New Techniques for Measuring Forces and Wear in Telephone Switching Apparatus

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One of the main problems in obtaining long life in telephone switching equipment is the wear caused by large momentary forces. In order to investigate this problem several new techniques have been devised for measuring normal and tangential forces and for producing and controlling normal and tangential motions for wear studies. The forces are measured by inserting small barium titanate ceramics between the points of application of the forces and observing the voltages generated on a cathode ray oscillograph. Barium titanate ceramic is about fifty times as sensitive as quartz and has a high enough dielectric constant so that with conventional amplifiers time intervals as long as a tenth second can be measured. Both normal and tangential forces can be measured by using properly poled ceramics. By using weights on top of the crystals, normal and tangential accelerations can be measured. With these ceramics, forces have been measured for relays and for frictional sliding of a wire over a plastic. By employing a barium titanate transducer capable of a large amplitude at 18,000 cycles it has been shown that no wear occurs for normal forces, and that all the wear observed in a relay is due to tangential sliding. Quantitative measurements of wear have been made for a variety of materials, and it has been shown that materials with a large elastic strain limit will wear better than materials with a small elastic strain limit even though the latter have a higher yield stress; materials such as plastics and rubber will outwear materials such as metals or glasses.

As the length of slide is reduced there is a threshold of motion for which there is no gross slide and very little wear. This region is determined by the condition that the tangential force is smaller than the normal force times the coefficient of friction. Theoretical and experimental results are obtained for this region and an equation is derived which determines the possible displacement without gross slide. The stress strain curve occurs in the *form of a hysteresis loop whose area varies approximately in proportion to the square of the strain amplitude. This region is important for relays for by introducing damping, long repeated vibrations—which are responsible for considerable wear—are quickly brought down to the low wear, no gross slide region with a corresponding reduction in wear. The mechanical resistance associated with the stress strain loop is of the same type that occurs in an assemblage of granular particles such as in a telephone transmitter where the motion is small enough so that no gross slide occurs.*

I. INTRODUCTION

In obtaining long life in telephone equipment such as relays, switches, selectors and other mechanical devices subject to large momentary forces, one of the main problems is the wear encountered in various parts. This is particularly true in such small motion devices as relays where even a few mil inches of wear increases the distance that the armature has to travel and may eventually cause the relay to fail to make contact. To obtain a design objective of one billion operations requires a very careful minimizing of deleterious forces and a careful selection of the best wearing materials.

As a step toward investigating this problem several new techniques have been devised for measuring normal and tangential forces and for producing and controlling normal and tangential motions for wear studies. These methods have been applied to relays and have given considerable information on the types of motion to be avoided and on the best types of materials to select for various parts of the relay to obtain long life. Specifically they have shown that normal forces cause very little wear and that tangential sliding of one part over another is the principal cause of wear. Fortunately, by designing the motion of the armature and contacts correctly, tangential sliding can be largely eliminated with a corresponding reduction in wear.

To aid in the quantitative evaluation of wear produced by tangential sliding two devices have been used. One is an electromechanical vibrator[1] driven at 500 cycles per second which is capable of several mil inches of motion and the other is a barium titanate longitudinal vibrator coupled to a metal "horn"[2] which is capable of a two mil inch motion at 18,000 cycles. Wires connected to these transducers are dragged over materials whose wearing properties are to be tested. The normal forces between the wire and material are varied as well as the length of the stroke. The wear by both methods is comparable showing that the accelerated wear testing method gives about the same wear as the slower method. With the barium titanate transducer a billion cycles can be obtained in 17 hours and a very rapid wear test is obtained.

If lubrication is not used, wear tests show that materials having a large elastic strain limit will in general wear better than materials which have a smaller elastic strain limit even though the latter may have a higher yield stress; materials such as plastics and rubbers will outwear materials such as metals and glasses. The volume of wear for one billion operations is porportional to the product of static force times the length of the stroke. The initial rate of wear is several times as large as the final rate. Calculations show that only about one part in 10^9 of the energy goes into producing wear, the rest going into heat production.

As the length of slide is reduced, calculations and measurements show that there is a threshold of motion for which no gross slide occurs. This condition occurs when the tangential force is less than the product of the normal force times the coefficient of friction. The limiting displacement for no slide increases as the two-thirds power of the normal load and inversely as the two-thirds power of the shear stiffness. Hence a heavily loaded material with a small shear elastic constant—such as rubber—will have a large displacement for which no slide occurs, and hence will wear considerably better than a stiff material such as a metal. Wear tests in the region of no gross slide show that the rate of wear is considerably less in proportion to the energy dissipated than in regions of gross slide.

A quantitative experimental and theoretical study of the region of no gross slide has been made.[3] Experimentally the results have been obtained by moving a glass lens with a large radius of curvature on both surfaces between two glass lenses when the lenses are pressed together with known normal forces. It was shown theoretically that slip should occur between these lenses over a circular annulus and experiments verify this prediction quantitatively. Force-displacement curves have been measured and it has been shown that the relation is a hysteresis type loop whose area varies approximately as the square of the strain amplitude. The small wear observed is related to the wear found in ball bearings, where no gross slide occurs. This region is important in relays for by introducing damping, long repeated vibrations—which are responsible for considerable wear—are quickly brought down to the low wear, no gross slide region with a corresponding reduction in wear. The mechanical resistance associated with the stress strain hysteresis curve is of the same type that occurs in an assemblage of granular particles such as in a telephone transmitter, where the motion is small enough so that no gross slide occurs.

II. METHODS FOR MEASURING NORMAL AND TANGENTIAL FORCES

In order to investigate the performance of a mechanical device and the causes of wear in it, it is desirable to be able to measure the forces occurring in various parts of the device. To measure the complete performance it is necessary to measure not only the slowly applied forces but also the very short time dynamic forces that occur when various parts of the device impinge on each other.

The most common method for measuring such forces is by means of a piezoelectric crystal such as quartz. Quartz, however, has the disadvantage that it is not very sensitive and also that it has such a low dielectric constant that the input impedance of any amplifier associated with it has to be prohibitively high if forces varying as slowly as a one-tenth of a second are to be measured. Since the impedance of the oscillograph or amplifier is usually lower than that of the crystal, the crystal having the greatest sensitivity will be the one which generates the most charge for a given force, which corresponds to the crystal having the largest d piezoelectric constant. Table I shows a tabulation of the d constants for compression and shear for several of the most common piezoelectric crystals and for the ceramic barium titanate. The dielectric constants are also given.

Of these materials the only ones that have sufficient mechanical strength to withstand the mechanical shocks they are subjected to in the measurements of forces are quartz, tourmaline and barium titanate ceramic. The crushing strength of the ceramic has been found[4] to be from 60,000 to 80,000 pounds per square inch. From the values of the d piezoelectric constants it is seen that the barium titanate ceramic is about 50 times as sensitive as quartz or tourmaline and it is possible to use small pieces of the ceramic to work directly into cathode ray oscillo-

TABLE I

Crystal	Compression Constant in cgs units	Shear Constant in cgs units	Dielectric Constant
Quartz	$d_{11} = 6.76 \times 10^{-8}$ stat. coulombs/dyne	$d_{26} = 13.5 \times 10^{-8}$ stat. coulombs/dyne	4.55
Tourmaline	$d_{33} = 5.5 \times 10^{-8}$	$d_{15} = 10.9 \times 10^{-8}$	8.0
ADP	$d_{31}' = 74 \times 10^{-8}$	$d_{36} = 148 \times 10^{-8}$	15.6
Rochelle Salt Y Cut	$d_{21}' = 84.5 \times 10^{-8}$	$d_{25} = 169 \times 10^{-8}$	11.1
EDT	$d_{21} = 34 \times 10^{-8}$	$d_{36} = 50 \times 10^{-8}$	8.0
Barium Titanate Ceramic	$d_{33} = 300$ to 400×10^{-8}	$d_{16} = 500$ to 650×10^{-8}	900 to 1500

graphs with the use of only the amplifiers that are included with such oscillographs. To work down to time intervals in the order of one-tenth second, the leakage resistance of the load across the polarized ceramic—which for small sizes may have a capacitance as low as 20-micro-micro-farads—has to be higher than is usually available in oscillographs. Fig. 1 shows a vacuum tube circuit[5] capable of giving a 750-megohm input resistance and when used with a barium titanate ceramic having a capacity of 20 $\mu\mu$f, allows measurements of forces for time intervals up to 0.015 seconds with no corrections. This time is usually sufficient to obtain all the force variations in a relay operation. The upper frequency limitation in the measurements of forces is caused by the setting up of natural vibrations in the ceramic block. The lowest frequency vibrations that can be set up in a ceramic block are the flexural vibrations. For a block 0.04 inch x 0.04 inch in cross section and 0.02 inch thick, such as have been used in relay force measurements, the lowest flexural frequencies are in the order of 1.6×10^6 cycles. The next lowest frequencies are the radial mode vibrations[6] which have frequencies above 4 megacycles for the block considered. Hence the measurements of force should be valid up to times in the order of a microsecond.

The properties of barium titanate and their stability with time and with large voltages applied in the opposite direction to the poling voltage depend to a large extent on the method of baking the ceramic and on

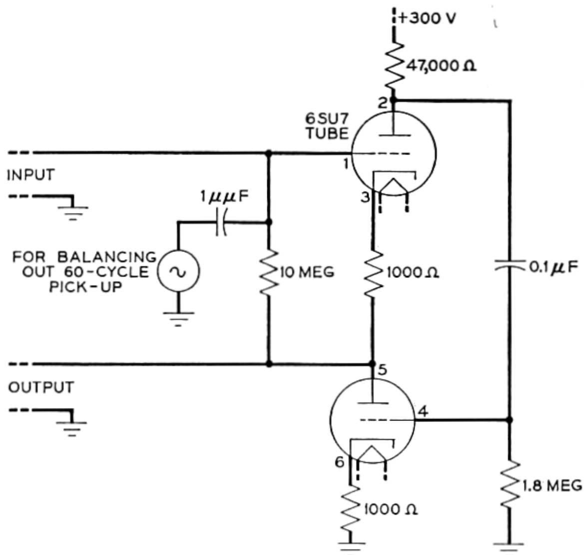

Fig. 1—High input resistance amplifier tube.

