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Ferrite Core Inductors

By H. A. STONE, JR.

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This paper describes the use of ferrite materials as cores for inductors and develops methods for taking maximum advantage of their properties in the design of inductors for communication circuits.

INTRODUCTION

The extent to which the theoretical capabilities of wave filters and networks can be realized in practice usually depends on how high a Q, ratio of reactance to resistance, can be obtained in the inductors. In the voice and carrier telephone frequency ranges the dissipation in mica capacitors is so small compared to that in inductors that it can generally be neglected. Even paper capacitors, in the lower frequency ranges, compare favorably in Q with the best available coils. Consequently, there has been considerable incentive to develop improved magnetic materials that will permit the realization of higher Q inductors for filter and network use.

Work along this line has resulted in the development of the permalloys, and later the molybdenum permalloys which, powdered, insulated and pressed into shapes, have become the standard materials for wave filter coils in the voice and carrier frequency ranges.[1, 2]

Although the permalloys represent a vast improvement over the soft iron that preceded them they share the fundamental disadvantage of all metals, that they are good conductors. Any conductor in the vicinity of an alternating magnetic field has eddy currents induced in it, and, if the conductor is the core of a coil, the power loss associated with these currents appears as added resistance in the coil windings. To restrict the eddy current paths it is customary to powder the material and insulate the particles with some kind of inorganic filler. However, there are two practical limitations on this procedure. First, there is the mechanical difficulty of grinding particles to smaller than the few microns diameter that now represents the commercial limit. Second, as the size of the individual particles is made smaller more air gaps are introduced in the magnetic circuit by the insulating material, and the effective over-all permeability of the core is reduced. If carried too far the advantages in using the magnetic material in the first place are largely lost.

The need for high resistivity magnetic materials has been recognized for many years, and, in fact, the naturally occurring magnetic iron oxide, Fe_3O_4, or lodestone, is such a material. Its permeability, however, is only about 3 or 4 and its use in coil work has been very restricted. The problem of producing an oxide or mixture of oxides having higher permeabilities was investigated by Philips Gloeilampenfabrieken in Holland during the late war and they were successful in producing oxide mixtures, or ferrites, having permeabilities of 1500 and higher.[3]

After the war, Bell Telephone Laboratories initiated a program of ferrite development, and at the present time ferrite cores as well as inductors and transformers which take advantage of their unique properties are in commercial production.

The most commonly used ferrites consist of solid solutions of oxides of manganese, zinc and iron, or nickel, zinc and iron. They are prepared by mixing the materials, pressing them into the required shape, and heat treating them under carefully controlled conditions. The resulting ferrite parts are hard and brittle but they can be machined with diamond tools.

Although ferrites belong in the class of so-called semiconductors their conductivities are of the order of a millionth of those of metals and the eddy current losses are proportionately smaller. There are frequency limitations in ferrites but these occur at higher frequencies than are ordinarily of concern in magnetic core inductor work.

Besides offering the coil designer new possibilities because of low eddy current losses the ferrites have a secondary advantage in that they do not have to be subdivided to keep those losses low. The powdered metallic materials are fragile, are difficult to press except in simple shapes such as rings or cylindrical plugs, and are difficult to machine. As a result the coil designer, for virtually every use, is restricted to the toroidal type coil which, although it has some important advantages, is inherently expensive to wind and adjust. With ferrites, on the other hand, a variety of shapes can be produced. They have ample strength and can be ground and machined.

The combined magnetic and mechanical advantages of ferrites put them in an entirely new class as core materials for inductors. To exploit these advantages properly it is necessary to reconsider some of the fundamental aspects of coil design, since some of the assumptions that applied to powdered core coils are no longer valid, and others have to be modified.

DISSIPATION FACTOR

Legg has shown that the core loss characteristics of metallic magnetic materials can be described by three constants, defined as eddy current (e), hysteresis (a), and residual (c), coefficients, respectively.[1] The total increment of resistance (R_m), due to core losses is given by

$$R_m = e\mu f^2 L + a\mu B_m f L + c\mu f L \tag{1}$$

where μ = effective permeability of the core structure

f = frequency in c.p.s.

L = inductance in henrys

B_m = max. flux density in gausses

This expression is valid for practical applications so long as the flux density is low.

Measurements on ferrite cores indicate that the same formula can be used, with appropriate constants, at least for frequencies up to 200 or 300 kc, and, again, for low flux density applications. However, the emphasis on the terms becomes quite different. The residual loss, as its name implies, is usually of negligible importance in metallic magnetic materials, but in ferrites because of the very low value of the eddy current constant, the residual loss often emerges as a controlling factor.

Beside the core losses the other factors in an inductor that contribute to the total measured resistance are the dc resistance of the winding, ac losses in the winding and the parasitic capacitances within the structure. The latter two, especially the capacitance effects, have always had to be taken into account in higher frequency work, but their contribution to the total loss becomes a matter of first order importance when ferrites are used since they are no longer small in comparison with the eddy current loss in the core.

The objective in the design of inductors for wave filter and similar applications is to provide a specified inductance with as low an associated resistance as practicable. The quality factor, Q, the ratio of reactance to resistance, is the traditional measure of the extent to which this has been achieved. In the discussion that follows, however, it will be more convenient to use the reciprocal of Q as the measure of quality. This permits combining the loss components in a simple additive manner. Thus

$$\text{Dissipation Factor} = D = \frac{1}{Q} = \frac{R_{dc} + R_e + R_h + R_r + R_c + R_s}{\omega L} \quad (2)$$

where R_{dc} = dc resistance of the winding

R_e = eddy current loss

R_h = hysteresis loss

R_r = residual loss

R_c = increment of measured resistance due to distributed capacitance

R_s = ac loss in wire of the winding.

or

$$D = D_{dc} + D_e + D_h + D_r + D_c + D_s \quad (3)$$

where $D_n = \dfrac{R_n}{\omega L}$ (n being any of the above subscripts).

Having been given the inductance, and the operating frequency and current, for a desired inductor, and having chosen the core material that will be used, each of the D's above will be functions of the following variables: Permeability, Volume and Proportions of the core structure. We will now investigate how each of these factors can be manipulated to yield the highest Q (or lowest D) coil.

EFFECT OF PERMEABILITY ON DISSIPATION FACTOR

The permeability of a permalloy powder core is determined by the fineness of the powder and the amount of insulation with which it is mixed before firing. Obviously it is commercially impracticable to manufacture cores having a great variety of permeabilities, and it has happened that four values have been standardized for commercial use; 125, 60, 26 and 14. The coil designer working with permalloy powder, beyond choosing the most appropriate of these four values, seldom finds it practicable to control the permeability. This is due to the difficulty of pressing shapes suitable for use with air gaps, and the fragility of the parts. With ferrite, on the other hand, the parts can readily be adapted to the control of permeability by insertion of air gaps, and thus establishment of the optimum permeability becomes an important step toward achieving the desired coil performance.

The first four of the "D's" in (3), dc resistance and the core losses, are direct functions of the permeability and can be discussed to a certain extent independently of the last two.

