Instantaneous Compandors

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(*Manuscript Received Apr. 9, 1951*)

Instantaneous compandors have found useful application in time-division systems. This paper discusses the theory of the instantaneous compandor and evaluates the noise advantage when instantaneous companding is applied to telephone channels. The noise advantage depends upon the noise standard of the system. If the standard corresponds to that of a toll telephone system, a noise advantage of about 20 db is possible.

INTRODUCTION

A compandor is characterized by compression followed by expansion. To achieve noise reduction by compandor action,* compression is applied before and expansion after the noise exposure. By compression one means that the effective gain which is applied to a signal varies as a function of the magnitude of the signal, the effective gain being greater for small than for large signals. In the process of expansion the effective gain also varies as a function of the signal but is greater for large than for small signals.

There are two general classes[1] of compandors, "syllabic" and "instantaneous." For many years, because of theoretical and practical reasons, only the syllabic type was used to any appreciable extent. Although utilized primarily in special situations[2-5], syllabic compandors have in these instances served to improve telephone operation by providing a substantial noise advantage. More recently[6-11] the instantaneous type also has begun to find important applications to time-division systems. Since an instantaneous compandor produces effective gain variations in response to instantaneous values of the signal wave, the instantaneous type is well adapted to pulse systems. Moreover, in time-division pulse-modulation systems, one instantaneous compandor usually serves a plurality of channels thereby affording additional economies.

THEORY

Noise advantage due to compandor action arises primarily because it is the weak signals that are most susceptible to degradation by noise or other unwanted interference. Accordingly, weak signals are highly amplified by the compressor and are carried at a relatively high level through the noise

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* For all numbered references, see list at end of paper.

exposure. Stronger signals are amplified less highly. Loss, therefore, is removed from the expandor as the signal increases and the noise increases correspondingly.

When the signals are conventional speech signals, loss is removed from the expandor as the speech volume increases and consequently the noise volume increases correspondingly. An instantaneous compandor has the important advantage that level adjustments are frequent, for example, in a pulse-modulation system at the rate of about 8000 times per second for a message channel whose bandwidth approaches 4000 cycles. Consequently, the increased noise will be continuously masked by increased speech sound. During all silent periods, unwanted noise and interference receive maximum noise suppression in the expandor. For an ordinary message channel these advantages are substantial.

Viewed broadly, an instantaneous compandor provides a ready means for making the noise susceptibility a function of the magnitude of the signal. If the noise susceptibility is made less than that of a linear system in one portion of the range, then it must be greater than that of a linear system in some other portion of the range. Whether an over-all improvement results depends entirely upon the nature of the signal. For example, in certain types of picture transmission systems a given value of noise produces about as much harm whether the signal be weak or strong. In this instance no benefit would accrue from making the noise susceptibility a function of signal strength.

An important consideration, therefore, is the evaluation of the noise advantage due to instantaneous companding. The theoretical treatment will give relationships for signal-to-noise ratio and noise susceptibility. Application of the theory to a particular example including a numerical evaluation of the noise advantage will be deferred to the last section.

Method of Analysis

The analysis is based upon deductions[12] related to the sampling principle and is illustrated by Fig. 1 which shows a schematic of one channel of a multichannel time-division system.

The incoming signal (Fig. 1) is filtered by a low-pass filter designated F_1. At the output of F_1 the signal should be regarded as an arbitrary signal occupying the band of all frequencies slightly less than B. Brief samples of the signal are taken uniformly at the rate of $2B$ samples per second. In this manner the signal is converted into a series of PAM (pulse amplitude modulated) pulses as indicated in Fig. 2. There is a unique relationship[12] between signals and samples (PAM pulses); if we are given the signal wave we can determine the samples; if we are given the samples we can determine the signal wave.

In Fig. 1, if we ignore the compressor, the samples are immediately filtered by another low-pass filter designated F_2 which temporarily will be assumed to be similar to F_1. If F_2 attenuates all frequencies higher than B and if each filter includes accurate in-band equalization including correction for phase distortion, then the wave at the output of F_2 except for delay will be an attenuated replica[12] of the wave at the output of F_1.

