Interaxial Spacing and Dielectric Constant of Pairs in Multipaired Cables

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A major handicap in the evaluation of different designs and manufacturing processes, in respect to efficiency of space utilization inside the sheath of multi-pair telephone cable, has been the lack of a simple and accurate method of measuring the dielectric constant of such cable pairs. This paper describes a simple non-destructive method of determining both the interaxial spacing between conductors of a cable pair and the dielectric constant. An important by-product of the work is the demonstration of the fact that $\epsilon = L \times C$ is not a valid means of determining the dielectric constant of cable pairs.

Introduction

A cable pair consists of two individually-insulated conductors, of nominally equal circular cross-section, which have been twisted together in a long helix and stranded into a cable core with similar pairs. It has not been possible to analyze rigorously the electrical characteristics of such a circuit in terms of its rather complex physical configuration. For this reason, methods largely of an empirical nature have been used in the past to correlate physical and electrical characteristics of multipaired cables.

The capacitance of any system of conductors immersed in a homogeneous medium is directly proportional to the dielectric constant of the medium. The dielectric of a cable pair is not homogeneous, but it can be described in terms of a homogeneous dielectric which would produce the same capacitance. In addition to the dielectric properties of the insulating medium, the capacitance of a cable pair is determined by the disposition of the paired conductors with respect to each other and with respect to the surrounding pairs or sheath.

In particular, the interaxial separation between the wires of a pair has a critical effect on capacitance. The interaxial separation is determined by the ability of the insulation to resist deformation due to compressive forces encountered in cabling operations. Thus, the capacitance of a cable pair is largely dependent on the mechanical and dielectric properties of the conductor insulation, and a criterion of the relative efficiency of an insulation is the capacitance level, for a particular conductor gauge, resulting from a given cable space allowance or space-per-pair.

Experience with paper ribbon and paper pulp insulated cables has shown that occasional wide deviations in capacitance can occur even though the space-per-pair allowance is substantially constant. The aforementioned empirical methods of capacitance-space analysis do not provide any insight as to whether these deviations are due to anomalies in the mechanical properties or in the dielectric properties of the insulation, or both.

With respect to some electrical and physical characteristics, it is evident that there is a close analogy between the cable pair and an "ideal" balanced shielded pair consisting of two straight and parallel solid cylindrical conductors enclosed in a cylindrical conducting shield, with the center line of the pair coinciding with the axis of the shield. A cross-section of such a circuit is shown in Fig. 1. The conductors are insulated from one another and from the shield by a homogeneous dielectric.

Rigorous mathematical expressions for the capacitance and inductance of the ideal pair in terms of its dimensions and dielectric constant have been

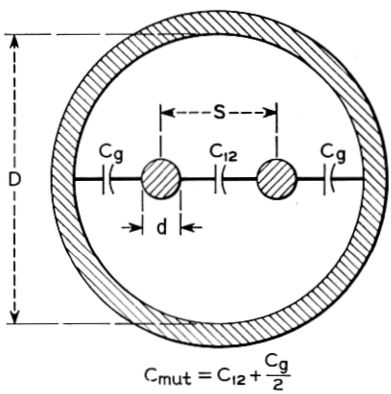

Fig. 1—A balanced, shielded pair.

derived by the Mathematical Research Group of the Laboratories and others. Measured values of the capacitance and inductance of a given cable pair, when substituted into these expressions, determine a set of dimensions and a dielectric constant value which describe an ideal pair having the same capacitance and inductance as the cable pair. The extent to which these idealized values represent actual cable conditions depends on the accuracy of the assumed equivalence of the two structures.

The purpose of this paper is to describe a few simple but direct experiments, the results of which demonstrate that these idealized parameters are closely representative of actual cable conditions in so far as interaxial spacing and dielectric constant are concerned. Thus emerges a simple technique, based on easy-to-make low-frequency measurements, for quantitative evaluation of these two important cable pair parameters. Applications of the method to commercial designs of multipaired cable are included in what follows.

Low Frequency Inductance and Interaxial Spacing

The self-inductance of a pair of straight parallel wires in free space or within any non-magnetic shield is a function only of the ratio of conductor diameter to the interaxial spacing if the frequency is so low that proximity, shielding, and skin effects are negligible. The formula is as follows:

$$L = 1.482 \log \frac{2S}{d} + 0.1609 \text{ (mh/mi)} \tag{1}$$

S = interaxial spacing

d = conductor diameter

The first term gives the self inductance due to net external flux-linkage and the 0.1609 constant represents flux-linkage within the non-magnetic conductors.

