Coaxial Impedance Standards

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(*Manuscript Received Mar. 7, 1951*)

The calibrations of bridge networks used in developmental tests on coaxial cable are obtained by comparison of the networks with calculable standards of impedance consisting of a group of short-length precision copper coaxial lines. The standards are calculable by reason of the availability of precise formulae relating the distributed primary constants to the measurable physical constants and dimensions of the coaxials. This paper outlines the constructional problems and design features of a group of such standards of impedance which provide a range of values over a broad band of frequencies.

INTRODUCTION

The "mile of standard cable" was for a long time the basis for rating the transmission qualities of telephonic apparatus and networks.[1, 2] The title of this paper suggests that a return to the old standard has been accomplished. This is true in a restricted sense, but with important differences. The standards here described consist of varying lengths of a rigid coaxial transmission line structure. Their sole function is to supply primary references of resistance, inductance, capacitance and conductance which are numerically comparable to typical unknowns encountered in laboratory cable measurements. Unlike the mile of standard cable, the rigid coaxial is simple structurally, its physical constants and dimensions may be determined accurately, and precise formulae are available for translating these properties into electrical constants at any frequency. It is thus an excellent means for the objective—calculable radio-frequency laboratory standards of R, L, G, and C of the restricted numerical range needed to calibrate the bridge networks used in measurements on the short lengths of cable available to the cable development engineer.

Because developmental cables are not usually available in the longer lengths on which the secondary constants of attenuation, phase, and characteristic impedance may be measured directly, laboratory measurements on a cable sample are usually confined to determination of the four distributed primary parameters or constants. From these the secondary constants may then be calculated.

Measurement of the distributed primary constants of a given line structure is an indirect process, except under limited or restricted circumstances.

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[1] R. V. L. Hartley, "The Transmission Unit," *Electrical Communication*, Vol. *1*, No. 1, July, 1924.

[2] W. L. Everitt, Communication Engineering, pp. 101–2.

This is because (1) an impedance bridge can do no more than measure the unknown impedance which may be placed across its terminals; and (2) the line structure can be measured only by making bridge readings at its input or output terminals, from which points the true distributed series properties of the line appear to be altered by the shunt properties, and vice-versa. Statement (2) applies to all observations at the end of a section of transmission line, except when the line is very short electrically.

Assuming that accurate bridge measurements of the impedance at the terminals of a line are available, standard transmission formulae may be used to calculate rigorously the distributed primary constants. Uncertainty as to the accuracy of impedance bridge measurements led to the development of coaxial impedance standards as a means of checking the accuracy of test apparatus. As this work progressed, and the merit of the standards was more fully appreciated, it came about that the bridges were not merely checked against the coaxial standards, but instead the bridge calibrations were derived from the coaxial standards.

TABLE I

DIMENSIONS AND PHYSICAL PROPERTIES WHICH DETERMINE THE PRIMARY ELECTRICAL CONSTANTS OF ANY COAXIAL TRANSMISSION LINE

Distributed Primary Electrical Constants	Determining Physically Measurable Quantities
Series Constants, R and L	
Center Conductor	ρ, d, F
Outer Conductor	ρ, ID, t, F
Shunt Constants, G and C	
Capacitance	d, ID, ϵ
Conductance	d, ID, F_p, ϵ, F

R, L, G, and C are distributed resistance, inductance, conductance and capacitance, respectively, at any frequency, F.

ρ is the dc volume resistivity of the copper conductors which have diameters ID and d, and wall thickness t.

ϵ and F_p are the composite dielectric constant and power factor respectively, assumed independent of frequency.

Input Impedance of a Coaxial

The distributed primary constants of any coaxial with a uniform structure may be precisely computed in terms of dimensions and physical constants using formulae which have been developed by Schelkunoff[3] and others. Table I indicates the physically measurable quantities used to compute the respective distributed electrical constants, R, L, G, and C.

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[3] S. A. Schelkunoff, "Electromagnetic Theory of Coaxial Transmission Lines and Cylindrical Shields," *B.S.T.J.*, Oct. 1934.

It is the open (Z_{op}) and short circuit (Z_{sh}) input impedances which are of utility for bridge calibration work, however, and except for lines much less than quarter wave in length, Z_{op} and Z_{sh} must be computed from the distributed constants using the transmission line equations.

The propagation constant and characteristic impedance may be computed from the distributed constants by means of the equations:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}, \text{ and} \tag{1}$$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}, \tag{2}$$

where the numerical values of the distributed constants are, of course, dependent on the appropriate quantities as in Table I and the length of the line. Further, for any coaxial line terminated in an open circuit or a short circuit, respectively:

$$\frac{1}{Z_{op}} = G' + j\omega C' = \frac{\tanh \gamma}{Z_o}, \text{ and} \tag{3}$$

$$Z_{sh} = R' + j\omega L' = Z_o \tanh \gamma \tag{4}$$

Equations (3) and (4) rigorously relate the input impedances for open and short circuited far-end conditions to γ and Z_o in (1) and (2), and thus back to the physically measurable quantities of the coaxial structure. Precise values of the apparent distributed primary constants R', L', G' and C' are the final objective, as these quantities comprise the standards. As is shown, they are computed from basic data on the dielectric of the coaxials and on a single metal comprising the conductors of the coaxials.

