A Study of Non-Blocking Switching Networks

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(Manuscript received October 30, 1952)

This paper describes a method of designing arrays of crosspoints for use in telephone switching systems in which it will always be possible to establish a connection from an idle inlet to an idle outlet regardless of the number of calls served by the system.

INTRODUCTION

The impact of recent discoveries and developments in the electronic art is being felt in the telephone switching field. This is evidenced by the fact that many laboratories here and abroad have research and development programs for arriving at economic electronic switching systems. In some of these systems, such as the ECASS System,* the role of the switching crossnet array becomes much more important than in present day commercial telephone systems. In that system the common control equipment is less expensive, whereas the crosspoints which assume some of the control functions are more expensive. The requirements for such a system are that the crosspoints be kept at a minimum and yet be able to permit the establishment of as many simultaneous connections through the system as possible. These are opposing requirements and an economical system must of necessity accept a compromise. In the search for this compromise, a convenient starting point is to study the design of crossnet arrays where it is always possible to establish a connection from an idle inlet to an idle outlet regardless of the amount of traffic on the system. Because a simple square array with N inputs, N outputs and N^2 crosspoints meets this requirement, it can be taken as an upper design limit. Hence, this paper considers non-blocking arrays where less than N^2 crosspoints are required. Specifically, this paper describes for an implicit set of conditions, crossnet arrays of three, five,

* Malthaner, W. A., and H. Earle Vaughan, An Experimental Electronically Controlled Switching System. Bell Sys. Tech. J., 31, pp. 443–468, May, 1952.

etc., switching stages where less than N^2 crosspoints are required. It then deals with conditions for obtaining a minimum number of crosspoints, cases where the N inputs and N outputs can not be uniformly assigned to the switches, switching arrays where the inputs do not equal the outputs, and arrays where some or all of the inputs are also outputs.

SQUARE ARRAY

A simple square array having N inputs and N outputs is shown in Fig. 1. The number of crosspoints equals N^2 and any combination of N or less simultaneous connections can exist without blocking between the inputs and the outputs. The number of switching stages, s, is equal to 1. The number of crosspoints, $C(s)$, is:

$$C(1) = N^2 \tag{1}$$

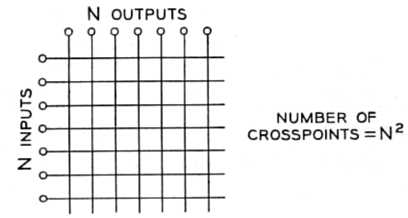

Fig. 1 — Square Array.

THREE-STAGE SWITCHING ARRAY

An array where less than N^2 crosspoints are required is shown in Fig. 2. This array has $N = 36$ inputs and $N = 36$ outputs. There are three switching stages, namely, an input stage (a), an intermediary stage (b), and an output stage (c). In stage (a) there are six 6 x 11 switches; in stage (b) there are eleven 6 x 6 switches; and in stage (c) there are six 6 x 11 switches. In total, there are 1188 crosspoints which are less than the 1296 crosspoints required by equation (1).

Of interest are the derivations of the various quantities and sizes of switches. In stage (a) the number, n, of inputs per switch was assumed to be equal to $N^{1/2}$, thus giving six switches and six inputs per switch. In a similar manner stage (c) was assigned six switches and six outputs per switch. The number of switches required in stage (b) must be sufficient to avoid blocking under the worst set of conditions. The worst case occurs when between a given switch in stage (a) and a given switch in stage (c): (1) five links from the switch in stage (a) to five corresponding switches in stage (b) are busy; (2) five links from the switch in stage (c) are busy to five additional switches in stage (b); and (3) a connection is desired between the given switches. Thus eleven switches are required in stage (b). The remaining requirements, namely, eleven verticals per switch in stages (a) and (c) and six by six switches in stage (b) are then easily derived.

The number of crosspoints required for three stages, where $n = N^{1/2}$, is summarized by the following formula:

$$C(3) = (2N^{1/2} - 1)\,(3N) \tag{2}$$

$$= 6N^{3/2} - 3N \tag{2a}$$

In Table I it may be noted that the number of crosspoints is less than N^2 for all cases of $N \geqq 36$.

PRINCIPLE INVOLVED

The principle involved for determining the number of switches required in the intermediary stage is illustrated in Fig. 3. The figure is for a specific case from which one can generalize for n inputs on a given input switch and m outputs on a given output switch. In the figure it is desired to establish a connection from input B to output H. A sufficient number of intermediary switches are required to permit the $(n - 1)$ inputs other than B on the particular input switch and the $(m - 1)$ outputs other than H on the particular output switch to have connections to separate intermediary switches plus one more switch for the desired connection between B and H. Thus $n + m - 1$ intermediary switches are required.

