Electrical Noise In Semiconductors

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Transistors, diodes, and single crystal filaments of germanium have common noise properties: a spectrum varying inversely with frequency, and strong dependence on the biasing current. Theoretical attempts to explain this noise are reviewed briefly. Experiments with single crystal filaments indicate that the noise resides in the behavior of the minority carrier. In one type of experiment, the correlation of noise voltages in adjacent portions of a filament is quantitatively related to the lifetime and transit time of minority carrier. In another, the effect of a magnetic field on the noise is found in accord with calculated changes in lifetime of the minority carrier.

In the development of the transistor it was recognized quite early that electrical noise in the device was considerably in excess of Johnson noise, particularly at low frequencies. Noise having a similar spectrum had been observed many years earlier in microphonic carbon contacts carrying a current, and in copper oxide rectifiers, composition resistors, and crystal diodes. Flicker noise in vacuum tubes appears to be a related phenomenon. A number of attempts have been made to determine the mechanism of production of noise of this sort, but none have been particularly successful.

In this paper we will first survey the more important characteristics of noise in germanium diodes and transistors. This will be followed by a partial hypothesis as to the nature of the noise mechanism. We will then discuss experimental work on noise in filaments of single crystal germanium carrying a dc current. These experiments strongly support the hypothesis, and in fact led to its formulation in the first place.

I. NOISE IN DIODES AND TRANSISTORS

There are many similarities in the noise phenomena found in diodes and transistors of both the point contact and junction type. It seems likely that the noise mechanism is similar in all these devices.

One of the most characteristic features of the noise in such structures is the spectrum. The spectral density (power per unit bandwidth) varies inversely as the frequency, according to the relation

$$dW = f^{-n}\, df$$

where the exponent lies between 1 and 1.5 with an average about 1.2. This type of spectrum will be referred to as a 1/f spectrum. Measurements of the spectra of silicon point contact diodes have been reported by P. H. Miller[1] for the frequency range 20 cycles to 300 kilocycles. Spectra of point contact transistors measured by the author have been reported elsewhere[2, 3] for the range 20 to 15,000 cycles. Typical spectra for *p-n* junction type diodes and transistors are shown in Fig. 1. Almost without exception, our measurements and those reported in the literature have shown the 1/f spectrum over most of the frequency range covered. There is some evidence from the related fields of flicker noise and carbon microphone noise that the 1/f spectrum may extend to frequencies well below 0.1 cycle per second. Some departures from this type of spectrum have been noted in the neighborhood of 100 kc, as shown in the curves.

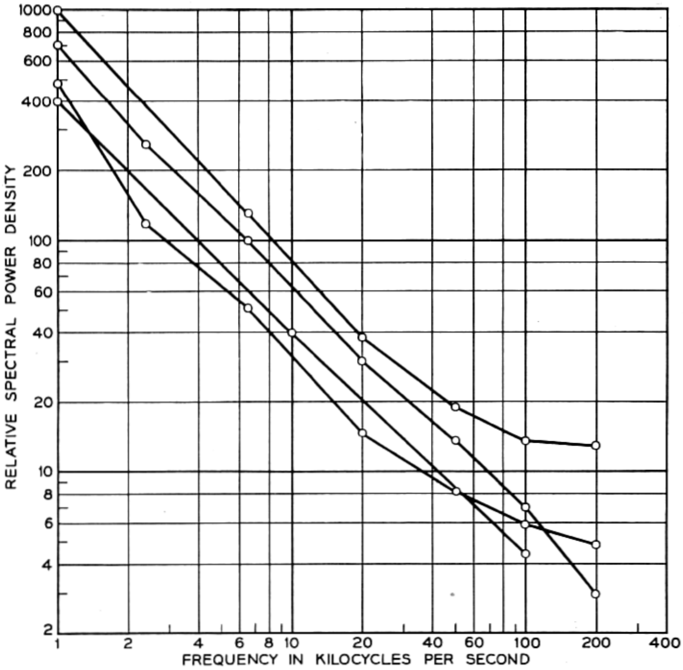

Fig. 1—The spectrum of noise in *n-p-n* transistors varies inversely with frequency.

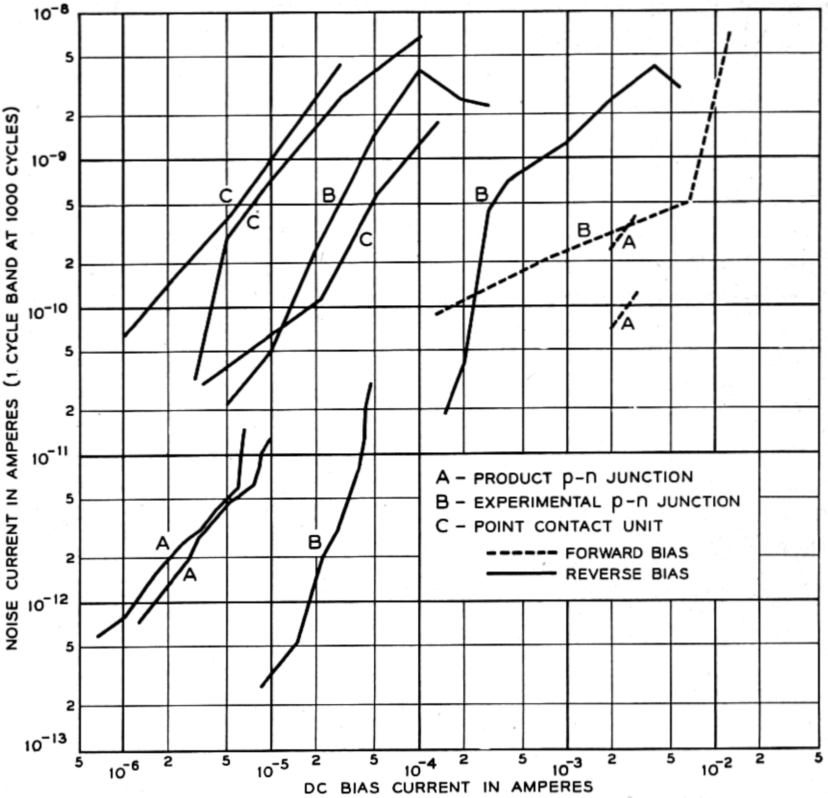

Fig. 2—The short-circuit noise current from a point contact or junction diode generally increases with dc bias current.

A second characteristic feature of noise in all semiconductor devices is that it is current dependent. In the absence of biasing current only Johnson noise is observed. When biasing current is present the noise power may be as much as three or four orders of magnitude above Johnson noise. As a general thing the noise increases as the bias is increased, although some minor exceptions to this rule are noted, usually at bias values where the slope of the current-voltage curve is changing rapidly.

To illustrate the bias-dependent behavior, the noise properties of some germanium diodes of various types are shown in Fig. 2. The short circuit noise current in a 1-cycle band at 1000 cycles is plotted as a function of dc bias current, some of the data being for forward bias, but most for reverse bias. Several curves are shown for each type of unit, and these are typical of the variations encountered. There is a general tendency for noise current to increase in proportion to bias current, but in limited regions the individual units may have slopes considerably different from unity. It would perhaps be more logical to plot current densities rather than total currents, but because of the general form of the relations this makes little difference in the overall picture, and there is some difficulty in estimating the appropriate area for the point contact units. There is an almost unlimited number of different ways of representing noise data. For example, noise current, current density, voltage, or available power may be expressed as a function of various bias parameters. Of a good many combinations tried, none gave an outstandingly simple picture of noise behavior, and the representation used in Fig. 2 is probably as good as any for an overall picture of diode noise.