It is of interest to note that, because of the mutual effects of the distributed constants on each other, the conductance component of the input admittance of a coaxial line becomes increasingly a function of the dimensions and resistance of the conductors of the line, as frequency is increased. Thus calculable standards of conductance are obtained which are essentially independent of losses in the insulating material used to support the center conductor.

Coaxial Standards for Laboratory Use

General Description

Although short-length coaxials have been used by the Laboratories for some years as impedance standards for cable measurements, refinements in measurement techniques have made it desirable to construct a new set of standards with very uniform components and improved structural qualities.

It is with the new set of impedance standards that this paper is chiefly concerned.

The new standards have been constructed in lengths varying from six inches to thirteen feet in increments of length so that, at a given frequency and far-end condition, eighteen standard impedances are available.

The completed standards have been provided with a permanent storage cabinet, Fig. 1, located in an air-conditioned cable development laboratory adjacent to measurement facilities. The special tools developed for construction, assembly, and use of the standards are also stored in the cabinet together with spare materials for maintenance.

Each standard consists of a seamless hard drawn copper tube $\frac{3}{8}''$ I. D. as outer conductor and a straight hard drawn copper wire, nominally No. 10 A.W.G., as center conductor. The insulation is expanded polystyrene in the form of spaced cylinders. An aluminum tube is used over the copper tube for mechanical protection but is insulated from it. Stainless steel fittings are provided at each end to exclude dust, to facilitate connection for circulation of dry air, and to provide the short-circuit necessary for Z_{sh} measurements. The properties, selection, and preparation of the three components—wire, tubing, and insulation and the provision of a repeatable method for short circuiting the coaxials are the basic problems in construction, and are discussed in the following paragraphs.

Physical Constants

The measured physical constants of the copper wire and tubing which are of interest in this application are given in Table II, and those of the expanded polystyrene insulation in Table III. Wherever practicable the absolute accuracies of the measuring instruments were checked against secondary standards of weights and measures, periodically referred to the U. S. Bureau of Standards laboratory for calibration.

Dimensions

The I. D. dimension quoted in Table II is the average of a number of tests on end samples of tubing and was obtained from dimensional and weight relationships as expressed by the equation:

$$D = \sqrt{D_o^2 - \frac{4V}{\pi \ell}}, \tag{5}$$

where V is the volume of copper in the sample as measured by the displacement technique, D_o is the measured O. D. of the sample and ℓ is its length.

The I. D. was also determined by direct measurements of O. D. and wall thickness, and agreement obtained with the figure quoted for the above