TABLE I — CROSSPOINTS FOR SEVERAL VALUES OF N

N	Square Array N^2	Three-Stage Array $6N^{3/2} - 3N$
4	16	36
9	81	135
16	256	336
25	625	675
36	1,296	1,188
49	2,401	1,911
64	4,096	2,880
81	6,561	4,131
100	10,000	5,700
...	......	......
1,000	1,000,000	186,737
10,000	100,000,000	5,970,000

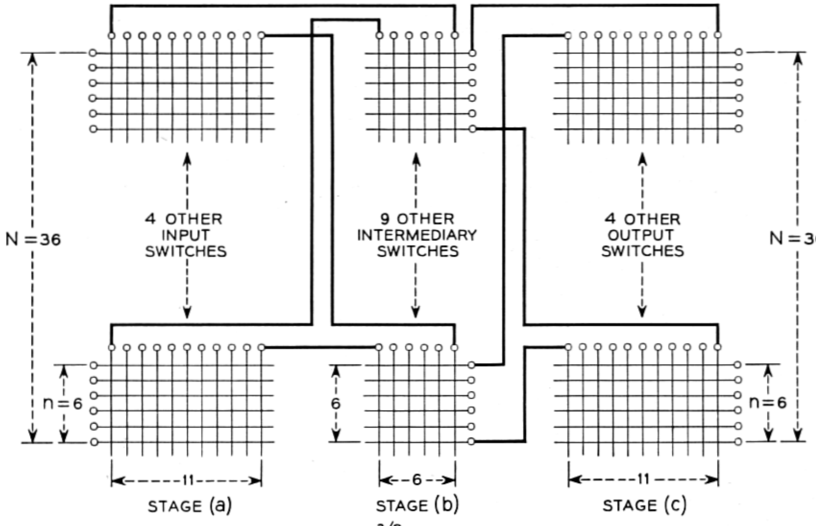

Fig. 2 — Three-stage switching array.

FIVE-STAGE SWITCHING ARRAY

A five-stage switching array is illustrated in Fig. 4. The analysis of this array can be made in the following manner. Each input and output switch is assumed to have $n = N^{1/3}$ inputs or outputs, respectively. Connection between a given input switch and a given output switch

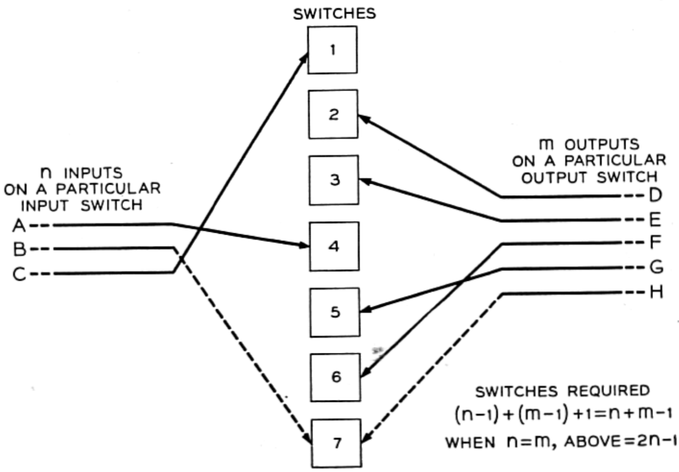

Fig. 3 — Principle involved.

is made via levels, a level consisting of three intermediary switching stages. The number of levels required is $(2N^{1/3} - 1)$. Each level has $N^{2/3}$ inputs and the same number of outputs. The number of crosspoints for a three-stage non-blocking array for $N^{2/3}$ inputs and $N^{2/3}$ outputs can be obtained from equation (2) by substituting $N^{2/3}$ for N in that equation. The total number of crosspoints required for the five-stage array is:

$$C(5) = (2N^{1/3} - 1)^2\, 3N^{2/3} + (2N^{1/3} - 1)\, 2N \quad (3)$$

$$= 16N^{4/3} - 14N + 3N^{2/3} \quad (3a)$$

The number of crosspoints required for several sizes of the five-stage array is given in Table II. The results are compared to the square and three-stage arrays.

SEVEN-STAGE SWITCHING ARRAY

A seven-stage switching array can be analyzed by considering paths requiring five intermediary switching stages as paths via switching aggregates. The number of such aggregates is $(2N^{1/4} - 1)$. Each aggregate has $N^{3/4}$ inputs and a like number of outputs. From equation (3) the crosspoints for each aggregate can be obtained by substituting

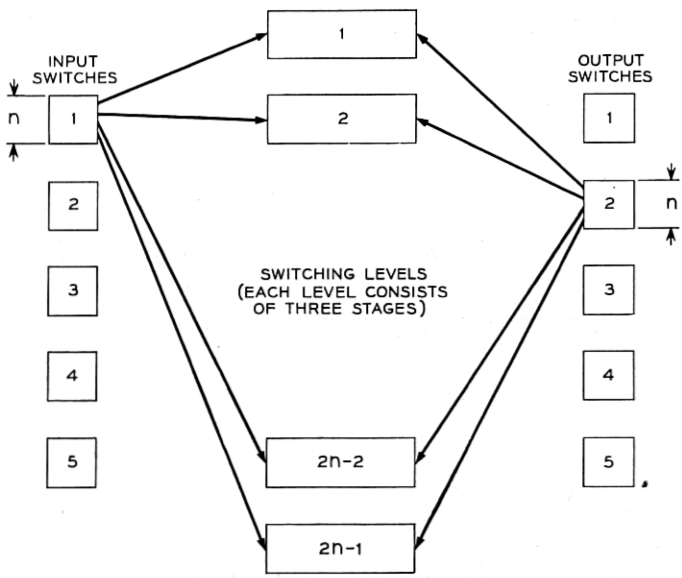

Fig. 4 — Five-stage switching array.