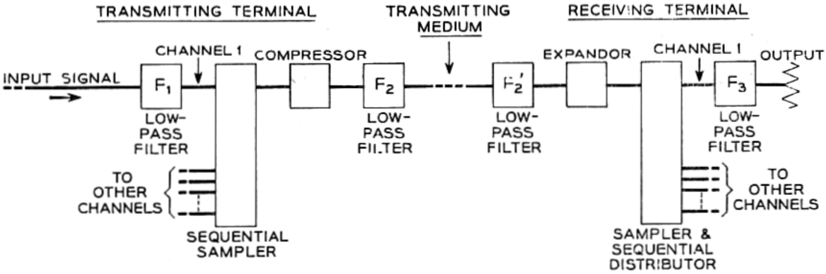

Fig. 1—Block schematic of multi-channel PAM system.

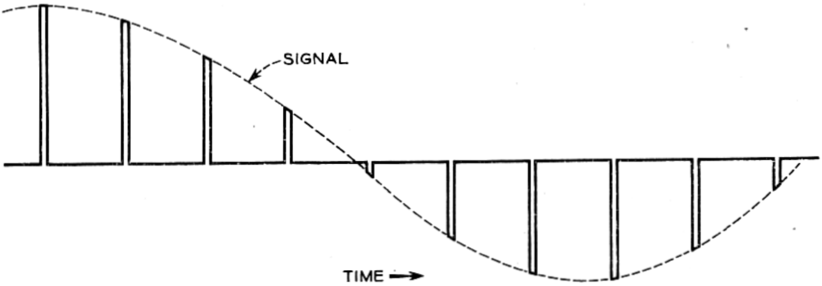

Fig. 2—PAM pulses.

If the cut-off frequency of F_2 is raised sufficiently, then in the output of this filter one finds PAM pulses clearly separated in time. These pulses are samples, on an enlarged scale, of the signal that would have existed had the cut-off frequency been in the neighborhood of B.

If now the compressor (Fig. 1) is taken into account, then the PAM pulses at the output of the sequential sampler will be impressed upon the input of the compressor. The general form of a compressor characteristic is indicated in Fig. 3. The compressor is essentially instantaneous if its bandwidth is wide enough so that it can effect the required change in the magnitude of each pulse without increasing its duration. It is also significant to note that, theoretically, no more bandwidth[12] is needed to transmit the samples after they have been compressed than before. Clearly, if the samples are compressed in accordance with an arbitrary but known law, and if the receiver expands them by an exactly inverse operation, the wanted information can be recovered.

Pulses from the sending end are fed to the transmitting medium (Fig. 1) and conveyed to the receiving terminal. This might be done by any of a number of different ways and the details of this portion of the system are not important to this discussion. What is important is that the analysis will assume that the signal at the input to the receiving end, except for noise accumulated along the way, is an exact but delayed copy of the signal leaving the transmitting end. The low-pass filter F_2' (Fig. 1) is similar to F_2 and has been inserted to reject unwanted high-frequency noise.

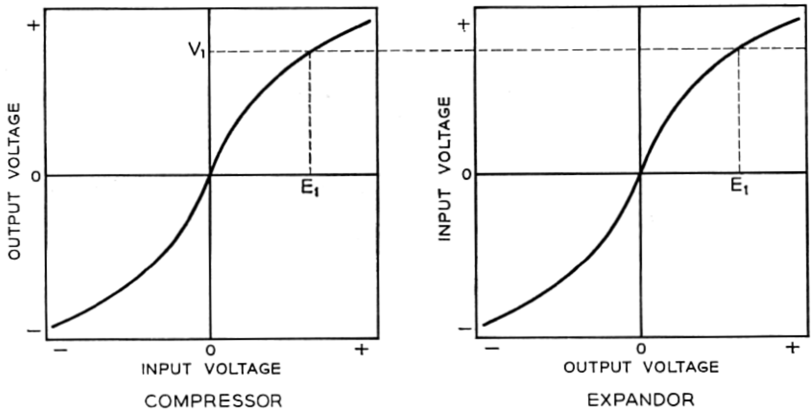

Fig. 3—Instantaneous compandor.

