically the signal pulse, the unused mode pulse and the reconverted pulse amplitudes after n trips past the conversion point. It is interesting to note that the most recent (the n^{th}) conversion to the unused mode appears in a time position close to the signal pulse whereas the most recent reconversion appears at a time far removed from the signal pulse.

Let us investigate the ratios (5) through (8) under conditions representative of those in the 5″ diameter waveguide line. Row 2 of Figure 15 shows that after 40 trips down and back on the 500-foot line, i.e., after 80 trips past the center of this line (where there is assumed a single conversion point), the amplitude of the reconverted pulses which appear just before and just after the signal pulse are about equal to the signal

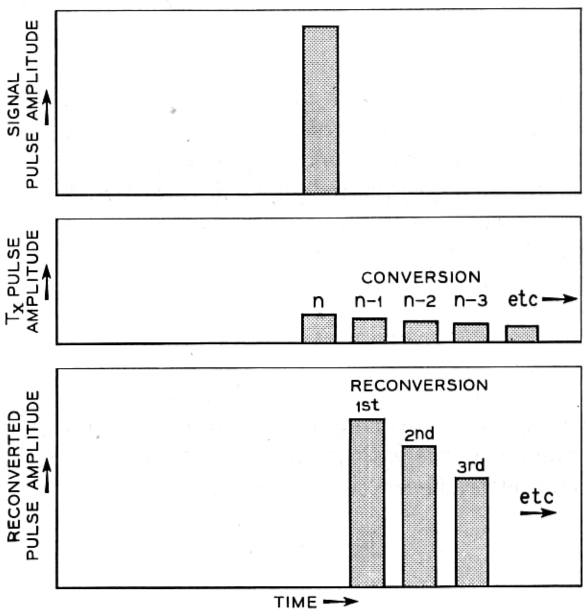

Fig. 19 — Sketch of pulse amplitudes in a line having equally spaced conversion points.

pulse. We may then set the ratio of signal-wave amplitude to the reconverted-wave amplitude, as given by (5), equal to unity at $n = 80$. We know the theoretical values of the heat loss coefficients α_x and α_1 for the unused and signal modes respectively. Allowing modest increase for surface roughness effects, we will assume the heat loss for the signal (TE_{01}) wave to be 0.2 db between conversion points (about 500 feet apart) and the loss for the unused mode to be about five times this or 1 db between conversion points. Substituting these numbers into (5) we can solve for the conversion coefficient k which is necessary to obtain the observed equality between reconversion and signal wave amplitudes at the $n = 80$ point. Such a calculation yields a value of k of 0.117 and this implies a conversion loss which is 32 per cent of the heat loss for the signal wave. Direct observations of mode conversion (to be described) show that the conversion losses in the 5″ diameter line must be at least as large as this. Therefore, we have confirmed that the basic mechanism which we are discussing can indeed account for a break-up of the signal pulse of the general type actually observed.

The calculated ratio of the second reconverted pulse compared to the first reconverted pulse from (6) for the $n = 80$ condition described above is only -0.5 db, which shows that if we really had a single conversion point which could add exactly in-phase in the manner assumed, we would have in our photographs an even worse series of reconverted pulses than actually do appear.

Again for the $n = 80$ condition in the 5″ line the calculated amplitude of the unused mode component relative to the signal component at the same point is -18.5 db for the n^{th} conversion and, with the square law response of the display system, it is to be expected that such an amplitude would not be observed. This fits satisfactorily with the observation that the pulses in the 5″ line, after 40 round trips, are all in the circular electric mode even though the conversion-reconversion process is significant.

As already implied, the mode conversion situation for the actual 500-foot line, which is represented by the photographs of Fig. 15, is more complicated than the idealized line just analyzed. First, note that in the case of the nonoverlapping pulses in the idealized line the position of the far end piston (i.e., the spacing between the conversion points) has no critical effect on the conversion and reconversion amplitudes. In the experiment (Fig. 15), far-end piston movements of a foot or so caused very significant changes in the observed conversion and reconversion. This is a consequence of the fact that in the physical 500-foot line, there were a large number of conversion points, and the pulse width was not small enough to produce non-overlapping effects. If all of the conversions in the 500-foot length were between TE_{01} and a single additional mode, then it may be shown that the piston position which would cause in-phase addition of components from one conversion point on successive traversals of the 500-foot line would also cause in-phase addition for the components from all of the other conversion points. However, in the physical line significant conversion takes place between TE_{01} and three or more other modes, each of which has a different phase constant and each of which requires a different piston setting for in-phase (or out-of-phase) addition of components generated on successive traversals. Whereas it is not possible to relate the shuttle-pulse observations of Fig. 15 to a simple quantitative theory, the general behavior of the experimental line is in qualitative agreement with the conversion-reconversion explanation.

The favorable piston setting in Fig. 15 represents an infinite line in which approximate amplitude cancellation of mode conversion effects is achieved, whereas the unfavorable piston setting corresponds to an infinite line in which approximate amplitude addition of mode conversion effects is experienced. Since the former is optimistic and the latter is pessimistic compared to what might be expected in a physical long line, it has been our practice to average the loss data obtained at the favorable and unfavorable piston positions.

The expression (5) for the ratio of the signal pulse to the reconversion pulse shows that appreciable loss between the conversion points, represented by the factor $\epsilon^{-(\alpha_x - \alpha_1)z}$, is an effective way of reducing the reconverted wave effects. During the early experiments it was found that mode filters did reduce the influence of the far end piston. In Fig. 15, rows 3 and 4, show records of the 5″ line pulse transmission with a single mode filter at the sending end. The mode filter did not completely eliminate the phasing effects of the piston, and this may be expected for at least two reasons: (1) the mode filter itself may cause some mode conversion, and this mode conversion component will be influenced by the piston setting, and (2) the attenuation introduced by the mode filter is not sufficient to completely eliminate the wave components in the unused mode.

Even in the presence of the best mode filter now available, row 4 of Fig. 15 shows that the conversion-reconversion phenomenon does take place. The reconverted pulse amplitudes show up at 7,000 yards in row 4 as small distortions on the right hand side of the signal pulse, and these distortions grow in magnitude as we proceed to the right in the series of pictures representing more trips past the conversion points.

In the 5″ line there are several modes for which the delay factor $z(1/v_x - 1/v_1)$ does not result in a separation between the reconverted pulse and the signal pulse until z is on the order of 5,000 feet. Thus, we might expect the conversion-reconversion phenomenon to broaden the signal pulse. This does indeed take place even for the favorable piston setting of the bare waveguide, as shown by the top line of pulses in Fig. 15. The pulses at the distance 3,500 yards are sharper than those pulses at the distance 27,700 yards. However, addition of the mode filter (which introduces negligible signal attenuation) does appreciably sharpen the pulse at the 27,700 yard distance (row 3).

Thus, on the basis of pulse transmission observations on the 5″ line and a simple theoretical analysis, we conclude that the conversion-reconversion phenomenon will be important in a waveguide system, and that it is important to have as much dissipation as possible present in the unused modes of propagation.

ANALYSIS FOR CONTINUOUS MODE CONVERSION

The traveling pulse type of theoretical analysis utilized in the preceding section can be extended to describe a more realistic spatial distribution of conversion points and to include a series of unused modes instead of only one. An extension of this type is required in order to calculate directly the behavior which might be expected in a waveguide composed of randomly disposed irregularities.

A much simpler mathematical treatment, originally suggested to the writer by J. R. Pierce, is to assume uniform mode conversion along the direction of propagation and to represent this condition by a differential equation. An analysis of this type is attached as an appendix. The work includes the assumption of quadrature addition of conversion and reconversion components, and the total magnitude of such components given by the analysis may be thought of as the rms average of the conversion magnitudes in waveguide lines containing randomly located imperfections. Any single line might show somewhat more or less conversion effects, a factor of ± 10 db probably being adequate to cover most lines containing randomly located imperfections. If practical lines show appreciable correlation between the spacings of the conversion points, the reconverted-wave magnitude would become greater. The analysis has the advantage of being simple and understandable and should give overall trends accurately.