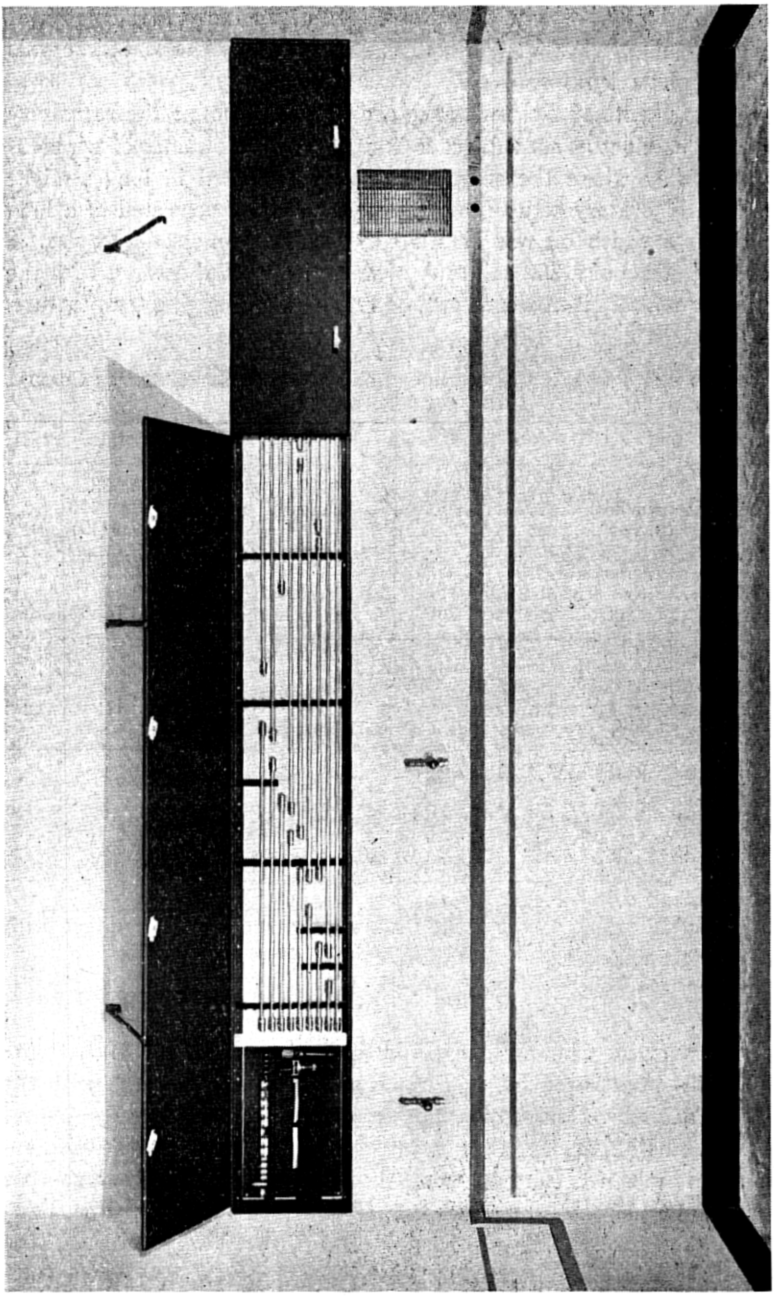

Fig. 1—Coaxial impedance standards and storage cabinet.

method. Variations of weight and dimensions along individual lengths of either wire or tubing are not significant as determined by checks at intervals on a number of the lengths.

The measurements of d-c. resistance needed to determine the resistivity of the copper conductors are subject to limitations of knowledge of temperature, particularly where the samples must be measured in long straight lengths. The laboratory setup used to obtain the data consisted of a long trough through which oil was circulated over the immersed samples. A feature was the use of a 200 gal. tank of water in thermal contact with the oil circuit to maintain it at a constant, or very slowly changing temperature.

TABLE II

MEASURED AVERAGE PHYSICAL CONSTANTS OF COPPER TUBING AND WIRE FOR COAXIAL IMPEDANCE STANDARDS

	Wire	Tubing
Density (gm/cc)	8.89	8.938
Outside Diameter (inches)	0.10042	0.50016
Inside Diameter (inches)	—	0.37137
Wall Thickness (inches)	—	0.06439
Volume Resistivity (microhm cm)	1.7480	1.7209
Mass Conductivity (per cent of IACS)	98.76	99.76
Temperature Coefficient of Resistance, 20°C.	0.003886	0.003938

TABLE III

MEASURED AVERAGE PHYSICAL CONSTANTS OF EXPANDED POLYSTYRENE INSULATING CYLINDERS FOR COAXIAL STANDARDS

Volume Expansion Ratio	41
Weight (gm/disc)	0.1789
Length (inches)	0.396
Density (gm/cc)	0.0257
Dielectric Constant	1.033

A Kelvin double bridge used for the tests was operated in accordance with minimum-error principles.[4]

Components-Wire

The center conductor of each coaxial must be drawn staright so that only light spaced support need be used to keep it in axial alignment with the tube. Since available commercial wire drawing machines normally depend on a driven small-diameter capstan to pull the wire through the final reducing die, it was necessary to draw the wire in the laboratory so that straight-out drawing could be achieved. Commercial machines were, however, used to reduce the supply from $\frac{3}{8}''$ rod to $0.110''$ dia. wire without

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[4] Electrical Measurements, Laws, McGraw-Hill, 1917.

intermediate annealing. The laboratory operation consisted of drawing the 0.110″ stock straight out through a 0.1004″ I. D. diamond die pre-selected as to circularity and finish of the bore.

The copper which was used is known in the trade as "electrolytic tough pitch" chemically composed of 99.95 per cent copper, 0.02 per cent oxygen and 0.03 per cent divided between six other minor contaminants.

There has always been some concern as to the possibility that a wire drawn from annealed stock might well have a thin full hard shell over a relatively annealed core. The d-c. resistivity measurements would then determine a weighted average resistivity, instead of the surface resistivity needed for calculations. To settle this point, tests were made on full-hard wire, semi-hard wire as used in commercial coaxials, fully annealed wire, and on annealed wire plated with a 0.2 mil layer of silver. The tests consisted of (a) comparing the a-c. resistance of the wires by precise methods at 1 mc where transmission is at a skin depth of 2 mils, and (b) microscopic study of grain structure of polished-etched cross sections of the samples at magnifications to 2000 diameters. The conclusion from both studies was that no thin skin exists. The wire is therefore treated as homogeneous throughout its cross section in computation of d-c. resistivity and in computation of a-c. resistance.