TABLE II — CROSSPOINTS FOR SEVERAL VALUES OF N

N	Square Array	Three-Stage Array	Five-Stage Array
64	4,096	2,880	3,248
729	531,441	115,911	95,013
1,000	1,000,000	186,737	146,300
10,000	100,000,000	5,970,000	3,434,488

$N^{3/4}$ for N in that equation. The total number of crosspoints required for the seven-stage array is:

$$C(7) = (2N^{1/4} - 1)^3\, 3N^{1/2} + (2N^{1/4} - 1)^2\, 2N^{3/4} + (2N^{1/4} - 1)2N \quad (4)$$

$$= 36N^{5/4} - 46N + 20N^{3/4} - 3N^{1/2} \quad (4a)$$

GENERAL MULTI-STAGE SWITCHING ARRAY

Equations (1), (2a), (3a) and (4a) are herewith tabulated as a series of polynomials together with the next polynomial:

$$C(1) = N^2 \quad (1)$$

$$C(3) = 6N^{3/2} - 3N \quad (2a)$$

$$C(5) = 16N^{4/3} - 14N + 3N^{2/3} \quad (3a)$$

$$C(7) = 36N^{5/4} - 46N + 20N^{3/4} - 3N^{1/2} \quad (4a)$$

$$C(9) = 76N^{6/5} - 130N + 86N^{4/5} - 26N^{3/5} + 3N^{2/5} \quad (5)$$

These polynomials can be determined for any number of switching stages from the following formula where s is an odd integer:

$$C(s) = 2\Sigma_{k=2}^{k=\frac{s+1}{2}} N^{\frac{2k}{s+1}} \left(2N^{\frac{2}{s+1}} - 1\right)^{\frac{s+3}{2}-k} + N^{\frac{4}{s+1}} \left(2N^{\frac{2}{s+1}} - 1\right)^{\frac{s-1}{2}} \quad (6)$$

An alternative expression equivalent to equation (6) has been suggested by S. O. Rice and J. Riordan. The recurrence relation used in individually deriving the foregoing polynomials can be used to directly derive the following formula:

$$C(2t + 1) = \frac{n^2(2n - 1)}{n - 1} [(5n - 3)(2n - 1)^{t-1} - 2n^t] \quad (6a)$$

where $s = 2t + 1$

$N = n^{t+1}$

Table III gives comparative numbers of crosspoints for various num-

Table III — Crosspoints for Various Numbers of Switching Stages, s, and Values of N

N	$s = 1$	$s = 3$	$s = 5$	$s = 7$	$s = 9$
100	10,000	5,700	6,092	7,386	9,121
200	40,000	16,370	16,017	18,898	23,219
500	250,000	65,582	56,685	64,165	78,058
1,000	1,000,000	186,737	146,300	159,904	192,571
2,000	4,000,000	530,656	375,651	395,340	470,292
5,000	25,000,000	2,106,320	1,298,858	1,295,294	1,511,331
10,000	100,000,000	5,970,000	3,308,487	3,159,700	3,625,165

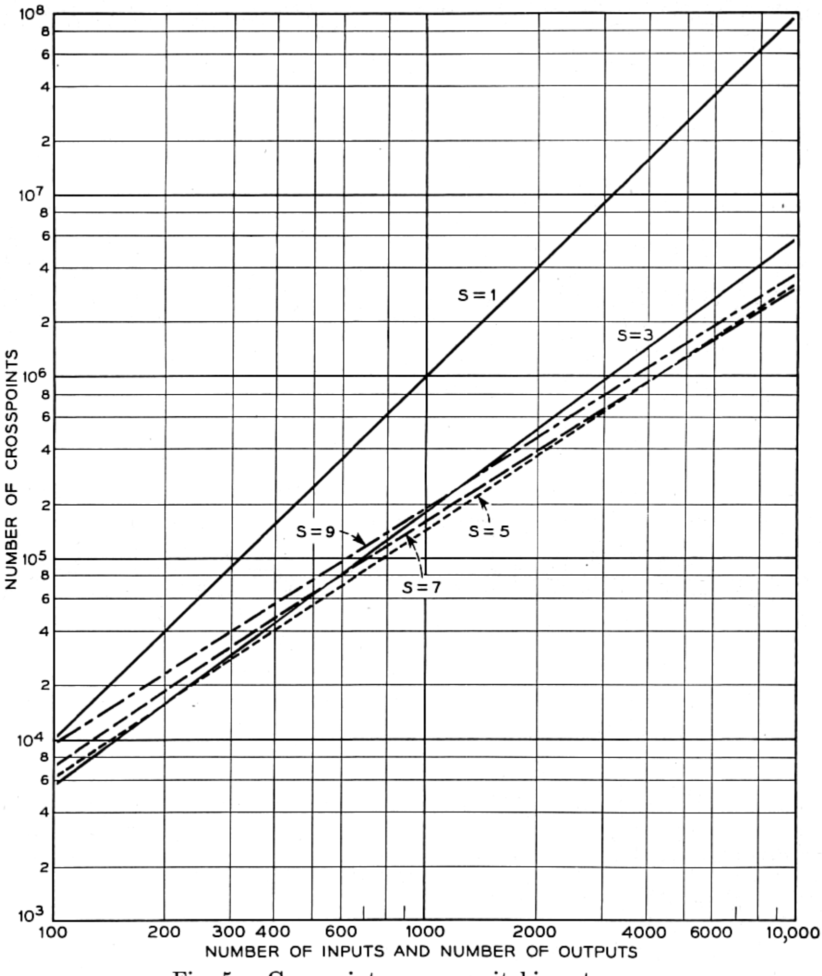

Fig. 5 — Crosspoints versus switching stages.

bers of switching stages and sizes of N. The data of Table III are plotted on Figure 5. The series of curves appear to be bounded by an envelope, representing a minumum of crosspoints. The next section dealing with minima indicates that points exist below this envelope.