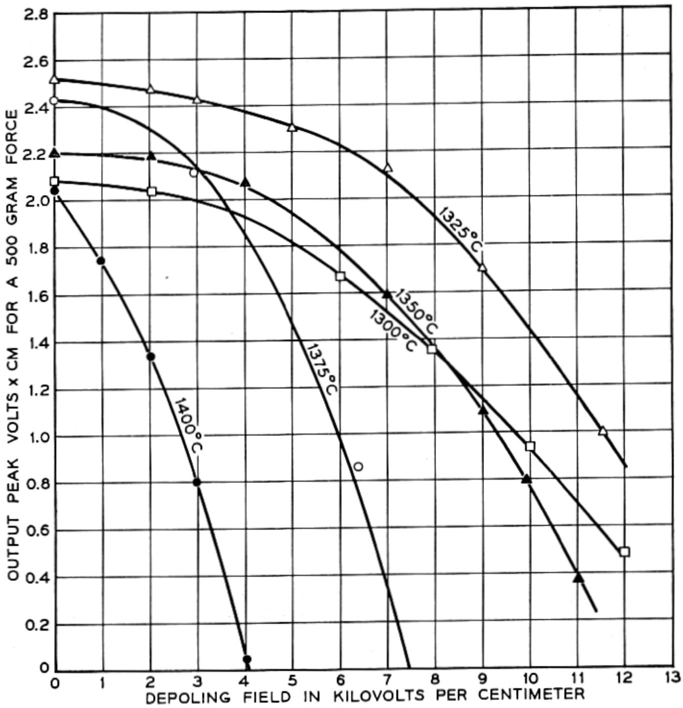

Fig. 2—Open circuit voltage for 500 grams force for normal barium titanate under depoling voltages.

the effect of additives. The data of Fig. 2 show[7] the effect of firing temperature on the initial open circuit voltage for sample disks 0.775 cm in diameter and approximately 0.15 cm thick. The variation of voltage with thickness and area was taken account of by multiplying the measured voltage by the area and dividing by the thickness.

The open circuit voltage was measured by using the circuit of Fig. 3. A barium titanate cylinder and metal horn described in a previous paper,[2] vibrating at 18,000 cycles, strikes the sample a blow at its central position. The voltage generated is applied to the input of a high resistance tube similar to the one shown by Fig. 1, and then actuates a cathode ray tube. The voltage corresponding to the height of the peak is calibrated by putting a known voltage in series with the ceramic across a small resistance R and hence the magnitude of the open circuit voltage can be quantitatively determined. The value of the mechanical blow applied to the polarized ceramic can be adjusted by controlling the drive on the ceramic cylinder. With the feed back circuit described in the previous paper,[2] this value can be held very constant and can be controlled by controlling the bias on the limiting device. The magnitude of the force can be determined by comparing the voltage with that obtained by suddenly lifting a weight off the ceramic and has been adjusted to equal 500 grams. The voltages shown then correspond to the open circuit voltages generated by applying 500 grams to a point at the center of the ceramic.

As shown in the appendix, the effect of applying a force at a point in a ceramic is not the same as that caused by distributing the force uniformly over the surface due to the fact that radial strains are generated and these act through the radial piezoelectric constant to reduce the value generated by the thickness piezoelectric constant. It is shown that the point application of stress generates only 40 per cent as much as would

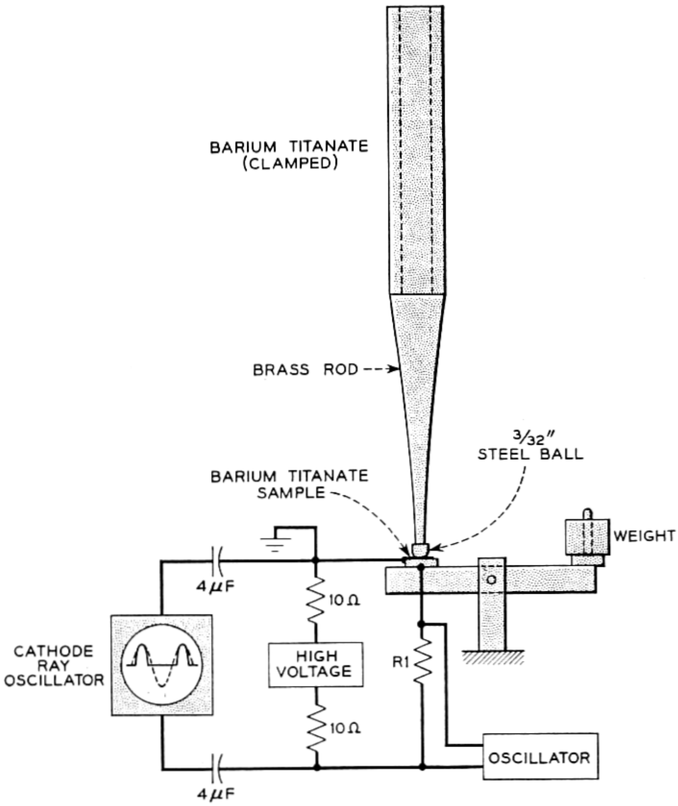

Fig. 3—Circuit used to measure open circuit voltages for barium titanate samples.

be generated in a disk with the stress applied uniformly. With this factor the open circuit voltage per unit of force—which determines the effective g_{33} piezoelectric constant of the ceramic—agrees well with that obtained by other methods of measurement. For the most desirable ceramic obtained for the 1325°C baking temperature the value of g_{33} equals

$$g_{33} = 3.25 \times 10^{-8} \frac{\text{cm}^2}{\text{stat-coulomb}} \text{ in c.g.s. units} = 0.98 \times 10^{-2} \frac{\text{meters}}{\text{Newton}} \text{ in m.k.s. units} \tag{1}$$

The dielectric constant ε of this material is about 1500 so that the d_{33} piezoelectric constant is

$$d_{33} = \frac{g_{33}\varepsilon}{4\pi} = 390 \times 10^{-8} \frac{\text{stat-coulombs}}{\text{dyne}} = 130 \times 10^{-12} \frac{\text{coulombs}}{\text{Newton}} \tag{2}$$

Since for some applications in this paper, high voltage gradients of opposite sign to the poling voltage are applied to the ceramic, it is a matter of importance to find out whether the ceramic will become depoled by the action of this voltage. To test out this feature the circuit of Fig. 3 is equipped with a high voltage generator, which is applied to the ceramic through 10-megohm resistors and the high voltage is kept out of the measuring circuit by 4-microfarad condensers. The procedure was to apply a negative voltage for 3 minutes, then to recalibrate the voltage due to impact. This was repeated with a higher voltage each time until the range was covered.

The curves of Fig. 2 show that there is an optimum baking temperature for a large coercive field. Above this temperature larger sized crystals grow in the ceramic and the coercive field decreases markedly. It is thought that the smaller crystal size corresponds to a more strained condition in the individual crystallites and it requires a higher field to overcome the mechanical bias and change the direction of the ferroelectric axis. A similar condition[8] has been found by x-ray techniques for single crystals where it has been found impossible to make a single domain out of a multidomain crystal by the application of a field, if the crystal is too highly strained.

The effects of additives are also very marked on the properties of the polarized ceramics. It has previously been reported[9] that the addition of 4 per cent of lead titanate to the commercial barium titanate increases the coercive field. This is confirmed by the curves of Fig. 4 which show the open circuit voltage for a 4 per cent lead titanate barium titanate ceramic for various baking temperatures and negative biasing voltages. The optimum temperature for a small grain size structure is lowered about 50°C by the addition of the lead titanate. As can be seen the coercive field is considerably increased and it appears safe to use a negative field of 6000 volts per centimeter without any depolarization. In Section IV a system is described for which an ac voltage of this magnitude was successfully used for many days with no change in sensitivity of the ceramic. The open circuit piezoelectric constant for the optimum ceramic of Fig. 4 is

$$g_{33} = 3.82 \times 10^{-8} \frac{\text{stat-coulombs}}{\text{dyne}} = 1.15 \times 10^{-2} \frac{\text{meters}}{\text{Newton}} \qquad (3)$$

Since the dielectric constant is about 1000, the effective d_{33} piezoelectric constant is about

$$d_{33} = 310 \times 10^{-8} \frac{\text{stat coulombs}}{\text{dyne}} = 104 \times 10^{-12} \frac{\text{coulombs}}{\text{Newton}} \qquad (4)$$

Another property of interest is the stability of the piezoelectric properties of the ceramic over a long period of time. While no very good comparisons have been made between the various baking conditions and between barium titanate with and without additions, some long time measurements have been made on four samples of the optimum 4 per cent lead titanate used in the transducer of Fig. 3. Over a period of two years during which they have been continuously used in a calibrated oscillator, the calibration has not changed noticeably, i.e. less than 5 per cent. On account of the superior voltage and time stability of the lead titanate, barium titanate mixture, all of the elements used have had this composition.