Copper Tubing

The effects on the resistance of a coaxial traceable to the physical constants of the outer conductor are scaled down about 5:1 so that the requirements on the tubing are not so severe as on the wire. However, stock copper tubing from a distributor's warehouse cannot normally be used. Such tubing may have unacceptable inside surface roughness, ellipticity of bore, and a high and variable resistivity. Roughness and ellipticity are the result of worn plug dies frequently used in the drawing of commercially acceptable tubing, or omission of the plug die in drawing the tubing to final diameter. Most stock tubing contains phosphorous and, even though the percentage of phosphorous may be very small, the effect in increasing the resistivity is marked. The full-hard tubing used in the coaxial standards here described was procured directly from a mill, and was largely drawn in consecutive lengths from a single casting of oxygen-free electrolytic copper using selected dies.

Insulation

Expanded polystyrene is the dielectric material used in the standards and its applicable properties are shown in Table III. In solid form it has a dielectric constant, ϵ', about 3% greater than that of air and, when used to insulate the coaxial standards in the form of spaced cylinders, the composite dielectric constant is increased only about 0.4% over theoretical for air. Errors in determining the constant of the expanded material are thus scaled down by a factor of nearly 10:1 in their effect on the composite constant. The figure quoted for dielectric constant in Table III was obtained by adding incremental amounts of dielectric to a 12″ length of standard coaxial and plotting the measured capacitance as increments above the computed capacitance for air dielectric, as in Fig. 2.

The distributed conductance, G, is derived from the power factor of the dielectric which, in the case of any reasonably good material, is so small that experimental determination is subject to large error. The apparent conductance G′ from (4) is the value at the input terminals of the coaxial. An important feature of the standards is that G′ becomes independent of G at high frequencies and, therefore, it is desirable to reduce G (representative of the loss in the dielectric) to as small a value as possible in order that G′ may become independent at the lowest possible frequency. This is accomplished by the use of expanded polystyrene as the dielectric of the standards.

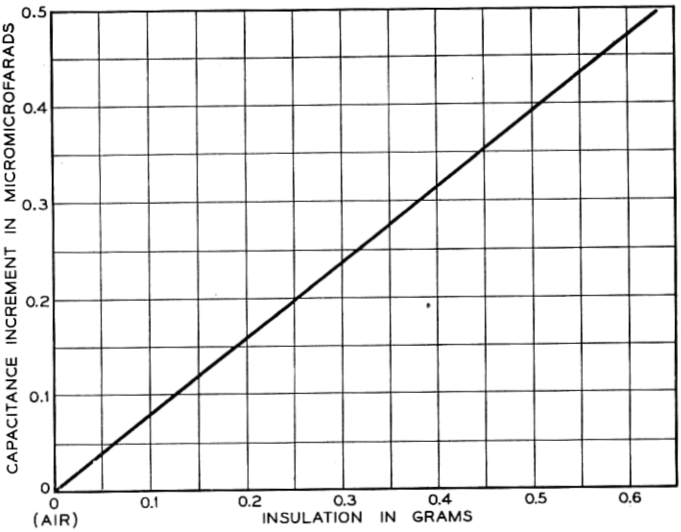

Fig. 2—Data for determination of dielectric constant of expanded polystyrene.

Polystyrene is of such molecular structure that it is not hygroscopic. However, under certain conditions, water vapor may condense in cells of the expanded material in sufficient amount to increase the dielectric constant and the conductance losses. For this reason all insulating cylinders have been pierced longitudinally so that low pressure dry air may be circulated through the assembled coaxials when they are in use.

The cylinders were cut with a high-speed fly cutter, and the center hole drilled in the same operation. It was determined by sensitive electrical tests that centering precision in the assembled coaxials was equivalent to that obtained in coaxials with lathe turned discs of solid materials.

Cylinder spacing of 3″ on centers was determined as about the maximum permissible to prevent detectable sag in the center wire between points of support. There is no specific strength requirement on the cylinders except that they must support the weight of the straight drawn wire. However, a single polyethylene disc is used at the test end to resist radial thrusts which may occur in making test connections.

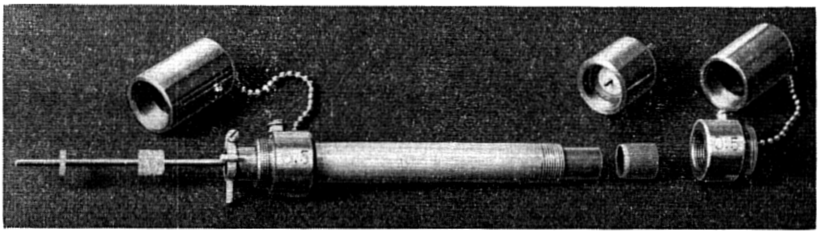

Fig. 3—Partially assembled coaxial standard.

