Transistors in Switching Circuits

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The general transistor properties of small size and weight, low power and voltage, and potential long life suggest extensive application of transistors to pulse or switching type systems of computer or computer-like nature. It is possible to devise simple regenerative circuits which perform the normally employed functions of waveform generation, level restoration, delay, storage (registry or memory), and counting. The discussion is limited to point contact type transistors in which the alpha or current gain is in excess of unity and to a particular feedback configuration. Such circuits, which are of the so-called trigger type, are postulated to involve negative resistance. On this basis an analysis, which approximates the negative resistance characteristic by three intersecting broken lines, is developed. Conclusions which are useful to circuit and device design are reached. The analysis is deemed sufficiently accurate for the first order equilibrium calculations. Transistors having properties specifically intended for pulse service in the circuits described have been developed. Their properties, and limitations, and parameter characterizations are discussed at some length.

INTRODUCTION

It is proposed to discuss some of the properties of transistors which are applicable to switching or pulse-type circuits, to develop elementary analysis methods and to describe a few circuits.

The bounds or limits of the field of switching are difficult to define. The common thread usually involves definite states of being as "open or closed", "off or on", "0 or 1", and so on, rather than a continuum of conditions. Even when consideration is given to more than two states, the thought involves distinct recognition of each state. The field is termed to be non-linear in distinction to linear manipulation of information. Any number of anomalies in definition may be raised.

Without attempting either to define or to limit the field, some of the functions which are often employed are: wave form generation, as rectangular pulses, sawtooth waves, etc.; memory or storage which may be for short, intermediate or long periods and involves the retention of information for subsequent use; operations involving addition, subtraction, multiplication and division; translation of information from one form or code to another; gating, involving the routing of signals according to a predetermined pattern or set of conditions; regeneration of signals in amplitude and wave form; delay, which may be thought of as a form of storage; and timing. Some of these functions are simple; others result from fairly complex structures of simpler functions.

Present trends in electronic switching systems are toward complicated automata as exemplified by digital computers.[1] The reliability, power consumption and physical size of the electron devices employed largely determines the degree of realizability of such systems. It is believed that the transistor will find a significant application in this field.

The transistor can reduce power consumption by the elimination of heater or filament power. In addition, particularly in broadband applications as in high speed pulse systems, the "B" power may be reduced by the order of one or two decades if not more. Transistor circuits with 0.02 μs rise time have been made to operate with an input power of 20 milliwatts which compares with approximately 2.5 watts (1-watt heater, 1.5-watt plate) for an equivalent tube circuit. Transistors have operated with less than one microwatt input power.[2]

Such power reductions result from the low operating voltages, low internal resistances and low capacitances of transistors. Low internal impedances greatly reduce the importance of stray wiring capacitances thereby making mechanical design much simpler and often eliminating the need for isolating or buffer amplifiers.

The transistor can contribute definite reduction in size directly. Fig. 1 shows a "bead" transistor which has a volume of approximately 1/1000 of a cubic inch and a weight of 5/1000 ounce. Indirectly the transistor can contribute to size reduction through the use of smaller, lower voltage, lower dissipation components. The reduction of power supply requirements in terms of size, regulation and capacity is also quite appreciable.

Transistors have been subjected to shocks in excess of 20,000 G without change in characteristics. Vibration tests have shown no resonances in the transistor shown in Fig. 1 to several thousand cycles. Harmonic accelerations of 100 G at 1000 cycles have produced no detectable current modulation.

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[1] L. N. Ridenour, "High Speed Digital Computers", *J. Appl. Phys.*, **21,** pp. 263–270, April, 1950.

[2] R. L. Wallace, Jr. and W. J. Pietenpol, "Some Circuit Properties and Applications of *n-p-n* Transistors", *Bell System Tech. J.*, **30,** pp. 530–563, July, 1951.

Life reliability and expectancy are difficult to determine due to the relative infancy of transistors, the definite finiteness of time, the many variables involved and the rate of development progress. Average life is presently estimated to be in excess of 70,000 hours. Life is a function of the operating conditions and may be materially reduced accordingly.

Transistors also have limitations. Noise at present is high for point-contact types as compared to electron tubes; input impedances are low, which may be either advantageous or disadvantageous; power output may be limited; frequency response is relatively low; circuit instability may cause design difficulties; and the devices are sensitive to temperature changes. There is also an absence of a long practical experience with a consequent art background in both devices and circuits.

A comprehensive review of transistor properties is given in the paper by J. A. Morton.[3]

While it is difficult to define the switching field, it is no less difficult to discuss circuit and device properties on a general basis. This is related to the non-linear nature of the circuits and devices in distinction to the virtually classical linear small-signal field. The lack of a classical method of analysis is a serious handicap in the synthesis of contemporary circuits and devices. When new devices, as the transistor, are to be considered,

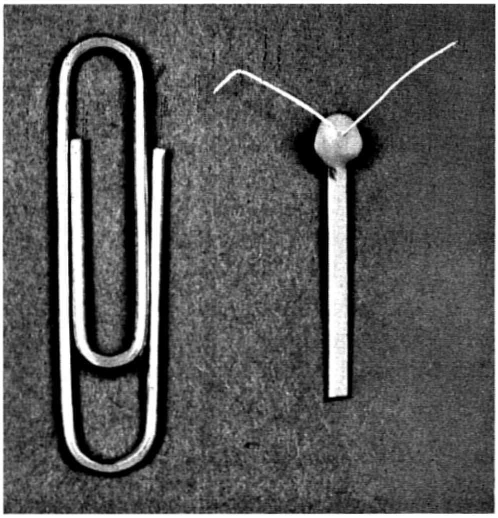


Fig. 1—A miniature switching-type transistor (M1689).

[3] J. A. Morton, "Present Status of Transistor Development", *Bell System Tech. J.*, **31,** pp. 411–442, May, 1952.

the problem is multiplied due to the lack of a long background of experience.

It has been assumed that negative resistance is a common thread among "trigger circuits" and oscillators regardless of the device employed — electron tube, gas tube, transistor, mechanical structures, etc. This is not a new or novel idea and there is no intent to present it as such.[4] Rather, it is used as a pattern upon which a certain degree of transistor analysis may be based, leading to simple understanding. The analysis assumes that the negative resistance characteristic can be broken into three regions; each region is then considered on a linear basis.

Section I will deal with simple circuit properties; Section II with analysis and Section III with device properties.

I—Simple Circuit Properties

The common property ascribable to switching functions is that of definiteness of state. The condition of the function is either "off" or "on". Switches are either open or closed; relays are operated or not; tubes are in cutoff or overload; doors are open or closed and so on. This is common regardless of the phenomena being exploited.

There is an intermediate region between these two conditions usually characterized by a time which is related to how fast the function may go from one state to another. Functionally the times of closing and opening are taken to be zero; practically, they are of determining importance. Relays replace hand-operated switches and electronic devices replace relays as speed becomes important. Obviously, no function or system can be faster than its state-devices.

All such state-devices will have separate attendant properties such as the degree of reverse coupling between the controlling signal and the controlled signal. Separated into families, however, there are those which are passive and those which are active. The latter are threshold devices in which a small amount of signal or control energy causes the translation of a relatively larger amount of stored energy into dynamic energy which consummates the change in state. As long as the control

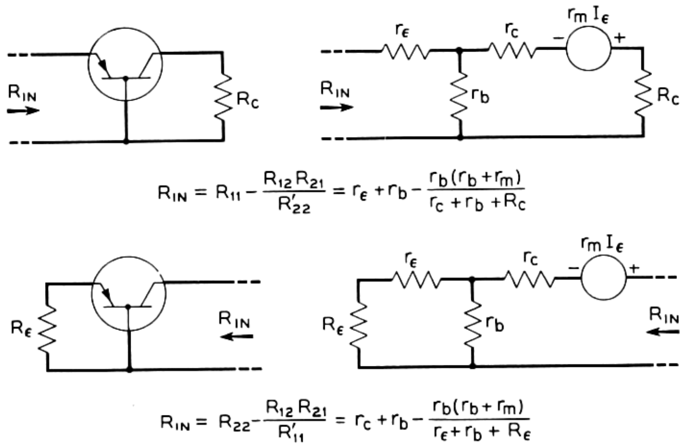
[4] See for example "Negative Resistances, Their Characteristics and Effects. Sinusoids, Relaxation Oscillations and Relaxation Discontinuities", Walter Reichardt, *Elektrische Nachtrichen-Technik*, **20**, pp. 76–87, March, 1943; "Uniform Relationship Between Sinusoids, Relaxation Vibrations and Discontinuities", Walter Reichardt; *Elektrische Nachtrichten-Technik*, **20**, pp. 213–225, Sept., 1943. For transistors: "Counter Circuits Using Transistors", E. Eberhard, R. O. Endres and R. P. Moore, *RCA Review*, pp. 459–476, Dec. 1949; "A Transistor Trigger Circuit", H. J. Reich and Ungvary, *Rev. Sci. Instr.*, **20**, p. 8, p. 586, Aug., 1949; and "Some Transistor Trigger Circuits", *Proc. Inst. Radio Engrs.*, **39**, pp. 627–632, June, 1951, P. M. Schultheiss and H. J. Reich.

signal is below the initial threshold there is no response and any change is directly related to the passive transmission of the control signal alone. When the signal exceeds the threshold the second state is assumed. Watch escapements, thyratrons, and the whole family of oscillators fall into this category. When the simplest cases of such functions are analyzed, they are found to involve in one way or another two stable states separated by a region in which there is positive feedback and gain in excess of unity with a resultant equivalent negative resistance. The proposition that a negative resistance characteristic is common to trigger or threshold switching circuits is tacitly assumed. The next step is to examine the transistor for such behavior and to classify the properties.

NEGATIVE RESISTANCE IN THE TRANSISTOR

That the transistor* can exhibit negative resistance has been demonstrated analytically[5] and experimentally. The resistances seen looking into the emitter and collector of the transistor with grounded base are shown in Fig. 2.

In the equations and discussion to follow, the symbol conventions are as follows: External circuit elements are capitalized as R_ϵ, R_b, and R_c. The symbols R_{11}, R_{12}, R_{22} and R_{21} define the open-circuit transistor resistances; the symbols r_ϵ, r_c, r_m, and r_b define the equivalent circuit



$$R_{IN} = R_{11} - \frac{R_{12}R_{21}}{R'_{22}} = r_\epsilon + r_b - \frac{r_b(r_b + r_m)}{r_c + r_b + R_c}$$

$$R_{IN} = R_{22} - \frac{R_{12}R_{21}}{R'_{11}} = r_c + r_b - \frac{r_b(r_b + r_m)}{r_\epsilon + r_b + R_\epsilon}$$

Fig. 2—Emitter and collector driving point resistances.

* Discussion is limited primarily to point contact transistors with α's or current gains greater than unity.

[5] R. M. Ryder and R. J. Kircher, "Some Circuit Aspects of the Transistor", *Bell System Tech. J.*, **28,** pp. 367–400, July, 1949.

element values. Network resistances which contain both device and external elements are primed. For example, $R'_{22} = R_{22} + R_c + R_b$, where $R_{22} = r_c + r_b$. See also references 3 and 5.

Taking the collector or output resistance, Fig. 2, for example,

$$R_{in} = (r_c + r_b) - \frac{r_b(r_b + r_m)}{r_\epsilon + r_b + R_\epsilon} \tag{1}$$

R_{in} can be negative or positive depending upon the relative magnitudes of the two terms. Actually, of course, r_m has a phase factor and so is frequency dependent. Frequencies wherein r_m is essentially resistive will be assumed. For negative resistance, r_m must be large, R_ϵ small and r_b not too small or else augmented by external resistance. Negative resistance is thus predicted on a small-signal linear basis. The large-signal behavior may be studied experimentally by adding sufficient resistance as R_c to the first or positive term to insure stability. This is shown in Fig. 3 with the resultant characteristic. External base resistance R_b has been added and R_ϵ is zero.

Fig. 3 illustrates the pattern of a three-valued characteristic: Regions I and III are portions with positive slope, indicating stable operating

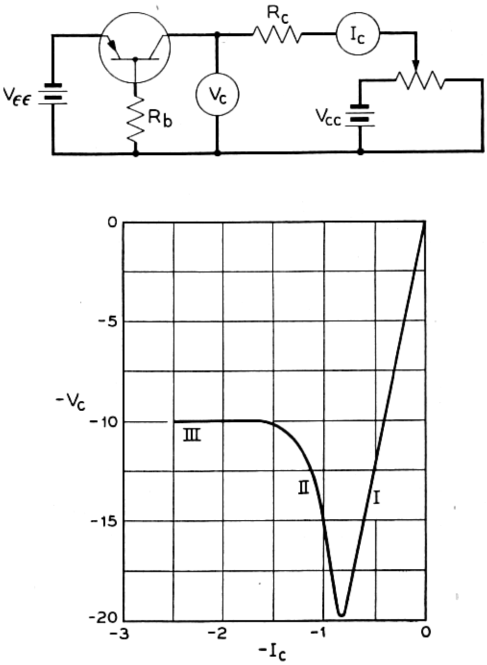


Fig. 3—Collector large-signal negative resistance characteristic.

regions, separated by Region II, a region of negative slope, indicating the possibility of instability. In this particular case, Region I has high resistance and Region III very low resistance.