MOST FAVORABLE SIZE OF INPUT AND OUTPUT SWITCHES IN THE THREE-STAGE ARRAY

The foregoing derivations were for implicit relationships between n and N, namely, n being the $\left(\frac{s+1}{2}\right)th$ root of N. To obtain minimum number of crosspoints a more general relationship is required. For the three stage switching array this is:

$$C(3) = (2n - 1)\left(2N + \frac{N^2}{n^2}\right) \tag{7}$$

When $n = N^{1/2}$ equation (7) reduces to equation (2).

For a given value of N, the minimum number of crosspoints occurs when $dC/dn = 0$ which gives:

$$2n^3 - nN + N = 0 \tag{8}$$

This equation has the following two pairs of integral values:

$$n = 2, \quad N = 16 \quad \text{and} \quad n = 3, \quad N = 27$$

As N approaches large values equation (8) can be approximated by:

$$N \doteq 2n^2 \tag{9}$$

Graphs of equations (8) and (9) are shown in Fig. 6. In Table IV the numbers of crosspoints are based on the nearest integral values of n for given values of N.

Where comparisons can be made, Table IV indicates fewer crosspoints than does Table I. This fact can be realized in another manner. By eliminating n in equations (7) and (9), the result for large values of N is:

$$C(3) \doteq 4\ (2)^{1/2}N^{3/2} - 4N \tag{10}$$

Equation (10) indicates fewer crosspoints than does equation (2).

MOST FAVORABLE SWITCH SIZES IN THE FIVE-STAGE ARRAY

If n be the number of inputs per input switch and outputs per output switch, and m be the number of inputs per switch in the second stage

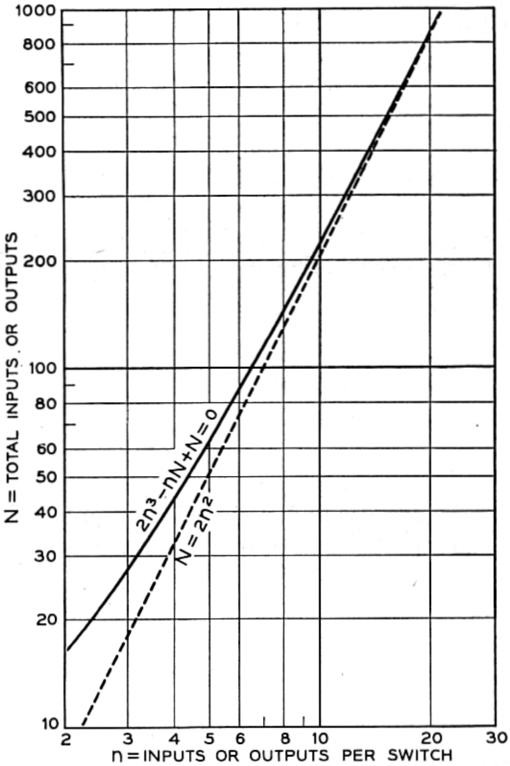

Fig. 6 — Relationship between N and n for minima in crosspoints. Three-stage array.

TABLE IV — CROSSPOINTS FOR SEVERAL VALUES OF N

N	Nearest Integral Value of n	Number of Crosspoints	
		N^2	Equation (7)
16	2	256	288
27	3	729	675
40	4	1,600	1,260
44	4	1,936	1,463
55	5	3,025	2,079
60	5	3,600	2,376
65	5	4,225	2,691
78	6	6,084	3,575
84	6	7,056	4,004
98	7	9,604	5,096
105	7	11,025	5,655

and outputs per switch in the fourth stage, then the following equation gives the total number of crosspoints:

$$C(5) = (2n - 1)\left[2N + (2m - 1)\left(\frac{2N}{n} + \frac{N^2}{n^2m^2}\right)\right] \tag{11}$$

The partial derivative of this equation with respect to m when set equal to zero yields:

$$n = \frac{N(m - 1)}{2m^3} \tag{12}$$

The partial derivative of this equation with respect to n when set equal to zero yields the following equation:

$$N = \frac{nm^2(2n^2 + 2m - 1)}{(2m - 1)(n - 1)} \tag{13}$$

Equations (12) and (13) can be solved for n and m in terms of given values of N. For example for $N = 240$, we obtain $n = 6.81$ and $m = 3.56$.

SEARCH FOR THE SMALLEST N FOR A GIVEN n FOR THE THREE-STAGE ARRAY

For a given value of n, equation (7) furnishes a means for locating that size of three-stage switching array which has N^2 or fewer crosspoints. This can be done by setting equation (7) equal to N^2:

$$N^2 = (2n - 1)\left(2N + \frac{N^2}{n^2}\right) \tag{14}$$

and solving for N in terms of n. The solution is:

$$N \geqq \frac{2n^2(2n - 1)}{(n - 1)^2} \tag{15}$$

Minimum values of N for given values of n are listed in Table V. This table also lists the next highest N exactly divisible by n. From this table it appears that when $N = 24$, we have the smallest switching array for which it may be possible to have less than N^2 crosspoints. However for $N = 25$, as shown in Table I, equation (2) gives more than N^2 crosspoints. The problem is one of finding an array for $N = 25$ with fewer than N^2 crosspoints. For this and all cases beyond, the next section indicates that it is profitable to consider situations where N is not exactly divisible by n.