The noise behavior of transistors depends on two bias parameters. Selection of the emitter current and collector voltage for the parameters usually leads to a rather simple representation. It often turns out that the noise behavior as an amplifier over the commonly used range of bias values depends largely on the collector voltage and is relatively independent of the emitter bias. Data of this sort were shown for point contact transistors in a previous reference,[3] and have been given for an *n-p-n* transistor by Wallace and Pietenpol.[4] A somewhat more complete family of curves is shown in Fig. 3 for a recent *n-p-n* transistor.

A few attempts have been made to determine the effect of tempera-

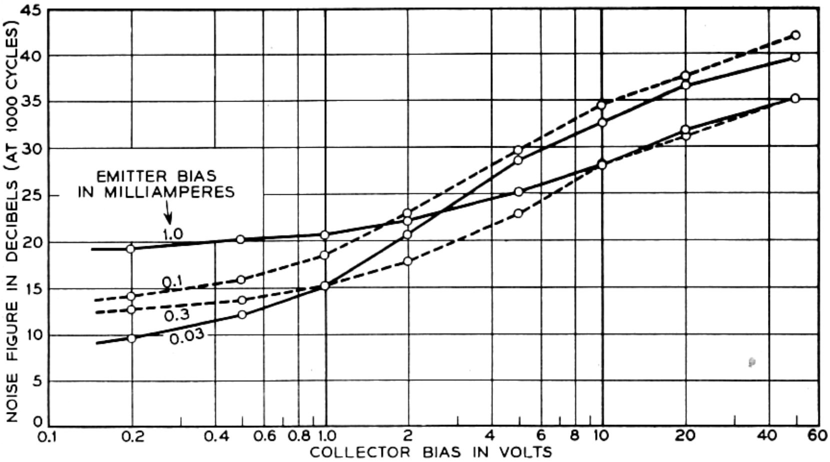

Fig. 3—The noise figure of an *n-p-n* transistor depends in a fairly simple way on emitter current and collector voltage.

ture on noise behavior. Such experiments have been rather unsatisfactory because the changes in impedance and gain characteristics as a function of frequency are of the same order as the changes in noise properties. This makes the interpretation ambiguous. By and large, such experiments suggest that changes in noise with temperature are rather small, perhaps of the order of the change in absolute temperature, and not at all like the exponential changes associated with a diffusion process. This observation does not necessarily rule out a diffusion-like noise process; it might indicate merely that we are not looking at the right part of the spectrum to observe exponential changes with temperature.

II. A HYPOTHESIS REGARDING THE NOISE MECHANISM

Considerable work has been done on the theory of current-dependent noise having a 1/f spectrum. Among the earliest was that of Schottky[5] in connection with flicker noise in vacuum tubes. He considered the arrival of foreign atoms on the emitting surface of the cathode as a random series of events governed by a diffusion law with a characteristic time constant, and arrived at a $1/f^2$ rather than a 1/f spectrum, and a highly temperature sensitive process. Surdin[6] pointed out that by postulating a series of decay processes with suitably distributed time constants a 1/f spectrum could be achieved. From physical arguments regarding the emission process from cathodes, Macfarlane[7] obtained a range of relaxation times and a 1/f spectrum, in a process which was highly temperature dependent. Richardson[8] gave a very general analysis of the noise properties of systems in which the conductivity was governed by a diffusion process. One conclusion was that a geometrically simple diffusion process in one, two, or three dimensions could not lead to 1/f spectrum, although by some highly specialized assumptions about the geometry of a contact surface he was able to obtain such a spectrum. DuPré,[9] in considering a hypothesis somewhat resembling that of Surdin, showed that the required range of activation energies was physically reasonable, and that the assumptions could be set up in such a way as to make the process relatively temperature independent. Several of the above authors and Van der Ziel[10] discuss the physical basis for applying flicker noise theory to the noise in semiconductors. Although this theoretical work has contributed a great deal to distinguishing between suitable and unsuitable mechanisms, there is still no specific physical theory of noise in semiconductors which can be tied in a quantitative manner to experimental results.

The experimental work described in the remainder of this paper has led to a hypothesis regarding the noise mechanism, which is by no means a complete explanation, but which may be a useful step in that direction. This hypothesis resulted largely from the experimental work, but it seems worth while to describe it first to help appreciate the significance of some of the experimental results.

It has been observed that in many semiconductor structures the noise voltage is approximately proportional to the dc bias current. This relation suggests that the noise is the result of fluctuations of the conductivity of the material, which modulate the bias current and produce a fluctuating voltage across the specimen. Such fluctuations in conductivity could result from variations in concentration of the minority carrier (holes in *n*-type material, electrons in *p*-type). The magnitude of the observed noise and the type of spectrum seem to demand that the fluctuation be coarse-grained in time to a much greater extent than could be accounted for by random statistical fluctuations of carrier density. Experiments of Haynes[11] on lifetime and transit of injected carriers in rods of germanium have occasionally indicated finite sources of minority carriers in the material. Our hypothesis is that such sources of carriers are rather generally distributed over the material (although mostly too small to be noticed in experiments of the Haynes type), and that their activity is being modified at a slow rate by some unspecified local influence in a suitable way to agree with the observed noise spectrum.

The experiments described below involving noise correlation phenomena and the effect of a magnetic field on noise point strongly to an important role for the minority carrier in the noise mechanism, and hence strongly suggest some such hypothesis as that just described.

III. NOISE IN SINGLE CRYSTAL FILAMENTS

It was found several years ago that a filament cut from single crystal germanium of high purity exhibits noise well above Johnson noise when a dc current is flowing in it. It is not clear whether this noise arises in the body of the material or on the surface, but to date no method of preparing the sample has eliminated this noise, and it is a prominent feature even at bias fields as low as 10 volts per centimeter. This noise seems to have most of the characteristics of the noise in diodes and transistors: it has the $1/f$ spectrum, is current dependent, and is quite stable with time. It has been the subject of considerable study in the hope that a better understanding of it would illuminate the whole field of semiconductor noise.

Samples, referred to as "bridges", have been cut from thin slabs of single crystal germanium, by a technique devised by W. L. Bond,[12] often of a form shown in Fig. 4. Side arms for both the current and the noise measuring electrodes have been found necessary to avoid spurious noise at the electrodes. A large inductance in the bias circuit greatly reduces the effect of any noise voltage generated at the bias electrodes. The spurious noise power from this source is seldom more than a few per cent of that being measured. It should be noted that the contact area for the noise measuring electrodes should not be on a portion of the specimen carrying bias current, otherwise spurious noise may be generated at these electrodes. Typical dimensions for the straight central filmanent of the bridge are 0.05 x 0.05 x 0.7 cm. The side arms have sandblasted surfaces to suppress holes or electrons injected at the electrodes. The central portion may be etched, sandblasted, or otherwise treated at will. The enlarged circular areas are rhodium plated to provide good contacts to each side arm.

Measurements of the noise spectrum in such bridges with several different etching treatments and with sandblasted surfaces are characterized by the 1/f spectrum over a wide frequency range.* Fairly extensive measurements have been made in the audio frequency range, and a few covering the range from 20 cycles to 1 megacycle. A typical spectrum is shown in Fig. 5.