Two types of units have been used for force measurements, one type that responds to normal forces and the other to tangential forces. The type responding to normal forces as shown by Fig. 5 is poled in the thickness direction which is also the direction in which the force is applied. The sensitivities for forces applied at points are given by the values of Fig. 4. For example for typical units having the dimensions 0.1 cm by 0.1 cm in cross section and 0.05-cm thick will produce an open circuit voltage of 2.7 volts for 100 grams applied to the ceramic. Such ceramics have been used in measuring the dynamic forces when various parts of the relay close or open. Fig. 6(a)[10] shows the voltage generated when the two relay contacts come together. The dynamic stress is somewhat higher than the static stress and varies with time due to mechanical

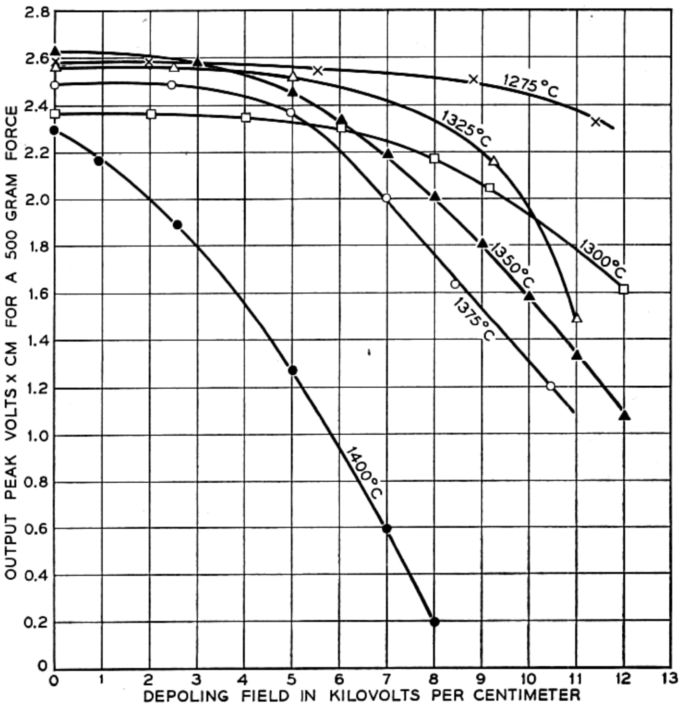

Fig. 4—Effect of 4 per cent lead titanate on the open circuit voltages generated for 500 grams force, and for depoling voltages.

vibrations of the relay structure. Fig. 6(b) shows the forces produced by opening the contacts. The large spikes are due to wire vibrations. By using such ceramics in various parts of the relay the points of high stress can be located.

The second type of structure which responds to tangential forces is poled as shown by Fig. 5 so that the poling direction lies along the direction for which the tangential force is applied and perpendicular to the

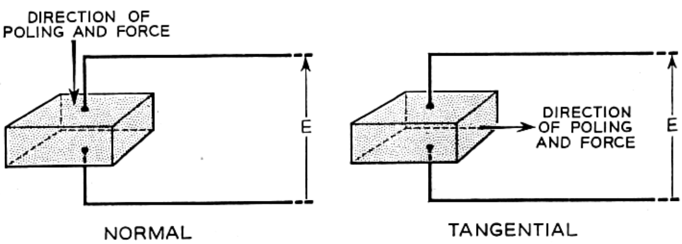

Fig. 5—Methods for polarizing barium titanate to respond to normal and tangential forces.

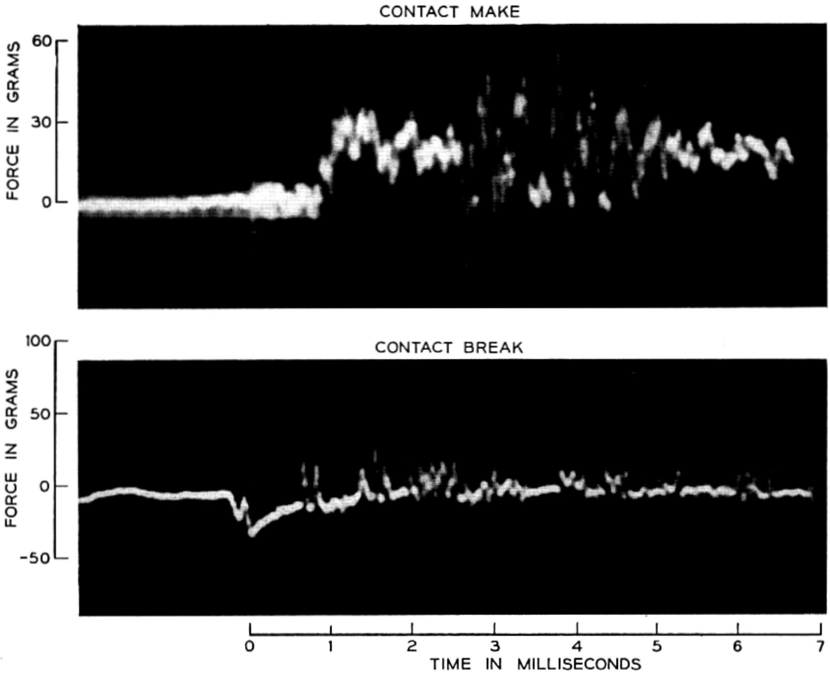

Fig. 6—Oscillograph tracings of forces generated in make and break operations.

direction of the electrodes. In this process, the crystal is first poled, after which the poling electrodes are ground or etched off and electrodes perpendicular to the poling direction are put on by using a polimerizing cement in which silver dust is mixed. The cement serves not only as an electrode but also holds the ceramic in the desired place. Fig. 7 shows an arrangement used for studying frictional forces. A small ceramic 0.1 by 0.1 cm in cross-sectional area is glued to a metal base while a thin specimen of the material whose frictional forces are to be studied is glued to the top surface. The forces caused by a wire drawn over the surface are transmitted to the crystal and generate a voltage which appears on the oscillograph. Pictures of such force generated voltages are

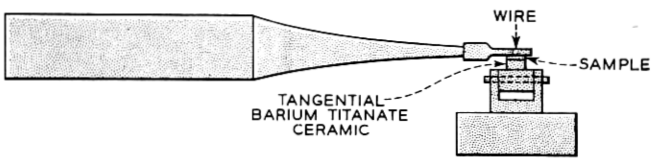

Fig. 7—Experimental arrangement for studying frictional forces.

shown by Fig. 9 of the next section and are discussed there. The sensitivity of this type of unit is higher than that for the normal force measuring unit. As shown in the appendix, the voltage generated is independent of the area of application and is about 9.7 volts for a unit the same size as discussed above which gave 2.7 volts for 100 grams applied at a point.

By placing weights on the upper surfaces both types of units can be used as accelerameters. They are cemented to the surface whose acceleration is to be measured and the force applied is equal to half the mass of the ceramic plus the added mass times the acceleration. By putting weights on the shear pickup ceramic types, tangential accelerations can be measured in the direction of the poling. By using three such accelerameters, the normal and two tangential components of acceleration of any surface can be measured.

III. METHODS FOR INVESTIGATING CAUSES OF WEAR

Wear in various parts of a relay is the limiting factor when a very large number of relay operations are desired. This wear opens up the spacing between contacts and causes the relay to lose its adjustment over a course of time.

A. Force Measurements and Wear Caused by Normal Forces

Since the forces operating on a material can be divided into normal and tangential forces, it appears desirable to separately determine the effects of each. Normal forces were produced by using the barium titanate, metal horn detail of Fig. 3. With a steel ball on the end of the metal horn, and a barium titanate specimen glued to the pivoted arm, the peak forces in grams are plotted against the volts used to drive the titanate unit for various static forces in Fig. 8. The pattern of voltage is approximately a rectified sine wave, since the ball is out of contact with the measuring titanate a part of each cycle. To observe the wear caused by normal forces a piece of material to be studied was glued to the pivoted arm on top of the barium titanate and the force was adjusted to the required value. For forces in the order of those measured in relays no wear at all was observed over a period of 18 hours which corresponds to a billion impacts, since the number per second is 18,000. For larger impulsive forces, it was found that the result of 60-million impacts against an insulator such as a phenolic was to produce a pit only a few tenths of a mil inch deep by a plastic flow. Since no wear of the type involved in relays was observed it was concluded that practically all of the wear was produced by tangential forces.

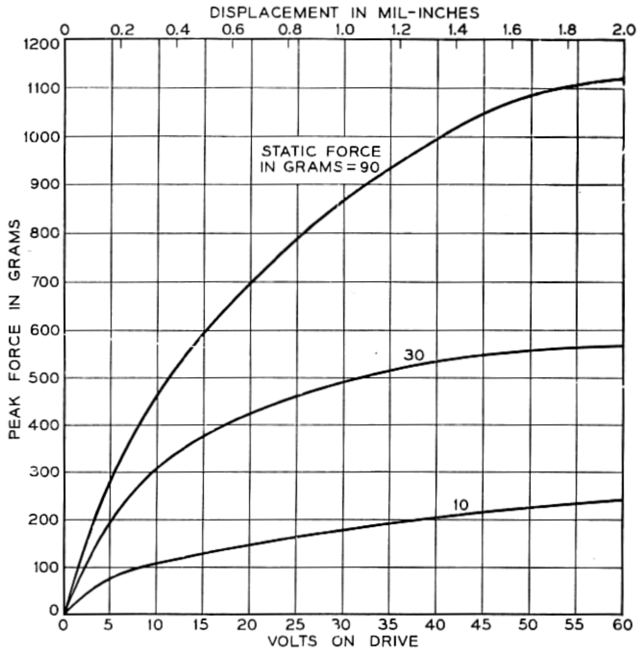

Fig. 8—Variation of normal impulse forces with drive voltages for three values of static force.

B. Tangential Force and Wear Measurements

To study the effect of tangential forces in producing wear, the transducer was mounted horizontally and the steel ball was replaced by a wire such as are used in some relays. The length of the wire was made short enough so that no lateral vibrations were generated and the motion was strictly tangential. Samples to be studied as shown by Fig. 7 were mounted on top of shear type ceramics which were glued to the pivoted arm in such a way that they responded to tangential forces applied perpendicular to the arm.