CASES IN THE THREE-STAGE SWITCHING ARRAY WHERE $N \equiv r(\text{MOD } n)$

Table I indicated that for $N = 25$ and $n = 5$ a total of 675 crosspoints were required. A square array requires only 625. Fig. 7 shows a layout of switches where $N = 25$ and $n = 3$. In this case one input is left over when 25 inputs are divided into threes. The lone input requires three paths to the intermediary switches. This is in accordance with Fig. 3. The lone output also requires three paths to the intermediary switches. Also from Fig. 3, the lone input to the lone output requires only one path. Hence there must be one switch capable of connecting the lone input to the lone output. The number of crosspoints required is 615 which is less than the 625 required by the square array. This scheme can be extended to any case where $N = kn + r$, where the remainder, r, is an integer greater than zero but less than n. The formula for the number of crosspoints where k input and k output switches of size n and one input and output switch of size r are used is:

$$C = 2(2n - 1)(N - r) + 2(n + r - 1)r + (n - r)\left(\frac{N - r}{n}\right)^2 + (n + r - 1)\left(\frac{N - r}{n} + 1\right)^2 - n + r \qquad (16)$$

I. G. Wilson has pointed out that for a lone input the crosspoints in the intermediary switches can be used to isolate its possible connections hence no crosspoints are required in the input stage. This likewise applies for a lone output. With this modification the array in Fig. 7 requires six fewer crosspoints. For this case, when $r = 1$, the number of crosspoints is:

$$C = 2(2n - 1)(N - 1) + (n - 1)\left(\frac{N - 1}{n}\right)^2 + n\left(\frac{N - 1}{n} + 1\right)^2 - n + 1 \qquad (16a)$$

TABLE V — MINIMUM VALUES OF N FOR GIVEN VALUES OF n

n	N per Equation 15	$N \equiv 0 \pmod n$
2	24	24
3	22.5	24
4	24.9	28
5	28.1	30
6	31.7	36

J. Riordan has found a more efficient arrangement for cases where $N = kn + r$. In place of using k switches of size n and one switch of size r, he proposes that $(k + 1 - n + r)$ switches of size n and $(n - r)$ switches of size $n - 1$ be used. For this case the number of crosspoints is:

$$C = 2(2n - 1)(k + 1 - n + r)n + 2(2n - 2)(n - r)(n - 1) + (2n - 3)(k + 1)^2 + 2(k + 1)(k + 1 - n + r) \quad (17)$$

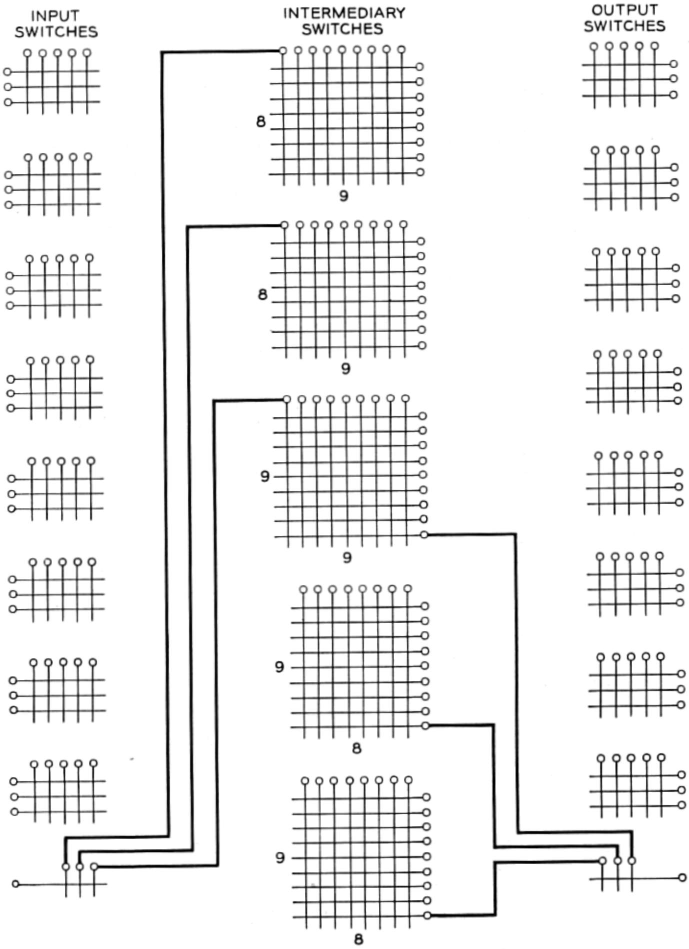

Fig. 7 — Three-stage array. $25 \equiv 1 \pmod 3$. An equivalent arrangement is to provide two 8 x 8 and three 9 x 9 intermediary switches. Two of the 9 x 9 switches need only 80 crosspoints.