An evaluation of the emitter or input characteristic leads to similar results, using the circuit of Fig. 4. R_b has been added here also and R_c taken as zero. The general pattern is again present. Region I has high, positive resistance; Region II, negative resistance; and Region III very low, positive resistance.

BIASES AND LOAD LINES—BISTABLE OPERATION

The negative resistances of Figs. 3 and 4 are both of the so-called open-circuit stable type. If loads are applied to the circuit terminals of Fig. 2 which are larger in magnitude than the negative resistances, the circuits will be stable; that is, there will be single operating points. This is shown in Fig. 4 by the dashed load lines marked, R'_ϵ , R''_ϵ , R'''_ϵ. A load resistance smaller in magnitude than the negative resistance may intersect the characteristic in three positions as shown by the load line R_ϵ .

The load line R_ϵ can be made to have single or multiple intersections by biasing properly as shown in Fig. 5, where the three possibilities are shown as R_ϵ , R'_ϵ , R''_ϵ . Single or multiple intersections result in accordance with the choice of emitter bias, $V_{\epsilon\epsilon}$, as shown. It can be shown

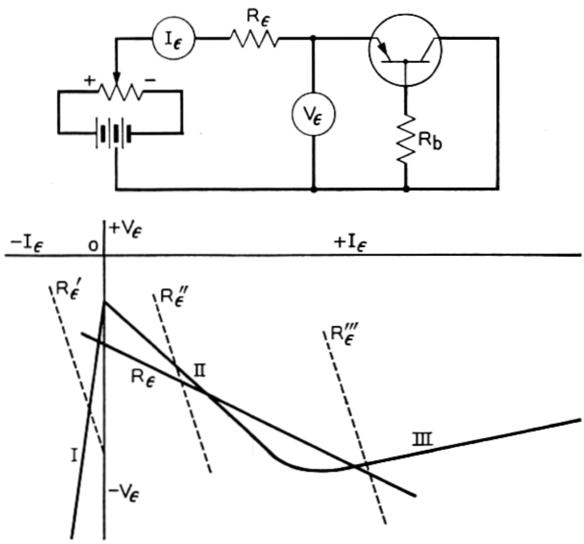


Fig. 4—Idealized emitter large-signal negative resistance characteristic.

that the intersection of load line R_ϵ with the characteristic at b in Fig. 5 is unstable whereas those at a and c are stable. Experiment in the multiple intersection case shows also that as $V_{\epsilon\epsilon}$ is slowly increased (decreased in absolute magnitude) the load line moves upward and that the assumed operating point, a, moves up along the Region I portion of the characteristic. At the turning point shown on the current axis, the operating point suddenly flips to the high current region, returning along the curve to c as $V_{\epsilon\epsilon}$ is returned to the original value.

A decrease in $V_{\epsilon\epsilon}$ toward $V_{\epsilon\epsilon}''$ moves the operating point at c downward along the characteristic until it "escapes" past the lower turning point and flips to the Region I portion, returning to a as $V_{\epsilon\epsilon}''$ is returned to the original value. This then is an elementary switching circuit, a bistable trigger circuit or "flip-flop". A positive emitter pulse will cause the circuit to flip to high current, a negative pulse to low current. The triggers may be applied to emitter, base or in combination with proper attention to polarity. Trigger sensitivities are shown in Fig. 6. Such a circuit is often used for register or storage purposes. It can store one bit of information for a potentially infinite period, be sampled for the presence of such information, and be cleared or restored to the original condition for reuse when the stored information is no longer useful.

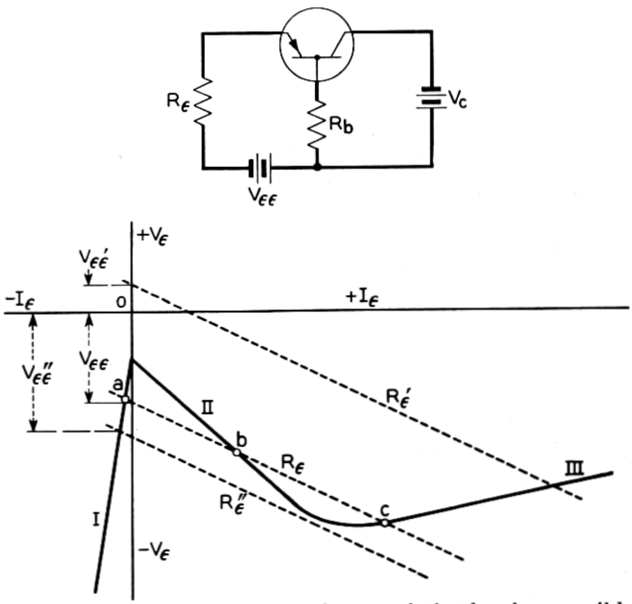


Fig. 5—Emitter negative resistance characteristic showing possible multiple operating points.

With the addition of suitable steering circuits it can be made to count by a scale of two.

MONOSTABLE AND ASTABLE CIRCUITS

The addition of a capacitor to the circuit as in Fig. 7(a) leads to either monostable or astable operation. In Fig. 7(b) the normal operating point is stable at *a* as discussed previously by virtue of the bias $V_{\epsilon\epsilon}$. As $V_{\epsilon\epsilon}$ is increased, as by a trigger, the load line is moved up and over the turning point. Without capacitor C in the circuit, the operating point would move to *b* with the resultant rapid change in voltage and current. However, a capacitor has in effect voltage inertia; this is equivalent to saying that a capacitor is a short-circuit to a voltage change. Both the capacitance and the rate of change of voltage are assumed high. Thus at the turning point the capacitor effectively short-circuits the emitter and the operating point snaps along dotted line (1) to intersect the characteristic. This point is quasi-stable and the capacitor is discharged along line (2) to the second turning point where the emitter is again effectively short-circuited and the operating point snaps along (3) to intersect the Region I portion of the characteristic. This point is also quasi-stable and the operating point moves slowly up to the initial or dc stable operating point. A single trigger thus causes a complete cycle of operation. The emitter current shifts

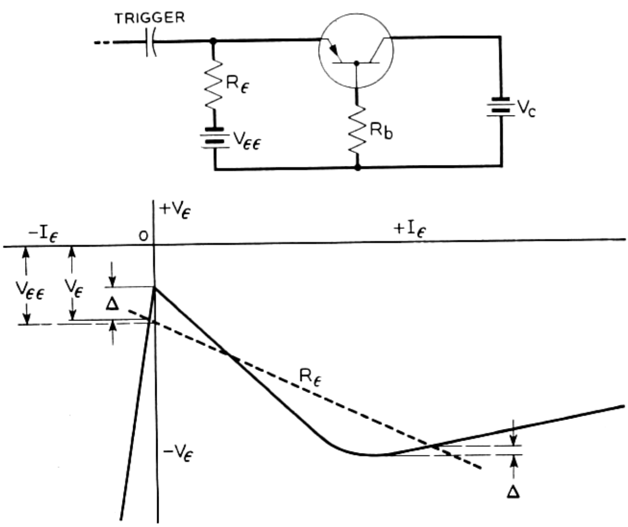


Fig. 6—Bistable circuit showing trigger sensitivity, Δ.

rapidly to a high value of current, falls relatively slowly to an intermediate value, then shifts rapidly to a small negative value and finally returns slowly to the original value. The emitter current and voltage are sketched in Fig. 8. It is a so-called "single-shot" circuit. Alternately the rest or dc stable point can be chosen to be in Region III, at high current, by choice of positive instead of negative bias $V_{\epsilon\epsilon}$. Practical considerations as ease in triggering and average power consumption usually indicate a preference for the Region I dc stable point.

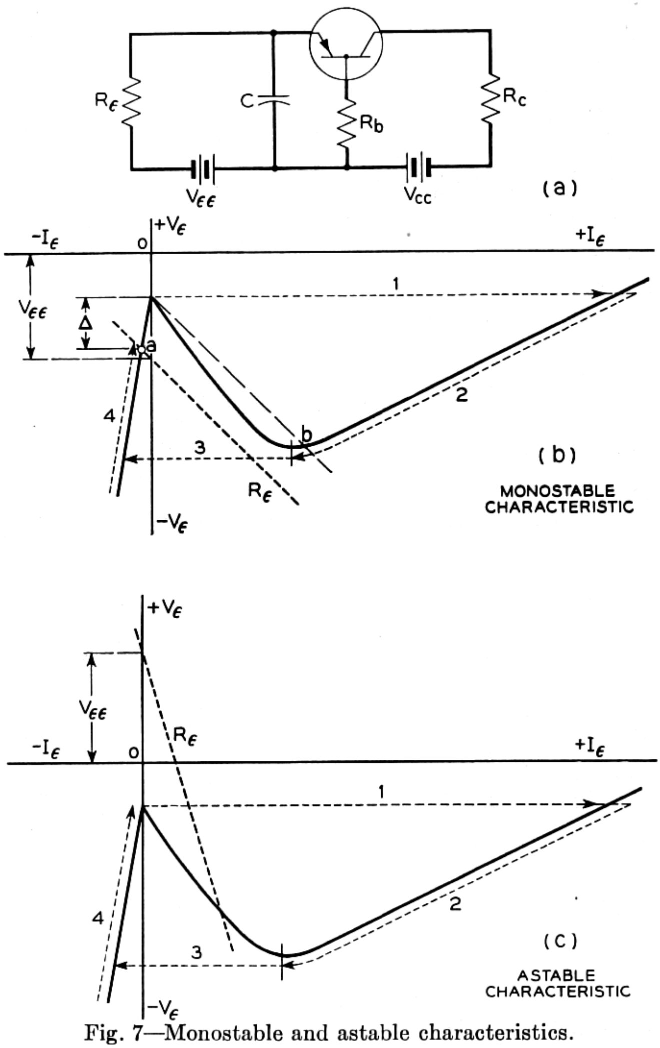


Fig. 7—Monostable and astable characteristics.

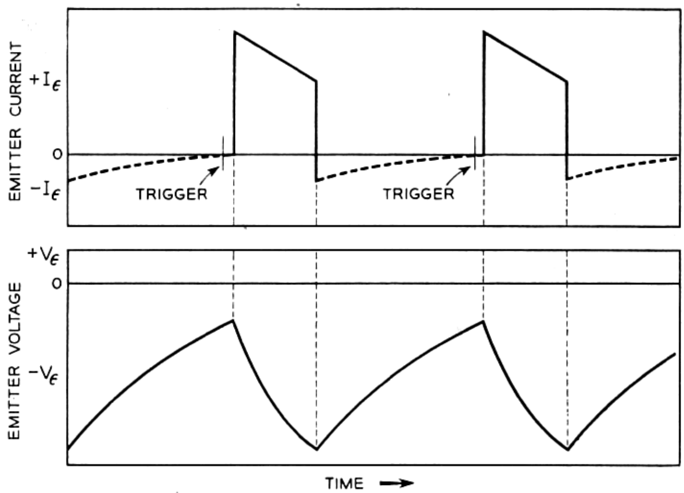


Fig. 8—Idealized monostable relaxation oscillator waveforms.

When the load line and bias are chosen to result in intersection in the negative resistance portion, astable operation or continuous oscillation results. This mode is illustrated in Fig. 7(c). Proper bias and $R_\epsilon > | -R_{in} |$, Region II, are required. The operating point formed by the intersection of the load line on the negative resistance portion of the characteristic would normally be stable. However, the capacitor provides an ac short-circuit in parallel with R_ϵ causing the path (1), (2), (3), (4) to be followed continuously. Another form of physical explanation of this relaxation oscillation, usually applied to gas tubes, is that the capacitor C is charged slowly throuch R_ϵ to a critical or breakdown value whereupon the tube or device rapidly discharges the capacitor. When the capacitor charge is dissipated, the device discharge can no longer be maintained due to the IR drop in R_ϵ and the tube or device open-circuits and the capacitor is recharged.

The above suggests a strong simularity to gas tube behavior and this is indeed so. In fact, the modes described above are common to all open-circuit stable negative resistance devices; only the parameters and device phenomena are different.