When a piece of A phenolic (which is a paper filled phenolic) was placed on top of the ceramic a series of oscillograph pictures were taken when the total displacement of the wire varied from 0.05 mil inch to 2.0 mil inches and the steady weight on the wire was 40 grams (0.0885 pound). These pictures are shown in Fig. 9. For amplitudes under 0.075 mil inch, the force is a good sine wave which increases with amplitude until the maximum force equals the product of normal force times the coefficient of friction. The force in this region is essentially elastic as is shown by the fact that the maximum force occurs at the time when the maximum displacement of the wire takes place. Above this amplitude the wire begins to slip on the plastic and for a travel of 0.3 mil inch there are indications that the point of contact between the wire and the plastic has changed from one position to another. This agrees with the idea that friction is due to a definite bonding between points of contact of the two materials which is broken by their relative motion. New points of contacts are then made and a stick-slip process occurs. At 0.5-mil-inch motion a number of small contacts occur during the

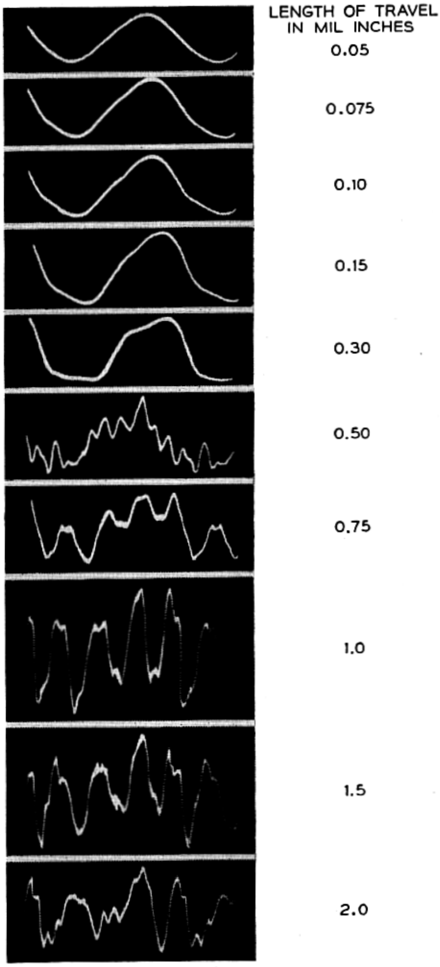

Fig. 9—Tangential forces measured for an 18,000 cycle oscillatory motion whose total displacements in mil inches are shown by the values given.

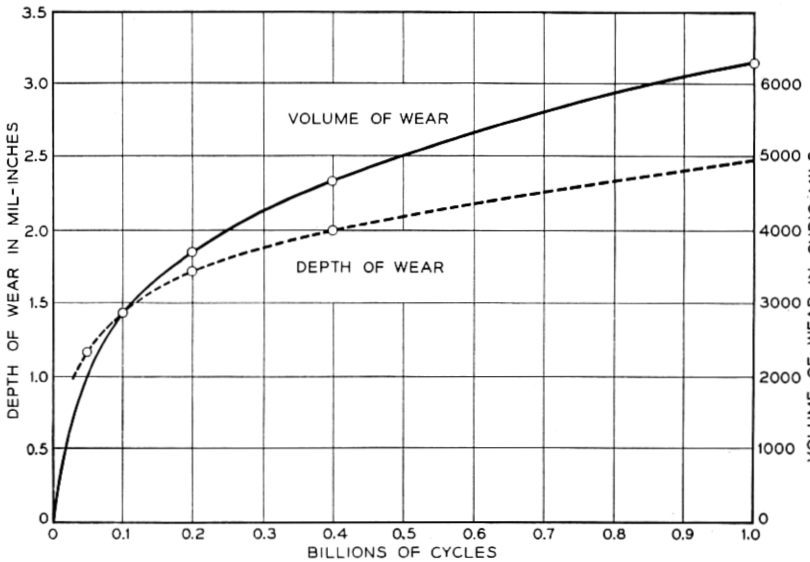

Fig. 10—Typical wear curve for A phenol fibre plotted as a function of the number of cycles.

travel. Since the picture is a trace of an oscillograph pattern which is being repeated 18,000 times a second and since a two second exposure is required to produce the picture it is obvious that the wire goes back and forth over the same points for a large number of times. Most of the energy is lost in producing elastic vibrations in the points of contact. These oscillations are produced by the bending of the areas of contact by the bonding force between them and by the motion. When the bond is broken the plastic forming the point is free to vibrate and the elastic energy goes into mechanical vibrations and eventually into heat. Since a pattern such as that for the 0.5-mil inch or the 0.75-mil inch displacement lasts unchanged for a number of minutes, it is obvious that very little of the energy goes into breaking the plastic points of contact and producing wear. This is confirmed by a rough calculation given later which shows that only about 1 part in 10^9 of the energy goes into producing wear. For displacements above a mil-inch motion it appears that groups of point contacts are broken at one time, and the pattern changes rather rapidly indicating that there is more wear at these amplitudes. Over a two-second interval the pattern is changing fast enough so that sharp pictures are not obtained.

Quantitative values of wear for various materials were obtained by running the barium titanate unit for various periods of time, different lengths of strokes and different normal forces. Fig. 10 shows a typical

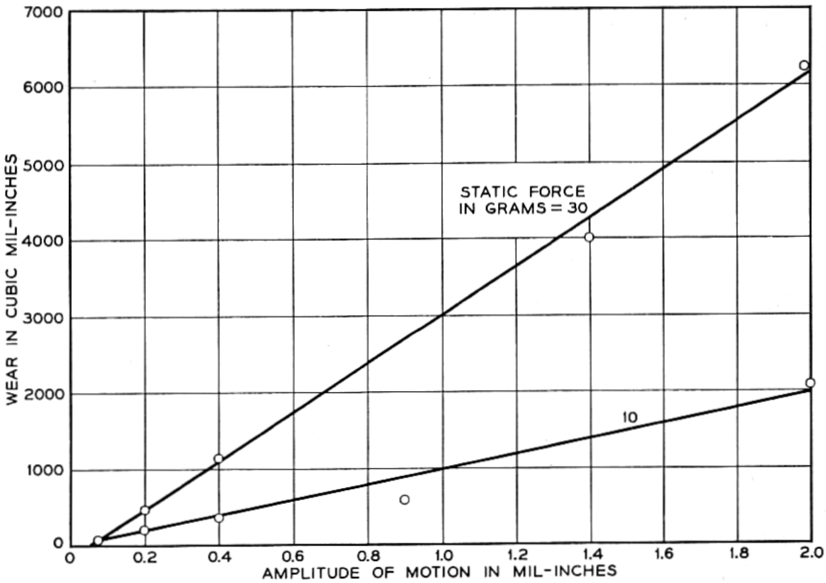

Fig. 11—Total wear for one billion cycles plotted against the length of stroke for two normal loads.

wear curve obtained for A phenolic (a paper filled phenolic) plotted as a function of the number of cycles. This wear was obtained by drawing a 0.025-inch nickel silver wire for a distance of 2.0 mil inches over the surface of the bar. The bar was $\frac{1}{4}$ inch wide. The normal force used was 30 grams (0.0665 pound). The wear was measured from the depth cut in the material and from this since the wire was round, the total volume of wear in cubic mil inches could be calculated. The rate of wear was faster at the start but approached a limiting rate with a large number of cycles.

A number of different lengths of stroke were employed and for the A phenolic the total wear for a billion operations is shown plotted by Fig. 11. The wear is approximately proportional to the slide but extrapolating down to small motions it appears that there is a threshold of motion below which the wear is very small. The values indicated are close to the no gross slide regions found from the force curves of Fig. 9 for both forces shown in Fig. 11. To check that the wear was definitely less in the no gross slide region an amplitude of motion of 0.075 mil inch for a normal force of 50 grams (0.11 pound) was run for a billion operations. The wear observed was so small that it could not be measured quantitatively, confirming the lower rate of wear in the elastic region.

Another type of wear measurement has also been employed. As shown by Fig. 12 the motor is a modification of the Western Electric 1A recorder, which was originally designed for cutting "hill and dale" phonograph records.[1] The moving system of this recorder consists of two coils (a drive coil and feedback coil) and a stylus, all rigidly coupled and coaxial. The drive coil is secured to the base of a cone shaped vibrating

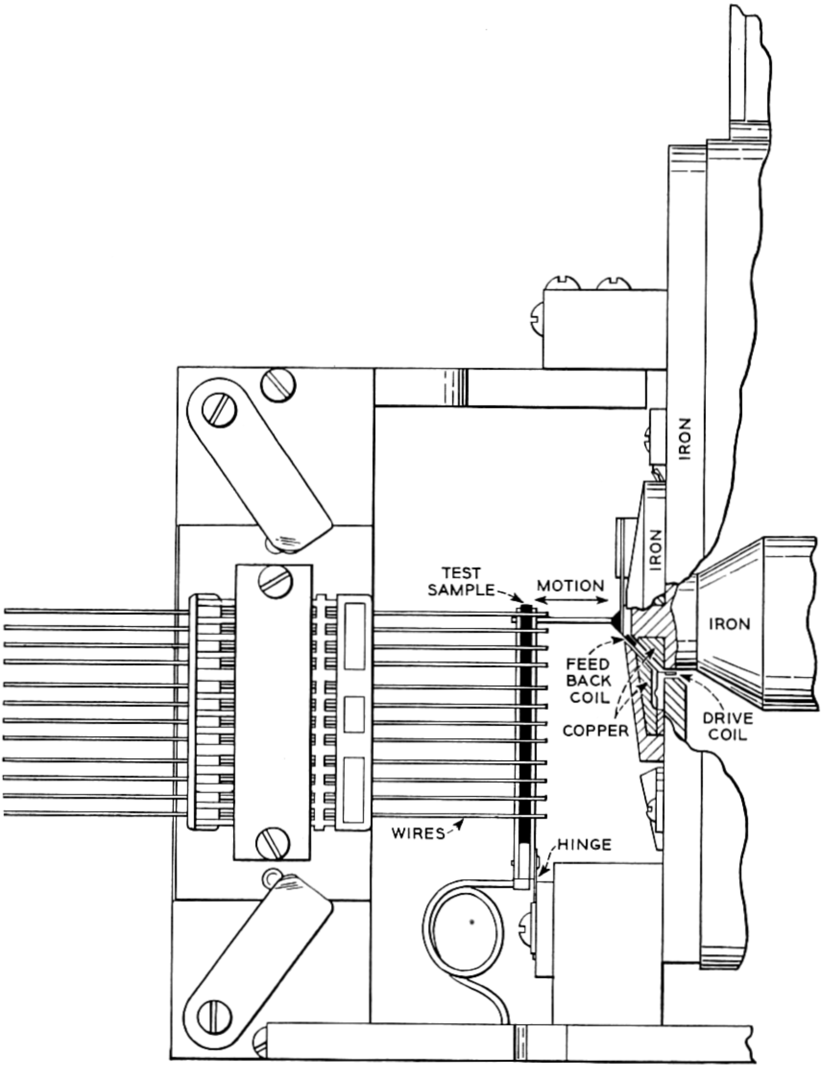

Fig. 12—Low frequency wear measuring device.

element which is carried at its base by a diaphragm and at its apex by cantilever springs. These furnish the restoring force and restrict the motion of the moving system to a single degree of freedom, motion parallel to the cone and coil axis. The second or feedback coil is secured to the cone near its apex, at which point the stylus or drive pin is attached. The coils move axially in annular air gaps polarized by a single magnet. In the space between the two coils, copper ring shielding (a shorted turn) is provided to minimize inductive coupling between them. The output of the driving amplifier is supplied to the drive coil, while the feedback coil is connected in proper (negative feedback) phase to the amplifier input.