The filtered PAM pulses go to the input of an instantaneous wide-band expandor whose characteristic is the inverse of that of the compressor. By interchanging the designations "input" and "output" on the compressor characteristic (Fig. 3) the characteristic becomes that of an expandor. The combination of compressor and expandor makes the over-all system linear as illustrated by the dotted lines in Fig. 3. The pulses from the expandor go to the sampler which is accurately synchronized[6, 7, 10, 11]. Channel 1 pulses from the sampler go to F_3, a low-pass filter similar to F_1, and produce in the output of F_3 a copy of the original signal[12] together with noise accumulated for the most part in the transmission medium.

Signal-to-Noise Ratio

To understand how the compandor affects the signal-to-noise ratio of the system, consider a single operation at the receiver. The magnitude of a particular pulse may be represented as $V_1 + v_1$, where V_1 corresponds to the signal voltage and v_1 to the noise voltage This pulse is impressed upon the input of a wide-band expandor (Fig. 4) and produces a new pulse at the output of the expandor. It will be assumed throughout that the maximum values of expandor input and output voltages are equal, and for convenience will be taken as unity. The solid curve is the positive portion of an assumed expandor characteristic. Since V_1 represents the magnitude that the input pulse would have if the noise voltage were zero, E_1 represents the corresponding magnitude of the output pulse. When the effect of noise is taken into account, the magnitude of the output pulse is $E_1 + \Delta E_1$. This goes by way of the sampler and distributor to the input of F_3.

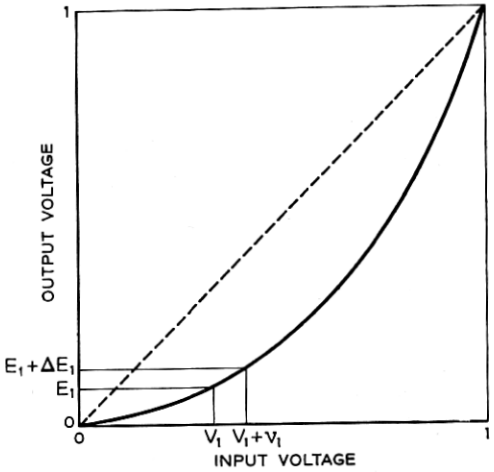

Fig. 4—Instantaneous expandor.

From the sampling principle one deduces[12] that each pulse which appears at the input to F_3 is directly proportional to the signal which occurs Δt seconds later at the output, where Δt is the delay in F_3. This delay will be neglected. Thus, in response to $E_1 + \Delta E_1$ at the input, the value of the voltage at the output of F_3 is $k(E_1 + \Delta E_1)$ where k depends upon the design details of the system. Represent instantaneous signal and noise voltages at the output of F_3 by S_1 and N_1. Then,

$$S_1 = kE_1 \tag{1}$$

and

$$N_1 = kv_1 \frac{\Delta E_1}{v_1}. \tag{2}$$

It is apparent (Fig. 4) that $\frac{\Delta E_1}{v_1}$ is a function of the slope of the expandor characteristic. Represented by s_1, this ratio is an important quantity and will be referred to as the "noise susceptibility of the system." Dividing (1) by (2) and dropping subscripts,

$$\frac{S}{N} = \frac{E}{vs} \tag{3}$$

where S/N is the ratio of instantaneous signal to instantaneous noise at the output of F_3 and $\frac{E}{v}$ the corresponding ratio without a compandor.