Table VI — Crosspoints for Various Values of N and n

N	Square Array	Three-Stage Array			
		$n = 2$	$n = 3$	$n = 4$	$n = 5$
23	529*	540	530	556	—
24	576	576	560*	588	—
25	625	625	609*	633	—
26	—	663	643*	667	—
27	—	716	675*	701	—
28	—	756	730*	735	—
29	—	813	766*	788	—
30	—	855	800*	824	864
31	—	—	861	860*	911
32	—	—	899	896*	951
33	—	—	935*	957	991
34	—	—	1002	995*	1031
35	—	—	1042	1033*	1071
36	—	—	1080	1071*	1128
37	—	—	1153	1140*	1170
38	—	—	1195	1180*	1212
39	—	—	1235	1220*	1254
40	—	—	1314	1260*	1296
	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$
50	1819	1800*	1879	—	—
60	2415	2376*	2420	—	—
70	3164	3024*	3056	—	—
80	3920	3744*	3764	—	—
90	—	4536	4455*	4499	—
100	—	5400	5291*	5315	—
110	—	—	6199	6100	6156
120	—	—	7040	7044	6975*
130	—	—	8076	7923*	7947
140	—	—	—	8840*	8860
150	—	—	—	9968	9811*
160	—	—	—	10979	10800*

* Minimum values.

Equation (17) is identical to equation (16) when $r = n - 1$. There are two cases, namely, when $n = 2$ and $n = 3$ where equation (16a) gives fewer crosspoints than does equation (17).

SEARCH FOR THE MINIMUM NUMBER OF CROSSPOINTS BETWEEN $N = 23$ AND $N = 160$

The equations of the preceding sections furnish a means for searching for minimum crossnet arrays. Table VI shows the results of such a search up to $N = 160$. Results are indicated in unit steps from $N = 23$ to $N = 40$ and for every tenth interval thereafter. At $N = 161$, a five-stage array requires the fewest crosspoints.

Table VI was computed by the use of finite differences. The equations were:

$$C[(k+1)n] - C(kn) = (2n-1)(2n+2k+1) \tag{18}$$

$$C(kn+r+1) - C(kn+r) = 2(k+3n-1) \tag{19}$$

$$C(kn+1) - C(kn) = 2kn+1 \tag{19a}$$

Equation (18) was derived from equation (7) with N being replaced by $(k+1)n$ and by kn as required. Equation (19) was derived from equation (17) with r being replaced by $r+1$ as required. This equation applies for all values of n greater than 3 and for the particular case of $n = 3$ and $r = 2$. Equation (19a) was derived from Equations (16a) and (7) and is for the particular case of $r = 1$, when $n = 2$ and $n = 3$.

SEARCH FOR THE MINIMUM NUMBER OF CROSSPOINTS FOR $N = 240$

For a case where N is large enough to require five-switching stages, the search for the minimum number of crosspoints should be based on equations (12) and (13) and on the use of Table VI. The method is suggested by means of Table VII. The data in a previous section indicate

Table VII — Crosspoints for $N = 240$ and Various Values of n

Input and Output Stages				Intermediary Stages				Total Crosspoints
n	No. of Switches	Size of Switches	Cross-points	No. of Levels	Inputs and Outputs	m	Cross-points	
2	120	2 x 3	1,440	3	120 x 120*	8	20,925	22,365
3	80	3 x 5	2,400	5	80 x 80*	5	18,720	21,120
4	60	4 x 7	3,360	7	60 x 60*	5	16,632	19,992
5	48	5 x 9	4,320	9	48 x 48	4	15,120	19,440
6	40	6 x 11	5,280	11	40 x 40*	4	13,860	19,140
7	{30	7 x 13	5,460	2	30 x 35	3	1,826}	19,369
	5	6 x 12	720	11	35 x 35*	4	11,363}	
8	30	8 x 15	7,200	15	30 x 30*	3	12,000	19,200
9	{24	9 x 17	7,344	2	24 x 27	3	1,230}	19,467
	3	8 x 16	768	15	27 x 27*	3	10,125}	
10	24	10 x 19	9,120	19	24 x 24*	3	10,640	19,760
11	{20	11 x 21	9,240	2	20 x 22	—	880}	20,116
	2	10 x 20	800	19	22 x 22	—	9,196}	
12	20	12 x 23	11,040	23	20 x 20	—	9,200	20,240
Crosspoints per equation (3) five-stage array								20,596
Crosspoints per equation (2) three-stage array								21,624
Crosspoints per equation (1) square array								57,600

* See Table VI for minimum number of crosspoints.

that a minimum should occur for $N = 240$, when $n = 6.81$ and $m = 3.56$. In Table VII the minimum occurs when $n = 6$ and $m = 4$. It fails to occur at $n = 7$ because 240 is not exactly divisible by 7. Except for this situation, the minimum would have occurred as predicted.

RECTANGULAR ARRAY

Referring to Fig. 1, if there were N inputs and M outputs, a simple rectangular array would result which would be capable of sustaining up to N or M, whichever is the lesser, simultaneous connections without blocking. The number of crosspoints is:

$$C(1) = NM \tag{20}$$

N INPUTS AND M OUTPUTS IN A THREE-STAGE ARRAY

For the case of a three-stage switching array with N inputs and M outputs, let there be n inputs per input switch and m outputs per output switch. A particular input to be able to connect without blocking under the worst set of conditions to a particular output will require $(n - 1) + (m - 1) + 1$ available paths. Thus by providing for that many intermediary switches, a non-blocking switching array is obtained. The number of crosspoints is:

$$C(3) = (n + m - 1)\left[N + M + \frac{NM}{nm}\right] \tag{21}$$

Differentiating this equation first with respect to n and then to m yields two partial differential equations whose solution indicates that a minimum is reached when $n = m$. Replacing m by n in equation (21), the equation for the number of crosspoints becomes:

$$C(3) = (2n - 1)\left[N + M + \frac{NM}{n^2}\right] \tag{22}$$

Solving for the minimum number of crosspoints gives the following expression:

$$n^3 - \frac{NM}{N + M}n + \frac{NM}{N + M} = 0 \tag{23}$$

When $N = M$ this equation reduces to equation (8).