The voltage generated in the feedback coil is proportional to the instantaneous velocity of the moving system, and by virtue of the negative feedback, the amplifier-recorder system becomes a high force, high mechanical impedance generator of mechanical motion, with the velocity very nearly proportional to the input voltage over a large range of frequency and mechanical load. Measurements of the voltage generated in the feedback coil provides a means of monitoring the velocity. Enough power capacity is present in the amplifier so that large changes in the load will not cause changes in the motion.

The samples of the materials to be tested for wear resistance are carried by a grooved aluminum beam, one end of which is hinged, the other being driven by the record stylus. The rubbing member, in this case 25-mil nickel silver wires, are tensioned against the test samples as they might be in switching apparatus. The wires can be removed for observation and measurements of the wear, and accurately replaced as the parts are dowelled together.

Fig. 13 shows a measurement of a number of materials for a normal force of 30 grams (0.0665 pounds) and a slide of 2 mil inches. The A phenolic, which is the same as that tested and recorded in Fig. 10 by the 18,000-cycle barium titanate transducer, produced essentially the same wear showing that the wear is approximately independent of the rapidity of motion for these materials. Nylon showed a rather erratic wear curve due to the fact that it has a low melting point and tends to ball up on the wires. This effect was considerably more pronounced at 18,000 cycles, where a very large indentation was found.

Only three materials show low wear at reasonably uniform rates out to a large number of cycles. These are C phenolic, a fabric filled phenolic, the B phenolic, a wood flour filled molding phenolic and the D phenolic, a cotton flock phenolic with graphite added. At lower forces and shorter slides the wear at 10^9 cycles is approximately proportional to the force

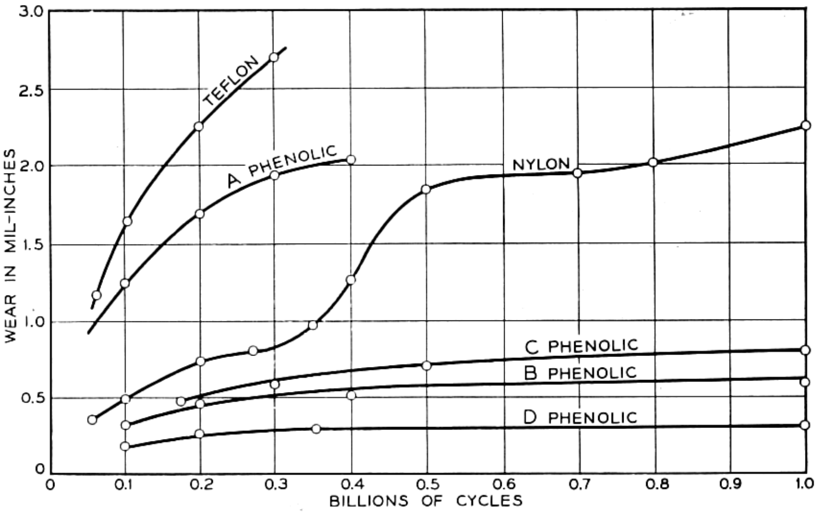

Fig. 13—Typical wear curves for a number of materials.

times the length of slide. Any of these three materials give sufficiently small wear to produce a long relay life, but the best performer under all conditions of force and slide appears to be the D cotton flock filled phenolic with graphite added.

In order to determine the causes of wear over a greater range of parameters a number of other materials were run by means of the barium titanate transducer. The wear for 2 mils motion, 30 grams (0.0665 pounds) force, and 10^9 cycles are shown by Table II.

C. Wearing Energy and Causes of Wear

A rough estimate of the energy required to break off pieces of the material shows that most of the energy goes into producing heat and very little into wear, i.e., into breaking pieces from the material. To show this let us consider a small cube fixed at one end and with a tangential force at the other. The force will cause the top surface to move with respect to the bottom surface as shown by Fig. 14, and a shearing strain S is set up in the material whose value is equal to

$$F = \mu S \, dx \, dy \tag{5}$$

where dx and dy are the cross section dimensions and μ the shear stiffness. In this displacement work is done by the sidewise displacement u equal to

$$W = \tfrac{1}{2} u F \tag{6}$$

But u the displacement is

$$u = \frac{\partial u}{\partial z} dz = S\, dz \tag{7}$$

and hence the total work done is

$$W = \tfrac{1}{2}\mu S^2\, dx\, dy\, dz = \tfrac{1}{2}(\mu S^2) \times \text{volume of material} \tag{8}$$

If the force is increased, the shearing strain S increases until it reaches the limiting strain that the material can stand. This limiting strain depends on the material and whether the strain is long repeated so that the material becomes fatigued. For most plastics this limiting strain is in the order of 1 per cent and for most metals the value is less than this.

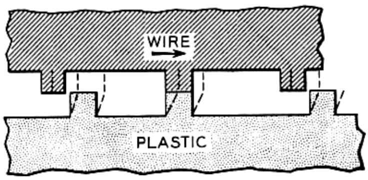

Fig. 14—Representation of points of contact and their displacements for plastic and wire.

Hence the energy to break up one cubic centimeter of material is

$$W = \tfrac{1}{2}\mu S_M^2 \tag{9}$$

where S_M is the breaking strain. For a plastic having a shear stiffness of $\mu = 2 \times 10^{10}$ dynes/cm^2 and a breaking strain of 0.01, the energy is 10^6 ergs per cubic centimeter.

This rough calculation and the amount of wear observed for various length strokes and forces allow a determination of the amount of energy going into wear production. The amount of work generated by a displacement of 0.002 inches or 0.005 cm with a normal force of 30 grams is

$$W = 0.005 \times 30 \times 980 \times f \text{ in ergs} \tag{10}$$

where f is the coefficient of friction. Since this is about 0.25 the work per stroke is 37 ergs. Twice this amount results from a complete cycle and for 10^9 cycles the work done is

$$W = 37 \times 2 \times 10^9 = 7.44 \times 10^{10} \text{ ergs} \tag{11}$$

The volume of wear observed for this condition is about 1×10^{-4} cubic cm for the A phenolic and hence we find that the part of the energy that goes into producing wear is

$$\frac{1 \times 10^{-4} \times 10^{6}}{7.4 \times 10^{10}} = 1.35 \times 10^{-9} \qquad (12)$$

or about 1 part in 10^9. This suggests that the wire goes back and forth over the same high points many millions of times until the material finally becomes fatigued and breaks off. This view is confirmed by the oscillograph pictures of Fig. 9 which are a stationary pattern for millions of oscillations.

According to this picture, the material that will wear the best is the one with the highest limiting shearing strain. If we assume that the limiting shearing strain is proportional to the limiting elongation strain under repeated vibrations—of which there are tables—the wear for various materials given in Table II agrees roughly with this concept. Table II shows the yield stresses, the Young's moduli, the per cent strains at the yield point and the relative wear at 10^9 cycles. It will be seen that the materials with the highest yield strain will in general wear longer than those with smaller yield strains.

An exception to this rule was nylon which had a large wear even though it has a large yield strain. However, nylon has a relatively low softening temperature and a low heat conductivity. Observations showed that the nylon was melted off rather than abraided off. According to this rule gum rubber should wear much better than any other material since it has such a high limiting shearing strain. A run was made with a two mil inch motion on a gum rubber specimen and no observable wear was found. The fact that a rubber tire will outwear a metal tire is also confirmation of this rule.

All the tests showed that the wear on the stainless steel or nickel silver

TABLE II

AMOUNT OF WEAR FOR VARIOUS MATERIALS CAUSED BY SLIDING A 0.025 INCH NICKEL SILVER WIRE FOR 2 MIL INCHES, 30 GRAMS NORMAL FORCE AND 10^9 CYCLES

Material	Yield Stress Dynes/Cm²	Youngs Modulus Dynes/Cm²	Per Cent Yield Strain	Wear, Cubic Cm, for 2-Mil Motion for 10^9 Cycles and 30-Gram Force
Lead Glass....	2.4 to 2.7×10^9	6.5×10^{11}	0.0037 to 0.0041	0.027
Brass..........	3.7 to 4.6×10^9	9×10^{11}	0.0041 to 0.0051	0.0075
Stainless Steel.	1.1 to 1.4×10^{10}	2×10^{12}	0.0055 to 0.007	0.00075
B Phenolic....	7.2×10^8	6.9×10^{10}	0.0105	0.000025

wire used to produce the wear on the plastic was always very much less than that of the plastic. The reason for this as seen from Fig. 14 is that since the displacement for a given force to break the bond between two high points is going to be inversely proportional to the shearing stiffnesses of the two materials, the displacement for stainless steel with a shear stiffness of 8 x 10^{11} dynes/cm^2 will be $\frac{1}{40}$ that of the plastic with a shear modulus of 2 x 10^{10} dynes/cm^2. Hence, the shearing strain for the stainless steel is much further below its limiting strain than is the shearing strain for the plastic. When the stainless steel wire was run against a bar of synthetic sapphire—which has a much higher shear constant—the stainless steel wire was soon worn through, while little wear occurred on the sapphire.