The current dependence of the noise is shown in Fig. 6 for a number of samples, mostly *n*-type, one *p*-type, and with various resistivities. The outstanding feature is that noise voltage always increases with dc bias voltage. In many cases there is direct proportionality at the lower bias values, increasing to a square law at higher biases. There are some

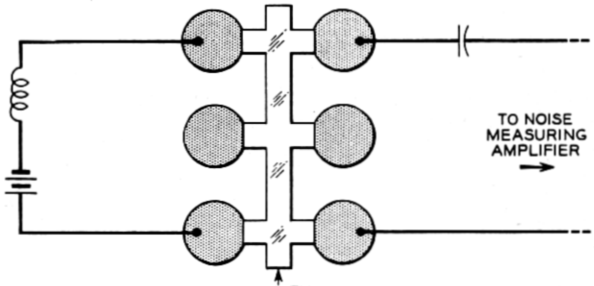

Fig. 4—Filament with side arms cut out of a single crystal of germanium.

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* Departures from the 1/f spectrum at frequencies of the order of 100 kilocycles and above were first discovered by G. B. Herzog and A. Van der Ziel. See Reference 13.

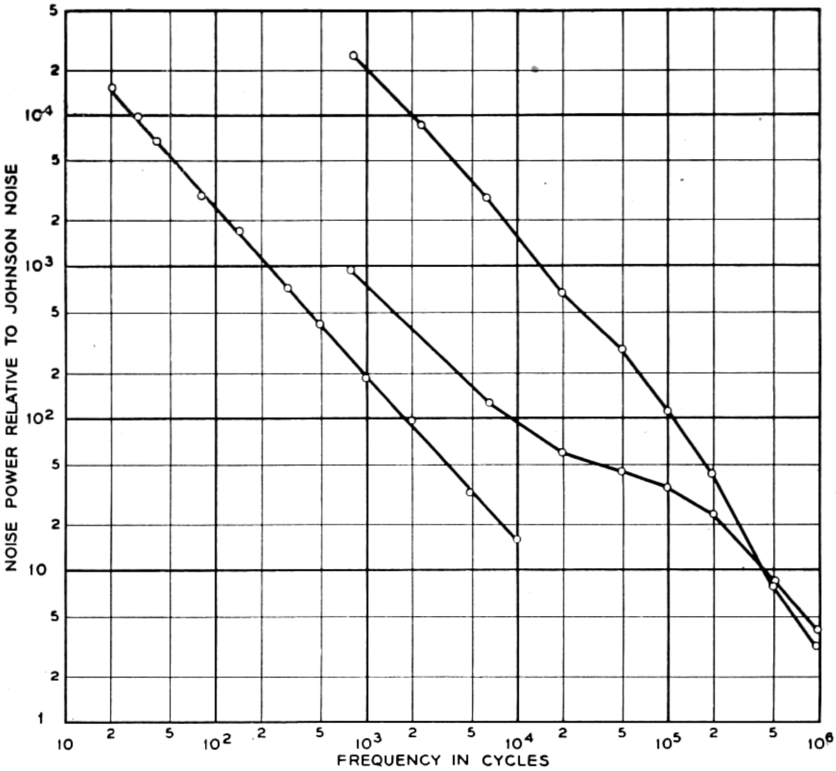

Fig. 5—Typical spectra of noise in single crystal filaments carrying a dc current.

exceptions to this trend. Also, there are large variations in the magnitude of the noise. An average unit shows a noise voltage about three times Johnson noise at a bias of 10 volts per centimeter.

The noise behavior at reduced temperatures has been investigated. Results on three different bridges are shown in Fig. 7. The open circuit noise voltage is shown as a function of temperature for constant bias voltage. Although the curves show rather large irregularities, there seems to be no general trend for noise to decrease with decreasing temperature over the range covered, from −200°C to room temperature.

The surface treatment applied to a bridge may affect the noise very substantially. A sandblasted surface usually gives the lowest noise. Etching the surface may raise the noise voltage by a factor of ten or more, though the dc resistance changes only a few per cent. The technique of washing and drying the surface may have an important effect

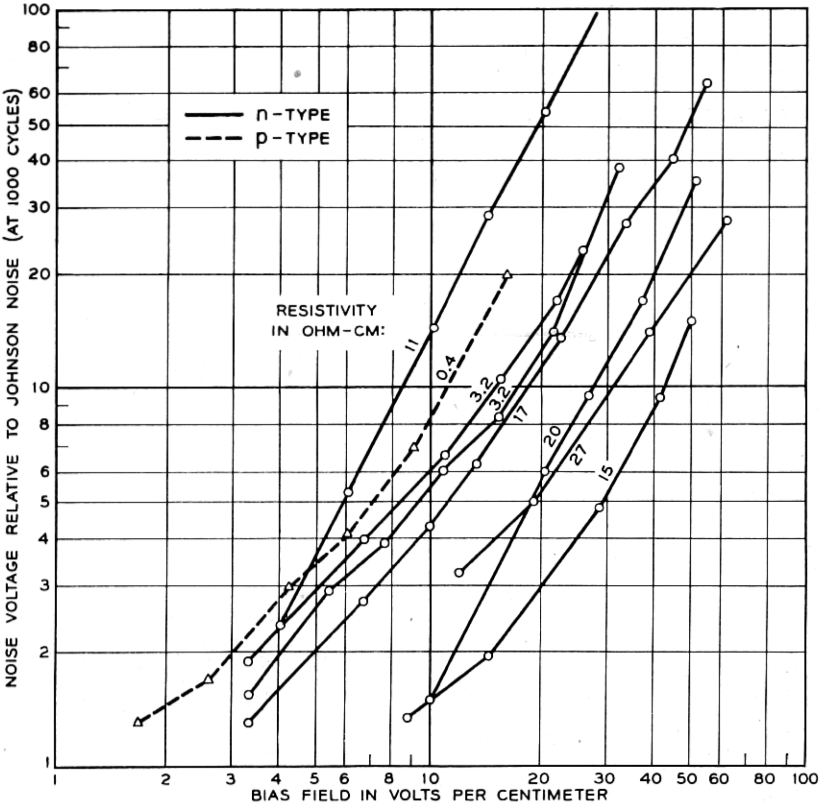

Fig. 6—Variation of noise with dc bias in single crystal filaments.

on the noise. Some of these processes also affect the lifetime of carriers in the bridge to a large extent. However, there seems to be no direct and simple relation between the two effects, since treatments have been found which change the noise by a large factor with almost no effect on lifetime, and vice versa.

Fig. 8 shows measurements of noise voltage on several dozen bridges at a uniform bias of 10 volts per centimeter, all having sandblasted surfaces, mostly of *n*-type but a few of *p*-type germanium, and with widely different values of resistivity, produced by varying impurity concentrations. There is considerable scatter in the results, but there is a fairly obvious tendency for noise voltage to increase in proportion to resistivity. Since Johnson noise also increases in proportion to resistivity in a structure of fixed dimensions, the conclusion is that with constant bias voltage the ratio of current induced noise to Johnson noise tends to be

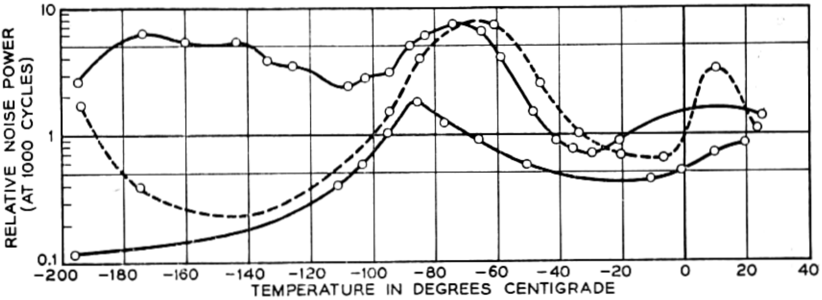

Fig. 7—Variation of noise with temperature in single crystal filaments.

independent of the resistivity of the material. From the data it also appears that there is no consistent difference between n- and p-type material.