The inductance of a magnetic core inductor is given by

$$L = \frac{kN^2A\mu}{\ell} \tag{4}$$

where k = constant

N = number of turns in winding

A = cross section of core

ℓ = mean length of magnetic path through core.

If the geometric proportions of the core have been prescribed, A and ℓ will bear a constant relationship to $V^{2/3}$ and $V^{1/3}$, respectively, and (4) may be written

$$L = k_1N^2V^{1/3}\mu \tag{5}$$

where k_1 = constant

V = core volume.

The dc resistance in the winding is

$$R_{dc} = \frac{\rho N^2\lambda}{k_wW} \tag{6}$$

where ρ = resistivity of the conductor

λ = mean length of turn

W = available area of cross section through which turns can be linked with the core

k_w = winding efficiency. This is the ratio of the actual cross section of conductor to the available area, W.

Again, for a core of given proportions (6) may be written

$$R_{dc} = \frac{k_2N^2}{V^{1/3}} \tag{7}$$

Eliminating N from (5) and (7)

$$R_{dc} = \frac{k_3 L}{V^{2/3}\mu} \tag{8}$$

and

$$D_{dc} = \frac{k_4}{V^{2/3}\mu f} \tag{9}$$

where k_2, k_3, k_4 are constants.

From (1) and (2) it is seen that

$$D_e = k_5 \mu f \tag{10}$$

$$D_h = k_6 B_m \mu = \frac{k_{10}\mu^{3/2} L^{1/2} i}{V^{1/2}} \tag{11}$$

$$D_r = k_7 \mu \tag{12}$$

in which the flux density,

$$B_m = \sqrt{2}\,\mu H = k_8 \mu \frac{Ni}{\ell} = k_9 \sqrt{\frac{\mu L i^2}{\ell A}} = k_9 \sqrt{\frac{\mu L i^2}{V}}. \tag{13}$$

H is the field strength due to the r.m.s. current, i, and k_5 to k_{10} are constants.

Combining (9) through (12)

$$D = \frac{k_4}{V^{2/3}\mu f} + k_5 \mu f + k_{10} \frac{\mu^{3/2} L^{1/2} i}{V^{1/2}} + k_7 \mu. \tag{14}$$

For a specified inductance at a given frequency and current the optimum permeability can be found from (14) by inserting the numerical values and solving graphically. It is very unlikely, however, that this cumbersome procedure would be necessary in practical work. Usually, depending on the design requirements, one or another of the core loss factors will be comparatively large and the others can be neglected. We will examine separately the three cases where residual, hysteresis and eddy current loss, respectively, predominate over the other core losses, and show how the permeability can be adjusted to minimize the dissipation factor.

1. *DC Resistance and Residual Loss Predominate*

This condition, which formerly was very seldom encountered, is becoming of increasing practical importance for two reasons. First, the eddy current losses in ferrite are extremely small. Second, the introduction of the transistor has created many new applications for inductors for use at very low power levels, and with proportionately lower hysteresis losses. For this condition we have, from (14)

$$D = \frac{k_4}{V^{2/3}\mu f} + K_7\mu \tag{15}$$

Solving for the value of μ, $\mu_{\text{opt.}}$, which will yield the lowest dissipation

$$\frac{dD}{d\mu} = -\frac{k^4}{V^{2/3}\mu^2 f} + k_7$$
$$\mu_{\text{opt.}} = \frac{k_4^{1/2}}{k_7^{1/2}V^{1/3}f^{1/2}} \tag{16}$$

and

$$D_{\text{opt.}} = 2\,\frac{k_4^{1/2}k_7^{1/2}}{V^{1/3}f^{1/2}}. \tag{17}$$

It may be noted from (16) and (17) that the optimum permeability is that which will result in the dc resistance and residual loss being equal.

2. *DC Resistance and Hysteresis Loss Predominate*

This condition, while it does not often apply to conventional size inductors for transmission networks, may occur when miniaturization of coils is contemplated. It will be noted from (14) that hysteresis is the only core loss component that is dependent on the volume of the core. We have, from (14)

$$D = \frac{k_4}{V^{2/3}\mu f} + \frac{k_{10}\mu^{3/2}L^{1/2}i}{V^{1/2}}. \tag{18}$$

The optimum permeability and dissipation factor derived from this equation are

$$\mu_{\text{opt.}} = \sqrt[5]{\frac{4k_4^2}{9k_{10}^2 f^2 L i^2 V^{1/3}}} \tag{19}$$

$$D_{\text{opt.}} = [(3/2)^{2/5} + (2/3)^{3/5}]\sqrt[5]{\frac{k_4^3 k_{10}^2 L i^2}{f^3 V^3}}. \tag{20}$$

Inspection shows that

(a) The optimum permeability is such that the ratio of dc resistance to hysteresis loss is 3/2.

(b) This value will depend not only on the frequency and volume of core, but on the inductance and current level as well.

3. *DC Resistance and Eddy Current Loss Predominate*

This condition is the most commonly occurring of all when metallic magnetic materials are used but is of very little importance for ferrite coil design. From (14)

$$D = \frac{k_4}{V^{2/3}\mu f} + k_5\mu f \tag{21}$$

$$\mu_{\text{opt.}} = \frac{k_4^{1/2}}{k_5^{1/2}V^{1/3}f} \tag{22}$$

$$D_{\text{opt.}} = \frac{2k_4^{1/2}k_5^{1/2}}{V^{1/3}}. \tag{23}$$

Here we note that, as in the case for residual loss, optimum permeability is that which assures that the core loss is equal to the dc resistance. If both eddy current and residual losses were significant the sum of these should be equal to the DC resistance. It is also seen from (22) and (23) that although the optimum permeability is a function of frequency the resulting dissipation factor is independent of the frequency.

We have developed expressions for optimum permeability for three conditions of core loss. It is now of interest to know how critical the adjustment of permeability is. Fig. 1 shows the effect on the dissipation factor of deviations from optimum in permeability. It is seen that for

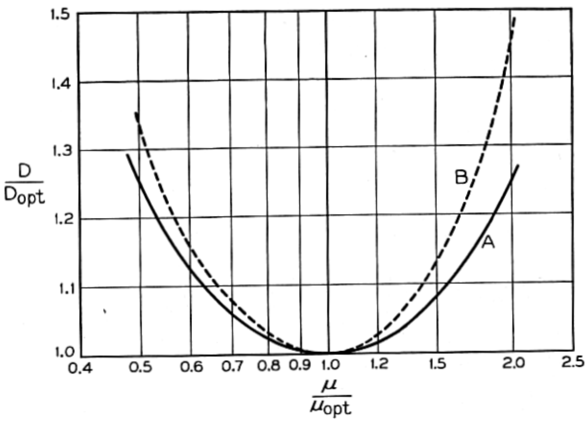

Fig. 1 — Effect of permeability on dissipation factor.