IV. THEORETICAL AND EXPERIMENTAL INVESTIGATION OF THE NO GROSS SLIDE REGION

Since in the no gross slide region, the shearing strain is less than in the gross slide region, the rate of wear should be considerably less. This is confirmed by direct tests of the wear as shown by Fig. 11, and by supplementary tests. Hence a further experimental and theoretical investigation has been made of this region which is defined by the condition that the tangential force is less than the product of the normal force by the coefficient of friction. If sliding motions can be kept small enough to be in this region, very little wear should occur.

Using a shear ceramic for measuring the tangential force, the static load was varied and the motion required to produce no gross slide was determined. Oscillograph figures of the type shown by Fig. 9 were used and when the figure was broadened out as shown by the third figure it was assumed that slide had occurred. Fig. 15, upper curve, shows the total motion in mil inches, plotted against the static force in grams, which will just cause gross slide. The bottom line shows the maximum shearing force in grams. This is slightly lower than the force determined by the coefficient of friction since the force becomes slightly larger as shown by the pictures of Fig. 9, when gross slide occurs. The total displacement for no slide increases as the two-thirds power of the static load.

Since neither the wire nor the plastic material is smooth, contact between the two is established at only a few points. To interpret the results obtained above, some calculations due to R. D. Mindlin[11] are used. These deal with the tangential forces and displacements of two balls pressed together, and are for conditions occurring before gross slide begins.

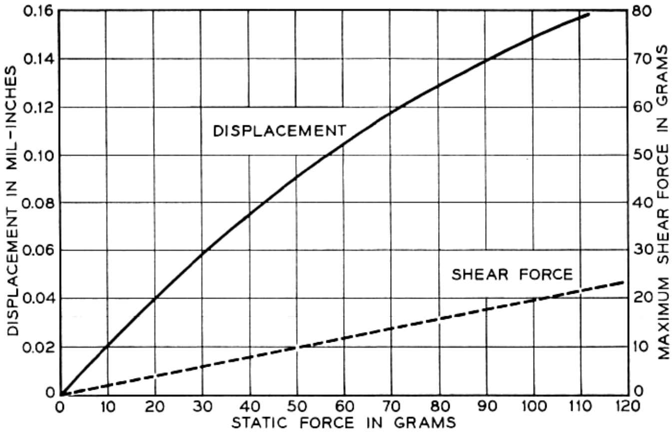

Fig. 15—Maximum total motion for no gross slide plotted against normal force. Low curve shows maximum tangential force.

From the Hertz theory of contacts,[12] the radius of contact a between two spheres is equal to

$$a = \sqrt[3]{\frac{3}{8} r N\left(\frac{1 - \sigma_1}{\mu_1} + \frac{1 - \sigma_2}{\mu_2}\right)} \tag{13}$$

where r is the radius of the spheres, N the normal force, μ_1 and σ_1 the shear elastic constant and Poisson's ratio for one sphere and μ_2 and σ_2 the same quantities for the second sphere. If now a tangential force T is applied to one of the spheres directed in the form of a couple, elastic theory shows that the tangential traction is everywhere parallel to the direction of the applied force and contours of constant tangential traction are concentric circles. The magnitude of the traction as shown by Fig. 16 rises from one half the average at the center to infinity at the edge of the circle of contact. The displacement of the circle of contact of one sphere with respect to its center is

$$\delta_x = \frac{2 - \sigma}{8\mu a} T \tag{14}$$

where a is the radius of the contact area which is given in terms of the normal force by Equation (13).

A feature of this solution that requires further study is the infinite traction at the edge of the circle of contact. Presumably the tangential component of traction cannot exceed the product of the coefficient of friction f and the normal component of traction p, which from the Hertz

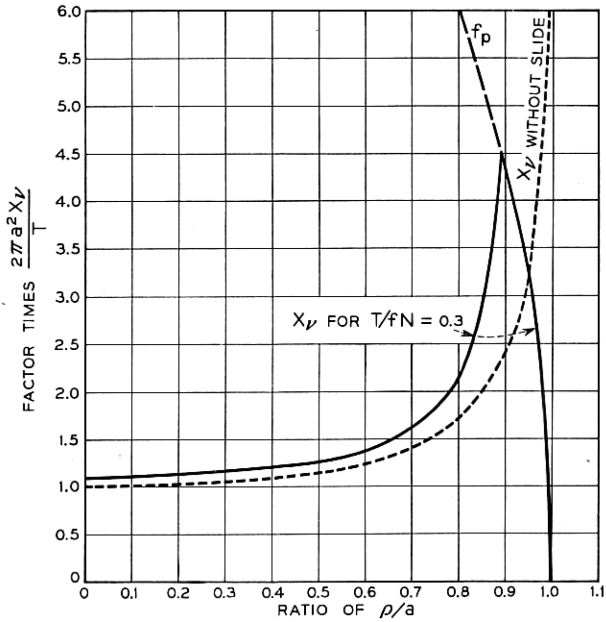

Fig. 16—Traction plotted against radius for elastic displacement and modification introduced by the effect of slip.

contact theory is

$$p = \frac{3N}{2\pi a^2} \sqrt{1 - \frac{r^2}{a^2}} \tag{15}$$

Mindlin assumes that slip takes place between the two surfaces until the tangential traction is equal to

$$X_v = \frac{3fN}{2\pi a^2} \sqrt{1 - \frac{\rho^2}{a^2}} \qquad \text{for } a' \lesseqqgtr \rho \lesseqqgtr a \tag{16}$$

and less than this for all interior points, where in this equation ρ is the radius vector and a' the inner radius for which slip stops. This corresponds to the introduction of a new system of forces and Mindlin has shown that equilibrium is reestablished when the surface tractions are given by Equation (16) when $a' \lesseqqgtr \rho \lesseqqgtr a$ and by

$$X_v = \frac{3fN}{2\pi a^2} \left[\sqrt{1 - \frac{\rho^2}{a^2}} - \frac{a'}{a} \sqrt{1 - \frac{\rho^2}{a'^2}} \right] \qquad \text{when } \rho \lesseqqgtr a' \tag{17}$$

Fig. 16 shows this distribution for the case $T/fN = 0.3$. The inner radius a' is given such a value that the integrated traction over the surface equals T and its value is found to be

$$a' = a \sqrt[3]{1 - \frac{T}{fN}} \tag{18}$$

The added slip increases the displacement δ_x and it is shown that the total displacement is equal to

$$\delta_x = \frac{3fN\ (2 - \sigma)}{16\mu a}\left[1 - \left(1 - \frac{T}{fN}\right)^{2/3}\right] \tag{19}$$

A plot of this curve is shown by the line OPQ of Fig. 17 and it is evident that the displacement before gross slip occurs is 1.5 times larger than the elastic displacement calculated on the assumption of no slip.

These calculations have been extended in a recent paper[3] to include the case of a cylically varying force $T \lesseqqgtr fN$ and it is shown that the force displacement curve is a hysteresis type loop whose end points lie

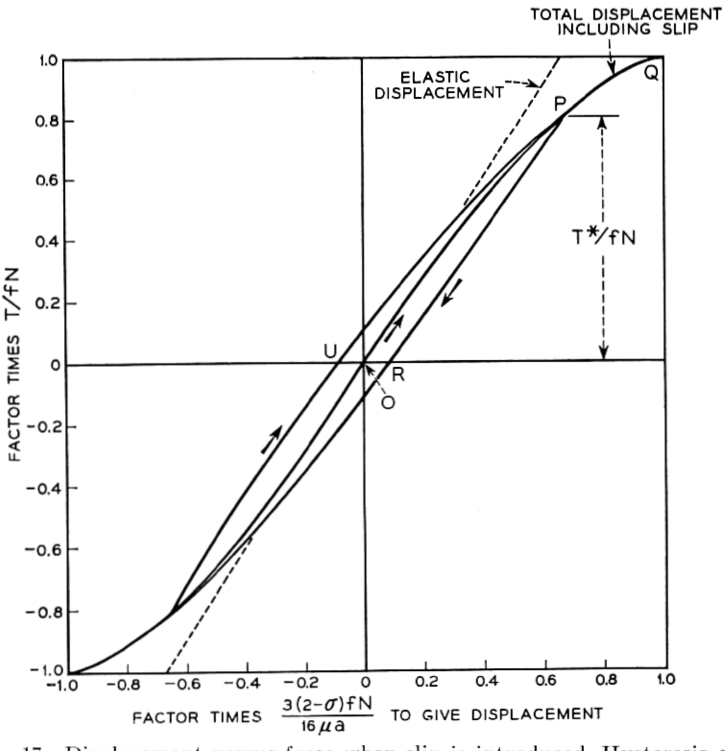

Fig. 17—Displacement versus force when slip is introduced. Hysteresis curve PRU shows displacement for an oscillating force.

on the OPQ curve of Fig. 17 and whose theoretical area W is

$$W = \frac{9(2 - \sigma) f^2 N^2}{10\mu a} \times \left[1 - \left(1 - \frac{T^*}{fN}\right)^{5/3} - \frac{5T^*}{6fN}\left[1 + \left(1 - \frac{T^*}{fN}\right)^{2/3}\right]\right] \tag{20}$$

where during the oscillation the tangential force T varies between the limits $\pm T^*$. Slip takes place as before between the radii a and a' given by

$$a' = a \sqrt[3]{1 - \frac{T^*}{fN}} \quad \text{or conversely} \quad \frac{T^*}{fN} = 1 - \frac{a'^3}{a^3} \tag{21}$$

Since the distribution of traction over the surface cannot be uniquely derived from elastic theory, the introduction of the slip function is an assumption that has to be justified by experiment. This assumption has been shown to correspond with experiment by employing the experimental arrangement shown by the photograph of Fig. 18. A barium titanate driver shown in more detail in Fig. 19 drives the middle of three glass lenses that are pressed together by a static force applied to the lever system as shown by Fig. 19. The central glass lens has a radius of curvature of 4.85 inches on each side while the other two lenses have the same radius of curvature on the sides touching the middle lens, but are flat on the other two sides and are rigidly attached to the lower platform and upper hinged lever by cement. In order to get sufficiently large

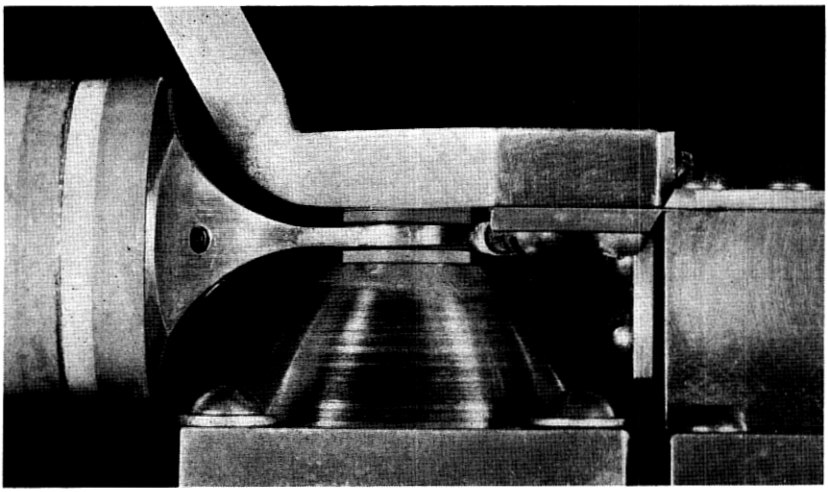

Fig. 18—Barium titanate driver, pick-up device and glass lenses.