Assembly Features

It is necessary to have mechanical protection for the copper tubes of the standards, and this is provided for each by an aluminum tube slipped over the copper tube, with insulation consisting of a helical wrap of 0.0015 x $\frac{1}{2}$″ paper ribbon first applied to the copper tube. The aluminum tube is locked on the copper tube by short lengths of fibre tubing wedged between at each end. The wedging action occurs when the end fittings are screwed to the aluminum tube. Figure 3 is a photograph of a partially assembled standard to illustrate further the construction. The presence of the aluminum tube may be disregarded in so far as its electrical effects are concerned because of the self-shielding qualities of the copper tube.

End Effects

Experimental data indicate that if a coaxial tube is longer than its center conductor and the wire is then lengthened incrementally until it is as long as the tube, the capacitance of the coaxial remains directly proportional to the length of the wire to better than 0.01 $\mu\mu$f. The "fringing effect" at an open-end of a coaxial is considered negligible.

The individual center wires of the standards have been cut 0.500″ longer than the tubes and the extra length utilized at the test end for connection to the test equipment. In computing the impedances, the length of the tube has been used. The 0.500″ center wire projection is considered as part of the test-equipment leads and is separately accountable.

Disc Short Circuiting of Coaxial Standards

A shortcoming of coaxial standards previously developed has been in the use of solder to attach a short circuiting disc at the far end when using the length to provide calculable series impedance (Z_{sh}). The use of a disc is the best means to short-circuit the inner and outer conductors with a minimum and calculable terminal impedance, and the accomplishment of this objective by repeatable mechanical means is an important feature of the new standards.

A very thin disc pressed against the end of a coaxial is effectively contacted along two concentric rings representing the peripheral edges of wire and tube of diameters *d* and *D* respectively. The d-c. resistance of the metallic area between the rings in terms of its resistance, *r*, in ohms per square is:

$$R = \frac{r}{2\pi} \int_{d/2}^{D/2} \frac{dx}{x} \tag{6}$$

(6) reduces to:

$$R = \frac{r}{2\pi} \ln \mathrm{D/d}. \tag{7}$$

Figure 4 shows a cross-section of the short-circuiting device developed for the standards. Figure 3 includes a view of the assembled device. It consists of a stainless steel housing carrying a trapped silver disc and other parts to effect a repeatable, minimum-impedance short circuit when the housing is screwed to the end fitting of the coaxial. The operation is as follows:

(a) When the housing is screwed on the end fitting, the disc is forced against the end of the copper tube, with pressure equalized by and derived from the rubber grommet.

(b) Assuming the housing is always screwed on the end fitting as far as it will go, the total pressure on the disc is regulated on initial assembly by control of the amount of projection of the tube beyond the end fitting.

(c) The center wire is drilled and tapped to a depth of $\frac{5}{16}''$ at one end to accept the 0–80 screw carried by the stud. After the disc is tightly pressed against the tube as in (a), the center wire screw stud is turned until the center of the disc is drawn tightly against the end of the center conductor. The stud projects from the end of the housing and is milled at its end to accept a torque wrench. The wrench provides the means for development of repeatable pressure of the end of the wire against the center of the disc.

Experimental data on the relative pressure versus d-c. resistance characteristics are illustrated by the curves of Fig. 5. Disc pressure on the center wire is controlling as would be expected. The total d-c. resistance introduced by the short circuit is of the order of 0.1 milliohm, which includes the contact resistance. On the basis of experimental observation the resistance at radio frequencies appears to be more stable because the area of contact becomes of less importance as frequency is increased. The inductance introduced by the disc is negligible for all lengths and frequencies. The a-c. resistance of the disc is equal to its d-c. resistance up to about 1 mc where complete wave penetration ceases and skin effect becomes apparent. Above 2 mc the disc resistance increases as the square root of frequency.

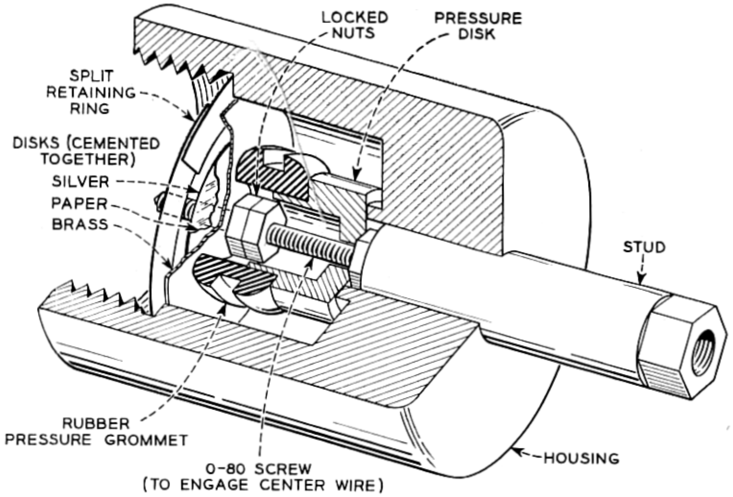

Fig. 4—Assembly detail, end-shorting device.