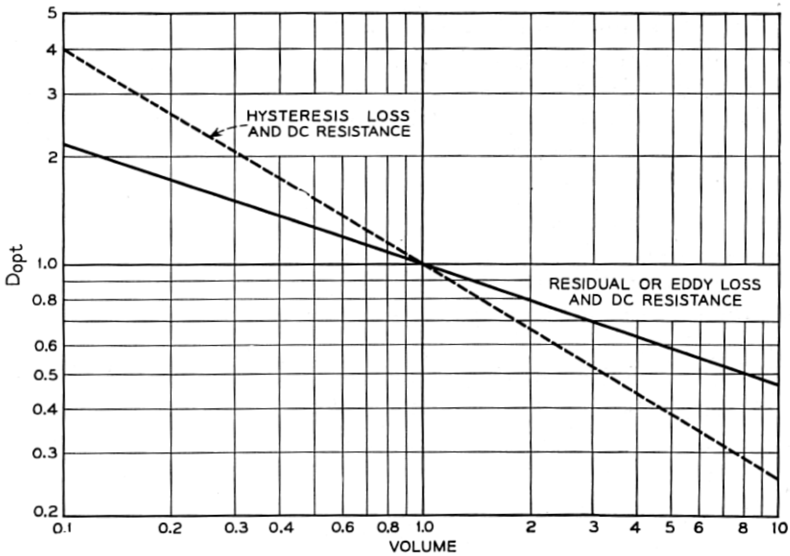

Fig. 2 — Relationship between core volume and dissipation factor.

deviations from optimum up to about 20 per cent the penalty in dissipation is not severe, but beyond this point the dissipation factor increases rapidly.

EFFECT OF VOLUME ON DISSIPATION FACTOR

It has been shown in the preceding section that the permeability can be adjusted for optimum coil quality but that the value to which it is adjusted depends on which of the loss factors predominates. It will also be seen from the foregoing that both the value of optimum permeability and the dissipation factor that can be realized by so adjusting the permeability depend on the volume of the core. The curves of Fig. 2 are derived from (17), (20) and (23), respectively, considering $D_{\text{opt.}}$ as a function of the independent variable, V. That is, they show how the dissipation factor will vary as a function of volume assuming that the permeability is adjusted as the volume is changed so that the lowest possible dissipation factor is always realized.

As would be expected, these curves show that, regardless of the relative magnitudes of the core losses, the dissipation factor can be decreased by increasing the size of the coil. There are serious limitations, however, on how far this process can be carried, and these are due to D_c and D_s, the resistive components resulting from capacitance and ac loss in the wire.

This ac loss is due to a combination of skin effect and eddy currents in the conductor. It is proportional to the sixth power of the diameter of each of the strands of wire that go to make up the conductor, and the square of the number of strands[4]. If the size of an inductor is increased, therefore, the ac loss may rapidly assume important proportions. Some help can be obtained by finer stranding of the wire and this has, in some cases, been carried to the extent of using 810 separately insulated strands in a single conductor. Even if fine stranding is sufficient to reduce the ac loss to a tolerable value, a penalty is paid in the form of higher dc resistance, due to the space occupied by the separate insulation on the strands. This amounts to a decrease in the winding efficiency, k_w, in (6).

The effect of distributed capacitance on the dissipation factor of a coil is a function not only of the volume, directly, but of the absolute value of the dissipation factor. If the dissipation factor is low compared to unity it is given, approximately, by

$$D_c = \frac{(D_0 + d)\dfrac{C}{C_g}}{1 - \dfrac{C}{C_g}} \tag{24}$$

where $D_0 = D - D_c$ is the dissipation factor that would obtain were it not for capacitance

d = dissipation factor of the distributed capacitance

C = distributed capacitance

C_g = resonating capacitance of the inductor.

If (24) is written in an alternative form

$$\frac{D}{D_0} = \frac{1 + \dfrac{d}{D_0}\dfrac{C}{C_g}}{1 - \dfrac{C}{C_g}} \tag{25}$$

it becomes evident that a coil with an inherently high Q is more seriously affected, proportionately, by distributed capacitance than is a low Q coil. Fig. 3 shows this information graphically, using the value 0.01 for d, which measurements on model inductors indicate to be of an appropriate magnitude.

Each of the curves in Fig. 3 is based on a constant value for C/C_g. In practice, however, as volume is increased it becomes more difficult to

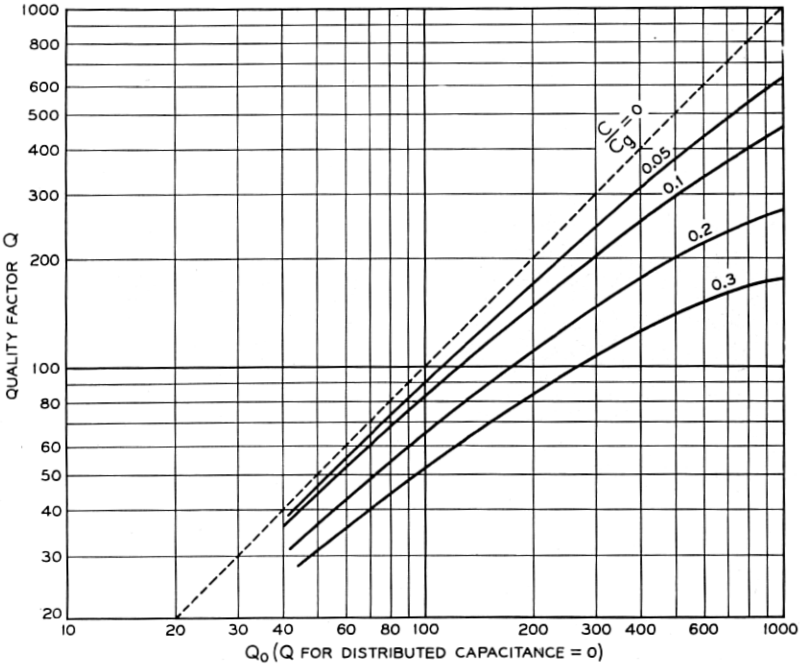

Fig. 3 — Effect of distributed capacitance on Q.

maintain small values of distributed capacitance. It will be necessary to increase the separation of windings from each other and from the core and this will result in a lower winding efficiency and a higher dc resistance.

Since the limitations imposed by ac losses in the wire and by distributed capacitance depend on the absolute magnitudes of the design parameters and the mechanical details of the inductor structure they do not lend themselves to representation in practicable generalized formulas as do the core losses. However, the following information on some model inductors will illustrate the magnitudes of these limitations.