The three-way relationships of n, N and M are shown in Fig. 8.

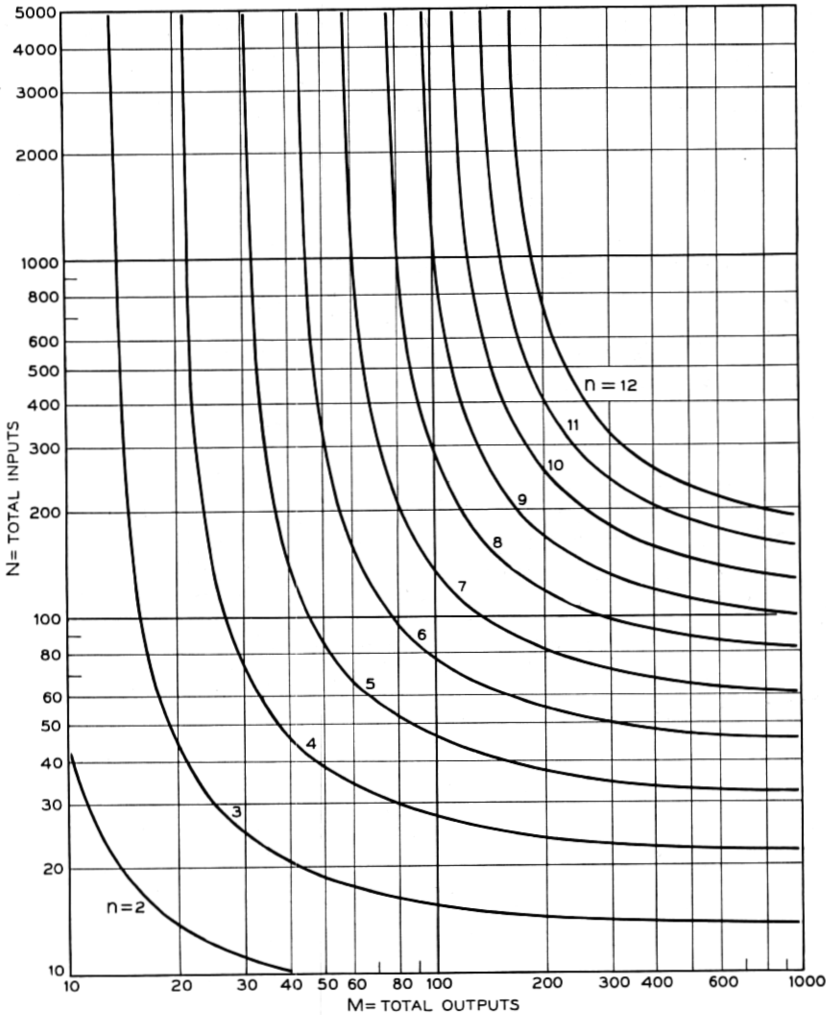

Fig. 8 — Relationship of n to N inputs and M outputs for a minimum in crosspoints in a three stage array.

TRIANGULAR ARRAY

If a case exists where all inputs are also the outputs, then an arrangement such as is shown in Fig. 9 can be used. The crosspoints in the intermediary switches permit connections between all switches on the left hand side. For connections between two trunks on the same switch it is assumed that one of the links to an intermediary switch can be used to establish the connection but without affecting any of the crosspoints on the intermediary switch. The number of crosspoints for this case is:

$$C = (2n - 1)\left(T + \frac{T^2}{2n^2} - \frac{T}{2n}\right) \quad (24)$$

where T = number of two-way trunks.

By differentiation, conditions for obtaining minimum numbers of crosspoints can be determined. The arrangement can also be extended to cases where extra switching stages are required.

ONE-WAY INCOMING, ONE-WAY OUTGOING AND TWO-WAY TRUNKS

A combination of the triangular array of Fig. 9 and of unequal inputs and outputs is shown in Fig. 10. In this figure, one-way incoming,

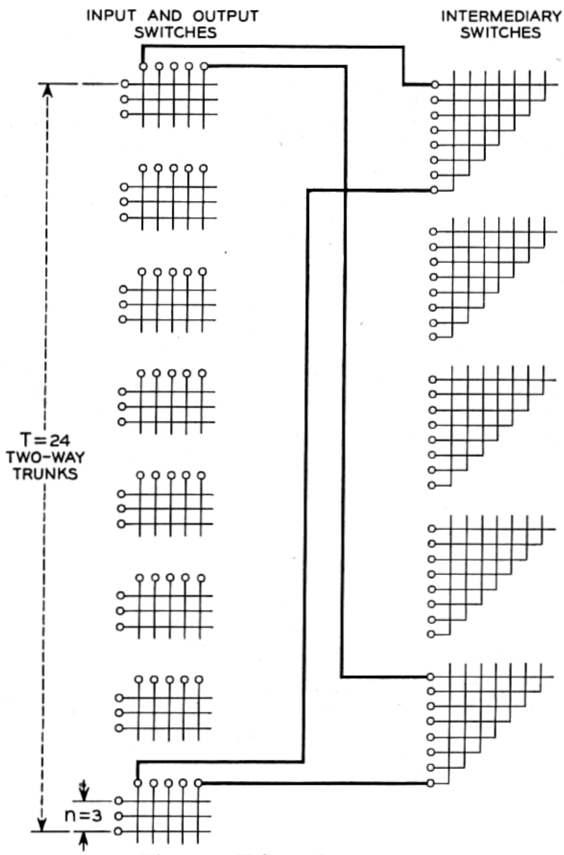

Fig. 9 — Triangular array.

one-way outgoing and two-way trunks can be freely interconnected without blocking. The number of crosspoints for this case is:

$$C = (2n - 1)\left[N + T + M + \frac{NT}{n^2} + \frac{MT}{n^2} + \frac{T^2}{2n^2} - \frac{T}{2n}\right] \quad (25)$$

The comments concerning the triangular array also apply for this case.