This analysis shows that the waveguide performance with regard to conversion-reconversion effects is completely specified with knowledge of (1) the ratio of the conversion coefficient (a_{1x}) to the true dissipation coefficient* in the signal mode (a_{1h}), (2) the ratio of the heat-loss coefficient of the unused-mode to that for the signal-mode (a_{xh}/a_{1h}), and (3) the length of the transmission line specified in terms of decibels of heat loss to the signal wave. For a given ratio of conversion-loss to heat-loss, the same ratio of signal-power to reconverted-wave power will be present for a long low-loss waveguide as for a short high-loss waveguide. This makes it important to determine the ratio of conversion-loss to heat-loss for waveguides of several nominal attenuation coefficients and to predict these effects theoretically insofar as it is possible.

Another result of this analysis is plotted in Fig. 20, which shows the ratio of the signal power to the power in the unused mode at the end of the line, with transmission-line heat loss as the abscissa. These curves have been plotted for a fixed magnitude of conversion loss coefficient (a_{1x}) equal to 50 per cent of the true heat loss coefficient (a_{1h}) and for ratios (a_{xh}/a_{1h}), heat loss in the unused mode to heat loss in the signal mode, between 2 and 100. These values are typical of solid round waveguide without mode filters. It is interesting to note in Fig. 20 that the magnitude of the unused mode power relative to the signal mode power reaches very nearly a constant value in a transmission line length of only ½ to 1 db, except for extremely low ratios a_{xh}/a_{1h}. Physically what is happening is that the unused mode power becomes dissipated through heat loss about as rapidly as it is created by mode conversion, after an initial short transmission line length.

Fig. 21 shows the ratio of signal power to reconverted wave power as a function of transmission line length for the same conditions described in connection with Fig. 20. A heat loss ratio on the order of 2 to 10 is typical of important modes in solid round waveguide without the addition of mode filters,† and Fig. 21 shows that the ratio of signal-to-reconverted wave power for such a medium becomes poorer than 20 db for transmission line lengths longer than 1.5 to 2 db. Although there is some uncertainty as to the precise interpretation which may be placed on the signal power to reconverted wave power calculated in this manner, since the time relations in connection with a definite modulation method are not included, it seems evident that a solid copper tube without mode

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* There is a very significant difference between the effects of signal power loss to other modes through conversion and signal power loss due to dissipation in the waveguide walls. However, it does not matter here whether the latter be due to surface roughness, chemical impurity or just the theoretical minimum heat loss for ideal copper. Therefore, all of the heat loss effects are combined into the single coefficient, a_{1h}.

† See the appendix for further discussion.

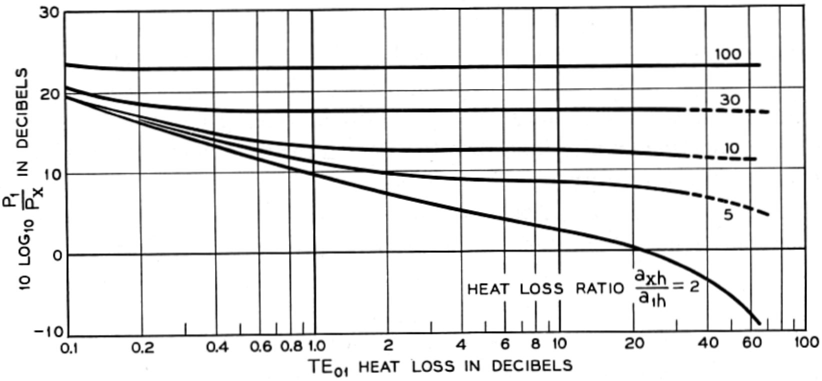

Fig. 20 — Ratio of TE_{01} signal power (P_1) to X-mode power (P_x) versus line length for conversion coefficients (a_{1x} and a_{x1}) equal to one-half the heat loss coefficient (a_{1h}).

filters is very unlikely to be satisfactory for long distances as a communication medium. However, the addition of mode filters will raise the heat loss to the undesired waves, and the latter improves the signal to reconverted-power ratio directly as the ratio of heat loss in the unused wave to heat loss in the signal wave. Thus, as shown on Figure 21, a heat loss ratio on the order of 500 would produce a ratio of signal power to reconverted wave power on the order of 20 db at a transmission line length of 60 db. It may be shown that the magnitude of the signal to reconverted wave power varies as the square of the conversion to heat loss coefficient ratio a_{1x}/a_{1h} .

The sharp break downward on the right hand end of the curves for $a_{xh}/a_{1h} = 2$ in Figs. 20 and 21 represents the condition wherein the power in the reconverted wave becomes comparable to the power remaining in the signal wave.

We next consider the improvement in signal fidelity which results from the introduction of ideal mode filters. We shall assume the ideal mode filters have a matched impedance for all modes, very high transmission loss to the unused modes, and no transmission loss for the signal mode. The improvement in the ratio of signal power to reconverted wave power due to the addition of such filters is shown in Fig. 22. This plot has been calculated for a total line length of 20 db heat loss, but the conclusions are valid for any line length wherein the signal wave power remains appreciably larger than the reconverted wave power. We observed that mode filters improve the signal-to-reconverted-wave powers very slowly when placed far apart, typical improvements ranging between 2 and 8 db for 1 db separation between mode filters. The larger improvement is obtained when the heat loss in the undesired mode is more nearly comparable to the heat loss in the signal mode before the addition of the mode filter. For very small spacing between the mode filters, addition of the ideal mode filters improves the signal-to-reconverted wave power by very large factors.

A somewhat different form of effect due to the addition of mode filters is observed if the line before the introduction of the mode filter has a reconverted wave power larger than the signal power. Under this condition, relatively large mode-filter spacings bring about a large improvement in signal-to-reconverted wave power, but this is due to the very poor condition present before filtering. It is doubtful whether the transmission line would become very useful without rather strong mode filtering of the type represented in Figure 22.

We may conclude that the mode filters should be placed very close together, preferably at spacings of less than .1 db heat loss to the signal wave. We may also conclude that the transmission of signals over distances corresponding to the order of 60 db heat loss will require either

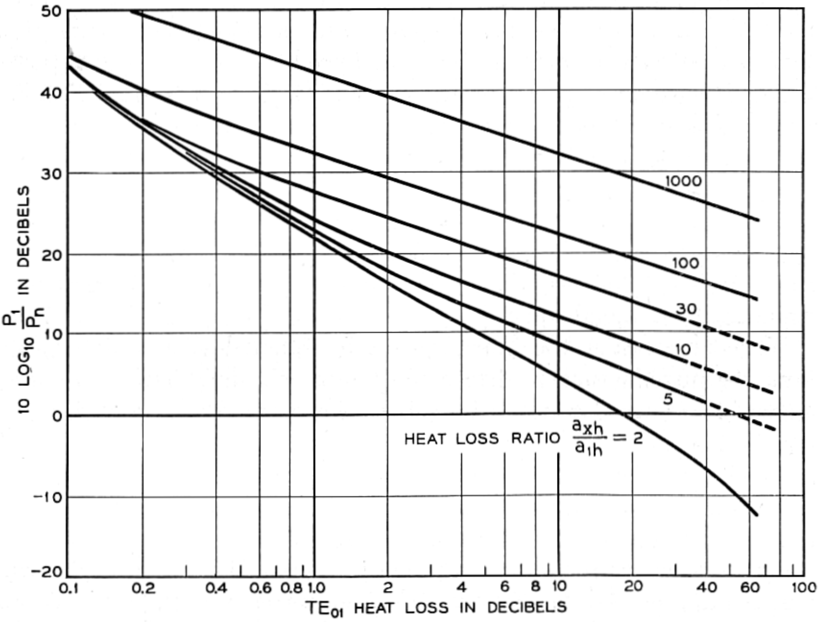

Fig. 21 — Ratio of TE_{01} signal power (P_1) to reconverted TE_{01} power (P_n) versus line length for $a_{1x} = 0.5\, a_{1h}$.