Noise Susceptibility

If it is assumed that the maximum signal is large compared to the noise, then

$$s = \frac{dE}{dV}. \tag{4}$$

Because the characteristic of the expandor is nonlinear, the noise susceptibility, s, varies as a function of signal input. When s is unity the noise susceptibility equals that of a linear system. The object of instantaneous companding is to make s a predetermined function of the magnitude of the signal. However, the predetermined choice is not entirely arbitrary. To avoid ambiguous signals at the receiver, as the input to the compressor varies continuously from zero to unity, the input-output characteristic must be single valued.

One notes that if s is averaged with respect to the expandor input voltage, the average value is unity regardless of the shape of the characteristic. Similarly, if $\frac{1}{s}$ is averaged with respect to expandor output voltage, the value obtained is always unity.

Important Difference Between Syllabic and Instantaneous Types of Compandors

At this point it seems advisable to emphasize an important difference between syllabic and instantaneous types of compandors. Signals compressed on a syllabic basis can be transmitted in a band not significantly different from that occupied by the original signal. Moreover, the requirements on the phase and attentuation-frequency characteristics of the path between compressor and expandor are about the same as if the signal were not compressed. Accordingly, syllabic compandors can and have been applied to a wide variety of existing types of systems.

This is not true of the instantaneous compandor. While instantaneous companding theoretically does not require an increase in bandwidth[12] between compressor and expandor, additional transmission requirements[13] which this path must satisfy usually would be regarded as very severe when this band is no more than the bandwidth of the signal entering the compressor.* This means, for example, that for practical reasons instantaneous compandors cannot be applied to existing types of single-sideband carrier telephone systems. On the other hand, if a pulse modulation or other type of multi-channel time-division system is capable of operating without compandors, the addition of instantaneous compandors will not alter matters. Of course, the net over-all transmission will change more than one db for each db change in the propagation from compressor output to expandor input, but this is true for either type of compandor and depends essentially upon the properties of the expandor.

APPLICATION OF THEORY

The theory will be used to evaluate the noise advantage of an instantaneous compandor in the PAM system shown in Fig. 1 when the signal is speech. The result is applicable[10] to other types of multi-channel pulse-modulation systems.

Choice of Expandor Characteristic

The first step is to select a suitable characteristic. If the characteristic of the compandor is logarithmic, the signal-to-noise ratio at the output of the system will be independent of speech volume. It appears reasonable for talkers of different volume to be treated alike and so a logarithmic characteristic will be chosen. Results of experimental observations on this type of characteristic will be discussed in a later paragraph.

A logarithmic compandor is one in which the output voltage of the compressor is a logarithmic function of its input voltage. Conversely, the output voltage of the expandor is an exponential function of its input voltage. This relationship may be written:

$$E = ae^{bV}$$

where a and b are arbitrary constants, E is the expandor output voltage, and V the expandor input voltage.

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* This implies suitable instrumentation which, for convenience, may utilize sampling. If a signal be compressed by an instantaneous compressor, the bandwidth occupied by the compressed signal obviously will be considerably increased. However, the information content of the compressed signal is no more than before and accordingly may be represented by another appropriate information signal whose frequency range is restricted to the bandwidth of the signal entering the compressor.

It is apparent that the characteristic cannot follow the exponential law at low values of input voltage, because, if the relationship is exponential, E is not zero when V is zero. This difficulty is avoided by using a characteristic which is linear for input voltages below a given value and exponential for input voltages above this value. A characteristic of this type is illustrated in Fig. 5. The point at which the characteristic changes from a linear to an exponential relationship is referred to as the "transition point." The transition from one function to the other occurs smoothly and the first derivative of the output with respect to the input is continuous at the transition point.