The primitive circuits of Fig. 7 have properties basic to several switching functions. These may be deduced from the waveforms of Fig. 8 which are essentially identical to both the monostable and astable cases. The emitter current has a rectangular waveform which suggests the generation of rectangular pulses; and, for the astable case, regenerative amplification for both amplitude and wave shape, pulse rate or frequency division and delay. As shown the current waveform is not particularly good, having neither a flat top nor a flat base line. Practically, the waveform may be derived from the collector by means of a small load resistor to obtain a flat base line. When the emitter current is negative there is sensibly no transfer action, hence, the collector current will be constant during the re-charge portion of the cycle instead of exponential as shown. The slope of the top is inherent and may be removed by clipping. Pulse rise time, the time required for transition from low current to high current, of 0.1 μs is quite easily obtainable; 0.02 μs with average input powers of 20 *mw* have been obtained. Fall time is usually longer than the rise time by factors of 3 or 4. It is to be noted in Fig. 8 that there has been shown a delay between the trigger application and the current transition. Such delay is not peculiar to transistors, but is common to all trigger type devices and circuits. The delay is shown here exaggerated in order to establish its existance and is associated with the static charging of the circuit and the dynamic delay of the device concerned. The trigger-transition delay with transistors is usually less than 0.1 μs.

The voltage waveform of Fig. 8 has a sawtooth form and may thus be employed to generate linear time bases or sweeps. The normal methods for linearization such as a high charging voltage $V_{\epsilon\epsilon}$ and a high charging resistance R_ϵ or other constant current means are applicable here as in other device circuitry. Free-running and driven sweeps may be obtained with the astable and monostable circuits respectively.

Since the collector characteristic shown in Fig. 3 is also open-circuit stable, the same sort of circuits can be constructed using the output characteristic. Bistable, monostable and astable circuits are shown in Fig. 9.

The resistances seen looking into the base are given in Fig. 10. These circuits are short-circuit stable. That is, high values of R_b result in instability. Bistable, monostable and astable circuits can be constructed also, but use is made of an inductor instead of a capacitor. The reactance of the inductor affords a quasi-open-circuit in the same manner as the capacitor afforded a quasi-short-circuit in the previous cases. Circuit examples are shown in Fig. 11.

SUMMARY

These simple circuits by no means exhaust the switching circuit possibilities of the transistor; rather, they are the simplest. The simple circuit is often satisfactory and may sometimes be employed with little more understanding than that given. More often, however, problems

relating to the sensitivity, constancy of sensitivity, operating currents and voltages, interchangeability and the like require a much more quantitative understanding in order to create circuit designs having specific properties.* An equal need also exists in transistor design for analytic circuit relationships. Such information is useful first, in the

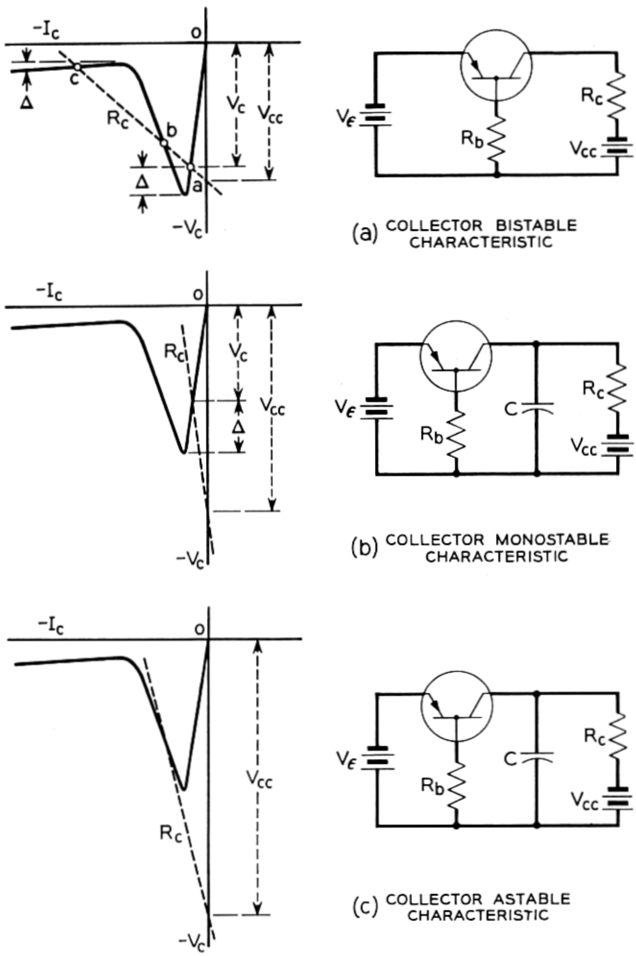


Fig. 9—Collector connection switching circuits.

* See, for example, J. R. Harris, "A Transistor Shift Register and Serial Adder", *Proc. IRE*, Nov., 1952; R. L. Trent, "A Transistor Reversible Binary Counter", *Proc. Inst. Radio Engr.*, Nov., 1952; H. G. Follingstad, J. N. Shive, R. E. Yaeger, "An Optical Position Encoder and Data Transmitter", *Proc. Inst. Radio Engr.*, Nov., 1952.

creation of optimized designs and, second, in the maintainance of proper parameter controls in manufacture. Finally, the more detailed the understanding, the more likely will be the creation of new circuits and new devices.

A complete analytical treatment will not be attempted here; consideration will be limited to the equilibrium case and in particular to the simple circuits described.

II—Analysis

In order to deal analytically with circuits and devices it is necessary to have analytic expressions for the device characteristics. For small signal analysis this is relatively easy. In large signal applications, as in switching, the situation is not so simple. The problems arise because of the high degree of nonlinearity wherein the simplifying assumptions employed in small signal analysis are by no means valid. Further, it is desirable to retain dc terms in many cases.

The method to be employed here is the so called broken-line method which involves approximating the negative resistance characteristic by three intersecting straight lines. The assumption is made that there are three distinct regions of operation in each of which the device is separately linear, but involving different parameter values for each region.

The approximation is shown in Fig. 12. The assumption that the negative resistance characteristic can be simulated by three straight lines is reasonably valid for gross considerations; for fine detail near the

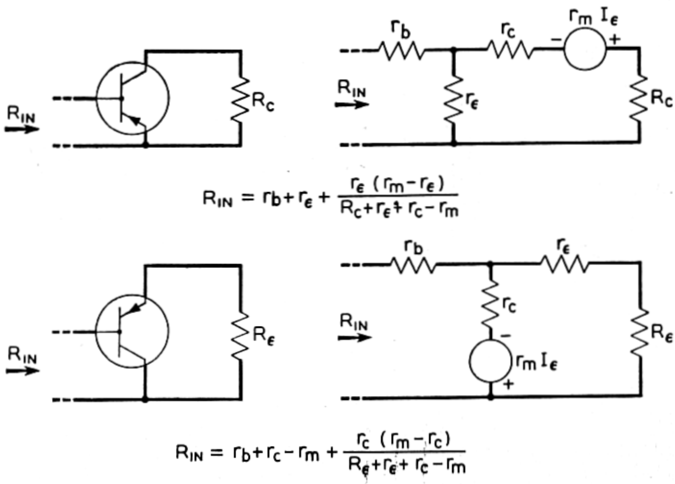


Fig. 10—Base driving point resistances.

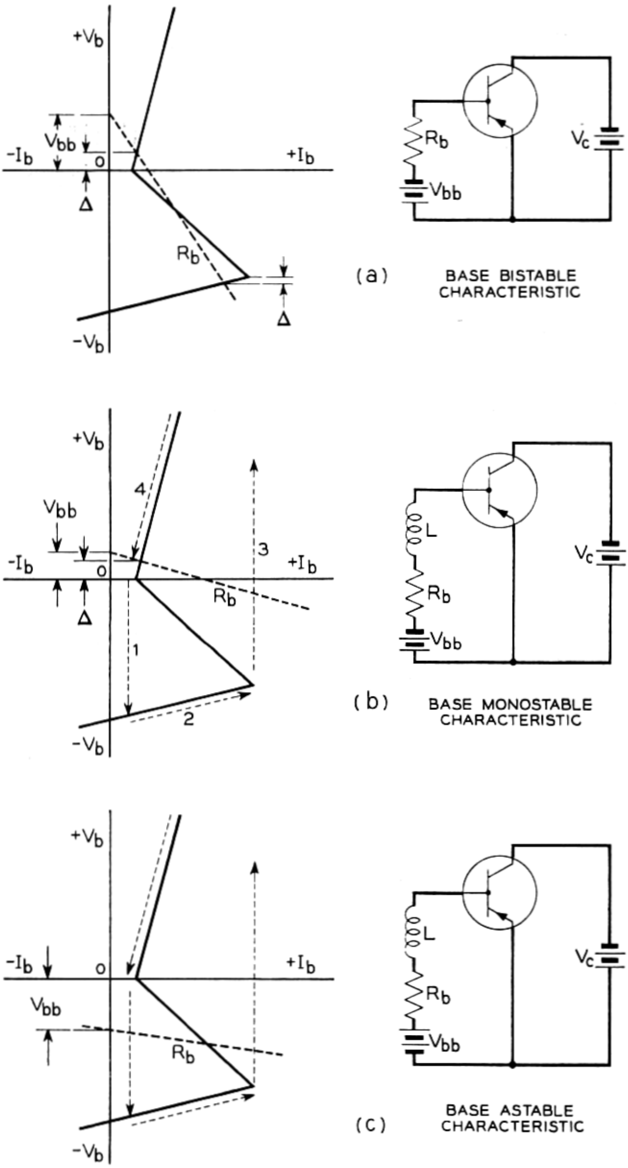


Fig. 11—Base connection switching circuits.

turning points the approximation is by no means accurate although affording zero order information.

Preparatory to the analysis of the negative resistance characteristics, it is necessary to obtain analytic expressions for the transistor currents and voltages. This in turn involves the following steps:

1. Identification of the three regions in terms of the device characteristics,
2. Idealization of the device characteristics to obtains simple, linear relations, and
3. Evaluation of the device parameters in each of the three regions.

Fig. 13 is a family of open circuit characteristics for a typical switching type transistor. Specifically, in small signal terms,

TABLE I

Parameter	*Equivalent Tee*
$R_{11} = \left.\dfrac{\partial V_\epsilon}{\partial I_\epsilon}\right]_{I_c}$	$R_{11} = r_\epsilon + r_b$
$R_{12} = \left.\dfrac{\partial V_\epsilon}{\partial I_c}\right]_{I_\epsilon}$	$R_{12} = r_b$
$R_{21} = \left.\dfrac{\partial V_c}{\partial I_\epsilon}\right]_{I_c}$	$R_{21} = r_m + r_b$
$R_{22} = \left.\dfrac{\partial V_c}{\partial I_c}\right]_{I_\epsilon}$	$R_{22} = r_c + r_b$

Also
$$\alpha = -\left.\frac{\partial I_c}{\partial I_\epsilon}\right]_{V_c} = \frac{R_{21}}{R_{22}} = \frac{r_m + r_b}{r_c + r_b}$$

The above set, normally employed for small signal analysis, will be assumed to be constant within a given region, but changing in value from region to region.

IDENTIFICATION OF THE THREE REGIONS

It may be recalled with the aid of Fig. 12 that the negative resistance characteristic consists of a negative resistance region bounded on each side by a region of positive resistance. Thus the device is first passive in nature with little or no gain, then very abruptly exhibits considerable gain with the resultant negative resistance, and finally becomes very abruptly passive again with little or no gain.

It would seem quite clear that abrupt changes in the transmission properties of a device should be associated with equally abrupt changes in the forward transfer characteristic. In the case of the transistor, the behavior of the forward transfer properties is given by the forward transfer impedance, R_{21}.

Examining the R_{21} family in Fig. 13, it is seen that in the normal, positive emitter current region the slope, R_{21}, is high indicating the possibility of high forward gain. When I_ϵ is negative, however, the slope is zero or nearly so, changing very abruptly at $I_\epsilon = 0$. Further, it is to be noted that as I_ϵ is made negative, the collector voltage is unaffected, remaining constant for further change in I_ϵ. Thus it may be said that the collector voltage is saturated.*

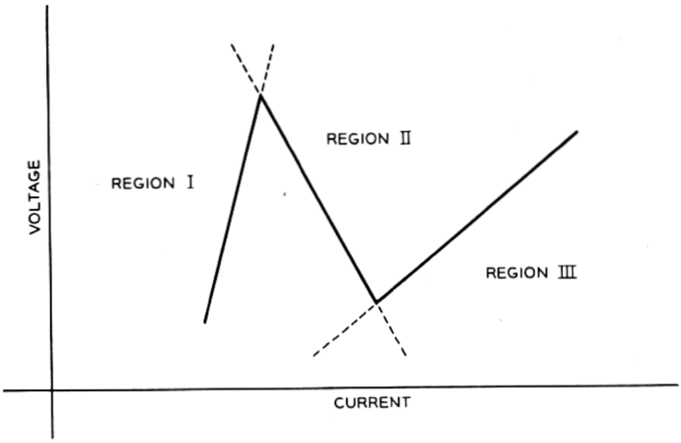


Fig. 12—Broken-line idealization of negative resistance characteristic—division into regions.