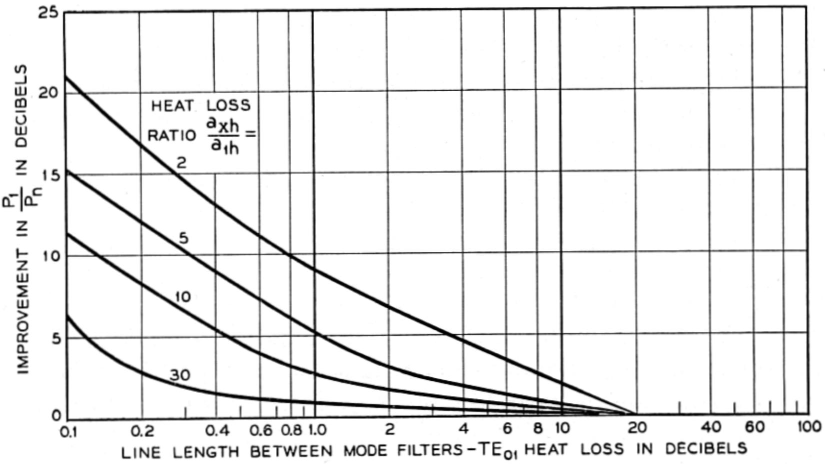

Fig. 22 — Improvement in P_1/P_n due to addition of mode filters. Total line length equals 20 db TE_{01} heat loss. (Plotted for $a_{1x} = 0.5\ a_{1h}$).

the spacing of mode filters at less than 0.1 db to the signal wave or use of a continuous form of transmission line having a ratio of heat loss in the unused mode to the heat loss in the used mode on the order of 500.

DIRECT EVALUATION OF MODE CONVERSION MAGNITUDES

The important influence that mode conversion effects are expected to exert on signal fidelity lead us to make direct evaluations of the conversion coefficients. The direct evaluation consisted of transmitting (actually or in imagination) a pure circular-electric wave into one end of a waveguide section and, by measurement or by calculation on the basis of known geometry, determining the relative magnitude of the power converted to the unused modes.

The simplest experimental technique for analyzing mode impurities consists of a short radial probe at the guide wall. The radial probe responds to energy in any mode of propagation except the circular electric family, and serves as a versatile instrument for measuring the *order of magnitude* of mode conversion effects. The limitations of the technique stem from (1) the fact that the probe responds to the vector sum of the amplitude of the radial electric field components of about 35 modes (in the 5″ line case), and this sum varies with circumferential and longitudinal position of the probe even though the power present in the modes is constant; and (2) the fact that the sensitivity of the probe response to a given magnitude of power in the guide is variable from mode to mode, being (in an extreme case) 25 db greater for TE_{91} than for TE_{13} in the 5″ pipe. For the majority of modes, however, the latter variation is ±3 db, and the maximum probe response as a function of curcumferential and (to a limited extent) longitudinal position can be determined without excessive labor.

The probe technique of mode conversion evaluation was first applied by M. Aronoff to the individual sections of the 5″ diameter experimental line. He found that the average indication of conversion for the (approximately) 20 ft. lengths of pipe was 29.5 db below the signal wave power; since the power loss due to dissipation in the walls for a 20 ft. section is about 27 db below the signal wave power, the individual pipe measurements gave an order-of-magnitude estimate of 0.55 for the ratio of conversion loss to heat loss (a_{1x}/a_{1h}). Four mechanically distorted sections of line, previously considered satisfactory, were identified and discarded on the basis of this approach.

A. C. Beck and M. Aronoff next applied the probe technique to the 5″ diameter line assembled into lengths of 145 feet, 270 feet, and 500 feet. The indications of converted power were −17 db, −10.5 db, and −13 db respectively (at wavelengths near 3.2 cm) which is compatible with the hypothesis of random addition of a number of conversion components. Since the heat-loss power for the 500-foot line is about −13 db compared to the incident signal power, the 500-foot line radial probe measurement gave an order of magnitude estimate of 1.0 for a_{1x}/a_{1h}, in fair agreement with the value of 0.55 from single pipe measurements. The probe indication of conversion as a function of frequency for the 500-foot line is plotted in Fig. 23, which shows that quite a number of

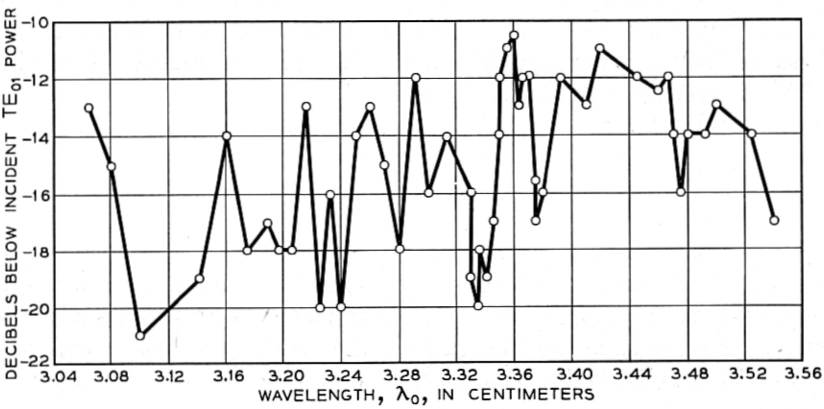

Fig. 23 — Probe recording of converted power in the 500-foot line.

conversion components were present; similar data were observed on the shorter lengths of line.

Azimuthal distributions of probe response showed conclusively that very little conversion was present to the 13 TE_{nm} and TM_{nm} modes having an index "n" of four or more.

A much more precise though more elaborate method of evaluating the conversion coefficients involves use of coupled wave transducers.[2] Such devices respond to only one mode and have known sensitivities, and therefore permit truly quantitative measurements. (At the present time this approach requires a separate transducer for each mode to be evaluated, a somewhat cumbersome procedure, but in principal the mode transducers can be made "tunable" for a series of modes.) A. C. Beck and M. Aronoff applied this more accurate method of conversion coefficient determination to several modes of the 500-foot line, and Table I shows the values averaged over the frequency band.

Table I — Average Ratio of Conversion to TE_{01} Heat Loss with TE_{01} Excitation

Mode	a_{1x}/a_{1h}
TE_{11}	0.21
TM_{11}	0.05
TE_{21}	0.14
TE_{31}	0.05
TM_{01}	<0.001
Total	0.47

The estimated absolute accuracy is between 10 per cent and 20 per cent for these ratios. The variation of the conversion coefficients as a function of frequency is shown in Fig. 24 for two of the important modes and for the total of the modes given in Table I. The total of the modes measured is consistent with the probe indications, though the accuracy of the latter is low enough that this should not be interpreted as proving there are no other important conversion contributors.

The above direct evaluation of mode conversion in the 500-foot line yielded magnitudes that are sufficient to explain the conversion-reconversion process as already outlined. We wished to extend our experience with this particular line to higher-frequency lines which might be built and to absolute tolerances that might be placed on the construction of a new line. Toward this end, theoretical relations were derived by S. P. Morgan, Jr., for the mode conversions to be expected due to waveguide ellipticity, and due to the tilt and offset which may be expected to occur at the junction of two sections. Experimental work was done by M.

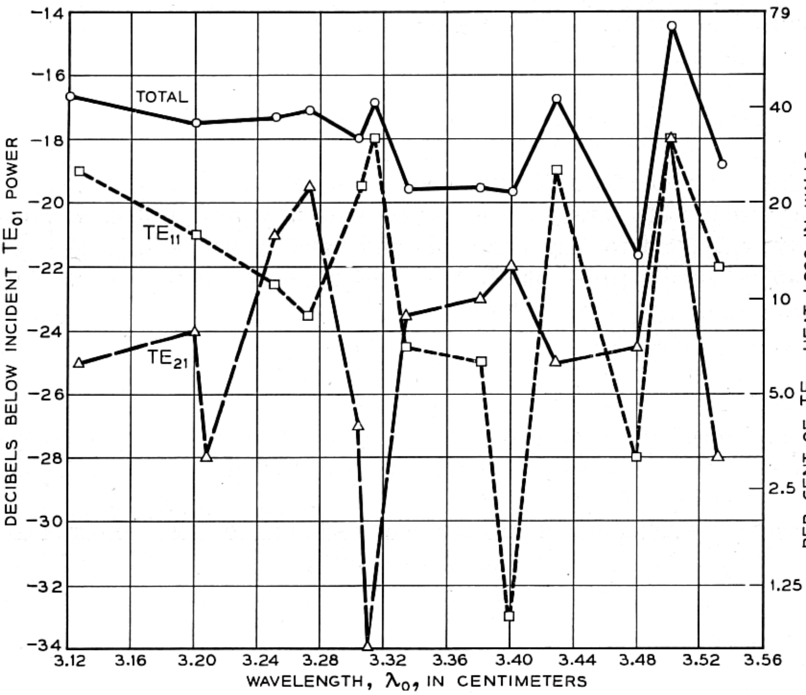

Fig. 24 — Observed conversion from TE_{01} to TE_{11}, TE_{21}, and the sum of TE_{11}, TE_{21}, TM_{11}, TM_{01}, and TE_{31} for the 500-foot line.

