

Addendum to
Delay Curves for Calls Served at Random

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Pages 100 to 119

I owe the following remarks, which help to complete the record, to Emile Vaultot:

1. The Erlang formula for delay with order of arrival service, for the proof of which reference has been made to a paper by E. C. Molina, was proved earlier by E. Vaultot (*Application du Calcul des Probabilités a l'exploitation telephonique. Revue Générale de l'Electricité, 16*, pp. 411-418, 1924). Indeed his seems to be the first proof.

Also, the associated Erlang C function $C(c, a)$, for which I said there was no extensive tabulation, is tabulated for $n = 1$ (1) 139 and an extensive but irregular set of a 's by Arne Jensen (*Moe's Principle, Table III, Copenhagen Telephone Co. Copenhagen, 1950*). Also the recurrence relation for this function given in a footnote has previously been given by Conny Palm (*Väntetider Vid Slumpvis Avverkad Kö, Tekniska Meddelanden Fran. Kungl. Telegrafstyrelsen, Specialnummer för Teletrafikteknik, pp. 109. Stockholm, 1946, see p. 43*).

2. The extensive treatment of delay by Conny Palm, just mentioned, includes a section on random service (section 4); it may be noticed that this is dated May 15, 1946, which is only a few months after Vaultot's article on the same subject (Jan. 28, 1946), and of course is an independent development.

I owe the following to my colleague S. O. Rice. Pollaczek, in the *Comptes Rendus* paper mentioned, has given an integral effectively for what I have called $F(u)$. Rice has put this in a slightly different form adapted to numerical computation and has obtained the following results for $F(u)$

$$v = u(1-\alpha)$$

α	1	2	4	6	8	10	12	14
0.8					0.0079	0.0039	0.0020	0.0011
0.9	0.2866	0.1388	0.0471	0.0198	0.0094	0.0049	0.0026	0.0015

Comparison with the tables of the papers shows a satisfying agreement and substantiates the conjecture that approximation by a relatively small number of exponentials is sufficient.

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