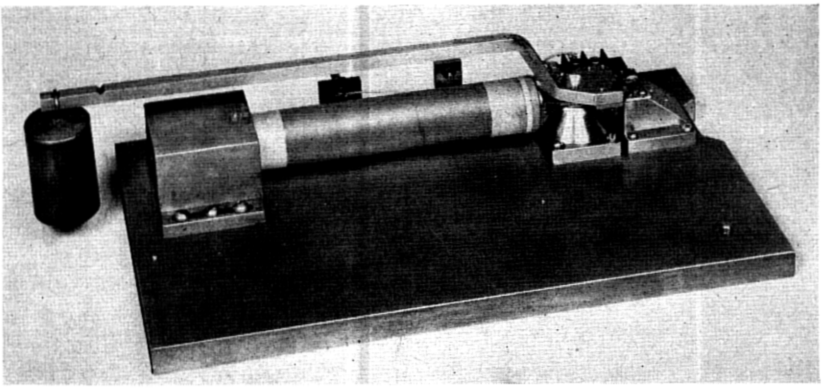

Fig. 19—The entire experimental arrangement.

circles of contact to be easily observed, normal loads on the lens system were 10 and 15 pounds which resulted in contact circles of 0.030 and 0.034 inches diameter.

With normal forces up to 15 pounds and two surfaces in contact, tangential forces up to 7.5 pounds are necessary in order to bring the central lens near the sliding point if the coefficient of friction is near one-quarter. This force was obtained by impressing voltages in the order of 3,000 rms volts on the barium titanate lead titanate hollow cylinder. This cylinder is $4\frac{3}{4}$ inches long, and has an outside diameter of 1 inch and an inside diameter of $\frac{1}{2}$ inch. The ceramic was poled in a radial direction and the constants of the material were such that a force of 167 pounds could be generated along the length for a clamped driver when a voltage of 3,000 volts (4,750 volts cm) was used. On the other hand if the driver works against no stress, the expansion in the plated length of 4 inches is 0.7×10^{-4} inches.

The actual force applied depends on how much the relative slip between the glass lenses amounts to. To measure this force, a poled lead titanate barium titanate disk is placed between the driver and the metallic bracket which clamps the middle lens as shown by Fig. 18. All the force exerted on the lens has to be exerted through the disk and hence the voltage generated by the disk is a measure of the force exerted on the middle lens. This voltage is calibrated by attaching a spring load of known constants and measuring the displacement of the load by means of a microscope.

Using a 60-cycle driving voltage, a number of sets of disks were run with varying tangential and normal loads and the wear patterns observed. Fig. 20 is a photograph (magnified 100 times) for a normal load

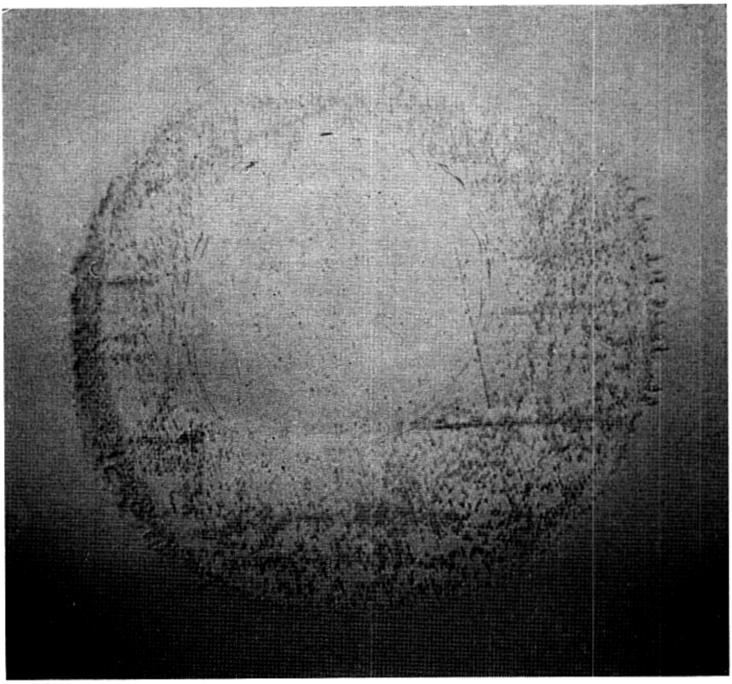

Fig. 20—Wear circles (magnified 100 times).

of 10 pounds and a maximum tangential load of 2.04 pounds per lens run for about 3 hours at 60 vibrations per second. The outer area of contact is seen to be 0.03 inches in diameter. The inner area of wear is a circle displaced slightly from a concentric form and has a diameter of 0.0175 inch. If we plot $1 - (a'/a)^3$ against the ratio of tangential to normal force, where a' is the inner radius and a the outer radius, as shown by Fig. 21, a point at 0.204 and 0.8 is obtained. A number of sets of lenses were run and as shown by Fig. 21 the results can be plotted on a straight line corresponding to a coefficient of friction of 0.25. This value agrees well with other determinations[13] of the coefficient of friction of glass on glass. Hence the assumption of slip between spheres under tangential forces appears to be verified. This type of slip may be responsible for some types of wear, such as in ball bearings, where no gross slide of one surface over another occurs.

An attempt was also made to check the area of the loop as determined theoretically by Equation (20). The applied force is measured directly by the barium titanate pickup and the displacement was measured by attaching a velocity microphone pickup to the transducer. The force voltage was placed on one set of plates of an oscillograph while the integrated output from the velocity pickup was placed on the other set. A series of oscillographs were taken for various amplitudes of motion and the pictures are shown by Fig. 22. Since the force and displacement measurements were separately calibrated, the area of the curves in inch pounds could be evaluated and are shown by Fig. 23. For amplitudes of motion near the gross slip amplitude, the area agree well with that calculated from Equation (20) from which the dotted line is obtained. For lower amplitudes the measured area is larger than the calculated area. Possibly a stick-slip process is causing the displacement to lag behind the applied force. The measured areas are nearly proportional to the square of the amplitude. The mechanical resistance associated with the stress-strain hysteresis curves of this sort is of the same type that

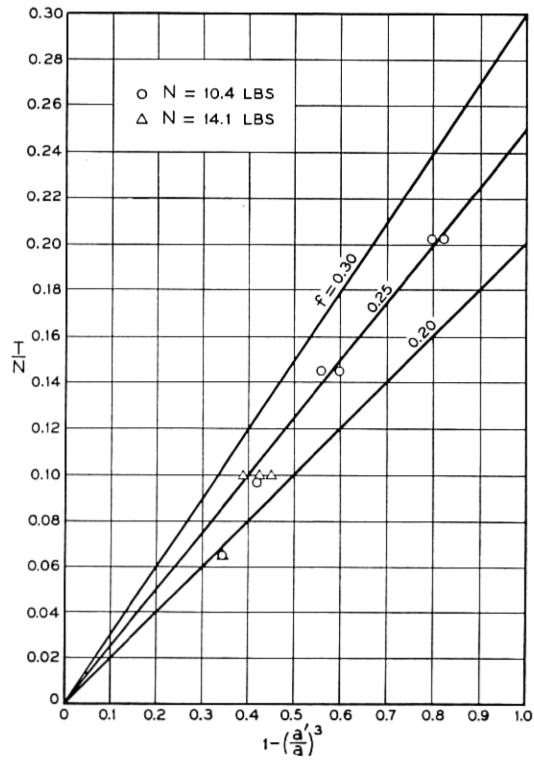

Fig. 21—Plot of $1 - (a'/a)^3$ against ratio of tangential and normal forces.

Fig. 22—Force displacement loops.

occurs in an assemblage of granular particles such as in a telephone transmitter for which the motion is so small that gross slide does not occur.

Since the theoretical displacement of Equation (19) has been verified by the glass lens experiment, we can use it to determine some of the quantities involved in the oscillographs of Fig. 9 and the displacement-normal force curve of Fig. 15. To obtain the relation between the total displacement δ_x and the normal force N, we have to eliminate a from Equation (17) since a is also a function of the normal force as shown by Equation (13). Introducing this equation, and neglecting $1/\mu_2$ as compared to $1/\mu_1$, since for the wire μ_2 is 40 times μ_1 of a plastic,

$$\delta_x = \frac{\left(\frac{3N}{\mu}\right)^{2/3} (2 - \sigma)f}{8\sqrt[3]{r}(1 - \sigma)} \left[1 - \left(1 - \frac{T^*}{fN}\right)^{2/3}\right] \qquad (22)$$

Hence in agreement with the data of Fig. 15, the displacement for no gross slide should vary as the two-thirds power of the normal force.