If, on the other hand, the emitter current is increased, at constant collector current, it is found that at a critical emitter current the slope again becomes zero or nearly so. There are also two further observations. First, the collector voltage is reduced to a very small value and second, that the critical emitter current is related to the collector current. From the small-signal relation,

$$V_c = R_{21}I_\epsilon + R_{22}I_c \tag{2}$$

or

$$V_c = R_{22}\alpha I_\epsilon + R_{22}I_c, \tag{3}$$

* It is tacitly assumed that in the relation $y = f(x)$ that there are extremes at which y becomes essentially constant and independent of further change in the independent variable x. The point farthest removed from the origin at which the dependent variable becomes constant is termed saturation. The point closest to the origin at which the dependent variable becomes constant is termed cutoff.

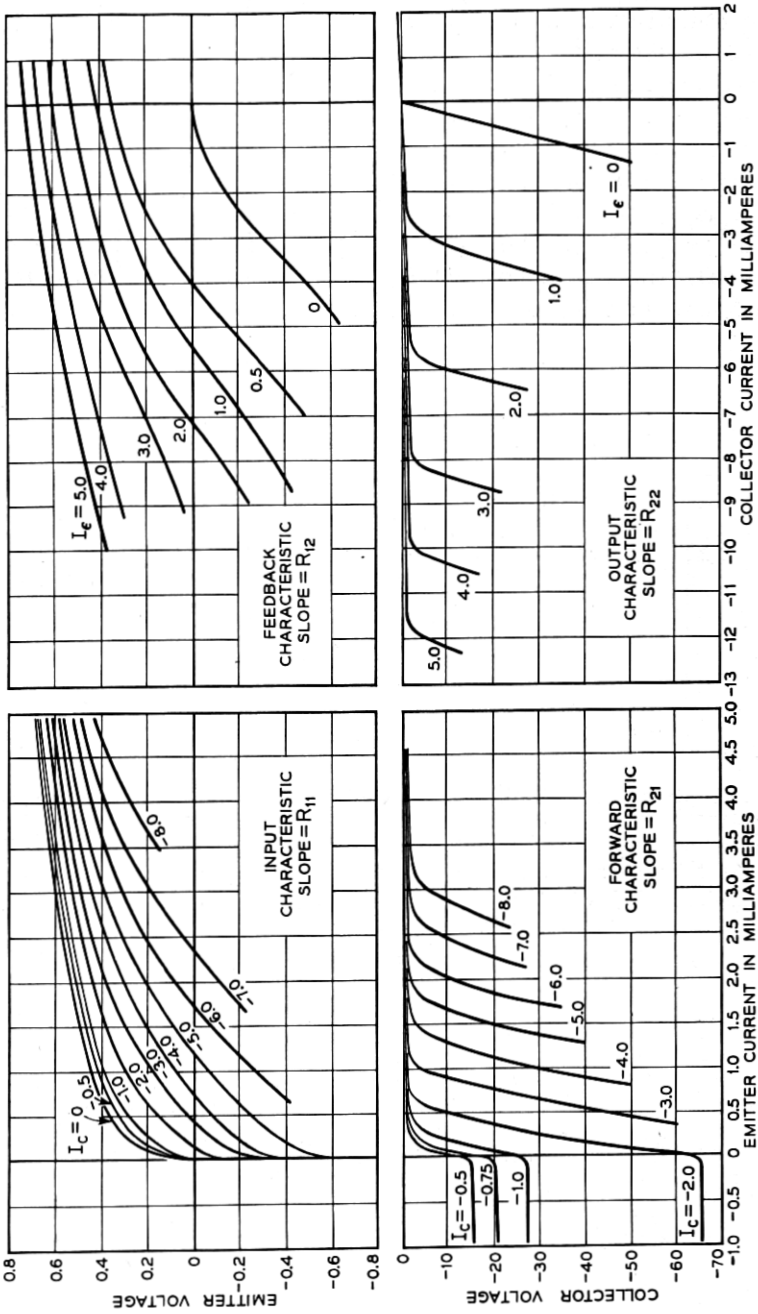


Fig. 13—Static characteristics of the M1689 developmental switching type transistor.

the critical emitter current for collector voltage cutoff may be obtained by setting $V_c = 0$, as,

$$I_\epsilon = -\frac{I_c}{\alpha} \tag{4}$$

This relationship is dual to the grid voltage-plate voltage relation in tubes for plate current cutoff as, $V_g = -(V_p/\mu)$. The criteria for defining the three regions are thus established as:

$$\text{Region I (Collector Voltage Saturation): } I_\epsilon < 0 \tag{5}$$

$$\text{Region II (Active): } 0 < I_\epsilon < -\frac{I_c}{\alpha} \tag{6}$$

$$\text{Region III (Collector Voltage Cutoff): } I_\epsilon > -\frac{I_c}{\alpha} \tag{7}$$

The identification of device parameters will be made for the several regions by a single prime for Region I as r_ϵ', none for Region II as r_ϵ, and three primes for Region III as r_ϵ'''.

LINEARIZATION OF THE CHARACTERISTICS AND APPROXIMATIONS

The next step is to linearize the characteristics and to make suitable approximations in order to obtain simple linear equations of the terminal currents and voltages. The relations which require linearization are the device parameters R_{11}, R_{12}, R_{21} and R_{22} which are in general functions of the currents as $R_{ij} = f(I_1, I_2)$.

LINEARIZATION OF R_{11} AND R_{12}

In terms of the equivalent tee circuit, which has been and will be employed, R_{11} is given as $R_{11} = r_\epsilon + r_b$. Also, $R_{12} = r_b$. It is convenient to separate r_ϵ and r_b and discuss each separately since r_b is fairly constant and r_ϵ will have widely different regional values.

In the R_{12} family of Fig. 13, it may be seen that R_{12} or r_b is fairly constant in all three regions and will be so taken here. Further, in the simple circuits under consideration, external base resistance R_b has been inserted so that minor variations in r_b in the total of $r_b + R_b$ are inconsequential since usually $R_b \gg r_b$. The approximation that r_b is constant is subject to review where finer detail is necessary, particularly at low emitter currents where the rate of change of r_b is at a maximum.

The emitter resistance r_ϵ is approximately the resistance of a diode in the forward direction. As such, r_ϵ is high when the emitter current is negative and low when the emitter current is positive. Experimentally, it is found convenient to give three values to r_ϵ and hence to R_{11}, one for each region as shown in Fig. 14. This recognizes the non-linearity with I_ϵ in the forward direction and assumes that a single value in the reverse direction is sufficient. As the circuitry becomes more sophisticated a more precise approximation will undoubtedly be required, particularly near $I_\epsilon = 0$.

It may be noted that in the functional relation $R_{11} = f(I_\epsilon, I_c)$ that R_{11} is taken to be a function of I_ϵ only. The contribution of I_c is to shift the characteristic in voltage by $r_b \Delta I_c$ increments. Thus the relationship of $V_\epsilon = f(I_\epsilon, I_c)$ can be written very simply as

$$V_\epsilon = R_{11} I_\epsilon + R_{12} I_c \tag{8}$$

Since the problem has been linearized to first order terms only, the currents and voltages are total instantaneous or dc values as indicated by the capital letters.

IDEALIZATION OF R_{21}

As indicated previously, R_{21} will be small in Regions I and III and large in Region II. Since $R_{21} = r_m + r_b$, R_{21} can be no less than r_b; the defining approximations will be applied to r_m. In Region I when the emitter current is negative, r_m is taken to be zero and reflects the device approximation that the emitter current under this condition is entirely electron current. This is not always a true approximation, particularly near $I_\epsilon = 0$, and limiting tests are employed in transistor testing.

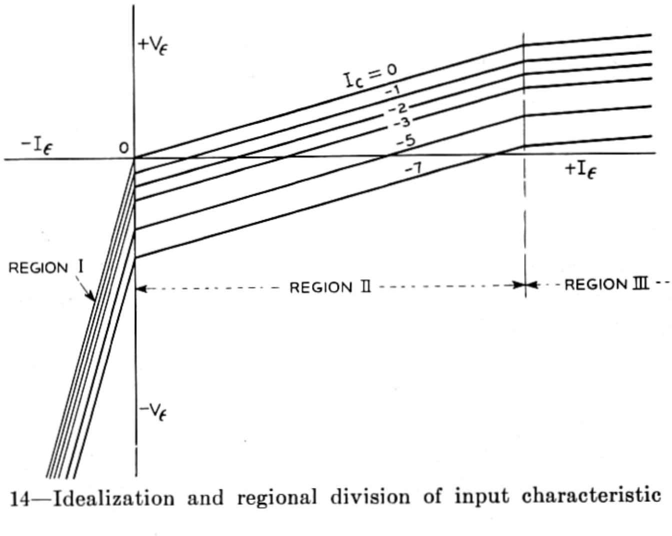


Fig. 14—Idealization and regional division of input characteristic (R_{11}).

In Region II, r_m has, of course, high values and in general $r_m \gg r_b$. Pending further investigation, r_m will be assumed finite, but very small in Region III.

IDEALIZATION OF R_{22}

In the output family of Fig. 13 it may be noted that R_{22} has two values, a high value for $I_c > -\alpha I_\epsilon$ and a low value for $I_c < -\alpha I_\epsilon$. The high value corresponds to Regions I and II and the low value to Region III. To a first order the two values are separately constant which was not true of earlier transistors in which R_{22} underwent extensive degradation in magnitude as I_c and I_ϵ increased.

The lower limit to which R_{22} can fall in Region III is r_b, since $R_{22} = r_c + r_b$, implying that r_c is zero in Region III. This is approximately, but not accurately true. As αI_ϵ approaches $-I_c$ in magnitude, the voltage across the collector barrier becomes nearly zero so that r_c has a low, but finite value. Under this condition, the hole current is very high and heavy conductivity modulation of the collector barrier resistance occurs. Thus the collector resistance in Region III is indeed quite low and may be neglected for many circuit computations.

In the functional relation $R_{22} = f(I_\epsilon, I_c)$ it has been assumed that R_{22} is a function of I_c alone. Further, the approximation involves first order terms only and hence the functional relation $V_c = f(I_\epsilon, I_c)$ may be written as:

$$V_c = R_{21}I_\epsilon + R_{22}I_c \tag{9}$$

Here again, as in the input case, the currents and voltages are total instantaneous or dc values as indicated by the capital letters.

It is believed desirable, however, to give one more consideration to the output relations. When $I_\epsilon = 0$, the collector characteristic is approximately that of a diode in the reverse direction. A diode has low reverse resistance until the voltage across the barrier exceeds a few tenths of a volt and then has quite high resistance, approaching infinite slope in the case of junction diodes.[6, 7] This effect is shown exaggerated in the idealized output family of Fig. 15. The current and voltage at the break in the $I_\epsilon = 0$ curve have been termed I_{c0} and V_{c0} respectively. I_{c0} and V_{c0} are quite evident in junction devices; in point contact devices they are not nearly so evident due to the lower value of R_{22} and the higher voltages and currents normally employed. Where currents and

[6] See Reference 2.
[7] *Holes and Electrons*, W. Shockley, Van Nostrand, p. 91, 1950.

voltages are of the order of several milliamperes and a few volts, I_{c0} and V_{c0} may normally be neglected. I_{c0} and V_{c0} do have considerable interest to the device designer, however. The net circuit interpretation of I_{c0} and V_{c0} is to effectively transfer the current-voltage axis from 0, 0 to I_{c0}, V_{c0}. Therefore,

$$V_c - V_{c0} = R_{21}I_\epsilon + (I_c - I_{c0})R_{22} \tag{10}$$

or

$$V_c - V_{c0} = (r_m + r_b)I_\epsilon + (I_c - I_{c0})\,(r_c + r_b) \tag{11}$$

Making the approximation that $V_{c0} = I_{c0}R_{22}'''$ and rearranging, equation (10) becomes,

$$V_c - I_{c0}R_{22}''' + I_{c0}R_{22} = R_{21}I_\epsilon + I_cR_{22} \tag{12}$$

or

$$V_c + I_{c0}(R_{22} - R_{22}''') = R_{21}I_\epsilon + I_cR_{22} \tag{13}$$

which is of the usual form except that a small dc generator of magnitude $I_{c0}(R_{22} - R_{22}''')$ has been added in series with the collector. Since $R_{22} = r_c + r_b$ and $R_{22}''' = r_c''' + r_b$,

$$I_{c0}(R_{22} - R_{22}''') = I_{c0}(r_c - r_c''') \tag{14}$$

The output family equation with equivalent circuit parameters is

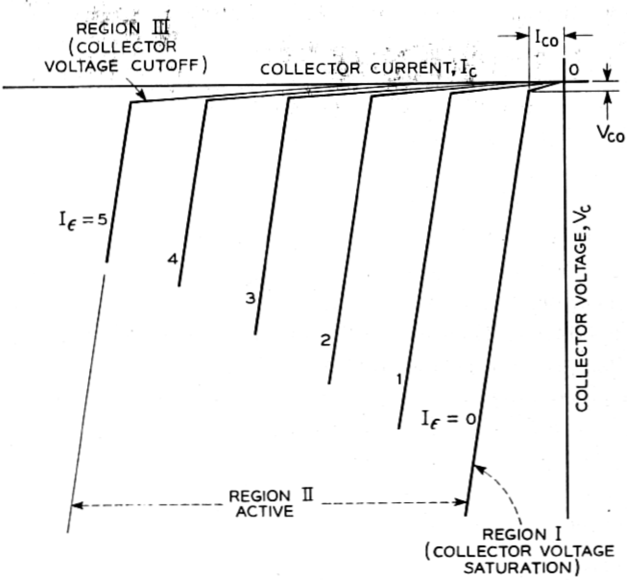


Fig. 15—Idealization and regional division of output characteristic (R_{22}).