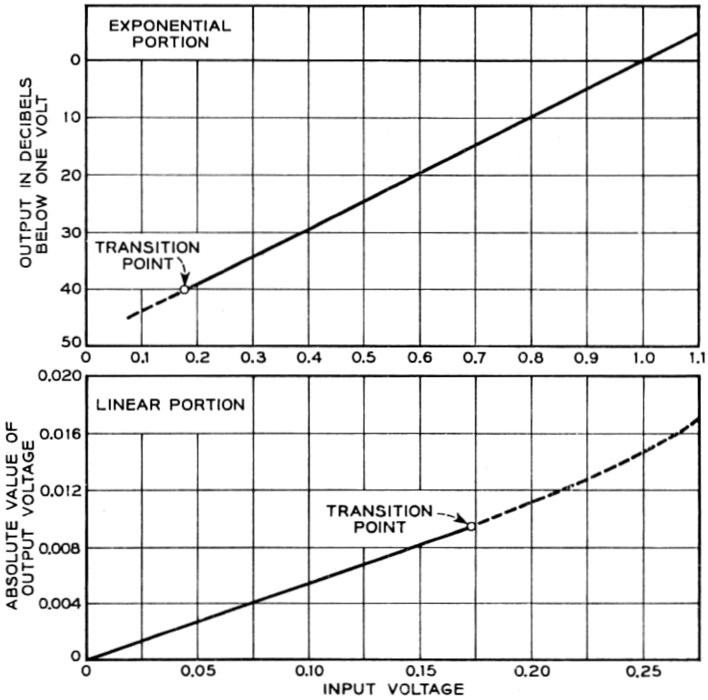

Fig. 5—Expandor characteristic.

Logarithmic Compandor

Since the characteristic of the compandor is an odd function, all formulas will be limited to the positive portion. The exponential portion of the expandor characteristic is given by

$$E = e^{(V-1)/V_s} \tag{5}$$

provided $E = 1$ when $V = 1$ and provided $\frac{dE}{dV} = \frac{E_t}{V_t}$ at the transition point where the expandor voltages are designated V_t and E_t. From (5) we write

$$E_t = e^{(V_t - 1)/V_t}. \tag{6}$$

Let "expansion ratio" be defined by the ratio of E_m/E_t to V_m/V_t where E_m and V_m are the maximum values of the expandor output and input voltages. Recall that $E_m = V_m = 1$, set K equal to the expansion ratio and write

$$K = \frac{V_t}{E_t}. \tag{7}$$

The expansion ratio may be represented as a function of V_t by replacing E_t in (7) by its value in (6), viz.,

$$K = V_t e^{(1 - V_t)/V_t}. \tag{8}$$

Expressed in decibels,

$$K(\text{in db}) = 20 \log_{10} (V_t e^{(1 - V_t)/V_t}). \tag{9}$$

When the value of K given by (9) refers to the compressor, it is called "compression ratio." This follows from the identical (Fig. 3) compressor and expandor characteristics after input and output designations are interchanged.

The manner in which K and E_t are related to V_t is given by (8) and (6) respectively. These relationships are plotted in Fig. 6. Clearly, if any one of the three parameters is fixed, the entire expandor characteristic is known.

Signal-to-Noise Ratio

Let s_2 represent the noise susceptibility of the system during intervals when the magnitude of the signal voltage is within the exponential range of the expandor. By differentiating (5) with respect to V and using (4) we get

$$s_2 = \frac{1}{V_t} e^{(V - 1)/V_t} \tag{10}$$

which relates noise susceptibility to expandor input voltage, V, which in turn equals the compressed signal voltage. To express s_2 as a function of the normal signal voltage, apply (5) to (10), viz.,

$$s_2 = \frac{E}{V_t}. \tag{11}$$

Noise susceptibility, therefore, is directly proportional to the magnitude of the signal voltage. When s in (3) is replaced by the right-hand side of (11) we get