A ferrite core coil was constructed similar to that shown in Fig. 4, but having a core volume of only 0.04 cubic inches. It was a 5 mh coil for use at 100 kc and had a distributed capacitance of 10 mmf. It was wound with a single strand conductor and the winding efficiency, k_w, was 0.4. Since it was intended for use at very low power levels the hysteresis loss was negligible. The permeability was optimized in accordance with case 1, above, for residual loss predominating. The measured Q was very close to 300. Thus, $D = 0.0033$. From the above data we note that $C/C_g = 0.02$, and from (25) we can calculate that $D_0 = 0.0030$,

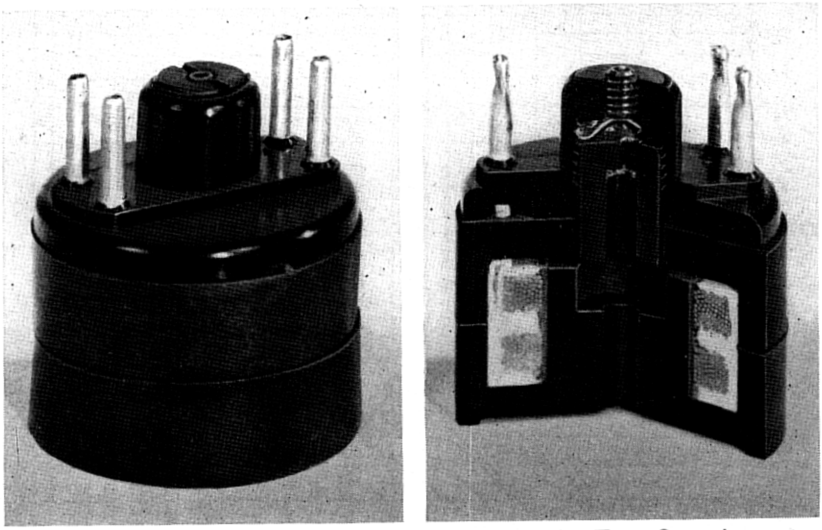

Fig. 4 — The 1509 type adjustable ferrite inductor for Type-O carrier system filters.

using the value 0.01 for d. Thus the "geometric Q", that which would obtain were it not for the distributed capacitance, is about 330.

The 1509 type inductor, shown in Fig. 4, was designed for use in the type-O carrier system.[5] It has a volume of 1.4 cubic inches, 35 times that of the small coil discussed above. From the standpoint of dc resistance and core loss alone, (7) indicates that the dissipation factor should be

$$D_0 = \frac{1}{35^{1/3}}(0.0030) = 0.00092,$$

or that the Q should be greater than 1000.

In order to keep the capacitance to the same order as that of the small coil, and to use stranded wire to avoid excessive ac loss, the winding efficiency, k_w, had to be reduced from 0.4 to 0.13. The effect of this on dissipation can be shown from (17), remembering that k_4 varies inversely as k_w:

$$\frac{D_0'}{D_0} = \frac{(3k_4)^{1/2}}{k_4^{1/2}} = 1.73$$

$$D_0' = (1.73)(0.0092) = 0.00159.$$

Thus, the Q (still neglecting the effect of the 10 mmf distributed capacitance that remains after reducing the winding efficiency) should be 630.

From (24)

$$D_c = \frac{(0.00159 + 0.01)0.02}{0.98} = 0.00024$$

and

$$D = D_0 + D_c = 0.00183$$

or the actual measured Q of the large inductor is 550, about half that indicated by core loss considerations alone.

EFFECT OF CORE PROPORTIONS ON DISSIPATION FACTOR

In the foregoing discussion of coil volume it has been assumed that the core porportions remained fixed as the volume was changed. It is now of interest to know what these proportions should be to insure the lowest dissipation.

The general type of structure under consideration consists of a closed cylindrical container and a center post of magnetic material, and an air gap which might be anywhere in the magnetic path. It is assumed that the thickness of the shells is such that the area of the magnetic path is uniform and equal to the area of cross section of the post. It is also assumed that the flux is uniformly distributed within the cross-section of the core. Fig. 5 represents such a core schematically.

Given a fixed over-all coil volume it is desired to know what proportions should apply to the outside diameter, the diameter of the center post and the axial height of the structure, to provide the lowest dissipation factor. This will be examined first for the case where residual or eddy

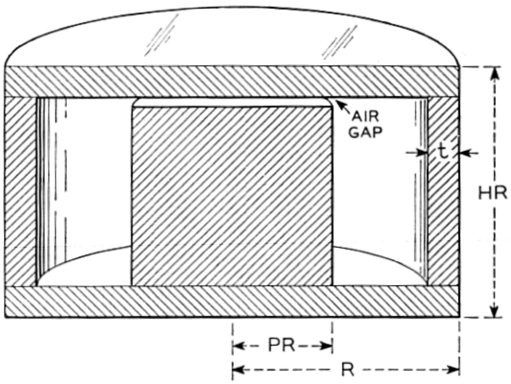

Fig. 5 — Optimum proportions for post and shell core assembly.

current losses are large compared to hysteresis loss, and then for the situation where hysteresis loss predominates. Referring to Fig. 5, the following relationships are noted.

$$t \text{ (thickness of shell) is given by } (R - t) = R\sqrt{1 - P^2} \tag{26}$$

$$A \text{ (magnetic cross-section)} = \pi P^2 R^2 \tag{27}$$

$$\ell \text{ (mean magnetic path)} = R(3\sqrt{1 - P^2} - 1 + 2H) \tag{28}$$

$$W \text{ (available winding cross-section)} = R^2(\sqrt{1 - P^2} - P)(2\sqrt{1 - P^2} - 2 + H) \tag{29}$$

$$\lambda \text{ (mean length of turn)} = \pi R(\sqrt{1 - P^2} + P) \tag{30}$$

$$V_c \text{ (coil volume)} = \pi R^3 H \tag{31}$$

1. *DC Resistance, and Residual or Eddy Current Losses, Predominate*

The dissipation factor, due to the dc resistance, is seen from (4) and (6) to be

$$D_{dc} = \frac{R_{dc}}{2\pi f L} = k_{11} \frac{\lambda \ell}{W A \mu} \tag{32}$$

where $k_{11} = (\rho/2\pi k k_w)$ is a constant with respect to the core proportions, assuming the winding efficiency is not materially affected by changes in shape.

Since, regardless of shape, we will want to adjust the air gap so that μ is optimum, and since we know from the previous discussion that this will occur when the dc resistance is equal to the sum of the core losses, we have, from (10) and (12)

$$D_{dc} = D_e + D_r = (k_5 f + k_7)\mu \tag{33}$$

Eliminating μ from (32) and (33):

$$D_{dc} = k_{12} \sqrt{\frac{\lambda \ell}{W A}} \tag{34}$$

where $k_{12} = \sqrt{k_{11}(k_5 f + k_7)}$ is again a constant with respect to the core dimensions. The dissipation factor is

$$D = 2D_{dc} = 2k_{12} \sqrt{\frac{\lambda \ell}{W A}} \tag{35}$$

Putting in the values, from (27) to (30), for λ, ℓ, W and A:

$$D = \frac{2k_{12}}{R}\sqrt{\frac{(\sqrt{1-P^2}+P)(3\sqrt{1-P^2}-1+2H)}{P^2(\sqrt{1-P^2}-P)(2\sqrt{1-P^2}-2+H)}} \quad (36)$$

Since the volume of the coil, given in (31), is assumed to be constant, we can eliminate R from the above equation:

$$D = k_{13}\sqrt{\frac{(\sqrt{1-P^2}+P)(3\sqrt{1-P^2}-1+2H)H^{2/3}}{P^2(\sqrt{1-P^2}-P)(2\sqrt{1-P^2}-2+H)}} \quad (37)$$

where $k_{13} = \dfrac{2\pi^{1/3}k_{12}}{V_c^{1/3}}$

We now have an expression for the dissipation factor in terms of the two variables, P and H, which determine the proportions of the coil structure. The effects of these proportions on the dissipation factor are shown in Fig. 6. It will be seen that best results are achieved when the radius of the post is approximately 0.45 of the outside radius and the axial height is about 1.2 times this radius. Fig. 5 is drawn to this scale, and the 1509 type coil shown in Fig. 4 approximates these proportions.