Connection—Test End

A connector currently in use at the test end of the outer conductor is shown in position in the photograph, Fig. 3. The projection normal to the axis of the assembly is used for clamp-type connection. A guillotine clamp for short-circuiting the input of the coaxial, as required in substitution type measurements, is placed across the center wire and the projection parallel to the axis of the assembly. The center conductor of each coaxial extends 0.500″ beyond the tube, and is thus available for clamp type connections. Soldered connections are thus completely eliminated in the use of the standards.

Eccentricity

Concentricity of the axes of wire and tube is assumed in most published formulae which relate the dimensions of a coaxial structure to its electrical constants. Consistent departures from concentricity, i.e. eccentricity, are subject to analysis as regards effects on the electrical constants. In the case of the standard coaxials, the use of straight, hard-drawn wire and tubing combined with close-spaced support of the wire are all factors which reduce eccentricity. However, it is obviously desirable to check the degree of residual eccentricity of the assembled coaxials point by point along each length. A method developed to do this makes use of Biot and Savart's law.[5]

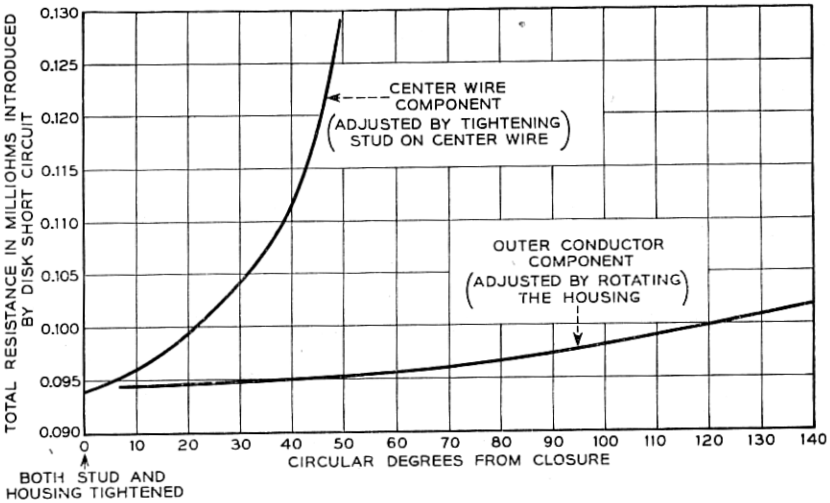

Fig. 5—Measured d-c resistance introduced by end shorting device as a function of adjustment.

The external field, at any point P, of a long circular wire or tube carrying a current I is given by

$$H = \frac{2I}{r} \tag{8}$$

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[5] W. R. Smythe, Static and Dynamic Electricity, p. 272.

where r is the distance from the axis of the wire or tube to point P. This assumes that the current density on any concentric circle is uniform.

If the wire and tube are carrying the same current I but in opposite directions and if the axes of the wire and tube coincide, there will be no magnetic field at any external point. Therefore an alternating current in the conductors will not induce a voltage in a pick-up coil placed external to the coaxial. If, however, the two axes do not coincide a voltage will be induced in the pick-up coil and will be a maximum when the pick-up coil is in the plane of the two axes. The magnitude of this maximum voltage will be proportional to the distance between the two axes if the measuring distance is large compared to this separation distance.