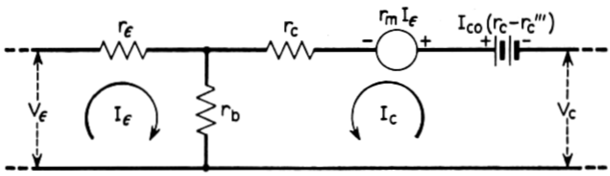
then:

$$V_c + I_{c0}(r_c - r_c''') = (r_m + r_b)I_\epsilon + (r_c + r_b)I_c \tag{15}$$

SUMMARY OF IDEALIZATION OF CHARACTERISTICS

The results of the idealization of the device characteristics are summarized in Fig. 16. Here are given analytic expressions for the input and output voltages in terms of the input and output currents; the regions are defined symbolically and by typical values; and an equivalent circuit is given. It may be noted that the equivalent circuit is identical to the small-signal equivalent tee, excepting the small dc generator $I_{c0}(r_c - r_c''')$ which usually may be neglected when dealing with contemporary point contact transistors.

To obtain any of the negative resistance characteristics it is only necessary first to solve the two equations simultaneously for the appropriate voltage in terms of the appropriate current, and then second to insert into the resultant equation the proper parameter values, region by region, to obtain three equations. These equations, when plotted, result in an idealized characteristic similar in form to that of Fig. 12. A detailed example plus synopsis of the properties of the several connections will be given in the following sub-section.



$$V_\epsilon = (r_\epsilon + r_b)I_\epsilon + r_b I_c$$
$$V_c + (r_c - r_c''')I_{co} = (r_m + r_b)I_\epsilon + (r_c + r_b)I_c$$

REGION	PARAMETER							
	r_ϵ		r_b		r_c		r_m	
	SYMBOL	TYPICAL	SYMBOL	TYPICAL	SYMBOL	TYPICAL	SYMBOL	TYPICAL
I	r_ϵ'	100 K	r_b	160	r_c	20 K	r_m'	0
II	r_ϵ	100	r_b	160	r_c	20 K	r_m	50 K
III	r_ϵ'''	25	r_b	50	r_c'''	70	r_m'''	30

$I_{co} \approx -50\mu A$

Fig. 16—Broken-line transistor equations, regional parameter values and equivalent tee circuit.

THE ALPHA OR CURRENT GAIN FACTOR*

The derivation just given has been in terms of the equivalent circuit parameters, r_e, r_b, r_c, and r_m. Another circuit factor, alpha or the short-circuit current gain, is also quite useful. Alpha has been defined in Table 1 as the negative ratio of the incremental change in output current to the incremental change in input current *under the condition of short-circuit output terminals.*

Thus alpha is restricted in interpretation to a specific device termination and care should be taken in the employment of alpha when other terminations are involved. For example, the circuit current gain under general conditions is given by R'_{21}/R'_{22}. The ratio R'_{21}/R'_{22} has been sometimes termed α_c. Thus,

$$\alpha_c = \frac{R'_{21}}{R'_{22}} = \frac{r_b + R_b + r_m}{r_b + R_b + r_c + R_c} \tag{16}$$

Depending upon the magnitudes of R_b and R_c, the two current gain ratios may be markedly different. In Region II where r_m and r_c are very large the effects of R_b and R_c in equation (16) often may be neglected. The circuit current gain, α_c, may then be taken as the device alpha. In Region I, r_m has been taken as zero; hence the current gain will be somewhat less than unity, given by $(r_b + R_b)/(r_b + R_b + r_c + R_c)$, and is definitely not zero. Equally, in Region III, the circuit current gain is not zero but rather approaches the ratio, $(r_b + R_b)/(r_b + R_b + R_c)$. If $R_b \gg R_c$, the ratio is nearly unity.

ANALYSIS OF NEGATIVE RESISTANCE CHARACTERISTIC

The objectives of the circuit analysis, as stated previously, are:

1. To determine operating conditions such as proper values of loads, biases, trigger sensitivities and operating currents and voltages,
2. To determine the relationships of the device parameters to the circuit behavior in order that these parameters may be optimized, properly characterized and controlled for required circuit performance.

For example, the trigger sensitivity may be given by the voltage difference between the load line intersection with the Region I portion of the characteristic and the upper peak or turning point of the characteristic as shown in Figs. 6, 7 and 9. The sensitivity Δ is thus determined by the nearness of the bias point to the peak of the characteristic. Since the bias is normally fixed, variations in the sensitivity will arise

* This section is inserted parenthetically as clarifying material due to the use of the α-factor in subsequent discussion.

from variations in the peak point. Thus it is necessary to know the relationships which determine the currents and voltages of the peak and valley points in order first to achieve a design and second, to establish controls on the proper device parameters.

In this example the emitter negative resistance characteristic will be solved and analyzed. The solutions for the other characteristics follow in the same manner and will be summarized.

EVALUATION OF EMITTER CHARACTERISTIC AS AN EXAMPLE

To obtain the emitter characteristic, it is necessary to solve the two equations of Fig. 16 for V_ϵ in terms of I_ϵ. The equations as given are for the short-circuit case. Since the general case will involve external parameters as R_ϵ and R_c, the equations will be modified to include these parameters.

The effects of external parameters may be applied very easily since,

$$V_\epsilon = V_{\epsilon\epsilon} - I_\epsilon R_\epsilon \tag{17}$$

and

$$V_c = V_{cc} - I_c R_c \tag{18}$$

where $V_{\epsilon\epsilon}$ and V_{cc} are supply voltages; V_ϵ and V_c are measured from the appropriate terminal to the far end of the external base resistor. External R_b adds directly to r_b. Thus the two equations become:

$$V_{\epsilon\epsilon} = (r_\epsilon + R_\epsilon + r_b + R_b)I_\epsilon + (r_b + R_b)I_c \tag{19}$$

$$V_{cc} + (r_c - r_c''')I_{c0} = (r_m + r_b + R_b)I_\epsilon + (r_b + R_b + r_c + R_c)I_c \tag{20}$$

In manipulation of equations (19) and (20) it is often more easy to do so in the functional form,

$$V_1 = R'_{11}I_1 + R'_{12}I_2 \tag{21}$$

$$V_2 = R'_{21}I_1 + R'_{22}I_2 \tag{22}$$

with substitution at the evaluation stage. The R''s here include both device and circuit parameters.*

Solving equations (19) and (20) simultaneously, the following rela-

* Here the primes indicate generalized open circuit driving point resistance rather than reference to Region I. The duplication of symbols is regretted.

tionship between V_ϵ and I_ϵ is obtained:

$$V_\epsilon = \left[r_\epsilon + R_\epsilon + r_b + R_b - \frac{(r_b + R_b)(r_b + R_b + r_m)}{r_b + R_b + r_c + R_c} \right] I_\epsilon + \frac{(V_{cc} + I_{c0}(r_c - r_c'''))}{r_b + R_b + r_c + R_c} (r_b + R_b) \tag{23}$$

Equation (23) is general for the given circuit; it suffers, however, in difficulty in interpretation due to the numerous terms. In the regional evaluation to follow, approximations will be made which bring out the significant factors although decreasing the accuracy somewhat. The $(r_c - r_c''')I_{c0}$ terms will be neglected. It is assumed also that large external base resistance R_b is employed.

EVALUATION IN REGION I

In Region I, from Fig. 16, r_m is zero and r_ϵ' is large so that $r_\epsilon' \gg (r_b + R_b)$. Also, by assumption, $r_b \ll R_b$. Applying these approximations, equation (23) becomes,

$$V_{\epsilon \text{I}} \approx r_\epsilon' I_\epsilon + \frac{V_{cc} R_b}{R_b + r_c + R_c} \tag{24}$$

Equation (24) is the equation of a straight line, having slope r_ϵ' and an intercept on the voltage axis at $(V_{cc}R_b)/(R_b + r_c + R_c)$. The small-signal input impedance is just the slope value or r_ϵ'.

The short-circuit case where R_c is zero is the most adverse device condition in the sense that the dc term will then be most dependent upon device parameters. When $R_c = 0$, equation (24) becomes

$$V_{\epsilon \text{I}} \approx r_\epsilon' I_\epsilon + \frac{V_c R_b}{R_b + r_c} \tag{25}$$

EVALUATION IN REGION II

In Region II all parameters are finite and the only approximations which may be made are $r_b \ll R_b$ and $r_\epsilon \ll R_b$. Thus,

$$V_{\epsilon \text{II}} \approx \left[R_b - \frac{R_b(R_b + r_m)}{R_b + r_m} \right] I_\epsilon + \frac{V_{cc} R_b}{R_b + r_c + R_c} \tag{26}$$

If R_b is not too large, it may be approximated that $(R_b + r_m)/(R_b + r_c) = \alpha$. Taking $R_c = 0$, thus,

$$V_{\epsilon \text{II}} \approx R_b(1 - \alpha) + \frac{V_c R_b}{R_b + r_c} \tag{27}$$

Equation (27) is also the equation of a straight line having the voltage axis intercept of $(V_c R_b)/(R_b + r_c)$ the same value as in Region I. The slope, $R_b(1 - \alpha)$, is negative provided $\alpha > 1$.

EVALUATION IN REGION III

In Region III it may be assumed that $r_b \ll R_b$, $r_c''' \ll R_b$ and $r_m''' \ll R_b$. Other suitable approximations will depend largely upon the magnitude of R_c. From equation (23)

$$V_{\epsilon \text{III}} \approx \left[r_\epsilon''' + R_b - \frac{R_b(R_b + r_m''')}{R_b + r_c''' + R_c} \right] I_\epsilon + \frac{V_{cc} R_b}{R_b + R_c} \tag{28}$$

If R_c is large, that is, large compared to r_ϵ''', but small compared to Region II r_c, then (28) becomes,

$$V_{\epsilon \text{III}} \approx \frac{R_b R_c}{R_b + R_c} I_\epsilon + \frac{V_{cc} R_b}{R_b + R_c} \tag{29}$$

Under these conditions, the circuit is essentially independent of device parameters. This is useful where a high independence of device parameters is required, but does not focus the attention upon the device parameters as does the $R_c \to 0$ case. This is the condition under which the transistor might be operated when it is desired to obtain the maximum ON current, or conversely the minimum internal switch resistance.

Where $R_c = 0$, equation (28) becomes,

$$V_{\epsilon \text{III}} = [r_\epsilon''' + r_c''' - r_m'''] I_\epsilon + V_c \tag{30}$$

Since r_ϵ''' and $(r_c''' - r_m''')$ are quite small the short-circuit currents may be very high. Where the transistor is considered as a switch between emitter and collector circuits, the "switch" voltage drop, as $V_{\epsilon c}$, is given by the first term of equation (30).

EVALUATION OF REGION I—REGION II TRANSITION

Earlier, trigger sensitivities were mentioned as being the small voltage and current differences between the turning points of the negative resistance characteristic and the stable operating points. The determination of the turning points and their stability is of great importance since it is usually desired to obtain maximum stable sensitivity. The voltage and current at the two turning points* have been given the subscript p and v for the low and high current conditions respectively as shown in the synopses of Fig. 17, 18 and 19. $V_{\epsilon p}$ and $I_{\epsilon p}$ in the short-

* Sometimes termed "peak" and "valley".