2. *DC Resistance and Hysteresis Loss Predominate*

We have noted that for this case optimum permeability is that which results in the following relationship between the dc and hysteresis dissipations:

$$3D_h = 2D_{dc} \quad (38)$$

From (11) and (13)

$$D_h = k_6 B_m \mu = \frac{k_6 k_9 \mu^{3/2} L^{1/2} i}{\ell^{1/2} A^{1/2}} \quad (39)$$

Putting the values from (32) and (39) in (38), we can eliminate μ and express D_{dc} in terms of the dimensional variables of the coil:

$$D_{dc} = k_{14}\frac{\lambda^{3/5}\ell^{2/5}}{W^{3/5}A^{4/5}} \quad (40)$$

where $k_{14} = (\tfrac{3}{2}k_6 k_9 k_{11}^{3/2} L^{1/2} i)^{2/5}$

The total dissipation factor is

$$D = D_{dc} + D_h = \tfrac{5}{3}D_{dc} = \tfrac{5}{3}k_{14}\frac{\lambda^{3/5}\ell^{2/5}}{W^{3/5}A^{4/5}} \quad (41)$$

Using (27) to (30) we can express D in terms of the coil proportions

$$D = \tfrac{5}{3}k_{14} \sqrt[5]{\frac{(\sqrt{1 - P^2} + P)^3(3\sqrt{1 - P^2} - 1 + 2H)^2}{R^9P^8\pi(\sqrt{1 - P^2} - P)^3(2\sqrt{1 - P^2} - 2 + H)^3}} \tag{42}$$

Again, eliminating R by use of the constant volume relationship, (31)

$$D = k_{15} \sqrt[5]{\frac{H^3(\sqrt{1 - P^2} + P)^3(3\sqrt{1 - P^2} - 1 + 2H)^2}{P^8(\sqrt{1 - P^2})^3(2\sqrt{1 - P^2} - 2 + H)^3}} \tag{43}$$

where $k_{15} = \dfrac{5\pi^{2/5}k_{14}}{3V_c^{3/5}}$

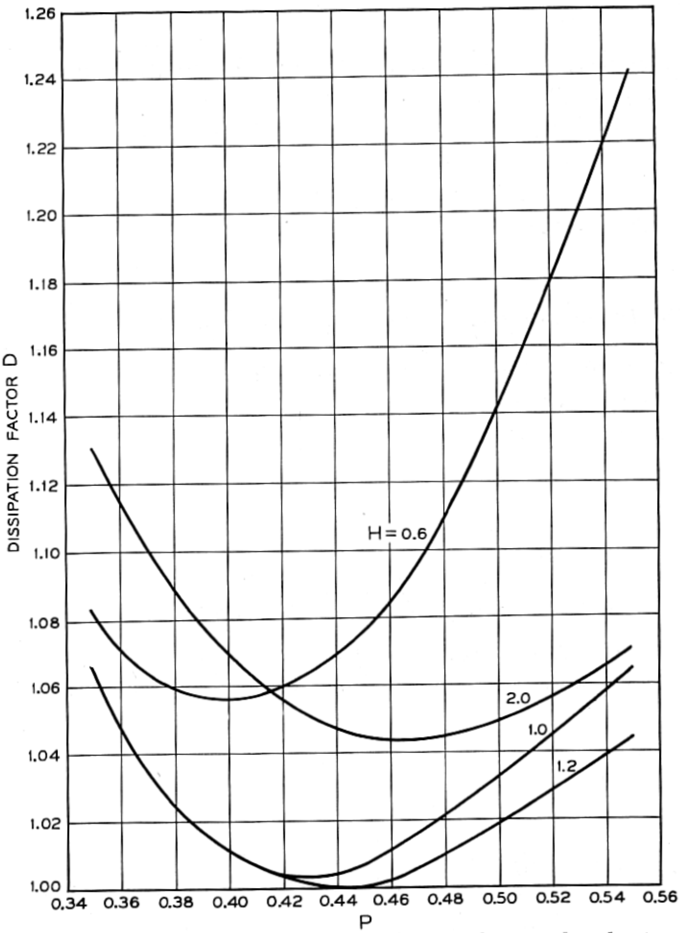

Fig. 6 — Effect of core proportions on dissipation factor when hysteresis loss can be neglected.

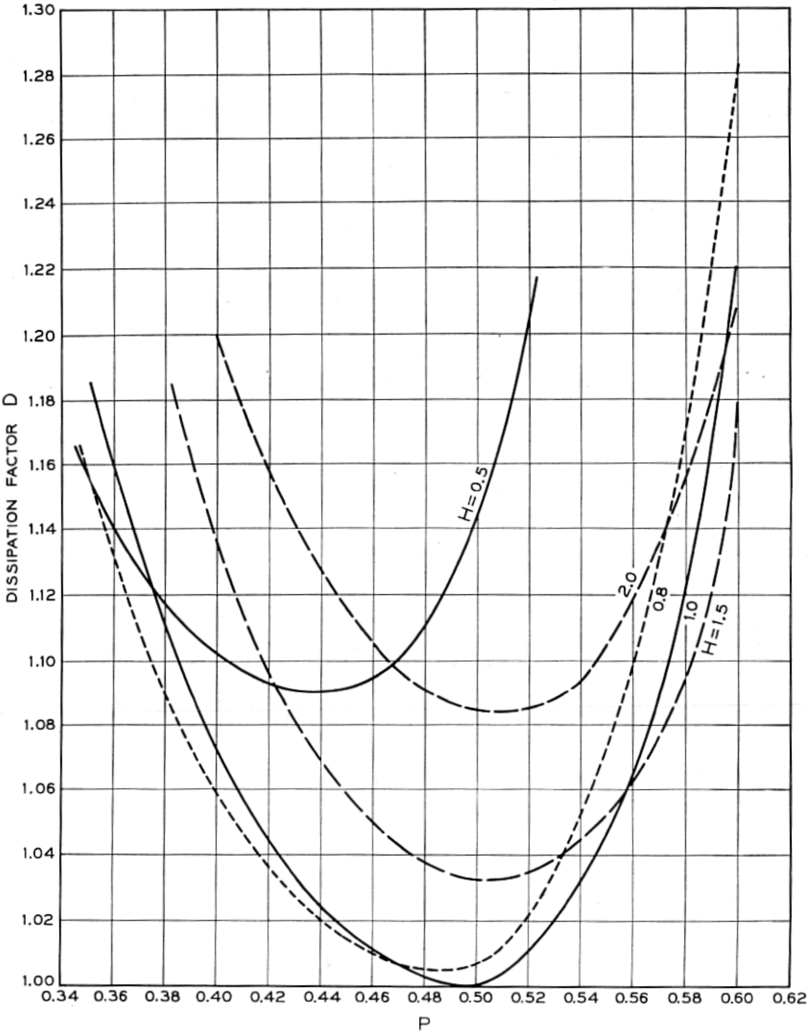

Fig. 7 — Effect of core proportions on dissipation factor when hysteresis loss is predominate.