Another deduction from Equation (22) is that the displacement for no slide should vary as the inverse two-thirds power of the shear stiffness constant μ. For example gum rubber with a shear stiffness of 2×10^7 dynes/cm^2 should give 100 times the displacement of a plastic with a shear stiffness of 2×10^{10} dynes/cm^2. A rough check of this deduction has been made by cementing a thin strip of gum rubber on the face of a shear responding ceramic and with a normal force of 30 grams (0.0665 pounds), vibrating the wire at its full amplitude of 2 mil inches. Over this range the voltage response was sinusoidal indicating that no gross slide took place. This is 33 times as large a motion as occurred for a plastic with an elastic stiffness 1,000 times that of the rubber and verifies the variation of δ_x with μ.

The other experimental quantity that can be obtained from Equation (22) is the radius r of the effective contact points of the plastic. If all

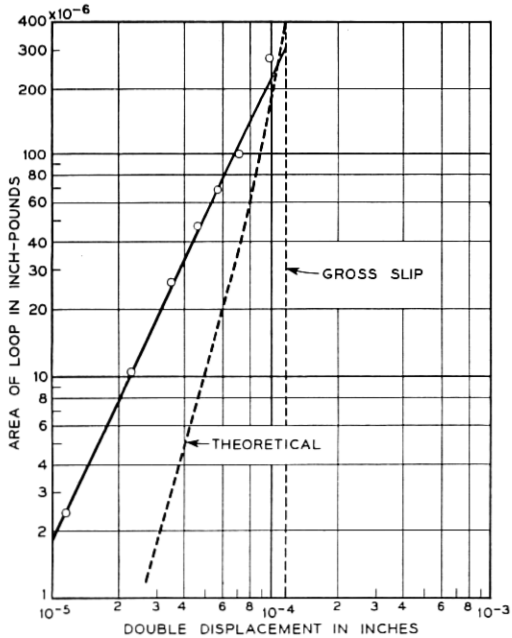

Fig. 23—Plot of area of force displacement loop against double displacement.

the force is supported by a single point at a time, then for

$$\begin{aligned} \delta_x &= \pm 3 \text{ x } 10^{-5} \text{ inches} = \pm 7.5 \text{ x } 10^{-5} \text{ cm}; \\ N &= 30 \text{ grams} = 2.94 \text{ x } 10^4 \text{ dynes}; \\ \mu &= 2 \text{ x } 10^{10} \text{ dynes/cm}^2; \\ \sigma &= 0.45 \end{aligned} \tag{23}$$

and the coefficient of friction $f = 0.25$, the value of r becomes 0.008 cm. If the weight were supported equally by n points the radius would be divided by n^2. Since the sidewise displacement would result in a strain of 0.009 for a single point and 0.036 for two points, the latter strain would be beyond the yield strain for the material. Hence the evidence seems to indicate that a single point supports the major part of the weight at any particular time.

While it is difficult to reduce the gross tangential slide of a relay to the values required for the low wear (no gross slide) region, the existence of such a region has considerable importance for other sources of wear in relays, namely long continued vibrations of component parts such as undamped wires. The tangential motions caused by such vibrations are small, but since they are repeated many times for each operation, the total integrated wear is considerable. By introducing damping so that the vibrations are quickly brought down to the low wear, no gross slide region, a considerable reduction in wear has been found for relays.

Appendix

VOLTAGE GENERATED BY COMPRESSIONAL AND TANGENTIAL CERAMICS BY FORCES APPLIED UNIFORMLY OR AT CONCENTRATED POINTS

When a stress is applied to a prepolarized barium titanate ceramic it has been shown[14] that the open circuit field generated along the Z axis is given by the equation

$$E_3 = -2[Q_{11}[\delta_{3_0} T_3 + \delta_{1_0} T_5 + \delta_{2_0} T_4] + Q_{12}[\delta_{3_0}(T_1 + T_2) - (\delta_{1_0} T_5 + \delta_{2_0} T_4)]] \tag{24}$$

where δ_{1_0}, δ_{2_0}, δ_{3_0} are the remanent values of polarization introduced along the three axes by the poling process, T_1, T_2, T_3, T_4, T_5, T_6 the three extensional stresses and the three shearing stresses, and Q_{11} and Q_{12} are the two electrostrictive constants for the ceramic. From the "effective" piezoelectric constants measured for these ceramics we find that

$$Q_{11}\delta_{3_0} = 2.2 \times 10^{-8}; \qquad Q_{12}\delta_{3_0} = -.8 \times 10^{-8} \text{ cgs units} \tag{25}$$

for pure barium titanate and

$$Q_{11}\delta_{3_0} = 2.4 \times 10^{-8}; \qquad Q_{12}\delta_{3_0} = -.9 \times 10^{-8} \text{ cgs units} \tag{26}$$

for 4 per cent lead titanate barium titanate ceramic.

If a force F is applied uniformly over the whole surface of a small barium titanate unit, then $T_3 = F/A$, where A is the area, and all the other stresses are zero. Under these circumstances when the permanent polarization δ_{3_0} is along the Z axis (normal poling), the open circuit potential is

$$E_3 = \frac{V_3}{l_t} = \frac{2Q_{11}\delta_{3_0}F}{l_w l} = \frac{2 \times 2.4 \times 10^{-8} \times F}{l_w l} \text{ cgs units} \tag{27}$$

where l_t is the thickness and l_w and l the cross-sectional dimensions. To get the number of volts generated this factor is multiplied by 300 and

$$V_3 = \frac{1.44 \times 10^{-5} F}{l_w l} \text{ volts} \tag{28}$$

where force F is expressed in dynes.

However for the data of Figs. 2 and 4, the voltage measured is that for a load applied at the center of the ceramic and for this case the stresses T_1 and T_2 of Equation (24) cannot be neglected. The solution[15] for the stresses occurring when a load F is applied at a point on the surface of a semi infinite solid is used to evaluate the corrections caused by the non-uniform load. In cylindrical coordinates the formulae for the three stresses T_{zz} and T_{rr} and $T_{\theta\theta}$ given by Timoshenko are

$$T_{rr} = \frac{F}{2\pi}\left[(1 - 2\sigma)\left[\frac{1}{r^2} - \frac{z}{r^2}(r^2 + z^2)^{-1/2}\right] - 3r^2 z(r^2 + z^2)^{-5/2}\right]$$

$$T_{\theta\theta} = \frac{F}{2\pi}(1 - 2\sigma)\left[-\frac{1}{r^2} + \frac{z}{r^2}(r^2 + z^2)^{-1/2} + z(r^2 + z^2)^{-3/2}\right] \tag{29}$$

$$T_{zz} = \frac{-3F}{2\pi} z^3(r^2 + z^2)^{-5/2}$$

where r is the radial distance from the point of contact, z the distance below the surface and σ = Poisson's ratio.

The response of a barium titanate unit in terms of cylindrical co-ordinates has been shown[16] to be for a unit polarized along the z axis

$$E_z = -2\left[Q_{11}\delta_{3_0} T_{zz} + Q_{12}\delta_{3_0}(T_{rr} + T_{\theta\theta})\right] \tag{30}$$

Now since the ceramic is plated, the major surface is an equipotential surface and hence E_z does not vary with r or θ. Hence integrating over the surface of the ceramic, we have for the open circuit field

$$E_z \int_0^\infty \int_0^{2\pi} r\, dr\, d\theta = -2\left[Q_{11}\delta_{3_0} \int_0^\infty \int_0^{2\pi} T_{zz} r\, dr\, d\theta + Q_{12}\delta_{3_0} \int_0^\infty \int_0^{2\pi} (T_{rr} + T_{\theta\theta}) r\, dr\, d\theta\right] \tag{31}$$

Introducing the values of T_{zz}, T_{rr} and $T_{\theta\theta}$ from Equation (29) and performing the integrations we find

$$E_z A = 2\,[Q_{11}\delta_{3_0} F + Q_{12}\delta_{3_0}\,(1 + 2\sigma)F] \tag{32}$$

where A is the cross-sectional area of the ceramic. The first term agrees with that for a uniform stress, but the second term shows that we have a correction due to the radial and tangential stresses generated by the application of the force at a point.

The amount of correction can be calculated by putting in the values of Q_{12} and σ the Poisson ratio. Recent measurements of the thickness resonance and the resonance of a torsional ceramic have shown that the best values of the Lame elastic constants are

$$\lambda = 5.8 \times 10^{11} \text{ dynes/cm}^2; \qquad \mu = 4 \times 10^{11} \text{ dynes/cm}^2 \tag{33}$$

With these values, Poisson's ratio becomes

$$\sigma = \frac{\lambda}{2(\lambda + \mu)} = \frac{5.8}{19.6} = 0.296 \tag{34}$$

For 4 per cent lead titanate barium titanate ceramic, introducing the values given above, the voltage generated by a force applied at a point is about 0.4 of that for a force applied uniformly, giving

$$V_z = \frac{0.575 \times 10^{-5} F l_t}{l_w l} \text{ volts} \tag{35}$$

This value corresponds reasonably well with the data of Fig. 4.

When the remanent polarization is applied along the Y axis and the voltage measured along the Z axis, Equation (24) shows that the open circuit voltage will be

$$E_3 = -2\,(Q_{11} - Q_{12})\delta_{2_0} T_4 \tag{36}$$

where $T_4 = Y_z$ is the stress in the direction of polarization (Y) applied to the surface of the ceramic. Since the single stress T_4 is involved, the open circuit voltage will be independent of whether the force is applied uniformly over the surface or at a point. This follows from the fact that E_3 is independent of x and y and hence

$$E_3 \int_0^{lw} \int_0^{l} dx\, dy = -2(Q_{11} - Q_{12})\delta_{2_0} \int_0^{lw} \int_0^{l} T_4\, dx\, dy \tag{37}$$

Integrating over the surface gives the total force F for the right side and hence

$$V_3 = \frac{2(Q_{11} - Q_{12})\delta_{2_0} l_t F}{l_w l} \text{ in cgs units} \tag{38}$$

$$= \frac{1.98 \times 10^{-5} l_t F}{l_w l} \text{ in volts}$$

For a ceramic 0.1 cm by 0.1 cm in cross-section and 0.05 cm thick a tangential force of 100 grams should generate a voltage of 9.7 volts.

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