Aronoff[3] at 9000 mc on accurately created imperfections of the above types, and he found excellent agreement. We proceed to use the theory to compare the present line to a hypothetical 50,000 mc line. The results are given in Table II.

These computations represent a single oval section or a single tilted or offset joint. The converted power varies as the square of the tilt angle, as the square of the offset distance, and as the square of the difference between the major and minor diameters. The total ratio of converted power to heat loss power depends on the number of conversions per unit length. For the accuracy of constructing a 2″ diameter line assumed in Table II, the amount of mode conversion to be expected is not appreciably different from what appears attributable to known mechanical imperfections in the 500-foot line already discussed.

Another waveguide property of interest is the way the mode conversion magnitudes vary across the frequency band in a fixed pipe, and Table III shows this for the 2″ diameter line.

TABLE II — MODE CONVERSION COMPARISON OF 9000-MC 4.732″ DIAMETER LINE WITH A HYPOTHETICAL 50,000-MC 2″ DIAMETER LINE

Imperfection		Pipe Diameter	Frequency	Magnitude of Converted Power	
Type	Magnitude			Percentage	Mode
		inches	*Mc*		
Ovality	16* mils	4.732	9000	0.53%‡	TE_{31}
Ovality	4*	2.0	50,000	0.18%‡	TE_{31}
Tilt	1°	4.732	9000	0.34%	TE_{12}
Tilt	1°	2.0	50,000	2.0%	TE_{12}
Offset	10† mils	4.732	9000	0.008%	TE_{12}
Offset	2.5†	2.0	50,000	0.003%	TE_{12}

* Difference between major and minor diameters.
† Separation of guide axes.
‡ These are upper-limit values, based on the length of the oval section of pipe which would produce maximum mode conversion, and based on a cross-sectional shape (trifoil) which would produce the maximum of mode conversion.

It is interesting that two of the three conversion effects are essentially independent of frequency. Tilt at a waveguide junction introduces a phase-front error and would be expected to cause greater conversion effects at increasing frequencies. We shall see that bends produce a similar mode conversion, also due to a phase front error, that increases with increasing frequency.

THE BEND PROBLEM

The problem of transmitting the circular electric wave around bends was recognized as being important at an early date, and contributions to its solution were made by M. Jouguet,[4,5] W. J. Albersheim,[6] S. O. Rice,[7] and the writer.[8] The essence of the problem is as follows: A bend in a

TABLE III — FREQUENCY VARIATION OF MODE CONVERSIONS IN 2″ DIAMETER GUIDE

Imperfection		Mode	Magnitude of Converted Power		
Type	Magnitude		f = 24,000 mc	f = 50,000 mc	f = 75,000 mc
Ovality	4 mils	TE_{31}	0.22%*	0.18%*	0.20%*
Tilt	1°	TE_{12}	0.41%	2.0%	4.8%
Offset	2.5 mils	TE_{12}	0.003%	0.003%	0.003%

* These are upper-limit values, based on the length of the oval section of pipe which would produce maximum mode conversion, and based on a cross-sectional shape (trifoil) which would produce the maximum of mode conversion.

round guide causes coupling between the circular electric wave (TE_{01}) and the TM_{11} wave, and in round solid pipe the TE_{01} and TM_{11} waves are degenerate, i.e., they have identical phase constants. This degeneracy has the effect of bringing in phase all components transferred to TM_{11} no matter how gradually the bend might take place. Theory neglecting dissipation[4,7] shows that in a round pipe bent with any radius of curvature, the power will flow from the TE_{01} mode to the TM_{11} mode and back as a function of the total bend angle θ according to the relations

$$TE_{01} \text{ amplitude } = \cos\left(\frac{\pi}{2}\frac{\theta}{\theta_c}\right) \tag{9}$$

$$TM_{11}'' \text{ amplitude } = \sin\left(\frac{\pi}{2}\frac{\theta}{\theta_c}\right) \tag{10}$$

where

$$\theta_c = \frac{\pi}{2.32}\frac{\lambda_0}{a}$$

λ_0 = free-space wavelength

a = waveguide raduis

A solid round pipe is unsatisfactory for transmission of the circular electric wave around bends in the broad bands we seek to use.

One method of making the guide satisfactory in bends is to break the degeneracy between the TE_{01} and TM_{11} waves. Use of an elliptical pipe has been shown theoretically to be one way of doing this. For a 2″ diameter guide at 50,000 mc an eccentricity of about 0.3 permits a bending radius of 700 feet with theoretical bend losses in the range 0 to 0.17 db for any total bend angle; the heat loss coefficient for such an elliptic guide is about 35 per cent higher than for a perfectly round guide.[8]

Another method of avoiding bend losses is to introduce dissipation to the TM_{11} wave without adding loss to the TE_{01} wave. It has been shown[2] that a large difference between the attenuation coefficients of two coupled waves reduces the power transferred from the low-loss wave to the high-loss wave. Applied to the bend problem, this means that a structure with increased TM_{11} loss may be bent with less signal (TE_{01}) loss even though the phase constants might be degenerate. The reader is referred to the earlier paper[8] for a more complete discussion. There exist several alternate forms of circular electric waveguide (to be discussed) which have an attenuation coefficient for TM_{11} more than 5,000 times the attenuation coefficient for TE_{01}. The calculated extra loss in the bend region for such structures and for solid round pipe has been plotted

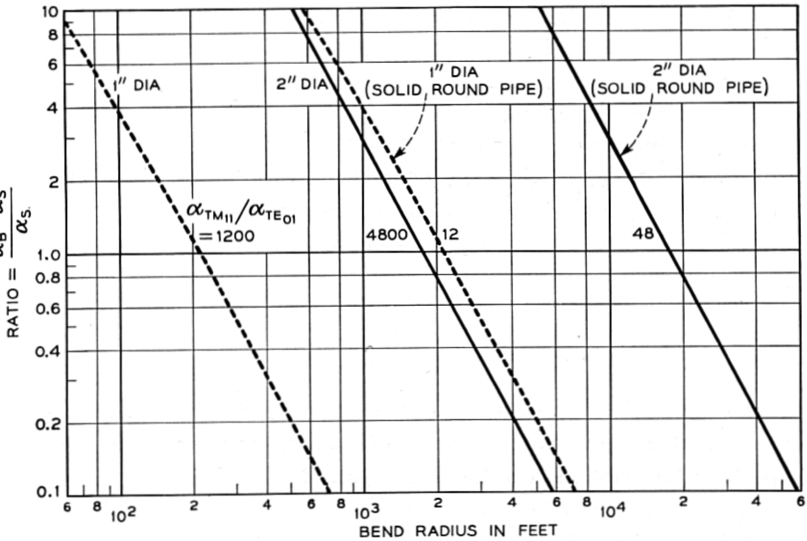

Fig. 25 — Computed change in the 50,000 mc TE_{01} attenuation coefficient due to bends in 1″ and 2″ diameter solid pipes and in modified guides having TM_{11} attenuation coefficients larger (than in solid pipe) by a factor of 100. α_B and α_s are the bend-region and straight-line attenuation coefficients, respectively.

as a function of bending radius in Fig. 25, assuming the TM_{11}-TE_{01} coupling due to the bend is the same in the altered guide as in solid round pipe. Whereas a bending radius of 17,500 feet causes a 100 per cent increase in TE_{01} heat loss for 50,000 mc waves in 2″ diameter solid pipe, the modified structure with a TM_{11} attenuation coefficient that is larger by a factor of 100 should tolerate a bending radius of 1,750 feet for the same heat loss increase. (The 50,000 mc attenuation coefficients for ideal 1″ and 2″ copper pipes are 14.8 and 1.79 db/mile respectively.)