$$\frac{S}{N} = \frac{V_t}{v}. \tag{12}$$

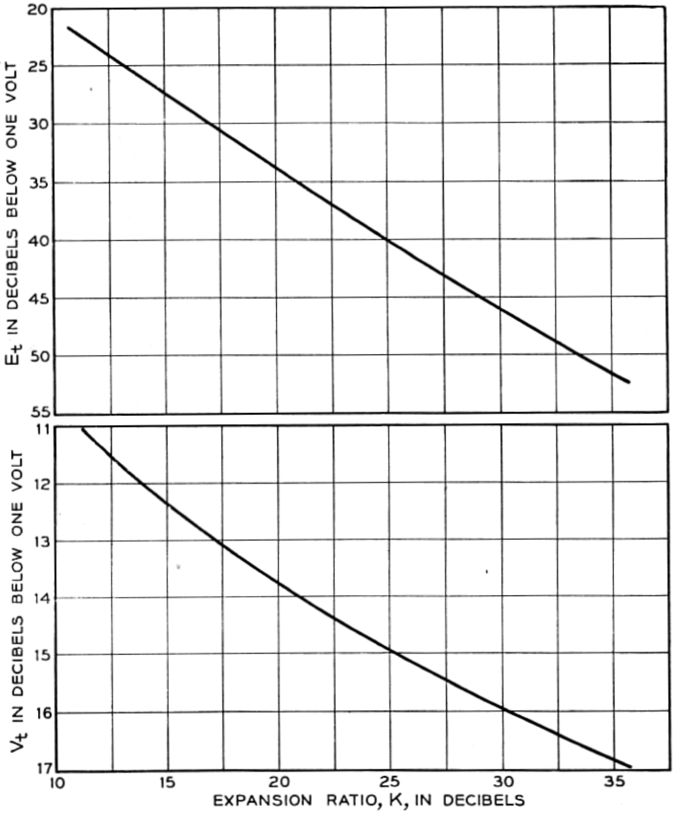

Fig. 6—Expandor parameters.

This shows that the ratio of instantaneous signal to instantaneous noise voltages at the output of the system is independent of the magnitude of the signal for voltages within the exponential range of the expandor.

Noise in the transmission medium is assumed to be fluctuation noise of uniform power density. It is convenient to replace v in (12) with the rms value of the noise voltage at the input to the expandor.[12] Designate this voltage v'_r, replace N by N_r, and (12) becomes

$$\frac{S}{N_r} = \frac{V_t}{v'_r}. \tag{13}$$

The physical significance of this ratio may be explained as follows. The voltage which appears at the output of the system after the occurrence of a PAM pulse may be represented as the sum of a noise voltage and a signal voltage. Suppose that a very large number of measurements were made of the noise voltage which occurs along with a preassigned value of signal voltage. If the magnitude of the signal voltage is within the exponential range of the expandor, then the ratio of signal voltage to the rms value of observed noise voltages would equal S/N_r in (13). The same value of S/N_r would be obtained if the measurements just described were repeated for any other preassigned magnitude of signal voltage within the exponential range of the expandor.

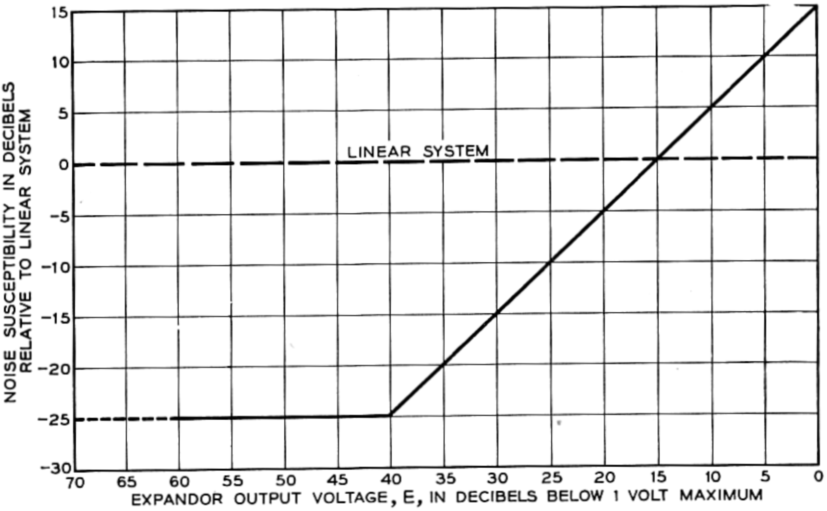

Fig. 7—Noise susceptibility.