Noise does not appear to depend on orientation of the filament with respect to the crystal axes. Filaments orientated along the 100, 110, and 111 directions and rotated in several ways about these directions showed

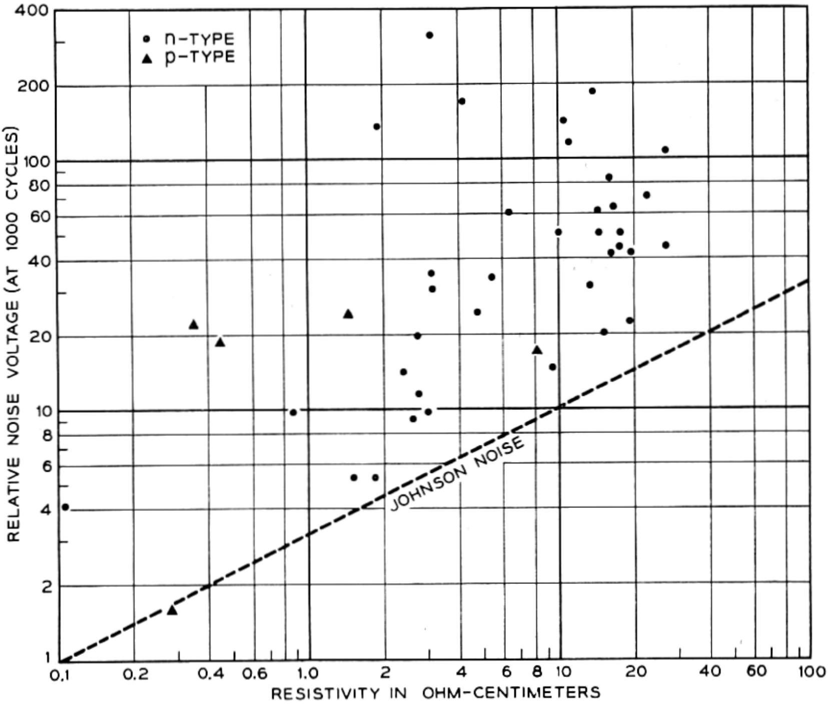

Fig. 8—Variation of noise with resistivity in single crystal filaments.

no significant differences in noise behavior. It should be noted, however, that small variations might be hidden in the large scatter in the data from undetermined causes.

IV. NOISE AND MAGNETIC FIELDS

An important role for the minority carrier in the noise mechanism was first clearly indicated in experiments on the effect of a magnetic field on noise in germanium filaments. It has been found experimentally that the noise in a single crystal filament may change by a substantial factor when the filament is subjected to a steady transverse magnetic field. The following discussion will show that this behavior is in harmony with the hypothesis of noisy injection of minority carriers, as set forth in a preceding section.*

The physical picture on which this treatment is based involves the random injection of holes into an n-type filament by hole sources which may be either in the interior or on the surface of the filament.† It is assumed that the spectrum of the noise arises from the fluctuating nature of the noise source. The effect which any source has will depend upon the lifetime of the holes which it emits. If these holes remain in the filament for a long time, they will produce more noise than if they remain in the filament for a short time. We shall be concerned with the effect of magnetic fields upon these lengths of time and shall not deal in this paper with the fluctuations of the noise sources themselves. If a transverse magnetic field is applied to an n-type germanium filament, a Hall effect voltage is set up and the holes will be deflected towards one surface of the filament. Since recombination takes place principally at the surfaces, this may cause a substantial change in the lifetime of the holes. In order to determine the effect of the magnetic field on the noise we proceed along the following lines.

(a) We assume that the observed noise is due to fluctuations in the conductivity of the filament produced by fluctuations in the hole concentration. Since these fluctuations are small, we may take the change in conducitivity to be proportional to the change in average hole den-

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* The following semi-quantitative theory of the dependence of noise on magnetic field is taken with some modification from unpublished work of W. Shockley and H. Suhl, on the basis of which the calculations leading to the curves of Figs. 10 and 11 were carried out. It is hoped that this work may be published in the near future.

† To simplify the terminology, the discussion is based on n-type material with holes as minority carrier. An exactly similar argument could be made for p-type material with electrons as the minority carrier. There is some experimental evidence of the similarity of behavior of n- and p-type germanium, though most of the experimental work has been done with n-type.

sity. (b) We restrict the noise measurements to frequencies low enough so that the period is long compared to the lifetime of a hole. It is then evident that the contribution of a hole source to the noise is proportional to the fluctuating hole current generated by the source and to the average lifetime of the holes. This lifetime depends on the position of the source in the filament, the absorption properties of the surfaces and the electric and magnetic fields.* (c) We assume that the generation properties of the sources are unaffected by the magnetic field, hence, the calculation of the effect of the field on the noise reduces to a problem of calculating the change in lifetime produced by the field. (d) We neglect body recombination in comparison with surface recombination. In germanium filaments of the size usually dealt with, this approximation causes only a small error in the lifetime. (e) Individual sources (or at any rate groups of sources over regions small compared to the dimensions of the filament) will be considered to be statistically independent; therefore, the total effect on the noise can be determined by summing the squares of the contributions from individual sources. Hence we wish to evaluate the following expression:

$$\text{Change in noise power at field } H = \langle \tau^2(H) \rangle / \langle \tau^2(0) \rangle \tag{1}$$

where the symbol $\langle\rangle$ indicates an average over all the noise sources. The statments (a) to (e) represent the principal assumptions in developing the theory.

In order to calculate τ as a function of the magnetic field, H, we consider a steady state case in which a current of holes J_0 is injected into a region in which the average lifetime is τ. If the density of holes in the region is $p(x, y, z)$, the total number is

$$P = \int p(x, y, z)\, dx\, dy\, dz.$$

However, $P = J_0\tau/q$, where q is the charge carried by a hole. Therefore,

$$\tau = \frac{q}{J_0} \int p(x, y, z)\, dx\, dy\, dz \tag{2}$$

This is the method of evaluating τ which is used in the qualitative discussion which follows, and also in the calculation of the curves of Figs. 10 and 11.

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* It should be pointed out that a consequence of the hole injection theory of noise in a filament is that marked frequency dispersion should occur when the frequency being studied is high enough so that a period is short compared to the lifetime of holes in the filament. However, we shall neglect this important and interesting aspect of the problem.

Three cases will be treated. In all of these it will be supposed that the width of the filament parallel to the magnetic field is relatively large, so that effects from the edges can be neglected. Also, we are concerned only with average effects over the long dimension of the filament. This permits us to deal with a one-dimensional problem. We shall consider first the case in which holes are supposed to be injected from the surfaces, and the two surfaces have equal and rather large recombination rates. In Fig. 9, part (a) shows how holes injected from each surface are distributed across the thickness in the absence of a magnetic field, and part (b) shows the distribution with a moderate field. The form of these distributions may be determined from the following arguments.

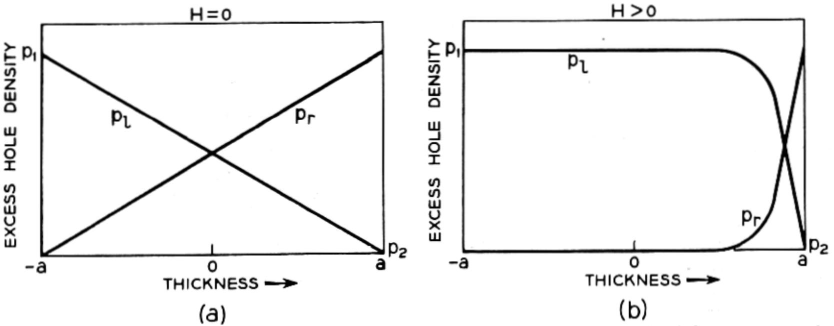

Fig. 9—Excess hole density across the thickness dimension, (a) with no magnetic field, (b) with moderate magnetic field.