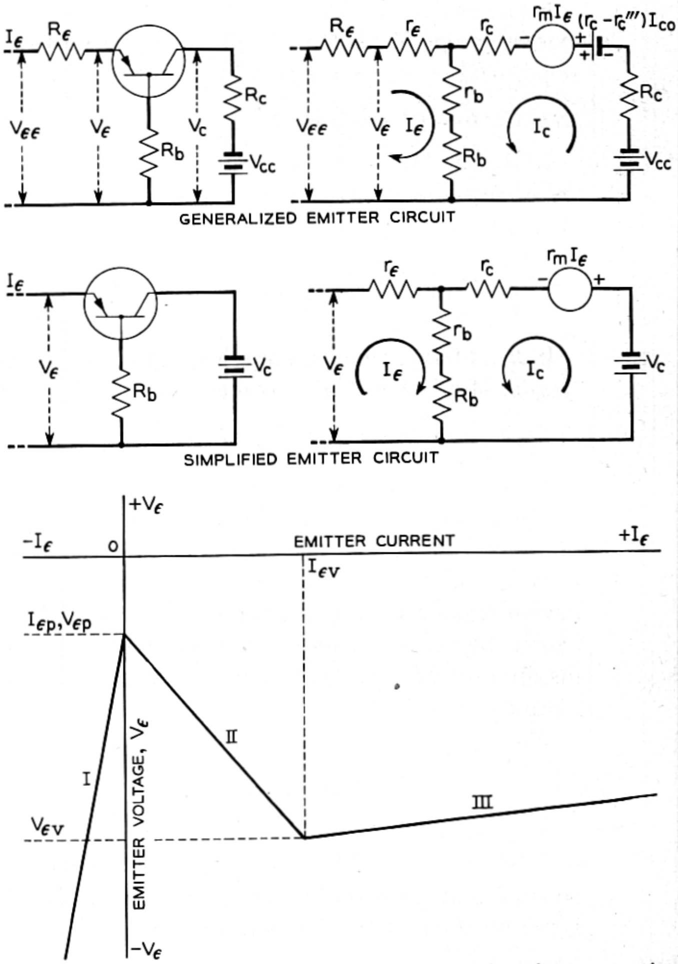


Fig. 17—Synopsis of emitter negative

circuit or $R_\epsilon = R_c = 0$ case for example may be obtained by a simultaneous solution of equation (24) for Region I and equation (27) for Region II. Thus

$$V_{\epsilon p} \approx \frac{V_c R_b}{R_b + r_c} \tag{31}$$

and

$$I_{\epsilon p} = 0. \tag{32}$$

That the low current turning point falls on the emitter current axis, i.e., $I_{\epsilon p} = 0$, is a consequence of the original assumption that $r_m = 0$

Synopsis

ieneral

$$V_{\epsilon\epsilon} = I_\epsilon \left[(r_\epsilon + r_b + R_b + R_\epsilon) - \frac{(r_b + R_b)(r_b + R_b + r_m)}{r_b + R_b + r_c + R_c} \right]$$

$$\frac{(V_{cc} + I_{co}(r_c - r_c'''))}{r_b + R_b + r_c + R_c} (r_b + R_b)$$

.pproximate Short Circuit Case

here

$$r_b \ll R_b; \qquad r_\epsilon, r_\epsilon''' \ll R_b; \qquad R_b \ll r_c, r_m; \qquad R_\epsilon = R_c = 0;$$

$$I_{co}(r_c - r_c''') \ll V_c$$

legion I

$$V_\epsilon \approx I_\epsilon r_\epsilon' + \frac{V_c R_b}{R_b + r_c}$$

legion II

$$V_\epsilon \approx I_\epsilon R_o (r - \alpha) + \frac{V_c R_b}{R_b + r_c}$$

legion III

$$V_\epsilon \approx I_\epsilon (r_\epsilon''' + r_c''' - r_m''') + V_c$$

$$I_{\epsilon p} \approx 0; \qquad V_{\epsilon p} \approx \frac{V_c R_b}{R_b + r_c}$$

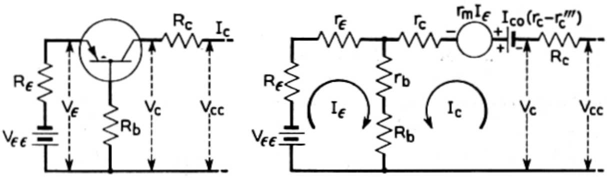
$$I_{\epsilon v} \approx \frac{V_c}{R_b(1 - \alpha)}; \qquad V_{\epsilon v} = V_c \left[1 + \frac{r_\epsilon''' + r_c''' - r_m'''}{R_b(1 - \alpha} \right]$$

$$\frac{V_{\epsilon v}}{V_{\epsilon p}} \approx \frac{R_b + r_c}{R_b}$$

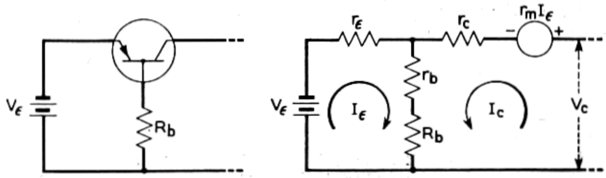
esistance characteristic and properties.

for $I_\epsilon < 0$ and $r_m > 0$ for $I_\epsilon > 0$. This is not a precise assumption and the turning point will usually lie slightly in the positive emitter current region. For very small triggers or more accurate calculations, consideration must be given to closer approximations of $r_m = f_1(I_\epsilon)$ and $R_{11} = f_2(I_\epsilon)$.

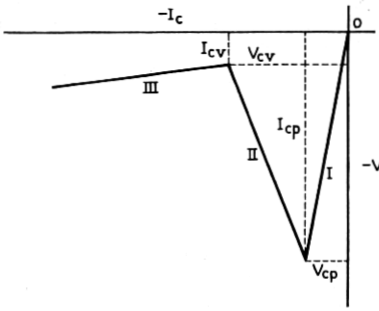
The consequences of equation (31) can be quite serious. V_c and R_b are of course fixed, but r_c is variable from unit to unit, with temperature and perhaps with life. The variability of $V_{\epsilon p}$ can result in failure to trigger, self-triggering or lock-up at high current.



GENERALIZED COLLECTOR CIRCUIT



SIMPLIFIED COLLECTOR CIRCUIT



Synopsis

General

$$V_{cc} + I_{co}(r_c - r_c''') = I_c\left[r_c + R_c + r_b + R_b - \frac{(r_b + R_b)(r_b + R_b + r_m)}{r_e + R_e + r_b + R_b}\right] + \frac{V_{ee}(r_b + R_b + r_m)}{r_e + R_e + r_b + R_b}$$

Approximate Short Circuit Case

where

$$R_e = R_c = 0\,; \qquad r_b \ll R_b\,; \qquad I_{co}(r_c - r_c''') \ll V_{cc}\,,\; R_b \ll r_m\,,\; r_c$$

Region I

$$V_c \approx I_c(r_c + R_b) + V_e\,\frac{R_b}{r_e'}$$

Region II

$$V_c \approx I_c[r_c(1 - \alpha)] + \frac{V_e(r_m + R_b)}{R_b}$$

Region III

$$V_c \approx I_c(r_e''' + r_c''' - r_m''') + V_e$$

$$V_{cp} \approx V_e\left(\frac{r_c + R_b}{R_b}\right) \qquad I_{cv} \approx \frac{V_e}{R_b}\left(\frac{\alpha}{\alpha - 1}\right)$$

$$I_{cp} \approx \frac{V_e}{R_b} \qquad \frac{V_{cp}}{V_{cv}} \approx \frac{r_c + R_b}{R_b}$$

$$V_{cv} \approx V_e\left[1 + \frac{\alpha(r_e''' + r_c''' - r_m''')}{R_b(\alpha - 1)}\right]$$

Fig. 18—Synopsis of collector negative resistance characteristic and properties.

EVALUATION OF REGION II—REGION III TRANSITION

The high-current turning point for the short-circuit case is determined from a simultaneous solution of the pertinent equations for Regions II and III, equations (27) and (30). Thus,

$$I_{\epsilon v} \approx \frac{V_c r_c}{R_b(1 - \alpha)(r_c + R_b)} \tag{33}$$

$$V_{\epsilon v} \approx V_c\left[1 + \frac{r_c(r_\epsilon''' + r_c''' - r_m''')}{(r_c + R_b)R_b(1 - \alpha)}\right] \tag{34}$$

Where it may be approximated that $r_c \gg R_b$, as has already been done in bringing in α, equations (33) and (34) become,

$$I_{\epsilon v} \approx \frac{V_c}{R_b(1 - \alpha)} \tag{35}$$

$$V_{\epsilon v} \approx V_c\left[1 + \frac{r_\epsilon''' + r_c''' - r_m'''}{R_b\ (1 - \alpha)}\right] \tag{36}$$

In this order of approximation, $V_{\epsilon v}$ is nearly equal to V_c. Any variation in the lower trigger point is primarily with $I_{\epsilon v}$, due chiefly to any change in α. It is interesting to note that the trigger point will move along the Region III curve given by (30).

The ratio of $V_{\epsilon v}$ to $V_{\epsilon p}$ is often of interest to estimate voltage swings or perhaps as a design objective in some switching circuits. Thus from (36) and (31),

$$\frac{V_{\epsilon v}}{V_{\epsilon p}} = \frac{[R_b(1 - \alpha) + r_\epsilon''' + r_c''' - r_m'''](R_b + r_c)}{R_b^2(1 - \alpha)} \tag{37}$$

If $r''' + r_c''' - r_m''' \ll R_b(1 - \alpha)$ then (37) becomes:

$$\frac{V_{\epsilon v}}{V_{\epsilon p}} = \frac{R_b + r_c}{R_b} \tag{38}$$

If R_b is very large, the ratio approaches unity with the implication of the existence of only two regions. This is equivalent to saying that the negative resistance becomes infinite over an infinitely short range. The proper choice of R_b in terms of (38) may well be a design problem where it is desired to have a high ratio of $V_{\epsilon p}$ to $V_{\epsilon v}$, as in lockout circuits.

SYNOPSIS OF NEGATIVE RESISTANCE CHARACTERISTICS

Synopsis for the three negative resistance characteristics are given in Figs. 17, 18 and 19. The solution and analysis procedures are the same as outlined for the emitter characteristic. It should be noted that the base characteristic is short-circuit stable in distinction to the emitter

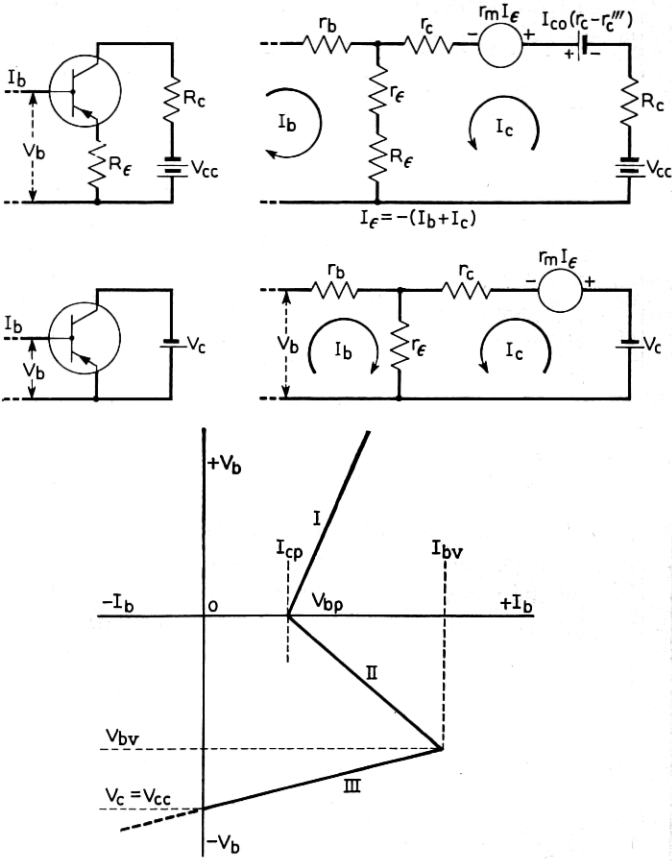


Fig. 19—Synopsis of base negative

and collector characteristics which are open-circuit stable. It would have been more appropriate to solve the base circuit in terms of conductances rather than resistances. The magnitudes of negative resistance obtained in this connection are quite low which may be misleading; the negative conductance is quite high, however, which is desired in short-circuit stable negative resistance circuits.