When the voltages impressed upon the input of the expandor are within the linear range of the characteristic, the noise susceptibility, designated s_3, is equal to $\frac{E_t}{V_t}$. From (7) we write

$$s_3 = \frac{1}{K}. \tag{14}$$

This shows that, within the linear range of the compandor, noise susceptibility is inversely proportional to expansion ratio.

Figure 7 illustrates the relationship between noise susceptibility and signal voltage. Noise susceptibility (Fig. 7) is expressed in db relative to that of a linear system.

Value of S/N_r

To evaluate the noise advantage which results from the use of an instantaneous compandor, it is necessary to know what requirement to place on S/N_r. This ratio refers to noise at the output of the system during intervals when the signal magnitude is within the exponential range of the expandor. During these intervals the noise susceptibility of the system is proportional to the signal magnitude, so that the character of the noise is entirely different from that encountered in a linear system. People listening to speech transmitted through a system equipped with an instantaneous compandor have mistaken this type of noise for the distortion produced by an overloaded amplifier. Accordingly, experiments were made to determine how small S/N_r could be in a telephone channel whose frequency range was 200 to 3500 cycles.

A test circuit was devised which simulated the noise performance of a system equipped with a logarithmic compandor, and arrangements were provided so that the signal-to-noise ratio S/N_r could be adjusted over a wide range of values. The test procedure was to allow an observer to listen, during two consecutive intervals of time, to speech from the output of a linear system and from the compandor system. Conditions were arranged so that the noise at the outputs of the two systems was the same when the signal voltages were within the linear range of the compandor. The sequence in which the two conditions were presented to the observers was changed in a random manner, so that there was no way of identifying the compandor system except for the effect of the enhanced noise susceptibility during intervals when the signal magnitude was within the exponential range of the expandor characteristic. Twenty-two observers participated in these tests and different speech volumes were used covering a range of 26 db.

Experimental results showed that the compandor system could be readily identified when S/N_r was 16 db or smaller, whereas the difference between the two systems was difficult to detect when S/N_r was 24 db or greater. An acceptable value of S/N_r for a typical telephone system is therefore somewhere between these two limits. A value of 22 db* was selected as a conservative estimate. To confirm this, several people experienced in rating the quality of telephone systems were asked to listen to the output of the test circuit with S/N_r adjusted to 22 db. The consensus was that the quality was satisfactory.

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* This value is in agreement with the one used by C. B. Feldman and W. R. Bennett in studies of bandwidth and transmission performance, reported in reference 10.

Another point brought out by these tests was that the difficulty or ease with which the difference between the two systems could be detected was substantially independent of speech volume.

Noise Advantage

The value of S/N_r given above may be used to evaluate the noise advantage. Basically the problem is to find the permissible db increase in noise at the output of the transmission medium when the PAM system of Fig. 1 is equipped with an instantaneous compandor instead of linear networks having characteristics indicated by the dotted line in Fig. 4. For the comparison to be valid, the noise at the output of the system during intervals when the signal voltage is zero must be the same for the two conditions, and when the compandor is used S/N_r must equal 22 db.

Recall that v_r' represents the rms value of the noise voltage at the output of the transmitting medium when the instantaneous compandor is used, and let v_r represent the corresponding value when the linear networks are used. The noise susceptibility of the linear system is unity. Therefore, the noise at the output of the system during intervals when the signal voltage is zero will be the same for the two conditions provided $v_r = v_r' s_3$. When s_3 is replaced by its value in (14) we require that

$$v_r = \frac{v_r'}{K}. \tag{15}$$

The equation which specifies that S/N_r equals 22 db is

$$12.59 = \frac{V_t}{v_r'} \tag{16}$$

obtained by replacing S/N_r in (13) with the voltage ratio corresponding to 22 db. As shown by the lower curve of Fig. 6, V_t is a function of the expansion ratio, K. The quality of the two systems will be the same provided (15) and (16) are satisfied simultaneously.