If we suppose a steady hole current J_0 emitted from the left-hand surface of the filament, then a relatively high concentration p_1 of holes will appear directly in front of the surface. Some of these holes will recombine upon the surface, the rate J_1 being given by

$$J_1 = p_1 S q$$

where S is the recombination constant for the surface. The balance of the holes will diffuse through the filament to recombine upon the right surface at a rate

$$J_2 = p_2 S q$$

and we note that $J_1 + J_2 = J$. Because of the high recombination rate, p_2 will be very small; hence, J_2 will be much smaller than J_1. In the absence of a magnetic field the gradient is uniform, and the concentrations will be linear, as shown in part (a) of the figure. An identical argument applies to p_r, the concentration of holes emitted from the right-hand side.

Under the influence of a magnetic field pushing holes toward the right, the concentrations will change to those shown in part (b) of the figure. The magnetic field will pull holes through the filament and tend to prevent diffusion from right to left. For some moderate value of field, the value of J_2 is not increased enough to change J_1 appreciably, so the value of p_1 is nearly the same as with no field. At the same time the effects of diffusion are suppressed by the field so that the concentration p_1 extends nearly to the right side of the figure. By the same action, the concentration of holes emitted from the right surface drops to zero very quickly.

From the curves of Fig. 2 and relation (2), we see that the area under the density curve, and hence the lifetime of holes injected at the left is at most doubled by the magnetic field, while the lifetime of those injected at the right is reduced nearly to zero. Recalling that the noise is proportional to a summation of the square of the lifetimes, we see that the noise power is at most doubled at a suitable value of magnetic field.

Higher values of field will sweep so many holes to the right-hand surface as to substantially reduce p_1, so at very high fields the noise decreases monotonically to zero.

Thus it is seen that the noise behavior is the result of competing tendencies. On the one hand, the magnetic field helps holes escape from the surface at which they are emitted, but on the other hand it tends to push these holes against the opposite surface and thereby reduce their lifetime. The relative importance of those two tendencies depends on the surface recombination properties and the strength of the magnetic field.

Calculation of the lifetime along the lines just discussed involves solution of the continuity equation

$$D \frac{d^2p}{dx^2} - \mu_p E_H \frac{dp}{dx} = 0$$

with suitable boundary conditions. The results of such a calculation carried out by Shockley and Suhl in the work already referred to are plotted in Fig. 10.

In order to make the results independent of sample dimensions, the following parameters are used. The first parameter is proportional to the applied magnetic field, and is defined as the effective transverse potential in units of kT/q:

$$\Phi = \frac{tE_H}{kT/q} = 172tEH \times 10^{-5} \tag{3}$$

where t = thickness of the filament (cm)

H = magnetic field (oersteds)

E = applied electric field (volts per cm)

E_H = effective transverse component due to Hall effect (volts per cm)

q = unit electronic charge

kT = Boltzman's constant × absolute temperature.

The constant may be derived by noting that kT/q is 1/40 volt at room temperature, and that the effective transverse field, E_H, may be expressed as follows. (See Reference 14, Section 8.8.)

$$\begin{aligned} E_H &= \theta E = (\theta_n + \theta_p)E \\ &= (\mu_n + \mu_p)HE \times 10^{-8} \\ &= 4.3 \times 10^{-5}HE \end{aligned}$$

where θ = Hall angle

μ_n = Hall mobility for electrons (2800 cm^2/volt-sec)

μ_p = Hall mobility for holes (1500 cm^2/volt-sec).

The other dimensionless parameter is proportional to the rate of surface recombination, and is defined as the ratio of the surface recombination velocity to the diffusion velocity from the center:

$$\psi = st/2D = st/86$$

where s = recombination velocity characteristic of the surface (cm/sec)

D = diffusion constant (cm^2/sec).

The numerical constant is given for holes at room temperature. The noise changes are expressed in decibles, that is, ten times the common logarithm of the ratio of noise powers with and without the magnetic field.

A second case is that in which generation and recombination are on the surfaces, but the two surfaces have unequal absorption properties. It might be expected that rather large increases in noise would result when the magnetic field was poled to pull holes away from the surface with high absorption properties, and this turns out to be the case when the calculations are carried out. The results are shown in Fig. 11 for a

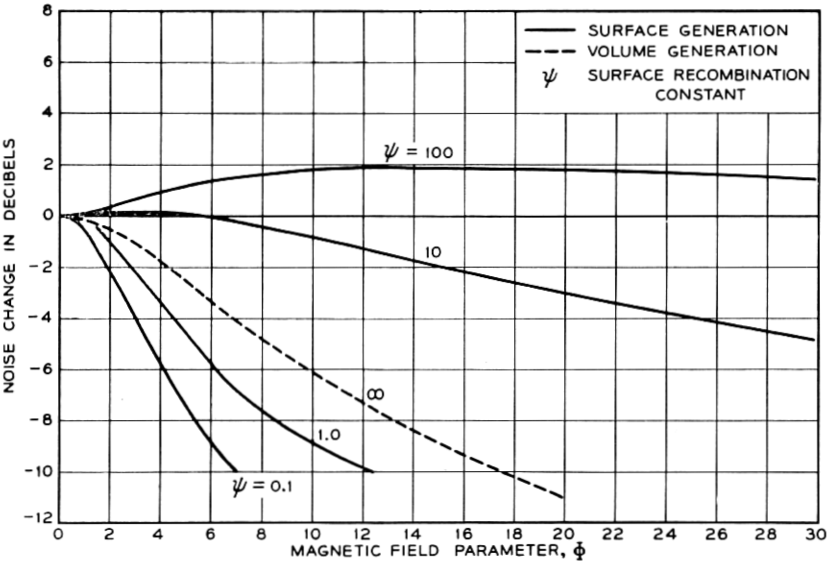

Fig. 10—Calculated magnetic effect for similar surfaces.

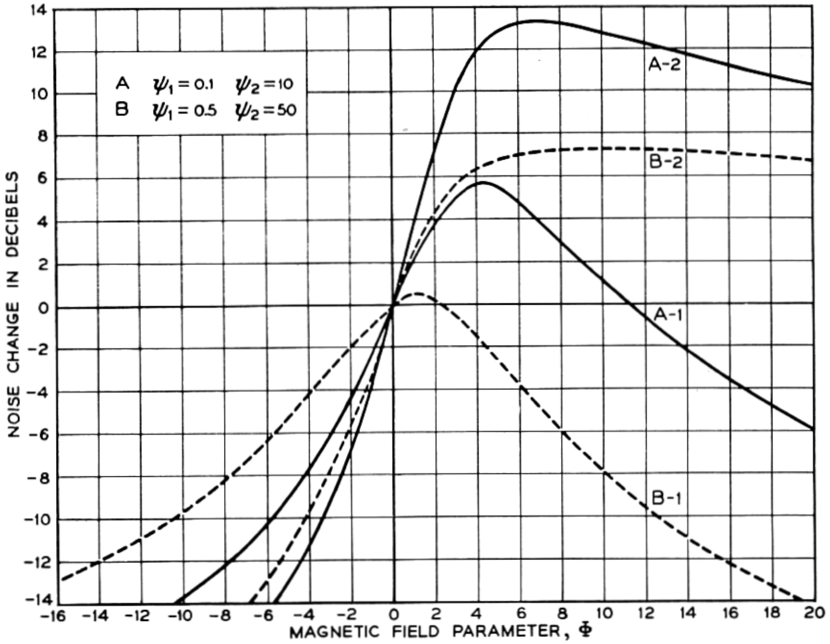

Fig. 11—Calculated magnetic effect for dissimilar surfaces. Curve 1 of each pair is for the contribution from the surface having the lower recombination constant.