This information is shown graphically in Fig. 7. It will be seen that when the hysteresis losses are important both the post diameter and the axial height should be about half of the overall diameter. These proportions are not far different from those derived as optimum from (37). This is fortunate since it means that cores of the same proportions will be suitable in different applications regardless of which core losses predominate.

METHODS OF INDUCTANCE ADJUSTMENT

The fact that ferrites can be molded in a variety of shapes, can be machined, and can be used with discrete air gaps permits considerable freedom in the mechanical methods that can be used to provide continuous adjustability of inductance. Whether the need is for factory adjustment to prescribed values or for adjustment after the coil is assembled in its equipment, the basic methods are the same.

We have noted, in (4) that the inductance of a magnetic core coil is given by

$$L = \frac{kN^2A\mu}{\ell} \tag{4}$$

in which k is a constant depending on the units used, and the other four quantities are design variables. Any of these, or any combinations of them, can be manipulated to produce variations in the inductance. They are: length of magnetic path, ℓ; cross section of magnetic path, A; number of turns, N; and effective permeability, μ. There is an additional variable hidden by the formula's assumption that perfect coupling exists among all the turns in the winding. Inductance can be adjusted by changing the coupling between parts of the winding. Each of these five variables, which are of widely differing practical value, will be commented upon at least briefly.

1. *Adjustment of Inductance by Change in Magnetic Path Length, ℓ*

Adjustment by a change in ℓ without an accompanying change in μ requires that the air gap also be changed since the effective permeability is a function of the ratio of the air gap dimensions to those of the total structure.

$$\mu = \frac{\mu_m}{1 + \mu_m \dfrac{g}{a}} \tag{44}$$

where μ_m = permeability of the core material

g = ratio of the length of air gap to total length, ℓ

a = ratio of effective cross section of gap to core cross section, A.

It would be mechanically possible to design an adjustable inductor of this sort but it is unlikely that there would be any practical advantage in maintaining constant permeability over the adjusting range. On the contrary, it would probably be more advantageous as well as mechanically simpler to have the air gap length constant so that the increase in inductance due to shortening the magnetic path would be augmented by an increase in permeability. In a device such as shown schematically in Fig. 8, as the two windings approach each other the over-all magnetic path decreases and at the same time the area of the air gap increases. Actually, the preponderant effect is due to the air gap change and the effect of variation in length of path becomes of secondary importance. This is likely to be the case generally, when both path length and air gap change simultaneously, since most of the reluctance is in the air gap.

2. *Adjustment of Inductance by Change in Magnetic Cross Section, A*

To provide for change of inductance by constriction of a part of the magnetic cross section is apt to be undesirable since it forces a concentration of flux in the constricted part of the magnetic circuit and may introduce unduly high hysteresis losses. Even if the levels are low enough so that this is not a consideration the amount of inductance variation that can be achieved even by a large constriction in part of the core is relatively small. To illustrate this we will consider a structure such as shown schematically in Fig. 9, in which one sector of the core can be varied in effective cross section. The reluctance of the structure is equal to the sum of the reluctances of the fixed part of the core, the sector of length $n\ell$, whose cross section aA can be varied, and the air gap:

$$\begin{aligned} R &= \frac{(1-n)\ell}{\mu_m A} + \frac{n\ell}{\mu_m a A} + \frac{g\ell}{A} \\ &= \frac{\ell}{\mu_m A}\left[1 + n\left(\frac{1}{a} - 1\right) + g\mu_m\right]. \end{aligned} \tag{45}$$

Let us assume the arbitrary but reasonable values of 2000 for μ_m, 0.01 for g, and 0.1 for n. Then approximately

$$R = \frac{\ell}{2000A}\left(20 + \frac{0.1}{a}\right).$$

It will be seen that as a is varied from 1.0, corresponding to the full cross section of the main core, to one-tenth of that, the change in inductance, which is inversely proportional to reluctance, will only amount to about 5 per cent.

3. *Adjustment of Inductance by Change in Number of Turns, N*

An adaptation of turns adjustment for use in a shell type structure is shown in Fig. 10.[6] The center post with the winding on it can revolve and turns can be removed by pulling on the outer lead, or added by rotating the knob at the end of the shaft. Continuous adjustment is possible since it is not necessary to add or remove integral numbers of turns. Although an inductor of this sort involves some mechanical complexity it has the advantage that the core parts do not have to be precisely machined. The turns adjustment can be used to compensate for sizable variations in the dimensions of the core parts and the resulting air gaps.

4. *Adjustment of Inductance by Change in Permeability, μ*

Adjustment by change in permeability, that is, change in length or area of air gaps, can be accomplished in a variety of ways to meet differing design needs and can be mechanically simple and economical. For most purposes permeability adjustment will offer more advantages than any of the other methods.

Whatever the method of adjustment used its effectiveness will depend on proper correlation between the mechanical motion that produces the change and the inductance itself. For most filter and network applications the following considerations will apply:

(a) The slope of the line showing inductance plotted against the displacement that produces the adjustment should be reasonably constant over the adjusting range.

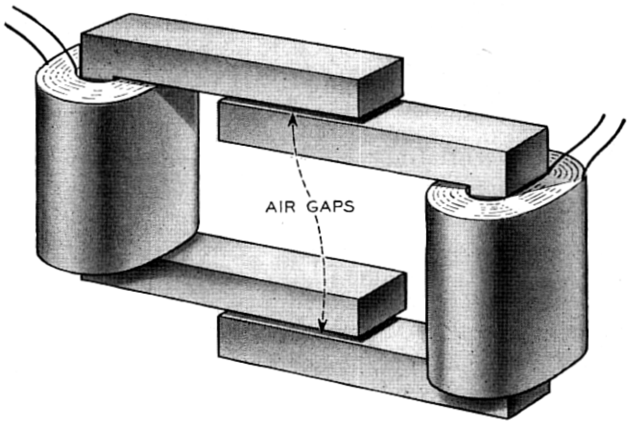

Fig. 8 — Inductance adjustment by decrease in magnetic path length and increase in air gap area.

(b) The slope should not be negative at any position in the adjusting range as this would introduce the possibility of false or double balance when the coil is being adjusted by null or peaking methods.

(c) The slope should not be so small that play in the mechanical parts will cause changes in slope of the same magnitude as the slope itself.

(d) Within the above limitation the smaller the slope the more precisely the adjustment can be made.

(e) Conversely, the greater the slope the greater the range of adjustment for a given amount of mechanical motion.

(f) It follows from (d) and (e) that the amount of mechanical motion available determines the product of range and precision. Where the mechanical motion is rotary, such as with adjustment by turning a screw, it is possible to adjust a coil to a precision of about 1/400 of a revolution without undue difficulty. If the range is covered by N revolutions of the adjusting screw, and the over-all range is $\pm R$ per cent of the mean value:

$$P = \frac{2R}{400N} = \frac{R}{200N}$$

where P = the precision of adjustment in per cent of the nominal value.