Values of v_r, v_r', and K which simultaneously satisfy (15) and (16) are plotted in Fig. 8. Larger values of K would yield values of S/N_r smaller than the specified value of 12.59 db. Smaller values of K correspond to less noise improvement and make S/N_r larger than assumed necessary. The use of these curves will be illustrated by the following example.

It will be assumed that the rms value of the noise voltage at the output of a typical telephone channel is approximately 56 db below the highest signal voltage which the system is called upon to transmit. In the PAM system of Fig. 1 one volt was arbitrarily taken as the peak signal voltage at the output of the transmitting medium so that v_r is 56 db below one volt. From the upper curve of Fig. 8, it is apparent that the noise at the output of the transmission medium, v_r', can be 35.8 db below one volt, and the corresponding value of K is 20.2 db. The noise advantage of the compandor equals K, and is about 20 db. Figure 6 shows that, in a 20 db expandor, V_t is 13.8 db below one volt. The noise voltages at the expandor input are well within the linear range of its characteristic. Otherwise (15) would not be valid.

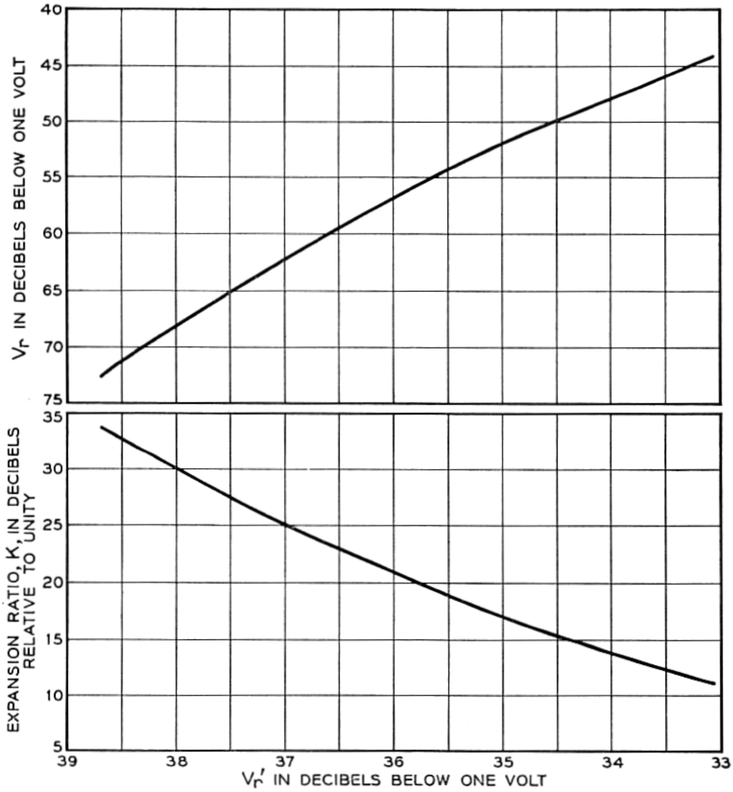

Fig. 8—System parameters used to evaluate noise advantage.

To illustrate that in the preceding example the noise advantage depends upon the noise standard, suppose the noise standard, v_r, had been 64 instead of 56 db below one volt. Proceeding as before, the noise advantage is about 26.7 instead of 20 db.

The general principles used in determining the noise advantage of instantaneous companding as applied to telephone channels presumaby may be applied with equal convenience to other systems also using instantaneous companding and operating under different conditions.

ACKNOWLEDGMENTS

The writer gratefully acknowledges the valuable assistance rendered to him in the preparation of this paper by Mr. H. S. Black, and the help of many others who participated in the experiments and offered valuable comments on the technical aspects of the analysis. These include: Messrs. C. B. Feldman, W. R. Bennett, J. O. Edson, and E. K. Van Tassel.

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