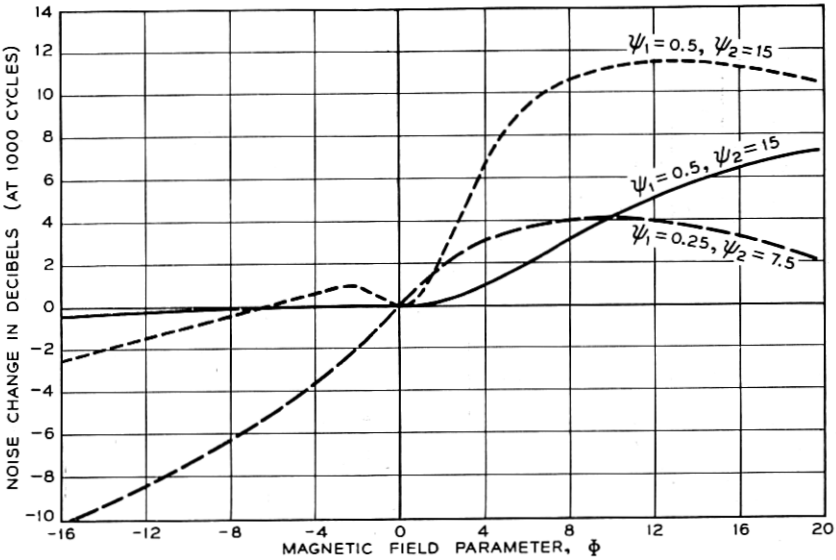

Fig. 12—Experimental magnetic effect for dissimilar surfaces.

case where the two recombination parameters are 0.1 and 10, and for a second case where the parameters are 0.5 and 50. In this figure, separate curves have been shown for noise due to holes generated on each of the two surfaces. The total noise would be gotten by adding the noise powers represented by the two curves after appropriate weighting for the contributions of the two surfaces. At present we do not see any way of determining the weighting factor.

A third case is that in which it is assumed that the noisy generation of holes is uniform throughout the body of the filament, but that recombination takes place on the surfaces only. These assumptions seem at first sight to be in contradiction to the statistical mechanical principle of detailed balancing, which states that under equilibrium conditions all processes occur with equal frequency in the forward and reverse directions. Thus it would seem that if holes are generated in the interior, we must consider recombination in the interior also. Actually this is not necessary under the non-equilibrium conditions which prevail during noise measurements. There is no necessity for the noise generated by a source and a sink for holes to be simply related to the strength of this source. Thus we may suppose there are relatively weak sources and sinks for holes in the interior, but that the hole absorption and generation of the sources is very noisy compared to the recombination and generation processes on the surfaces. If this is the state of affairs, most of the noise will be generated in the interior, but a hole generated in the interior will be much more likely to recombine on the surface. The dotted curve of Fig. 10 has been calculated assuming a uniform distribution of noise sources throughout the interior of the filament and equal and very large recombination constants for the two surfaces. It is seen that for this case the reduction of lifetime predominates, and there is a monotonic decrease in noise with increasing magnetic field.

Experimental work has given results which in most cases are in fair qualitative agreement with the calculated relations. Measurements for three filaments, each of which had one high recombination and one low recombination surface, are shown in Fig. 12. The recombination parameters, as shown on the curves, were of the order of $\psi = 10$ for one surface, and $\psi = 0.5$ for the other. The general shape of the curves is quite similar to the calculated curves of Fig. 11. The maxima are of the right order of magnitude, and occur at reasonable values of the field parameter Φ. The lack of detailed agreement between the measured and calculated curves is not surprising, because the experimental conditions did not fulfill the assumptions made for the calculations in several respects. The filaments were neither wide enough nor long enough so that edge and end effects could be overlooked. The recombination properties of the surfaces could not be measured directly, but had to be estimated from other filaments which had been similarly treated. One experimental curve shows a secondary maximum on the opposite side of the origin. This might indicate a defective portion of one surface having an anomalous recombination constant.

Experimental results are shown in Fig. 13 for four filaments, each of which had nominally equal recombination constants for the two surfaces. These may be compared with the calculated curves of Fig. 10. It will be noted that the experimental curves are not symmetrical about $\Phi = 0$. This lack of symmetry is probably due to dissymmetry in the

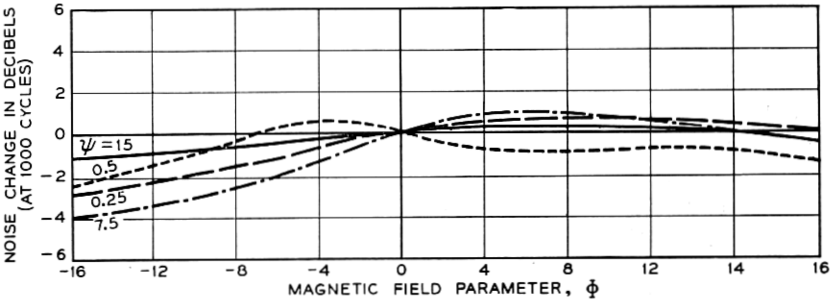

Fig. 13—Experimental magnetic effect for similar surfaces.

samples, particularly the fact that one surface of each filament was cemented to a support, which would probably change the surface recombination properties somewhat. Aside from the lack of symmetry, the behavior of the two filaments with the higher recombination constants is in reasonable agreement with the calculated curves. The filaments with the lower recombination constants are in poor agreement with calculated values, in that the noise does not fall off with increasing field nearly as fast as calculated. The cause of this behavior is not understood. These experimental curves may also be compared with the dotted curve of Fig. 10, calculated on the assumption of volume generation and surface recombination. The similarity is quite poor in all cases. The somewhat better agreement with the surface generation calculations than with the volume generation calculations is not the basis for anything more than a very tentative feeling that the experimental results support the surface generation viewpoint.

While there are many discrepancies in detail between the experimental and calculated relations between noise and magnetic field, these are at least partially understandable in terms of the differences between the experimental setup and the theoretical model. The high degree of qualitative agreement considerably strengthens the hypothesis of noisy injection of minority carriers as an important element in the noise process.

V. NOISE CORRELATION PHENOMENA

The noisy hole injection hypothesis leads one to expect certain correlation phenomena in the noise voltage observed in neighboring portions of a filament. Consider first noise measurements at a frequency so low that the transit time of a hole* along the filament is negligibly small. This might be a frequency of one kilocycle in a typical experiment. The holes have an average lifetime, from which can be determined an average life path, which is defined as the product of the lifetime by the drift velocity under the existing electric field. Noise voltage measurements across segments of the filament much shorter than a life path should be highly correlated, since nearly all the holes which make a transit of one segment will make an almost simultaneous transit of the other segment. On the other hand, noise voltages across segments much longer than a life path should show little correlation, because most of the holes appearing in the two segments are from different sources, and the sources have been assumed to be statistically independent.

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* As before, the concepts apply equally well to electrons in p-type material.

A second situation arises when noise is measured at frequencies high enough so that the transit time of holes between segments is not negligible. In this case we should expect the correlation between the noise voltages to be improved by incorporating in one channel of the measuring circuit a delay equal to the transit time between segments.