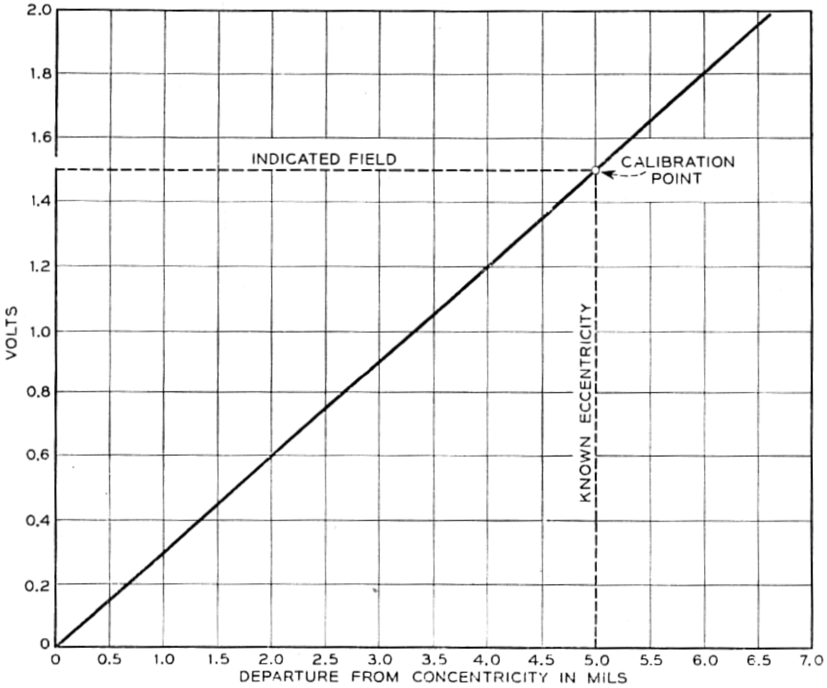

Fig. 6—Typical calibration curve in measurements of eccentricity of coaxial standards.

In practical use, the maximum field at P was measured in terms of volts relative to a known eccentricity obtained by insulating a coaxial with discs of the known eccentricity. Advantage was taken of the linear relationship of eccentricity and detectable field so that, with the distance to the tube maintained constant, the calibration curve of Fig. 6 was used to evaluate all standard coaxials after final assembly. The sensitivity was such that the effect of an 0.08 mil known deviation in diameter of the wire was detectable in coaxials with otherwise perfect symmetry. Such deviations in symmetry as were observed developed primarily from variations in wire diameter as already stated, and from deviations in wall thickness of the copper tubing.

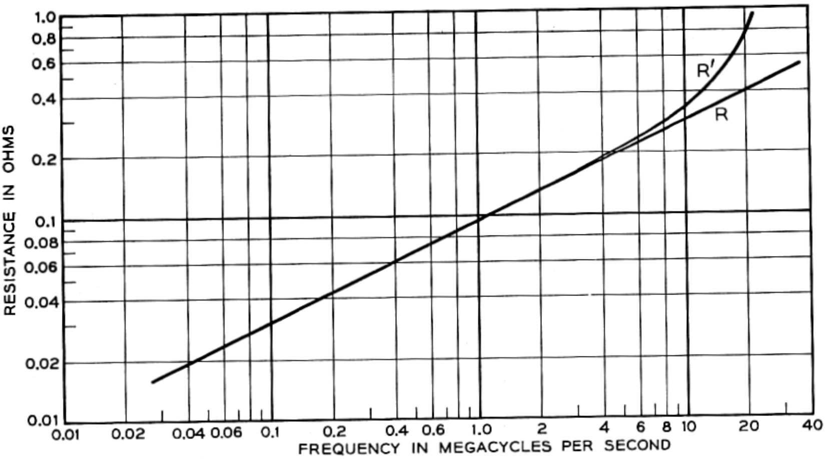

Fig. 7—Distributed resistance, R, and R' component of Z_{sh} of 7.0 ft. length, coaxial impedance standard.

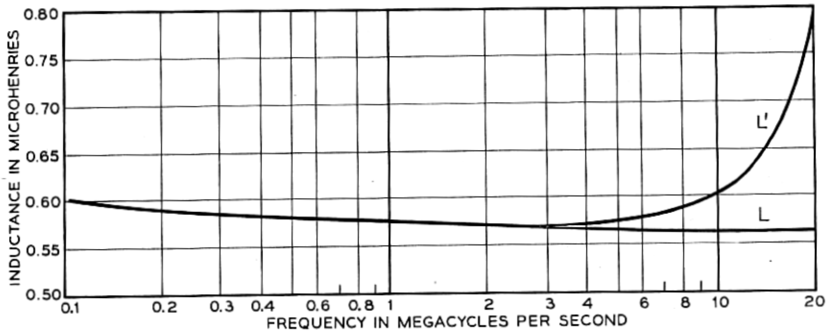

Fig. 8—Distributed inductance, L, and L' component of Z_{sh} of 7.0 ft. length, coaxial impedance standard.

Numerical Values of Standard Impedances

Graphical Example

As an example, values for the apparent series and shunt primary constants R′, L′, G′ and C′ for a length of 7.0 feet are presented graphically in Figs. 7–10. For comparison, the distributed primary constants have also