For estimating purposes, the ratio of the extra loss per unit line length in the bend region to the straight line loss may be calculated for solid round pipes from the relation

$$\frac{\alpha_B - \alpha_s}{\alpha_s} = \frac{2.5 \times 10^{10} a^6}{\lambda_0^3 R^2} \tag{11}$$

and the absolute increase in attenuation due to a bend is*

$$(\alpha_B - \alpha_s) = \frac{9.7 \times 10^5 a^3}{\lambda_0^{3/2} R^2} \text{ (db/meter for copper guide)} \tag{12}$$

* For small bend radii and bend angles less than θ_c, this relation gives a greater loss than the correct value. See Figs. 22 and 23 of Reference 8 and also see Reference 2.

where α_s = straight line attenuation coefficient
α_B = bend region attenuation coefficient
a = guide radius
R = bending radius
λ_0 = free space wavelength

The approximations used in deriving (11) and (12) are good when the operating frequency is at least 50 per cent greater than cut off for TE_{01}. For guides modified to have higher TM_{11} attenuation both (11) and (12) may be divided by the factor

$$\frac{\alpha^m_{TM_{11}}}{\alpha^m_{TE_{01}}} \cdot \frac{\alpha^0_{TE_{01}}}{\alpha^0_{TM_{11}}} \tag{13}$$

where α^m and α^0 denote attenuation coefficients for the modified guide and solid round guide respectively. On the assumption that the mode coupling is the same in the modified guide as in the solid round guide, use of (13) with (11) or (12) provides an estimate of bend losses in modified circular-electric waveguides.

For a fixed ratio of bend-region attenuation to straightline attenuation, the allowable bending radius varies inversely as the square root of the ratio of TM_{11} heat loss to TE_{01} heat loss, varies inversely as $\lambda_0^{3/2}$, and varies directly as the third power of guide radius.

For fixed bending radius, the *absolute* bend loss varies inversely as $\lambda_0^{3/2}$; since the straight line TE_{01} loss varies directly as $\lambda_0^{3/2}$, bend losses tend to equalize the overall heat loss versus frequency characteristic of the waveguide.

IMPROVED FORMS OF CIRCULAR ELECTRIC WAVEGUIDE

In the preceding discussion it has been indicated that added dissipation for the unused modes of propagation has the effect of decreasing signal losses and of reducing the interference effects associated with mode conversion. Dissipation can be introduced to the unused modes of propagation through the addition of mode filters at intervals along the line, but it appears very desirable to introduce the dissipation to the unused modes on a continuous basis. Several ways of making the line lossy to the non-circular electric modes have been found, and one is illustrated in Figure 26. The copper rings lie in planes transverse to the direction of propagation and provide the conductivity required as a boundary for the circular electric wave family. Successive rings are insulated from each other, however, and the guide provides very poor conductivity in the longitudinal direction. All modes other than the circular electric wave family have wall currents in the longitudinal direction and experience considerably increased loss in the spaced-ring structure compared to a solid-walled waveguide. A. G. Fox first observed that the spaced ring structure could be used to transmit circular electric waves around bends, and since that time additional work has been carried out by A. P. King and M. Aronoff. The observed loss for the spaced-ring structure under proper conditions was observed to be about 60 per cent more than the theoretical loss for an ideal copper tube, whereas the observed loss for the unused modes of propagation was on the order of 1,000 to 5,000 times the circular electric wave value.

The spaced-ring structure therefore has the electrical properties we seek. The higher-order circular waves exist with losses comparable to their values in a solid copper pipe, but fortunately the magnitudes of conversion between the waves of the circular-electric family have been found to be small. The spaced-ring structure does present some difficult problems with regard to fabrication.

An analogous structure composed of a continuous helical conductor supported within a lossy housing has electrical properties which approximate those of the spaced-ring structure, and the helix should be considerably easier to manufacture in long lengths. The helix might be expected to support a wave-type approximating the circular electric wave both from the standpoint of field distribution and loss when one observes that a helix of very small pitch presents almost circumferential conductivity as required by the circular-electric wave, and the very small longitudinal component necessary due to the finite wire size tends toward zero as the helix pitch tends toward zero. James A. Young of these laboratories has constructed helices in the 2 db/mile waveguide size (4.73″ diameter at 9,000 mc) and found a heat-loss coefficient on the order of 1.75 times the theoretical value for ideal copper pipe. These large ex-

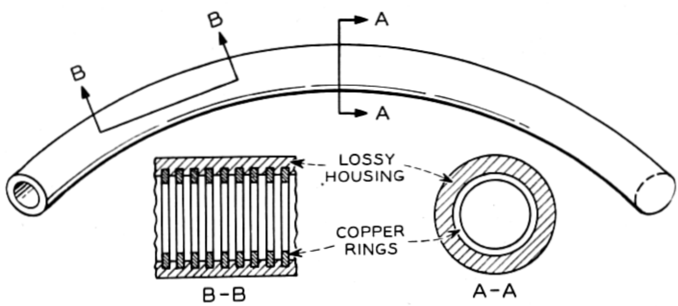

Fig. 26 — Spaced-ring circular electric waveguide.

perimental helices were known to be imperfect, and in a smaller diameter of helix, losses within 15 per cent of the theoretical values for perfect copper pipe were achieved. These results lead us to regard the helical line as a very promising medium for circular electric waves.

SURFACE ROUGHNESS

Since frequencies on the order of 50,000 mc are desirable for low-loss waveguide use, it was recognized that roughness of the surface at the waveguide walls might appreciably increase the heat loss in a practical waveguide. The first approach to this problem was made by W. A. Tyrrell using sections of 5 diameter copper pipe from the experimental line. Tyrrell measured the heat-loss coefficients of the pipe when used as a resonant cavity at 9,000 mc in lengths on the order of 4 to 8 feet. Carefully selected resonant conditions were employed to avoid bringing the unused modes of propagation into resonance at the same time that the circular electric wave was resonated. Whereas Tyrrell observed that the heat loss coefficient in the pipe as originally drawn was about 21 per cent higher than the value computed using the measured dc conductivity, he found that rotary* grinding and polishing the inner surface of the guide reduced the excess loss to about 12 per cent. Tyrrell also observed that commercially drawn brass and 2S aluminum tubing of approximately the same dimensions showed measured losses 11 per cent and 20 per cent respectively greater than the value predicted from the measured dc conductivity. Therefore, the indication from Tyrrell's work was that surface roughness did indeed account for increased losses even at 9,000 mc, and that the excess losses could be reduced either by polishing the surface or through the use of lower conductivities (which have the effect of increasing the skin depth).

A parallel approach to the measurement of surface-roughness effects was made by A. C. Beck and R. W. Dawson, also at 9,000 mc.[9] Beck and Dawson used small wire samples as the center conductor of a coaxial cavity† and found that commercially drawn copper, aluminum and silver wires showed loss values 10 per cent and 15 per cent higher than those expected from the measured dc conductivity. By mechanically polishing

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* Because the wall currents for the circular electric wave are circumferential, the longitudinal surface scratches produced by drawing are in the worst possible orientation. Polishing was carried out in a rotary manner so that the current would not cross the scratches so induced.

† In a coaxial, the currents are longitudinal, as are the scratches from drawing, so the measurements in the coaxial would be expected to show somewhat less excess loss due to surface roughness than the measurements made in circular-electric waveguide cavities.

these same wires the losses were reduced to 5 to 8 per cent above the dc values and, by electropolishing, copper wires were brought within 2 per cent of the theoretical value.

An investigation of the effects of aging on the 9,000 mc conductivity of copper surfaces was conducted jointly by Dawson, Tyrrell and Beck, using wires which were measured in the coaxial cavity.[9] They found (1) that the conductivity of untreated surfaces remained essentially stationary when stored indoors; (2) that the conductivity of bare electropolished surfaces deteriorated (over a period of months) to a value comparable to that for commercially drawn wire; and (3) that a tight bonding coating very greatly slows down the aging process.

Recent measurements made in the vicinity of 50,000 mc on a sample of commercially-drawn fine-silver waveguide have indicated that surface roughness effects increased the loss values by approximately 20 per cent.

The overall conclusion would appear to be that it is now feasible commercially to produce waveguide surfaces which have excess losses due to surface roughness no greater than 20 per cent even at frequencies near 50,000 mc, and that by refining the manufacturing method it may be possible to approach the ideal value more closely. In order to avoid aging effects, a tight-bonding coating and a protective atmosphere may be desirable.

CIRCULAR ELECTRIC WAVE AND MILLIMETER WAVE TECHNIQUES

A whole new line of components and techniques are required to carry out in the millimeter wave region the same functions that have always been required in a system application, i.e., power generation, amplification, frequency-band separation, hybrid division, amplitude and phase equalization, and so forth. With a view to ultimate use in a waveguide transmission system, basic research has been done on all of these elements over a period of years at these Laboratories.