In the coil shown in Fig. 4, six turns of the adjusting screw are effective in producing an over-all change of ± 15 per cent in the inductance. The precision with which the adjustment can be made is, therefore, very close to ± 0.01 per cent of the value desired.

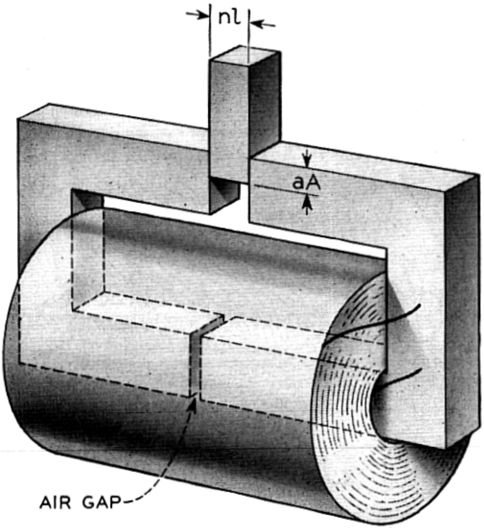

Fig. 9 — Inductance adjustment by constriction in magnetic cross section.

(g) It would appear from the above that by gearing or other means of increasing the mechanical motion that governs the inductance change it should be possible to improve the precision. This is true up to the point where consideration (c) is violated, to the point where play in the mechanical parts permits variations of the same order as the nominal precision.

An obvious form of permeability adjustment would consist of a means for varying the distance between two plane magnetic surfaces, as shown schematically in Fig. 11 (a). One disadvantage of this, or of any other means that involves a change in air gap length, is in the nonlinearity of adjustment. The effective permeability of a coil is given in (44). In most voice and carrier frequency applications of ferrite the magnitudes of g and a will be such that, approximately

$$\mu = \frac{a}{g} \tag{46}$$

and, if the cross section of the air gap is the same as that of the core

$$\mu = \frac{1}{g}.$$

A typical adjustment curve resulting from this inverse relationship is shown in Fig. 11 (b).

In addition to its nonlinearity the simple butted gap has a shortcoming in that the mechanical motions involved are of the order of only a few hundredths of an inch, which requires that parts be very accurately fitted. This can be somewhat alleviated by using cone or wedge shaped

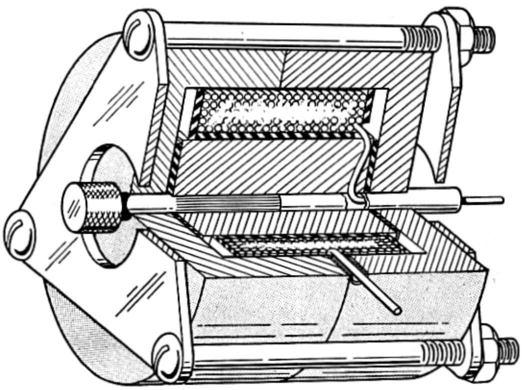

Fig. 10 — Inductance adjustment by addition or removal of turns.

gaps, as illustrated in Fig. 11 (c). Here the distance d, travelled by the screw is longer than the effective air gap g, by the amount

$$d = \frac{g}{\sin\,\theta}.$$

One means of overcoming the nonlinear characteristic of gaps such as these is to introduce compensation in the form of a secondary gap that opens as the main gap closes.[7] Such an arrangement is shown in Fig. 12 (a). When there are two gaps in series their reluctances are additive and their effect on permeability is given approximately by

$$\mu = \frac{1}{g_1 + g_2}. \tag{47}$$

Fig. 12 (b) shows the inductance characteristics that would result from either of the two gaps alone, and the effect of the two gaps in series.

From (46) it can be seen that the effective permeability varies ap-

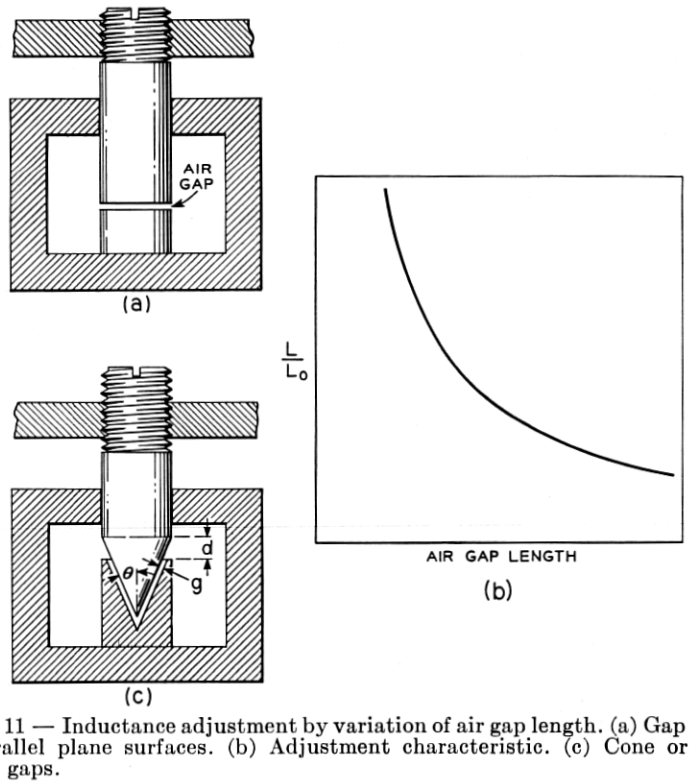

Fig. 11 — Inductance adjustment by variation of air gap length. (a) Gap formed by parallel plane surfaces. (b) Adjustment characteristic. (c) Cone or wedge shaped gaps.

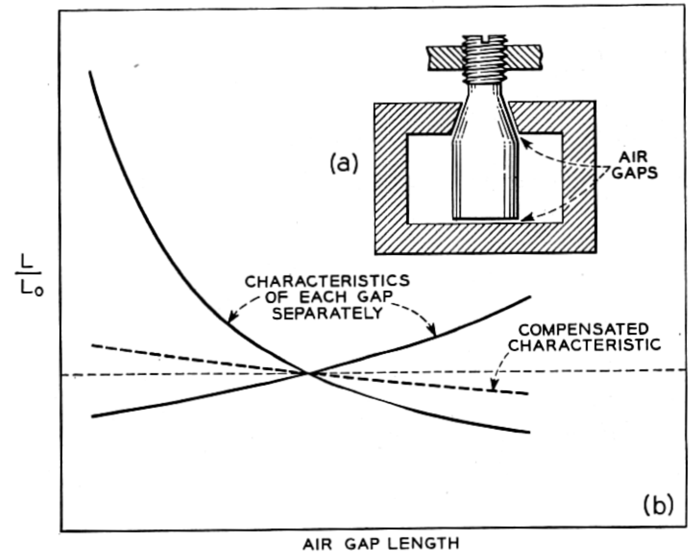

Fig. 12 — Inductance adjustment with partially compensating air gaps.