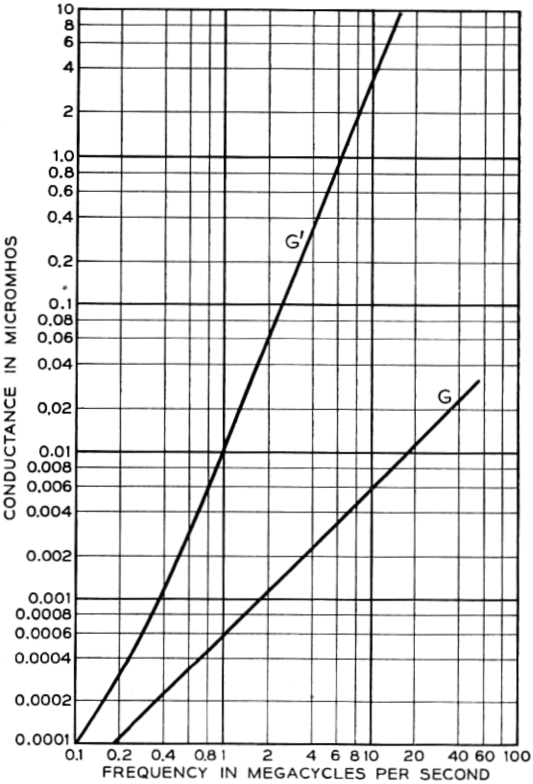

Fig. 9—Distributed conductance, *G*, and *G'* component of Z_{op} of 7.0 ft. length, coaxial impedance standard.

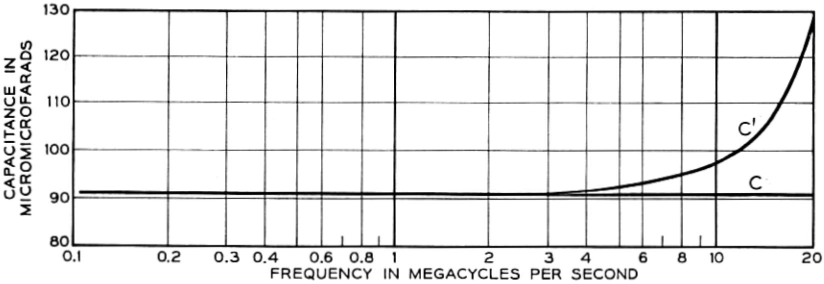

Fig. 10—Distributed capacitance, *C*, and *C'* component of Z_{op} of 7.0 ft. length coaxial impedance standard.

been plotted. It should again be emphasized that it is the "primes" which are available at the terminals of each standard for calibration purposes.

The plotted values of the primes were computed for a frequency range of 50 kc to 20 mc using equations (3) and (4). From an error standpoint it is not practicable to use lengths much longer than 7 ft. as standards at 20 mc. However, shorter lengths of which there are twelve may be used to provide standards at higher frequencies. There are six longer lengths, and the entire group of eighteen may be used to provide eighteen numerical values of each "prime" in the frequency range of 50 kc to 10 mc. There is the restriction, from the standpoint of application, that the impedance components are available only in "pairs"; each R′ has a definite L′ associated with it, or each G′ has a value of C′ in association.

Error Considerations

Examination of Figs. 7–10 shows that the "primes" as computed from (3) and (4) exceed the distributed constants by a gradually increasing amount as frequency is increased. Except for G′ the excess is largely proportional to the product $\omega^2 LC\ell^2$, where ℓ is the length of the line, and L and C are in terms of unit length. The total error in a "prime" at a given frequency is then approximately the error in the distributed value combined with the proportioned error in $\omega^2 LC\ell^2$. Table IV shows the computed errors for the 7.0 ft. standard at 1 mc where the contribution of $\omega^2 LC\ell^2$ is small and at 20 mc where it is relatively large. The errors quoted were computed from estimated errors involved in determination of the various physical quantities associated with the constants of Tables II and III.[6]

TABLE IV
ESTIMATED ERRORS IN COMPUTED INPUT IMPEDANCE COMPONENTS OF 7.0 FT. LENGTH COAXIAL STANDARD

Input Impedance Component	Per Cent Error (±)	
	1 mc	20 mc
R′	0.05	0.1
L′	0.04	0.1
G′*	2.	0.3
C′	0.06	0.1

* Assumes possible error of ± 25% in knowledge of power factor of the dielectric material.

G′ was mentioned above as an exception. This results from the fact that G′ is largely proportional to $\omega^2 C^2 R\ell^3$ above 1 mc. That is, the excess of G′ over G rapidly increases with frequency so that the value of G may be neglected above 1 mc. The precision of G′ is that of the determination of $\omega^2 C^2 R\ell^3$, a quantity which can be determined with very good precision as

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[6] "Electrical Measurements, and the Calculation of the Errors Involved," D. Karo, Macdonald & Co., London.

compared to determination of the power factor of the dielectric of the standard which is controlling in determination of G. Table IV shows the effect of these circumstances in the case of the 7.0 ft. length. The error in G' is computed as less than 2% at 1 mc. for 25% error in power factor determination. At 20 mc, G is less than one thousandth of G' and, as indicated in the table, G' has an error of 0.3% due entirely to errors in $\omega^2C^2R\ell^3$.

ACKNOWLEDGEMENT

The writer is greatly indebted to Mr. M. C. Biskeborn who contributed much valuable advice during the development of the standards.