The importance of primary oscillators and amplifiers to millimeter-wave systems is self-evident, and work has begun going forward in this difficult field under the direction of J. R. Pierce. The results have been published during the last several years. Pierce made the first 6 mm reflex klystron oscillator.[15] The first 6-mm amplifier, made by John Little,[10] was a travelling wave tube using a conventional helix and it provided 3 db gain. Later, S. Millman[11] introduced the space-harmonic type of travelling-wave amplifier and built several which had over 20 db gain near 6 mm. R. Kompfner[12] devised the backward-wave type of travelling-wave oscillator and built a tube which oscillated from about 5 mm to 8 mm. A. Karp[13] devised a simple structure for backward-wave oscillators which has already resulted in 5 to 6 mm oscillators and which appears suitable for use at shorter wavelengths.* Recently, E. D. Reed produced a 5 to 6 mm reflex klystron.*

R. S. Ohl has continued his pioneering work on point-contact crystal converters and has made units for use at 6 mm having conversion losses of less than 8 db and output noise ratios on the order of three times basic thermal noise. Ohl also made point contact silicon units for use as harmonic generators to permit the conversion of 24,000 mc power to 48,000 mc power. His harmonic generators have proven invaluable as a source of millimeter wave power — essentially all of the radio research work done to date has been carried out using them.

In order to evaluate crystals and millimeter wave oscillators, it is essential to have an absolute power reference in the millimeter region, and work* has been done by W. M. Sharpless to establish such a reference.

Up to the present time all of the amplifiers, oscillators and other circuit elements have employed dominant-mode rectangular waveguides in order to simplify the circuit design. Therefore, it is of importance to know how to transform a signal from a dominant-mode rectangular guide to the circular electric wave in round pipe. The first circular-electric-wave transducer made in these Laboratories was designed by A. P. King and had the form sketched in Fig. 27. This transducer is of the general type in which the metallic boundary of the waveguide is shaped to force the field lines in the cross section of the guide into the pattern characteristic of the desired output wave. In Fig. 27 the dominant-mode rectangular guide at the left end is gradually tapered to the sector of a circle; the size of this sector is small enough so that only one wave type can exist at this point, and the electric field lines are arcs of a circle. Next, the angle of the sector is gradually increased along the axis of propagation until at one point a cross section of the guide has the shape of a half circle. The size of the sectoral angle is continually increased, however, until finally the metallic sector of the circle disappears as a radial vane. When the taper is done gradually (an overall length of approximately 10 to 15 wavelengths) the electric field lines remain arcs of a circle as

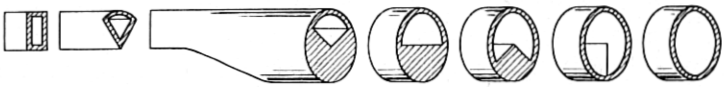

Fig. 27 — Circular-electric wave transducer (due to A. P. King).

* A portion of this work was carried out under Office of Naval Research Contract Nonr 687(00).

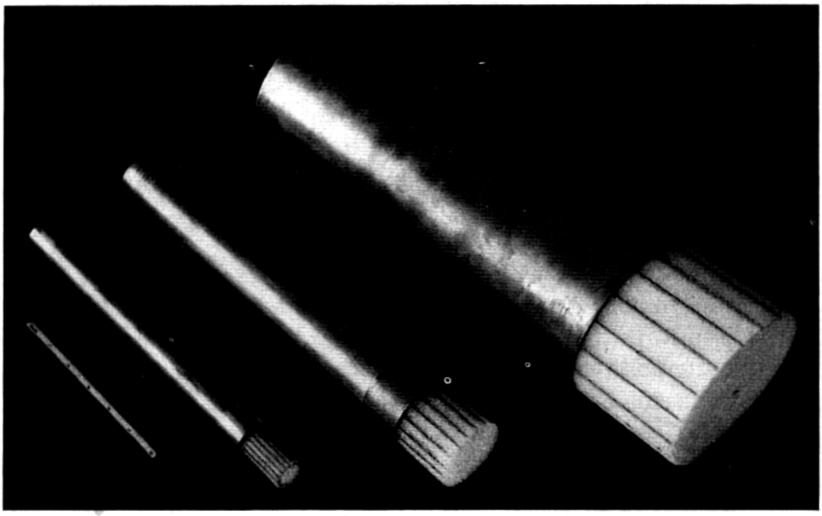

Fig. 28 — Mode filters which pass only circular electric waves.

they were in the sector at the left-hand end of Fig. 27, and the circular-electric wave emerges in the round pipe. This type of transducer has been shown to have transfer losses from dominant-mode rectangular guide to circular-electric wave in round pipe of approximately 0.3 db at 24,000 mc. Similar models have been made by A. G. Fox for use at 48,000 mc.

Another important component is the mode filter previously referred to and which attenuates all wave types other than the circular electric wave family. One type of mode filter to perform this function is the spaced ring structure of Fig. 26 and another type, due to A. P. King, consists of resistive sheets along radial planes as shown in the photograph of Fig. 28. The circular electric wave family has no electric field in a radial direction or in a longitudinal direction. All other wave types, however, have radial-electric or longitudinal-electric field components and experience attenuation due to the presence of the resistive sheets.

The coupled-wave type of transducer sketched in Fig. 29[2] is useful in connecting from dominant rectangular guides to the circular electric or other modes in round guide. This type of transducer makes use of the fact that the various modes in the multimode guide have unequal phase constants. The transfer of power from rectangular guide to the round guide takes place only to the particular round-guide mode whose phase constant is equal to that of the wave in the rectangular guide. This type of transducer has the same geometric appearance (aside from exact dimensions) for any mode in the round guide and is attractive in that it presents a matched impedance to all the modes of propagation. This property may be used to combine a series of signals onto different modes in a single transmission line. The coupled wave transducer may also be employed to multiplex a series of frequency bands into one pipe. We may wish to employ the long-distance waveguide over the frequency band from perhaps 35,000 mc to 75,000 mc and will require a series of transducers to go from dominant mode guides in various portions of this band to the circular-electric wave in the round guide. Frequency-selective coupled-wave transducers may be employed in the manner sketched in Fig. 30 to multiplex these frequency bands into the pipe for the long distance transmission.

A. G. Fox[14] has shown that dielectric waveguides are attractive as a flexible connecting link for terminal equipment in the millimeter wave region and may also be employed in circuits such as hybrids.

On all of these items of millimeter wave technique and multimode waveguide technique, individual publications will appear as soon as the work has reached the point where this becomes possible.

MODULATION METHODS

The modulation method to be used for the transmission of intelligence on a waveguide system will probably be dominated by the conversion-reconversion phenomenon already discussed. In order to evaluate the

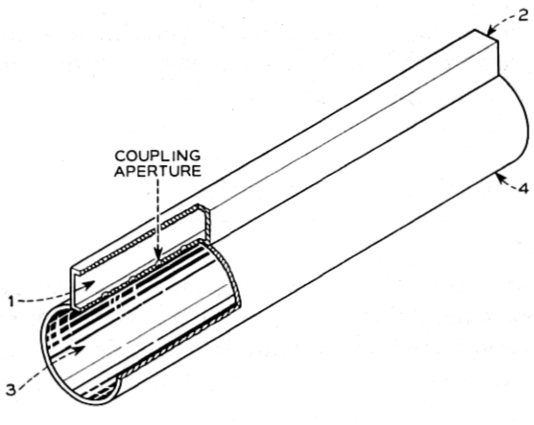

Fig. 29 — Coupled-wave transducer for generating circular-electric or other waveguide modes.

interfering effects of the conversion process, it is important to take into account the time relations between the signal components and the reconverted wave components. In general, it seems clear that reconverted energy returning to the signal mode within a time interval appreciably less than the reciprocal of the highest modulation frequencies will be less interfering than reconverted wave components separated from the signal components by time intervals on the same order as the reciprocal of the modulation frequencies. In practice, a distribution of reconversions will be obtained from a line containing randomly disposed imperfections. As discussed in connection with equation (1), an approximate upper limit can be placed on the time separation between the signal energy and the reconverted wave energy. Table IV records the "upper-limit" time delay for the 5″ diameter experimental 9000 megacycle waveguide