proximately directly as the ratio of air gap area to magnetic core area. Adjustment by variation of the area of the gap, therefore, is not subject to the nonlinearity that results from manipulating the air gap length. A simple method for providing adjustment by varying the effective area of the air gap is shown in Fig. 13. As one shell is rotated with respect to

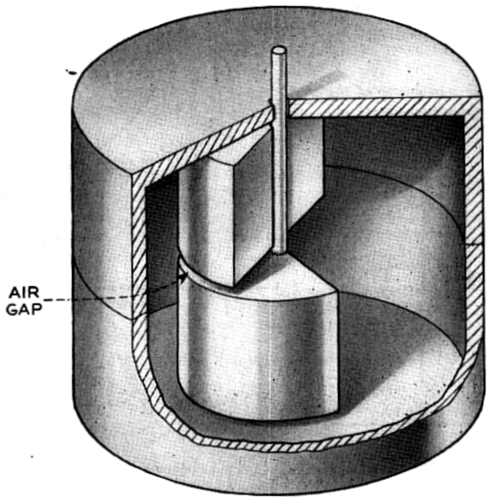

Fig. 13 — Inductance adjustment by variation in air gap area.

the other the area of registration between the semicircular faces of the center posts is reduced or increased. A simple arrangement such as this provides good linearity but has two limitations: (1) The total mechanical motion available for adjustment is only 1/2 revolution. (2) As the cores are rotated to reduce the air gap area and the inductance, the magnetic flux in the cores tends to concentrate more and more in the vicinity of this reduced area, and may under some conditions give rise to high hysteresis loss.

The method of adjustment used in the 1509 type inductor, Fig. 4, is essentially a variable area method but it is designed in such a way as to overcome these two limitations.[8] The air gap arrangement, visible in the cutaway view, consists of two gaps in parallel. The main annular gap is fixed and is large enough in area to insure that under no anticipated conditions of operation will the flux concentration be too high. The screw adjustment moves a cylindrical ferrite part into a depression in the center post, as shown. The effective cross section of this adjustable gap is approximately determined by the amount of surface of the cylinder within the depression. The total useful adjusting range corresponds to about six full turns of the adjusting screw.

5. *Adjustment of Inductance by Change in Coupling*

The overall inductance of two coils of equal inductance connected in series is

$$L = 2(1 + k)L_1 \tag{48}$$

where L = series inductance

L_1 = inductance of either coil

k = coupling coefficient

k may have any value between -1 and $+1$, these values corresponding to complete coupling and the windings connected in series opposing and series aiding, respectively. It is practicable to make inductors whose coupling can be continuously adjusted from very high positive values through zero to equally high negative values. This type of design is especially useful where a very wide range of inductance variation is desired. It will be seen from (48) that with couplings of plus and minus 90 per cent, respectively, in the extreme positions a range of 19 to 1 in inductance variation would result. A coil of this type is shown in Fig. 14.

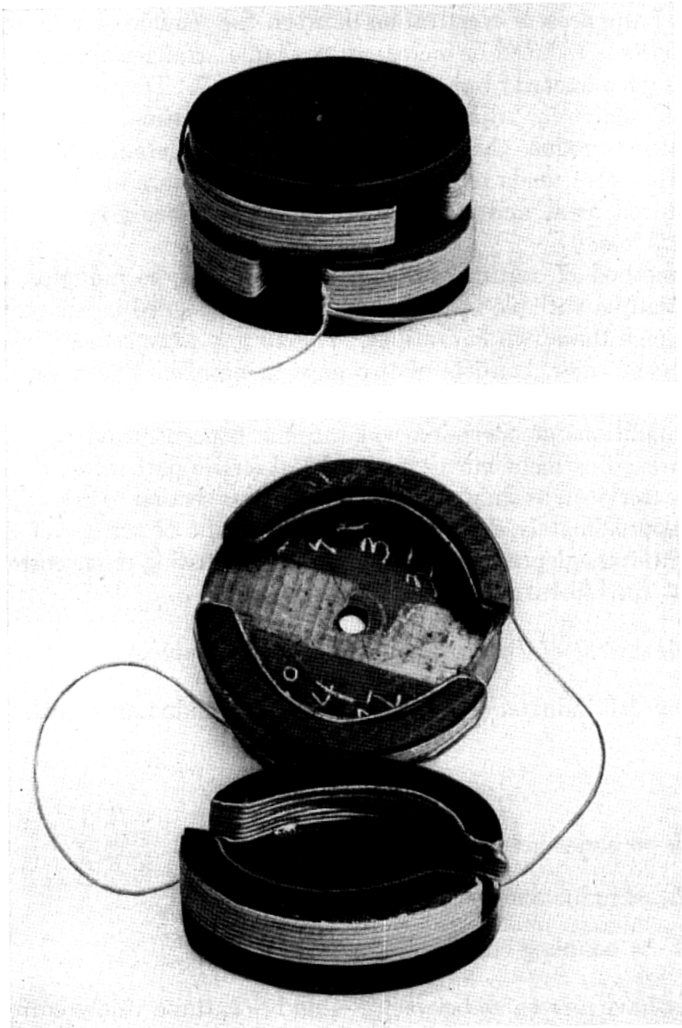

Fig. 14 — Magnetic variometer. Inductance adjustment by variation in mutual inductance.

CONCLUSION

The combined magnetic and mechanical characteritics of ferrite permit the design of inductors superior to those using metallic cores in at least three important respects:

1. Higher values of Q are obtainable than have ever before been practicable. In the range from about 50 to 200 kc, frequently used for telephone carrier, it is very difficult to realize a Q above about 300 in a

metallic core coil. Ferrite coils, on the other hand, have been made having Q's as high as 1000, and inductors with Q's between 500 and 600 are available commercially.

2. Formulas have been derived which show how ferrite can be used to realize the best inductor characteristics in the smallest volume. The 1509 type inductor, for instance, is only about 1/3 as large as the nearest equivalent permalloy core coil, yet its Q is over twice as high.

3. It is physically practicable in ferrite coil designs to include inductance adjustment facilities to meet a wide range of requirements.

It should not be concluded that the day of metallic cored inductors is over. For power applications, especially those involving direct current, present-day ferrites are inferior to silicon iron and permalloy. At voice frequencies there are many applications for which ferrite is, at best, no better than some of the older materials, although in others it has distinct advantages. In higher frequency ranges, however, and especially for low power level applications, the advantages of ferrites are outstanding enough to justify the expectation that they will largely replace the older iron and nickel-iron powders.

ACKNOWLEDGEMENTS

The development of ferrites and their application to inductors has been carried out by several teams in Bell Telephone Laboratories, including J. H. Scaff, F. J. Schnettler and their associates in the metallurgical department, V. E. Legg and C. D. Owens in the magnetic applications group, and S. G. Hale, R. S. Duncan and others in the inductor development area. I don't know who was first to conceive of using "dissipation factor" instead of "Q" to simplify his mathematics, but it was not the author. Equation (24) is derived by inserting $1/D_n$ for Q in an expression originally worked out by P. S. Darnell.

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