COMPARISON WITH EXISTING NETWORKS

Few existing crossnet arrays are non-blocking. An example is the four-wire intertoll trunk concentrating system. In one of its standard sizes 4,000 crosspoints are required for 100 incoming trunks and 40 outgoing intertoll trunks. From Fig. 8, for $N = 100$ and $M = 40$ it may be noted that the nearest integral value for n is 5. By substituting this value in equation (22), a non-blocking three-stage switching array of 2,700 cross-

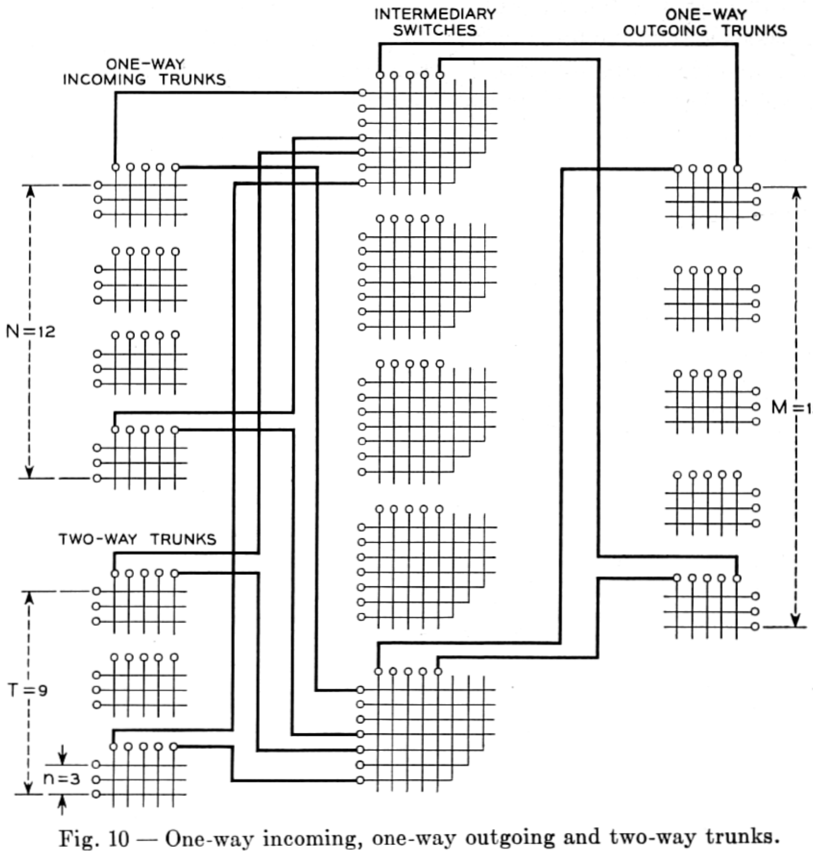

Fig. 10 — One-way incoming, one-way outgoing and two-way trunks.

points is found which could be used for the concentrating switch. In this case the new approach to the switching network problem may prove to be of value.

Comparisons with existing arrays having blocking are likely to be unfavorable because the grades of service are not the same. For instance, a No. 1 crossbar district-to-office layout of 1,000 district junctors and 1,000 trunks requires 80,000 crosspoints. This layout can handle 708 erlangs with a blocking loss of 0.0030. The minimum number of crosspoints with a non-blocking array is slightly less than 138,000. This, however, can handle 1,000 erlangs without blocking. By introducing blocking into the design methods described in this paper, a more favorable comparison with existing arrays having blocking can be made. This can be done by omitting certain of the paths. If done to an array requiring 1,000 inputs and 1,000 outputs a layout can be obtained requiring 79,900 crosspoints with a blocking loss of 0.0022 for a load of 708 erlangs. For this example, at least, it appears that the new design methods may prove to be valuable especially for use in the development of electronic switching systems where the control mechanism may not be dependent upon the particular switching array used.

CONCLUSION

In present day commercial telephone systems the use of non-blocking switching networks is rare. This may be due to the large number of crosspoints required. With the design methods described herein, a wider use of non-blocking networks may occur in future developments. For the usual case of networks with blocking, new systems have generally been designed by an indirect process. Several types and sizes of switching arrays are studied until the most economical one for a given level of blocking is found. With the new design methods, a straightforward approach is possible. Fig. 5 indicates that a region of minimum values exists. By first designing a non-blocking system with a reasonable number of switching stages and then omitting certain of the paths, the designer can arrive at a network with a given level of blocking and be very close to a minimum in crosspoints. The possibility of the adoption of this direct design method is important.

ACKNOWLEDGEMENT

In addition to those specifically mentioned in this paper, the author is also indebted to E. B. Ferrell, B. D. Holbrook, C. A. Lovell and E. F. Moore for suggestions and encouragement in the preparation of this paper.