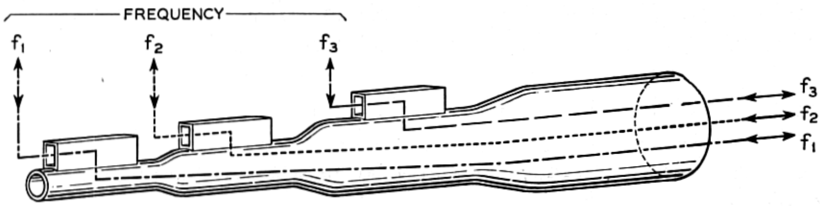

Fig. 30 — Proposed method of multiplexing several frequency bands in the circular electric waveguide.

and for a 2″ diameter pipe used at a frequency of 48,000 mc. The negative time delays represent reconverted-wave components which precede the signal at the receiving end of the system. These calculations suggest that reconverted wave energy might precede the signal by almost ½ microsecond or lag the signal by approximately 1 microsecond in the 5″ diameter 9,000-mc line. Photographs of the observed pulse transmission shown in Fig. 15 indicate negligible reconverted-wave energy at more than one-half these time intervals, and therefore it appears that the method of estimating (described in connection with equation 1) is pessimistic. However, the "upper limit" time delay does provide a convenient way of comparing the experimental line with a proposed 2″ diameter line used at 48,000 mc. Observe in Table IV that the 2″ diameter line will show appreciably less delay for the reconverted wave components than is to be expected from the 5″ diameter line. This is due to higher attenuation in the unused modes and to smaller differences between the group velocities of the various modes in the 2″ diameter pipe. "Upper limit" time delays in the range 1 to 100 millimicroseconds

TABLE IV — "UPPER LIMIT" TIME DELAY FOR RECONVERTED ENERGY COMPONENTS IN SOLID PIPE WITHOUT MODE FILTERS

Mode	"Upper Limit" Time Delay	
	4.732″ Dia. Pipe at 9000 Mc	2.0″ Dia. Pipe at 48,000 Mc
TE_{11}	-0.456×10^{-6} second	-0.0137×10^{-6} second
TE_{21}	−0.097	−0.0035
TE_{31}	+0.0357	+0.0014
TE_{12}	1.093	+0.111

are indicated for the 2″ diameter line and may also be regarded as pessimistic estimates by virtue of our experience with the 5″ diameter line.

It is to be emphasized that the time delays of Table IV apply to solid round guide without mode filters. The addition of mode filters in the solid pipe would increase the loss for the unused modes and appreciably reduce the "upper-limit" time delays for reconverted wave energy. For the improved forms of circular-electric waveguide the increased losses for the unused modes would result in similarly reduced time delays; typical factors of reduction might run between 100 and 1,000 for spaced-ring and helical waveguides compared to the data for solid pipe given in Table IV.

Taking into consideration both conversion-reconversion effects and some equalization of delay-distortion due to waveguide cut-off, it would appear likely that baseband widths on the order of 100 to 1000 mc could be employed in a 2″ diameter line.

The character of the reconverted wave interference will be as discussed in connection with Figs. 15 and 18 if the modulating wave of the signal carrier contains frequencies near the reciprocal of the "upper-limit" time delay for the particular waveguide used.

The magnitude of the reconverted wave energy has been discussed in connection with Fig. 21, and it has been observed that ratios of signal power to reconverted-wave power on the order of 20 db may be expected on a well engineered waveguide line having 60 db of heat loss for the signal wave. Although the time relations were not specifically taken into account in this discussion, it seems likely that the order of magnitude of the reconverted-wave power will be unaltered by more precise analysis.

One might therefore conclude that the modulation method to be used in a waveguide system must be one which will tolerate large amounts of signal interference. Pulse code modulation is one arrangement of signaling which will tolerate such interference, and undoubtedly there are others as well.

Another conclusion the writer has reached is that some form of signal regeneration is likely to be required at each repeater, and that the modulation process should be selected in such a way as to permit this regeneration with limited complexity.

CONCLUSION

Single-mode waveguide is unattractive as a long-distance communication medium due to limited bandwidth and either unreasonable size or excessive loss.

The circular-electric mode in a 2″ diameter round pipe has a theoretical attenuation coefficient of 2 db per mile for carrier frequencies near 50,000 mc. Delay distortion due to waveguide cut-off will require equalization if baseband widths on the order of 500 mc are to be provided on repeater spacings of 25 miles. Using frequency-division multiplex, such a waveguide might be exploited over the 40,000-mc band from 35,000 to 75,000 mc, for which the theoretical attenuation coefficients are 3 db/mile and 1 db/mile respectively.

Transmission experiments were conducted at 9,000 mc in a 500-foot copper pipe 4.73″ I.D. for which the theoretical circular-electric wave loss was 1.9 db/mile. Under favorable conditions the observed losses were within 25 per cent of the theoretical value; under unfavorable conditions (which are not likely to occur in practice) the observed losses were as high as 75 per cent greater than the theoretical value. Surface roughness accounted for losses about 20 per cent above the theoretical value, and the remaining excess losses were due to conversion of energy to other modes of propagation. Direct observation of the power transferred between modes in the 500-foot line confirmed the latter conclusion.

Other observations in the 500-foot line showed that the mode conversion process produced a signal-distortion or signal-crosstalk effect through reconversion of energy from the unused modes of propagation back to the signal (circular electric) mode. This type of interference seriously limits the capabilities of bare copper pipe for use as a long-distance communication medium. However, it has been shown experimentally and theoretically that the effects of the conversion-reconversion process are greatly reduced through the addition of mode filters, which absorb energy present in the unused modes of propagation.

The combination of solid pipe plus mode filters remains attractive as a communication medium. Average losses for the unused modes of propagation should be on the order of 500 times the loss to the signal wave, in order that the conversion-reconversion effects be tolerable in the presence of conversion losses of the magnitude observed in the 500-foot experimental line.

Improved forms of circular-electric waveguide have been demonstrated. These waveguides have the common property of providing very large attenuation coefficients for the unused modes of propagation while maintaining essentially the same circular-electric wave attenuation as in solid pipe. Thus, these guides are essentially continuous mode filters. Structurally, such a medium may consist of a series of metallic rings supported in a lossy housing (Fig. 26), or may consist of a helix supported in similar manner. The helical circular-electric waveguide appears more attractive from the standpoint of ease of fabrication.

Where bends must be sharp, they may be negotiated in a number of special ways. Gradual bends may be introduced in the improved forms of circular-electric waveguide with modest increase in heat loss. For the 2″ diameter guide at 48,000 mc, it is estimated that a bending radius of about 2,000 feet would double the heat loss of a helical or spaced-ring waveguide. For a 1″ diameter helical or spaced-ring guide (theoretical heat loss of 15 db/mile in solid pipe) the corresponding bending radius is about 200 feet. The extra heat loss due to bending varies inversely as the square of the bending radius.

The type of modulation to be used in a waveguide system will probably be dominated by conversion-reconversion effects. An upper limit on the time delay between the signal component and the associated reconverted-wave components lies in the range 1 to 100 millimicroseconds for 2″ pipe at 48,000 mc without mode filters; the addition of mode filters or use of the helical circular-electric guide should reduce these time delays by factors of 100 or more. Thus, it appears likely that basebands on the order of 500 mc or more should be usable.

It is concluded that a waveguide signalling method must be capable of withstanding large amounts of signal interference, and that some form of regeneration is likely to be required at each repeater.

ACKNOWLEDGMENT

The encouragement of R. Bown, H. T. Friis, and J. C. Schelleng is gratefully acknowledged. The contributions of numerous colleagues (as noted throughout the paper) form the building blocks without which the present understanding of waveguide transmission could not have been obtained.