Care should be taken in the literal employment of the approximate regional relationships in Figs. 17, 18 and 19. They are very definitely approximate and are intended to illustrate behavior and the limiting condition only to bring out the relative importance of device parameters. It is suggested that calculations be started with the general case and approximations be made as are valid. For example, the con-

Synopsis

eneral

$$V_b = I_b(r_b + R_b + r_\epsilon + R_\epsilon) + I_c(r_\epsilon + R_\epsilon)$$

$$V_{cc} + I_{co}(r_c - r_c''') = I_b(r_\epsilon + R_\epsilon - r_m) + I_c(r_\epsilon + R_\epsilon + R_c - r_m)$$

$$_b = I_b\left[r_b + R_b + R_\epsilon + r_\epsilon - \frac{(r_\epsilon + R_\epsilon)(r_\epsilon + R_\epsilon - r_m)}{r_\epsilon + R_\epsilon + r_c + R_c - r_m}\right] + \frac{[V_{cc} + I_{co}(r_c - r_c''')](r_\epsilon + R_\epsilon)}{r_\epsilon + R_\epsilon + r_c + R_c - r_m}$$

pproximate Short Circuit Case

$$R_\epsilon = R_c = 0\ ; \qquad I_{co}(r_c - r_c''') \ll V_c\ ; \qquad r_\epsilon \ll r_c(1 - \alpha)$$

egion I

$$V_b = I_b\left(\frac{r'_\epsilon r_c}{r_c + r'_\epsilon}\right) + \frac{V_c r'_\epsilon}{r'_\epsilon + r_c}$$

egion II

$$V_b \approx I_b\left(\frac{r_b + r_\epsilon}{1 - \alpha}\right) + \frac{V_\epsilon\, r_\epsilon}{r_c(1 - \alpha)}$$

egion III

$$V_b \approx I_b r_b''' + V_c$$

$$I_{bp} \approx \frac{V_c}{r_c}\ ; \qquad V_{bp} = 0$$

$$I_{bv} \approx V_c\left[\frac{1 - \alpha}{r_\epsilon}\right]; \qquad V_{bv} \approx V_c\left(1 - \frac{(\alpha - 1)r_b'''}{r_\epsilon}\right)$$

esistance characteristic and properties.

clusion is reached in the collector characteristic that the negative resistance (Region II) is independent of the base resistance or feedback. This is true for only the limited range where $r_\epsilon \ll R_b \ll r_c$.

EXAMPLE OF CALCULATED AND EXPERIMENTAL CHARACTERISTICS

An example to illustrate the analysis is shown in Fig. 20 where both experimental and calculated characteristics for the emitter circuit are given. In this example there is appreciable load resistance; hence r_c''', r_ϵ''' and r_m''' are of no consequence since they will all be very small compared to the R_c of 2.2K ohms. Also, $R_b = 6.8K$ ohms is much greater than r_b ; hence r_b can be neglected. Since V_c is -45 volts, the I_{c0} term may also be neglected.

Computing $V_{\epsilon p}$ first,

$$V_{\epsilon p} = \frac{V_c R_b}{R_b + r_c + R_c} = \frac{-45(6.8K)}{(6.8K + 19K = 2.2K)} = -10.9 \text{ volts} \quad (39)$$

The calculated value of -10.9 volts compares quite favorably to the measured -11.0 volts.

Region II is given in this case, approximately by,

$$V_\epsilon \approx \left[R_b + \frac{R_b(R_b + r_m)}{R_b + r_c + R_c} \right] I_\epsilon + \frac{V_c R_b}{R_b + r_c + R_c} \quad (40)$$

or

$$V_\epsilon \approx \left(6.8K + \frac{6.8K(6.8K + 50K)}{6.8K + 19K + 2.2K} \right) I_\epsilon - 10.9 \quad (41)$$

$$V_\epsilon \approx (-8.9K) I_\epsilon - 10.9 \quad (42)$$

The first term is of course the slope in Region II and is the magnitude of the negative resistance. The calculated value is -8900 ohms whereas the measured value was approximately -9200 ohms.

The Region III approximation, derived also from the general relationship is,

$$V_\epsilon = \frac{(R_b R_c)}{R_b + R_c} I_\epsilon + \frac{V_c R_b}{R_b + R_c} \quad (43)$$

$$= \frac{((6.8K)(2.2K))}{(6.8K + 2.2K)} I_\epsilon = \frac{45(6.8K)}{6.8K + 2.2K} \quad (44)$$

or

$$V_{\epsilon \mathrm{III}} = (1785) I_\epsilon - 34 \quad (45)$$

The relation for Region III agrees quite well in slope but not in dc value as may be seen in Fig. 20. Since in this example the Region III behavior is determined essentially by the circuit parameters, it is surmised that the nominal 45-volt battery employed in taking the data was actually 47 volts.

The Region I check is essentially perfect since the approximation given in Fig. 17 is quite good.

Note the error at the intersection of the Regions II and III. The broken-line method predicts a sharp transition whereas the actual case is gradual. The deviation is due to the gradual changes in r_m and r_c as the collector voltage approaches cutoff and is the largest gross error in the approximation.

It is believed that analysis of this sort will reasonably predict circuit behavior and lead to device requirements. There must be a thorough understanding of the approximations involved and the accuracy will be directly related to the degree to which the original idealized characteristics are approximated. Extended, by means of more than three broken

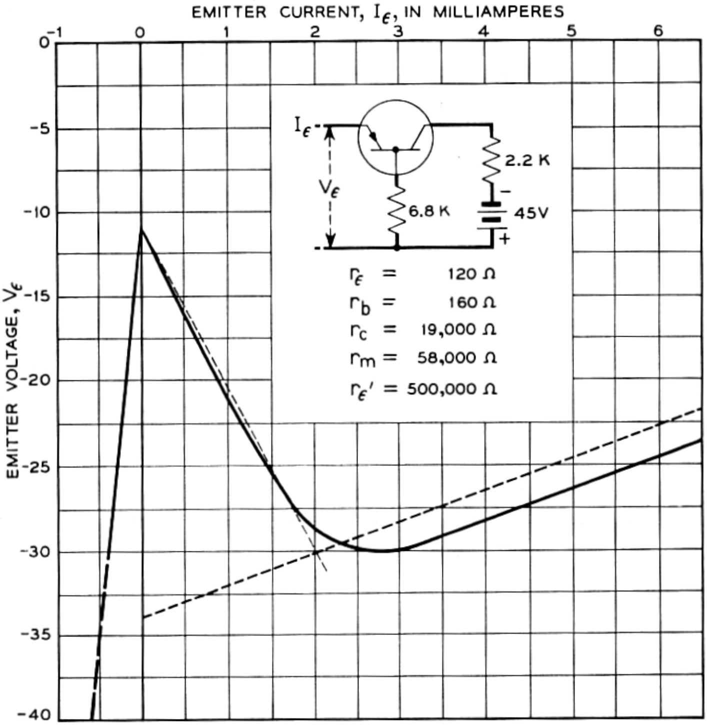


Fig. 20—Experimental and calculated emitter negative resistance characteristic.

lines, the method will yield fine detail to the degree to which device parameters are known and patience will permit. Transient behavior and analysis have not been discussed and are needed for a more complete understanding, particularly where transitional speeds are of concern.*

III—Switching Type Transistor Properties

An examination of the circuit approximations given in Figs. 17, 18, and 19 will reveal that the transistor and circuit designers will want to know nearly all there is to know about the device characteristics. This is not particularly surprising since the device is used over its entire range rather than over a limited portion as in the case of small-signal applications. The same examination of the circuit relations will also show that

* A treatment of the transient behavior between regions is given in B. G. Farley, "Dynamics of Transistor Negative Resistance", *Proc. Inst. Radio Engr.*, Nov., 1952. Analysis and the solution for the periods of the monostable and astable cases, assuming infinite region to region transition speed, are given in G. E. McDuffie, Jr., "Pulse Duration and Repetition Rate of a Transistor Multivibrator", *Proc. Inst. Radio Engr.*, Nov., 1952.

virtually all of the device parameters should be constant from unit to unit and with ambient conditions.

It can be shown that for small-signals a device may be uniquely characterized by five measurements. In terms of the parameters used here these might be R_{11}, R_{12}, R_{22}, R_{21} and the dc bias point or equally, r_ϵ, r_b, r_c, r_m and the bias point. Since the problem was linearized in the approximation, it follows that 15 such measurements, five in each region, are necessary for proper switching device characterization. The indicated extensive testing required may be reduced somewhat by suitable approximations. It is clear that the switching device designer and producer must reconcile themselves to making more tests for accurate characterization than when small-signal devices are concerned.

What will be given here is a description of typical developmental switching transistors in terms of the parameters which have evolved as a result of practical approximations. The method will be to discuss device properties and measurements region by region; then to discuss the properties at the transition points. Temperature, frequency and life behavior will be taken up separately.

REGION I PROPERTIES

In Region I, the emitter current is negative. Hence the emitter resistance r_ϵ' is large and is essentially that of a diode in the reverse direction. At present r_ϵ' is measured by a simple dc test of the current which flows at a nominal -10 volts. Both r_ϵ' and r_b will be discussed further under the Region I–Region II transition properties.

The Region I collector resistance is one of the most important parameters in switching. This is because of its determining nature in the turning point voltages in Figs. 17, 18, and 19. Actually, what is of concern is not the small-signal slope shown as r_c* in Fig. 21, but rather the dc current and voltage relationship shown as r_{c0}. For example in Fig. 17, it may be seen that $V_{\epsilon p}$ is given by the voltage drop determined by the product of R_b and the dc collector current.

Fig. 21 is an idealization of the R_{22} characteristic and has been designed to bring out the diode nature of the collector by emphasizing the saturation current and voltage, I_{c0} and V_{c0}. In junction devices the break in the $I_\epsilon = 0$ characteristic at I_{c0} is quite evident whereas in present point contact devices the transition is smooth due to the much lower values of r_c. The device significance is the same, however; I_{c0} varies rapidly with temperature whereas r_c varies at a considerably lower rate.

* The actual parameter is of course R_{22}, but since $R_{22} = r_c + r_b$ and $r_b \ll r_c$ R_{22} is taken as r_c.

In junction devices the proper measurements would be of I_{c0} and r_c. Since I_{c0} is difficult to define in point contact devices, r_{c0} has been measured as an approximation. In the idealization, r_c and r_{c0} are related as,

$$r_{c0} \doteq \frac{(I_c - I_{c0})}{I_c} r_c \tag{46}$$

The measurement of r_{c0} is made at a collector voltage which is typical of the applications in the range of perhaps -10 to -45 volts.

A constant dissipation line has been drawn on Fig. 21, which reveals the desirability of having r_{c0} very large in order to operate at higher voltages and to secure high efficiency through lower dissipation in the OFF or rest condition.

REGION II PROPERTIES

The Region II low frequency properties are essentially identical to those of transistors intended for small-signal applications. A possible exception is the somewhat less attention paid to the base resistance, r_b, which is critical to small-signal applications. The characterization consists of a normal small-signal set plus dc bias values.

REGION III PROPERTIES

The Region III properties have been defined largely by a figure of

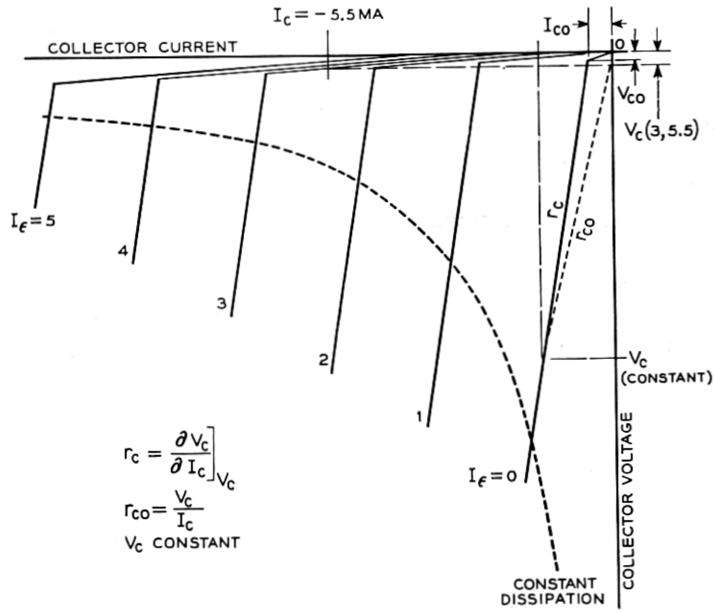


Fig. 21—Idealized output characteristic illustrating parameters.

merit measurement shown as $V_c(3, -5.5)$ in Fig. 21. This measurement is the voltage from collector to base under the condition that $I_\epsilon > -(I_c/\alpha)$. In this instance I_ϵ has been chosen to be 3 mA and I_c to be -5.5 mA. The collector current value is chosen on the basis of the smallest tolerable value of alpha expected so as to place the point of measurement near the R_{22} knee, but in Region III or overload.