In order to calculate the correlation resulting from the first situation, we set up a theoretical model based on a few simplifying assumptions: (a) The noise process may be represented by an array of noisy hole current generators which are statistically independent; (b) These generators are uniformly distributed along the filament over the segments where the noise is to be observed, and for a sufficient distance on either side to produce uniform conditions over the segments; (c) The hole currents from the generators decay exponentially with a decay constant determinable from the lifetime; (d) Measurements are made at low enough frequencies so that time of transit of holes may be neglected. We will consider later an alternative to the second assumption. The correlation coefficient between two voltages of instantaneous values v_1 and v_2 may be defined as

$$\rho_{12} = \overline{v_1 v_2}/(\overline{v_1^2} \times \overline{v_2^2})^{1/2}$$

where the bars represent time averages. To evaluate this expression, the contribution of a single generator to the noise voltage in each segment is determined by integrating over the appropriate portion of the decay curve. The total contribution from all generators to the mean voltage product and the mean squared voltages is then determined by integrating the product or square over all the generators. The details are carried out in the appendix, and lead to the solid curve of Figs. 14–16, in which the ordinates are the correlation between noise voltages in two segments of a filament and the abscissae are the ratio of life path of a hole to the segment length.

In an experiment the lifetime τ of holes remains fixed, determined chiefly by the recombination properties of the surface. Consequently the life path ℓ is proportional to the hole velocity, which is determined by the electric field, according to the relation

$$\ell = \tau \mu E$$

where E is the applied field in volts per centimeter and μ is the drift mobility of holes. Hence, by varying the biasing voltage a large range of life path values can be obtained.

The correlation is measured by carrying the noise voltages through separate amplifying channels having identical pass bands extending from 800 to 1300 cycles per second. A switching arrangement makes it possible to apply either of the output voltages or their sum or difference to a rectifier-meter combination. From the readings of the meter the correlation can be computed according to the relation

$$\rho_{12} = (S^2 - D^2)/4V_1V_2 . \tag{6}$$

V_1 and V_2 are rms values of the individual noise voltages, and S and D are the rms values of their sum and difference. The equivalence to expression (5) can be seen by noting that

$$S^2 - D^2 = \overline{(v_1 + v_2)^2} - \overline{(v_1 - v_2)^2} = 4\overline{v_1 v_2} .$$

Results of correlation measurements on three bridges are shown in Figs. 14–16. In each case the calculated curve is shown for reference. The values of ℓ were calculated from decay measurements on optically injected holes, as described by J. R. Haynes,[11] using a value for mobility of 1700 cm^2/volt-sec. In Fig. 14 the agreement with the theoretical model is very good. The scatter in the points is due to fluctuations in the noise, which are quite large in the band used for these measurements. In Fig. 15 the agreement could be made quite good with a lateral shift

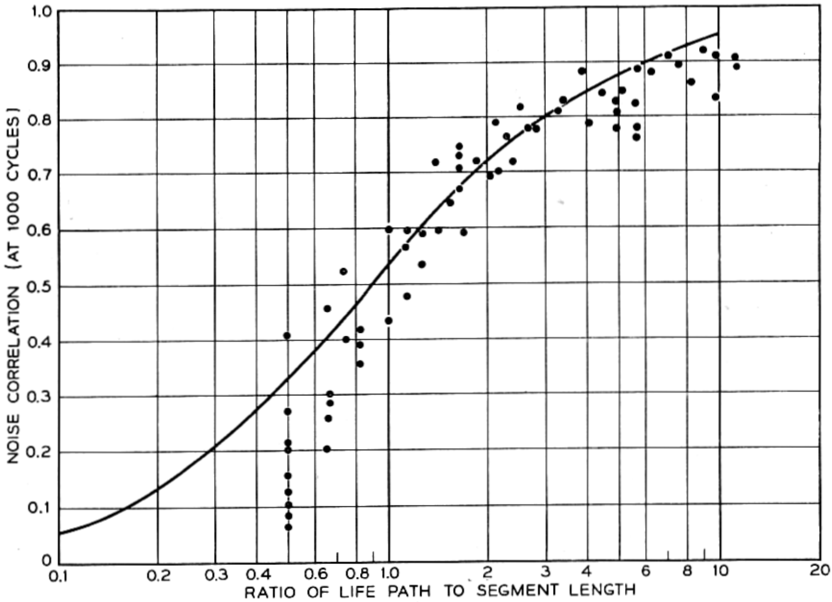

Fig. 14—Noise correlation. The solid curve is calculated, the points experimental.

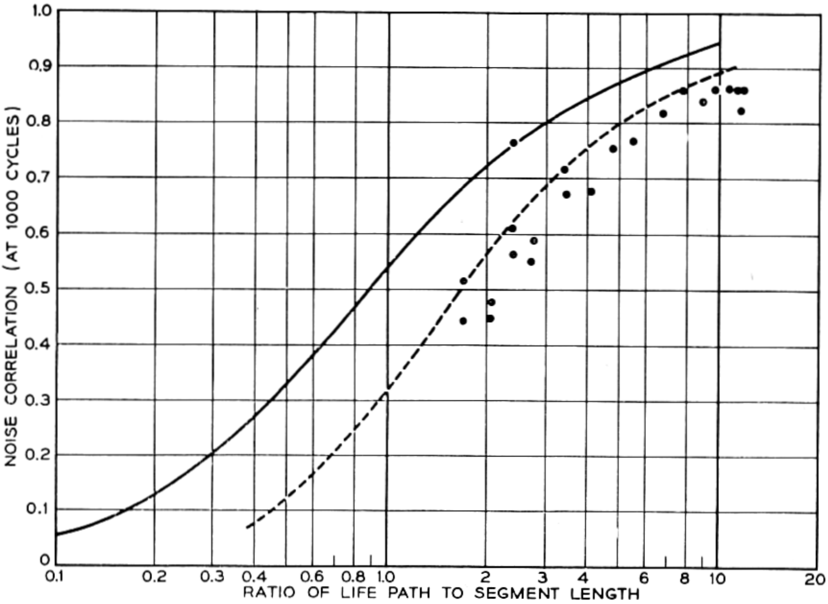

Fig. 15—Noise correlation. The dotted curve includes allowance for losses at the side arms.

by a factor of two. In Fig. 16 the form of the experimental curve seems different from that calculated. In particular, the slope is steeper, and the curve tends to level off at a correlation of about 0.8. It seems possible to explain the discrepancies between the experimental data and the calculations on the basis of two considerations which were not included in the model. (a) The pair of side arms separating the two segments of the filament serve to drain off some holes which would otherwise contribute to the correlation. The dashed curve in Fig. 15 shows the calculated effect, on the assumption that the absorption in the side arms is equivalent to an extra section of filament equal in length to half a segment. The actual distance across the side arms is only 20 per cent of a segment, but it is not hard to believe that the decay rate in this region might increase by a factor of two or three due to the reduced electric field and loss of holes down the side arms. (b) The model assumed a uniform distribution of noise sources along the filament. There is experimental evidence that the distribution may be quite spotty. This can have a substantial effect on the form of the correlation curve. For example, the dashed curve in Fig. 16 shows the curve calculated for noise sources lumped at the mid-point of each segment. Other assumed positions might shift the curve considerably along the horizontal axis.