Appendix

THEORETICAL ANALYSIS FOR CONTINUOUS MODE CONVERSION

Whereas the travelling-pulse type of theoretical analysis utilized in the body of this paper can be extended to a realistic spacial distribution of conversion points and to a series of modes instead of only one, J. R. Pierce suggested that the assumption of uniform mode conversion along the axis of propagation would lead to a solution in closed form and would probably show the general properties being sought. This suggestion was adopted and P_1 is designated as the signal power, P_x as the power in the unused mode, and P_n as the power which has transferred from mode-x back to the signal mode, mode 1. We assume quadrature addition of conversion components, and write a series of differential equations expressing the *power flow* between the modes along the axis of propagation, including the heat loss effects:

$$\frac{dP_1}{dz} = -a_{1h}P_1 - a_{1x}P_1 \tag{9}$$

$$\frac{dP_x}{dz} = -a_{xh}P_x - a_{x1}P_x + a_{1x}P_1 + a_{1x}P_n \tag{10}$$

$$\frac{dP_n}{dz} = -a_{1h}P_n - a_{1x}P_n + a_{x1}P_x \tag{11}$$

in which the symbols have the following definitions:

a_{1h} = the heat loss coefficient – mode 1
a_{1x} = the mode conversion coefficient from mode 1 to mode x
a_{xh} = the heat loss coefficient – mode x
a_{x1} = the mode conversion coefficient from mode x to mode 1
z = distance along the axis of propagation

Note that the above heat loss coefficients are those associated with power rather than attenuation coefficients associated with amplitudes, ($2\alpha_1 = a_{1h}$, $2\alpha_x = a_{xh}$). The above equations also imply mode conversion in the forward direction only.

In a phenomenological way, these equations represent the decay of power in the signal mode and the build up of power in both the unused mode x and reconverted energy P_n in mode 1. The general plan is to solve these equations for P_x and P_n in terms of the input wave power. P_n is maintained separate mathematically from P_1, even though both of them are in the same mode, so that we can clearly identify the energy which has been at one time in the unused mode x.

For mathematical solution, (9), (10) and (11) may be put in the following form:

$$\frac{dP_1}{dz} + \alpha P_1 = 0 \tag{12}$$

$$\frac{dP_x}{dz} + \beta P_x - a_{1x}P_1 - a_{1x}P_n = 0 \tag{13}$$

$$\frac{dP_n}{dz} + \alpha P_n - a_{x1}P_x = 0 \tag{14}$$

in which $\alpha = a_{1h} + a_{1x}$

$$\beta = a_{xh} + a_{x1}$$

The general solution for (12), (13) and (14) is given by the following

$$P_1 = k_1\, \varepsilon^{-\alpha z} \tag{15}$$

$$P_2 = k_2\, \varepsilon^{r_1 z} + k_3\, \varepsilon^{r_2 z} \tag{16}$$

$$P_n = -k_1\epsilon^{-\alpha z} + \frac{k_2}{a_{1x}}(r_1 + \beta)\epsilon^{r_1 z} + \frac{k_3}{a_{1x}}(r_2 + \beta)\epsilon^{r_2 z} \tag{17}$$

where

$$\begin{matrix} r_1 \\ r_2 \end{matrix} = \frac{-(\alpha + \beta) \pm \sqrt{(\alpha - \beta)^2 + 4a_{1x}a_{x1}}}{2} \tag{18}$$

The positive sign is to be associated with r_1 and the negative sign with r_2. For the boundary conditions, at $z = 0$,

$$P_1 = P_0$$

$$P_x = 0,$$

$$P_n = 0,$$

that is to say, the input to the transmission medium being zero in both the x mode and reconverted energy mode, then the solution takes the following form:

$$P_1 = P_0\epsilon^{-\alpha z} \tag{19}$$

$$P_x = \frac{a_{1x}P_0}{(r_1 - r_2)}\,\epsilon^{r_1 z} - \frac{a_{1x}P_0}{(r_1 - r_2)}\,\epsilon^{r_2 z} \tag{20}$$

$$P_n = -P_0\epsilon^{-\alpha z} + \frac{(r_1 + \beta)P_0}{(r_1 - r_2)}\epsilon^{r_1 z} - \frac{(r_2 + \beta)}{(r_1 - r_2)}\epsilon^{r_2 z} \tag{21}$$

It is informative to note that $(r_1 - r_2)$ is always positive and is equal to

$$\sqrt{(\alpha - \beta)^2 + 4a_{1x}a_{x1}}$$

We are usually interested in the ratio of the power in the x-mode to the signal power P_1 and the ratio of the reconverted energy P_n to the signal power P_1. These ratios are given by the following expressions

$$\frac{P_n}{P_1} = \frac{(r_1 + \beta)}{(r_1 - r_2)}\epsilon^{(r_1+\alpha)z} - \frac{(r_2 + \beta)}{(r_1 - r_2)}\epsilon^{(r_2+\alpha)z} - 1 \tag{22}*$$

$$\frac{P_x}{P_1} = \frac{a_{1x}\epsilon^{(r_1+\alpha)z}}{(r_1 - r_2)}[1 - \epsilon^{-(r_1-r_2)z}] \tag{23}*$$

Thus, we have explicit solutions for the uniform transmission medium containing mode conversions.

In order to make the most general study of these relations, we shall express the mode conversion coefficients in terms of the heat loss coefficient in the signal mode—i.e., as the ratio a_{1x}/a_{1h}. This is natural enough physically, for we are interested in the relative magnitudes of the heat loss and mode conversion effects. It is found that knowledge of the ratios a_{1x}/a_{1h}, a_{x1}/a_{1h} and a_{xh}/a_{1h} enables us to completely determine P_x/P_1 and P_n/P_1 in terms of the distance parameter $\epsilon^{-a_{1h}z}$. The latter is the heat loss in the signal mode, another familiar physical characteristic.

CHARACTERISTIC CONDITIONS IN BARE ROUND WAVEGUIDE

In order to use the theoretical relations derived in the preceding section, we need to know typical values of the parameters. In particular, we need to know the magnitude of typical conversion coefficients a_{1x} and values of the heat loss coefficients a_{1h} and a_{xh} for the modes of interest.

One set of heat loss coefficients which is of immediate interest may be made up from the calculated values for the 5-inch diameter round waveguide used in waveguide experiments at Holmdel. Table V shows the ratio of attenuation coefficients for several modes in this line. The circular electric wave TE_{01} has the lowest attenuation coefficient (absolute value

* When using these relations, it is helpful to note that $(r_1 + r_2) = -(\alpha + \beta)$ at all times. Hence $(r_1 + \alpha) = -(r_2 + \beta)$ and $(r_2 + \alpha) = -(r_1 + \beta)$.

TABLE V — RATIO OF ATTENUATION COEFFICIENTS FOR SEVERAL MODES IN 4.732″ DIAMETER. WAVEGUIDE AT $\lambda_0 = 3.0$ CM

Modes	Attenuation Coefficient Ratio α_x/α_1 or a_{xh}/a_{1h}	Modes	Attenuation Coefficient Ratio α_x/α_1 or a_{xh}/a_{1h}
TE_{12}/TE_{01}	2.45	TE_{31}/TE_{01}	12.5
TE_{02}/TE_{01}	3.82	TE_{41}/TE_{01}	17.
TE_{11}/TE_{01}	4.56	TE_{51}/TE_{01}	21.
TE_{22}/TE_{01}	4.63	TE_{61}/TE_{01}	27.
TE_{13}/TE_{01}	6.61	TE_{71}/TE_{01}	34.
TE_{21}/TE_{01}	8.51	TE_{81}/TE_{01}	44.
TM_{11}/TE_{01}	10.78	TE_{91}/TE_{01}	61.
TE_{03}/TE_{01}	11.3	$TE_{10,1}/TE_{01}$	100.

equal to 1.6 db/mile), and the a_{xh}/a_{1h} ratios for the other modes range from 2.5 to 100 times the TE_{01} value. Table V represents a selection of the various modes which can exist in the 5-inch diameter line, but the number tabulated is *not* an indication of the density of the ratio of attenuation coefficients near a given value. Actually, there are eight modes having a_{xh}/a_{1h} ratios in the range 2.5 to 10, 19 modes in the range 10 to 20, and 15 modes greater than 20.

Experimental work reported elsewhere shows that significant conversion takes place between TE_{01} and the TE_{11}, TE_{21}, TE_{31} and TM_{11} modes. There is some likelihood that conversion to TE_{12} takes place, but the magnitude has not been measured. The experience gained by measurement, therefore, shows that most typical conversions occur between TE_{01} and modes having attenuation ratios a_{xh}/a_{1h} in the range 2.5 to 12.

The absolute magnitudes of the conversion coefficients a_{1x} have in some cases been measured directly, and may also be inferred from measurements of total signal attenuation on the 500 ft. experimental line and separate knowledge of the heat-loss values; the inference is that a_{1x} must fall in the range 0.1 to 1.0 a_{1h} for the particular line studied.

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