The $V_c(3, -5.5)$ measurement is a good measurement for defining the general behavior. $V_c(3, -5.5)$ taken with the r_{c0} measurement constitute a very good defining set for checking the transistor as in re-measuring. For design purposes, the $V_c(3, -5.5)$ measurement is not sufficient. It provides an approximate value for r_c''', but does not define r_ϵ'' and r_m'''. A second dc measurement, the collector to emitter voltage drop, $V_{\epsilon c}$, has been employed experimentally also. An improved characterization will undoubtedly involve separate measurements of r_ϵ''', r_m''' and r_c'''.

REGION-TO-REGION TRANSITION PROPERTIES

The transition between Regions I and II is accompanied by abrupt changes in r_ϵ and r_m.

The theory assumes that both of these parameters change at an infinite rate at a fixed emitter current, taken as $I_\epsilon = 0$. Unfortunately neither of these assumptions is strictly true. r_ϵ undergoes a gradual change from high to low values which is only approximated by the three assigned values. In particular the behavior near $I_\epsilon = 0$ is of concern when dealing with small triggers.

The forward transfer impedance changes at a finite rate also. Further, the emitter current at which the maximum rate of change occurs will vary from unit to unit. Present practice also has been to measure α rather than r_m. The rational for doing so is not too good since r_m is quite likely the better parameter to characterize. Alpha has a strong physical appeal, fits well into the circuit problems and is easy to measure.

Since $\alpha = (r_b + r_m)/(r_b + r_c)$ it is necessary to assume that r_b and r_c are constant near $I_\epsilon = 0$, an only fair approximation. Having made the approximation, the typical α behavior shown in Fig. 22 may be taken as a measure of r_m. Three values are measured, the first of which, α_1, in Region II, is redundant to the Region II small-signal measurements. The two limits, α_2 and α_3, serve to place lower and upper limits on the absolute values of α at the Regions I–II transition. These limits in turn place a lower value on the rate of change in α within the $I_\epsilon \pm \Delta$ range shown.

It may be noted that α in Region I is finite. There is a lower limit

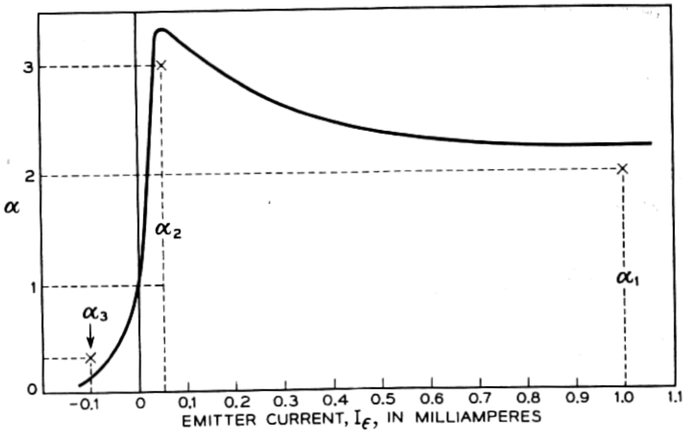


Fig. 22—Alpha characteristic.

even though r_m is zero since $\alpha_1 \rightarrow (r_b/r_b + r_c)$. The values normally encountered at $I_\epsilon = 0 - \Delta$ are usually in excess of this lower limit.

The feedback resistance r_b tends to rise as $I_\epsilon \rightarrow 0$ which may be important to some trigger circuits. As the circuitry becomes more sophisticated, it is expected that more attention will need to be paid to the behavior of r_ϵ, r_m and r_b at and near $I_\epsilon = 0$.

The transition from Region II to Region III is determined from the relation $I_\epsilon = -(I_c/\alpha)$. The problem is quite similar to the control of the μ factor in tubes where plate current cut-off is given by $V_g = -(V_p/\mu)$. Present practice has been to depend upon the α_1 values and upon the lower limit placed on alpha in the $V_c(3, -5.5)$ measurement. Further effort is needed here also.

TYPICAL PARAMETER VALUES AND DISTRIBUTIONS

Integrated distribution curves for the parameters of a typical developmental switching transistor are shown in Fig. 23. The unit-to-unit variations are deemed to compare favorably with those of commercial electron tubes. The parameter of most serious variability is r_{c0} which is unfortunate since r_{c0} is so important to trigger sensitivity stability.

TEMPERATURE, FREQUENCY AND SHOCK PROPERTIES

Transistor parameters are reasonably constant with temperatures below room temperature. Above room temperatures some of the parameters are variable. r_ϵ and r_b are fairly constant, changing very little to 70°C. r_c and r_m decrease fairly rapidly, maintaining a ratio such that alpha rises slightly. r_ϵ' and r_{c0} change most rapidly and, while both of

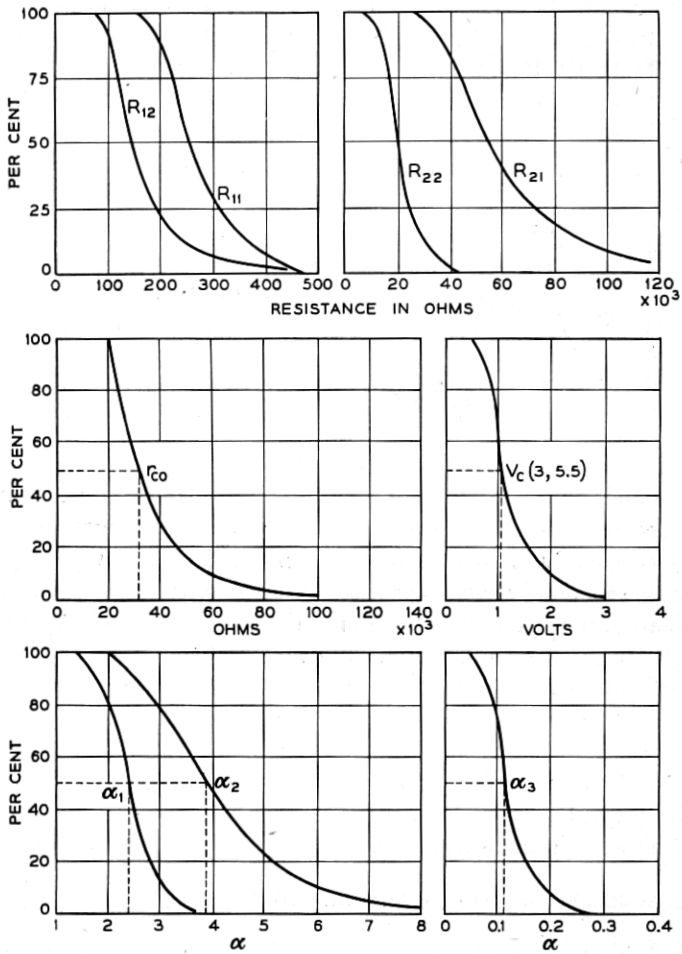


Fig. 23—Variation in parameters of developmental switching type transistor (M1698).

these parameters are of little consequence in small-signal applications, they are quite important in switching, particularly r_{c0} .

Early transistors might exhibit a change in r_{c0} at 60°C of 3 to 1 or more from room temperature values. The transistors of which the data in Figs. 13 and 23 are typical have an r_{c0} temperature coefficient of about -0.75 per cent/°C. That is, the room temperature value of r_{c0} might be reduced by 30 per cent at 70°C. The improved temperature behavior implies a corresponding reduction in variation in trigger sensitivity. Parameter values, large-signal and small-signal, are shown in Fig. 24 as a function of temperature.

Variation in characteristics will arise from self-engendered heat, that is, dissipation. Transistors may be thermally unstable under constant voltage conditions. Since the switching properties are exhibited under short-circuit or constant voltage terminations, thermal properties are of concern. The limitations involved are similar to those of any positive feedback circuit. If the thermal loss through radiation and conduction exceeds the heat input, the system will be stable. The practical significance is to place limitations on dissipation and to employ designs which result in rapid heat loss. Other design criteria such as miniaturization may limit the latter.

If perfect switching characteristics were obtainable, dissipation would be of little consequence in switching. This is akin to saying that neither a short-circuit nor an open-circuit dissipates any energy. Further, the perfect device has zero transition time and therefore involves no loss. The transistor has finite resistance both open and closed and a finite although rapid transition time. There is some advantage however. A constant dissipation curve shown as a dotted line has been included in Fig. 21. Small-signal operation at mid-range currents and voltages results in fairly low limitations on both current and voltage. The intersection with the R_{22} voltage saturation line ($I_\epsilon = 0$) is at fairly high voltage. Similarly, the intersection with the collector voltage cut off line is at high current. For constant dissipation, approximately,

Voltage saturation: $$P_d \approx \frac{V_c^2}{r_{c0}}$$

Voltage cutoff: $$P_d \approx I_c^2(r_\epsilon''' + r_c''' - r_m''')^*$$

Depending upon the circuit the assumed dissipation limit may or may not be exceeded during the transitions. Should the limit be exceeded,

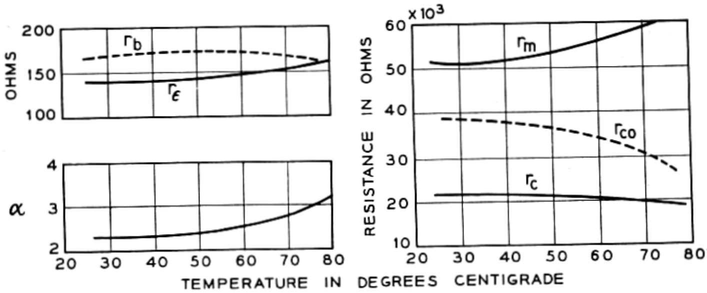


Fig. 24—Temperature behavior of characteristics of developmental switching-type transistor (M1689).

* This includes both emitter and collector dissipation. See equation (30).

and substantially so, there are normally no serious consequences due to the very rapid transitions and consequent low thermal energy generated.

Transistors may not be able to tolerate excess dissipation on this basis if the circuits are slow, that is with transition times in excess of perhaps a few tenths of a microsecond. Such conditions may arise, for example, if loads are inductive. In many such cases, shunting capacitor networks will often permit a rapid transition with consequent transfer of current to the inductive load.

The frequency response of point contact transistors can be sufficiently good to insure switching type operation with rise times of the order of 0.1 to 0.01 μs. Fall times may be somewhat longer due to the hole storage effect. In regenerative circuits, operating speeds are faster than might be imagined from the small-signal frequency cutoff. Reliable operation with rise times of 0.1 μs is obtained with only nominal attention to frequency cutoff. Speeds of the order of 0.02 μs require a 10 mc. lower limit. Present junction transistors are substantially slower.

Accurate life estimates are difficult to make due to the rapid rate of development, the relative age of the transistor and the number of parameters involved. A given device is quite likely to be obsolete and forced to give way to an improved version before sufficient models can be obtained for life tests. A small quantity of transistors having properties similar to those of Fig. 20 and 21 have been operated for over 6,000 hours with an indicated life of 30,000 hours. Other similar transistors with longer life histories have indicated lives of better than 70,000 hours. The pattern appears to be similar to that of electron tubes—an early failure and change rate followed by a very slow exponential rate. It is believed that life is extended by low power operation and is decreased by high temperature operation.

The relatively high noise level of transistors does not appear to be a significant problem at present when considered in terms of automata. Systems employing switching type circuits in pulse communication will of course be concerned. It is suggested that the non-concern for noise in non-transmission type systems is largely a reflection of the ease with which high magnitudes of state changes are obtained. With design trends toward low power and low operating levels, noise will undoubtedly set a lower limit of level operation in such systems also.

The extreme resistance of the transistor to shock and vibration with a consequent absence of microphonism may in some applications result in effective lower noise. Shocks in excess of 20,000G have resulted in no damage. No evidences of current modulation in excess of noise have been detected with vibrational forces of the order of 100G at frequencies as high as 1000 cycles in tests on the transistor of Fig. 1. Transistors have been included in plastic embedded circuits without change of characteristics.

SUMMARY—TRANSISTOR PROPERTIES

Transistors have been designed with properties expressly intended for switching applications.* The characteristics are acceptable for contemporary switching type circuits and sufficiently reproducible to permit interchangeability of devices in circuits of normal requirements. The characterization has been sufficiently unique to permit the calculation of first order circuit performance. The characterization is not sufficiently complete to permit determination of the complete transient behavior.

In terms of the circuits described, the major parameter limitation is concerned with the variability of the d-c collector resistance among units and with temperature. It is expected that future circuit development will place additional requirements on the transistor, particularly as related to the transitions between regions. It is also to be expected that future circuit designs may establish new or modify present device requirements.

A major consideration for computer or computer-like systems, reliability, particularly with respect to time and temperature, has not been established, but appears to be favorable.

ACKNOWLEDGMENT

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* See Reference 3 also.