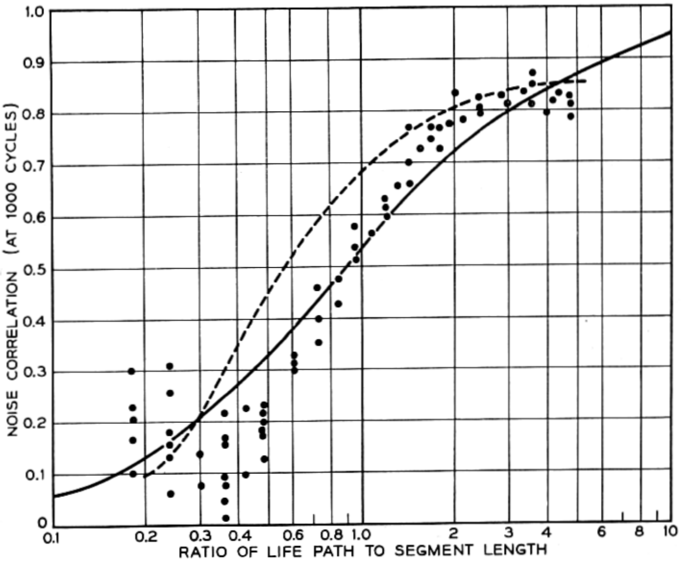

Fig. 16—Noise correlation. The dotted curve is calculated for lumped noise sources.

In view of these considerations there seems to be very satisfactory agreement between the experimental results and the model.

Another type of experiment involves noise measurements at frequencies high enough so that the transit time of a hole across a segment is an appreciable fraction of a cycle. In this case the correlation between noise voltages from adjacent segments can be improved by putting a time delay in one channel of the measuring circuit. Measurements were made by taking the noise voltages from the two segments through separate amplifying channels having identical pass bands extending from 17 to 24 kilocyles. The outputs of the two channels were put on the vertical and horizontal plates of a cathode ray oscilloscope, forming a sort of Lissajous pattern. The patterns differ from those obtained with sinusoidal voltages in that the elliptical figures are filled in solid, due to the continual variation in amplitude of the noise. A phase shifting device is included in one channel, and as the phase is shifted to give optimum correlation, the elliptical pattern narrows down and approaches a line inclined at 45°. For a quadrature phase shift, the pattern becomes circular, and in practice this setting can be determined more precisely than the in-phase setting, largely because the background noise in the circuit is less troublesome. With the phase shift for optimum correlation determined, the delay at the center of the pass band is easily calculated, and since the band is not very wide, the variation in delay over the band is not important. From the drift mobility of holes we may estimate the transit time between segments, according to the relation

$$t = L/\mu E$$

where t = transit time, seconds

L = distance between segment mid-points, cm

E = applied field, volts/cm

μ = mobility of holes, cm^2/volt-sec.

Data for a bridge of n-type germanium of resistivity about 20 ohm-cm are given in Table I. The transit distance, L, after a small correction for reduced field across the side arm, was taken as 0.305 cm. As noted in the table, the bridge temperature rose somewhat at the higher bias values, and the assumed values of mobility have been modified according to the inverse three-halves power of the absolute temperature. The delay required for optimum correlation is shown in the second column of the table, and the calculated transit time between segments in the last column. It is seen that the two are in reasonably good agreement, especially at low fields. When the direction of the field is reversed, an equal delay is required, but in the opposite channel of the measuring circuit, as would be expected. Here, again, we have experimental evidence supporting the noisy hole injection hypothesis. The cause of the discrepancy shown in the table at higher fields is not understood. It is possible that trapping phenomena increase the transit time over that calculated from the mobility. There is some evidence for this sort of behavior in lifetime experiments, but to date there does not seem to be enough information for any estimate of magnitude of such an effect.

TABLE I

E volt/cm	Delay micro sec.	Bridge Temp. °K.	Mobility cm^2/volt-sec.	Transit Time micro sec.
10	21.1	298	1700	18.0
14	15.7	299	1690	12.9
20	11.8	301	1670	9.1
30	9.2	305	1640	6.2
40	9.3	313	1580	4.8

VI. GENERAL COMMENTS

These studies of electrical noise in semiconductors leave little doubt that the noise is closely related to the behavior of the minority carriers.

It is not yet clear whether the noise is a surface or a volume property of the material, but it is well established that the surface properties have an important connection with the magnitude of the noise. From some of the experimental work it seems likely that the generation and recombination processes are separate and have different noise properties. Because of the nonequilibrium situation, this does not violate the principle of detailed balancing. It seems probable that a more complete understanding of the generation and recombination processes and a clearer picture of the origin of noise in semiconductors may be expected to develop together.

VII. ACKNOWLEDGEMENT

The analysis leading to the theoretical relations between noise and magnetic field is the work of W. Shockley and H. Suhl, under whose direction the calculations leading to the curves of Figs. 10–11 were carried out. The continued interest of Dr. Shockley in the experimental work has been invaluable. The author is indebted to many associates for helpful discussion of certain problems, and also for the construction of many of the devices and materials which entered into the experimental work.

APPENDIX

Suppose that a source of holes located at a point x_0 in a filament produces a fluctuating current of holes of rms value J_1 in a specified frequency band. The hole current is swept down the filament by a field E and is assumed to decay exponentially according to the relation

$$J = J_1 e^{-(x-x_0)/\ell} \tag{1}$$

where the life path ℓ may be expressed in terms of drift velocity v, hole mobility μ, and lifetime τ

$$\ell = v\tau = \mu E \tau.$$

Assuming that the frequency of measurement is low enough to justify neglecting the hole transit time, the noise voltage due to holes from a single source is proportional to the number of holes present in the segment. This is obtained by integrating (1) over an appropriate range

$$\begin{aligned} dv &= J_1 \int_a^b e^{-(x-x_0)/\ell}\, dx \\ &= \begin{cases} K_1 e^{x_0/\ell}[e^{-a/\ell} - e^{-b/\ell}] & x_0 < a \\ K_1[1 - e^{-(b-x_0)/\ell}] & a < x_0 < b \end{cases} \end{aligned} \tag{2}$$

where K_1 is an omnibus constant which will cancel out in the final result.

Under the assumption that the sources are statistically independent, the total voltage squared is obtained by integrating the square of (2) over all the sources.

$$\begin{aligned}\overline{v_1^2} = \overline{v_2^2} &= K_1 \int_{-\infty}^{a} e^{2x_0/\ell}[e^{-a/\ell} - e^{-(a+L)/\ell}]^2\,dx_0 \\ &\quad + K_1 \int_{a}^{a+L} [1 - e^{-(a+L-x_0)/\ell}]^2\,dx_0 \\ &= K_2\left[1 - \frac{\ell}{L} + \frac{\ell}{L}\,e^{-L/\ell}\right].\end{aligned}$$

Similarly, the cross product of voltages in two segments, extending say from 0 to L and L to $2L$, is

$$\begin{aligned}\overline{v_1v_2} &= K_1 \int_{-\infty}^{0} e^{2x_0/\ell}[1 - e^{-L/\ell}][e^{-L/\ell} - e^{-2L/\ell}]\,dx_0 \\ &\quad + K_1 \int_{0}^{L} e^{x_0/\ell}[1 - e^{-(L-x_0)/\ell}][e^{-L/\ell} - e^{-2L/\ell}]\,dx_0 \\ &= K_2\,\frac{\ell}{2L}\,[1 - e^{-L/\ell}]^2.\end{aligned}$$

From the definition of the correlation coefficient

$$\rho_{12} = \overline{v_1v_2}/(\overline{v_1^2} \times \overline{v_2^2})^{1/2} = \frac{\ell}{2L}\,\frac{(1 - e^{-L/\ell})^2}{1 - \dfrac{\ell}{L}\,(1 - e^{-L/\ell})}$$

which is the desired relation, from which the solid curves of Figs. 14–16 were calculated.

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