Some textbooks give the formula with $(2S\text{-}d)/d$ as the argument of the logarithm. This form is not valid when d is not small compared to S, as is the case with cable pairs. The derivation of formula (1) in Russell's "Alternating Currents" shows that it is valid for any S, provided only that the current is uniformly distributed across the conductor cross-section.

While it was believed that formula (1) was valid for cable pairs at, say, 1000 cps, it was desirable to establish experimentally whether or not the twist in the pair, and magnetic coupling between pairs, affected the measured inductance at a frequency in this range. The effect of coupling between pairs is greatest when all of the cable pairs except the pair under test are shorted together at both ends of the cable, thus providing a large number of closed loops for any induced currents. In making laboratory inductance tests at 1000 cps on lengths of 19-gauge cable with 0.084 μf/mi capacitance (Type CNB) it was found that opening or shorting the surrounding pairs did not affect the measured inductance. Also, grounding or floating the far end of the test pair made no difference. The tests were repeated with the same results on lengths ranging from 300′ to 5000′.

It should be noted that a correction term must be added to the measured 1000 cps inductance (L') to obtain the true distributed inductance (L) when the cable length is such that propagation effects become appreciable. When the correction term is included, the equation for L at 1000 cps becomes:

$$L = L' + 1/3(R')^2C'$$

Where L', R', and C' are measured inductance, resistance, and mutual capacitance, respectively.

Expressed as a percentage of L per length, the correction term varies as the square of length. It is only 0.5% for 750′ of CNB but is 20% for 5000′ of the same type of cable. The single correction term given is not sufficiently accurate for lengths of CNB greater than about 5000′. The comparable limiting length for smaller gauges would be less; it is about 1000′ for 26-gauge pairs.

It is also necessary to allow for stranding takeup effects when converting from per length to per mile values. Stranding takeup is defined as the increment in length of a given pair compared to the cable length, due to the helix introduced in the pair during the cabling operation. It depends upon the size of cable and stranding lay and is negligible for cables of 50 small gauge pairs or less but causes an increase of 2.7% in pair length over cable length for pairs in the outside layer of a full size CNB cable ($2\frac{3}{8}''$ diameter over the paper wrapped core).

TABLE I

COMPARISON OF MEASURED 1000 CPS INDUCTANCE WITH THAT CALCULATED FROM THE THEORETICAL FORMULA

Spacing (S) in mils	d/2S	Calculated L mh/mi	Measured L mh/mi	% Difference
13 Gauge Wire				
75.6	0.474	0.641	0.641	0.00
106.2	0.337	0.860	0.856	−0.46
145.1	0.247	1.060	1.063	+0.28
19 Gauge Wire				
38.8	0.464	0.656	0.656	0.00
69.0	0.261	1.027	1.021	−0.60
Theoretical Formula				
$L = 1.482 \log 2S/d + 0.1609$ (mh/mi)				

Some short lengths of pair with wires parallel and approximately straight and with accurately known dimensions were made up in the laboratory. Inductance measurements at 1000 cps on these pairs agree very closely with inductances calculated from formula (1), for $d/2S$ ratios from about 0.25, which is close to the nominal value for cable pairs, up to nearly 0.50, which is the highest possible value. These results are shown in Table I. The 19-gauge pair with a $d/2S$ ratio of 0.261 was twisted fixed carriage style under constant tension. There was no measurable change in inductance for twist lengths as short as 2″, compared with the straight, parallel condition. Measurement accuracy was about ±0.25%.

The foregoing results lead to the conclusion that the effective $d/2S$ ratio along the length of a cable pair can be determined from formula (1) using 1000 cps inductance, with an accuracy equal to that of the inductance tests. Since d is generally known or can be found with greater precision than the inductance, S can likewise be found with an accuracy dependent on the precision of the inductance tests. This conclusion applies regardless of the type of conductor insulation. However, it should be noted that formula (1) would not be accurate if there happened to be magnetic materials in close proximity with the pair under test, such as, for instance, pairs surrounding a core of steel tape bound coaxials.

The Dielectric Constant

The dielectric of a cable pair consists of a non-homogeneous mixture of solid insulating material and air. The dielectric constant of a cable pair can be defined as the ratio of the actual mutual capacitance of the pair to the mutual capacitance which would result if the solid insulating material were removed leaving a 100% air dielectric in the otherwise undisturbed cable. The problem is, of course, to find out what the mutual capacitance would be with an air dielectric, or with any other homogeneous dielectric of known dielectric constant. Because of the complex configuration of the cable structure it is not calculable.

The shielded balanced pair (referred to herein as the "ideal" pair) structure and its components of mutual capacitance are illustrated in Fig. 1. Although the cable pair structure is different from that of the ideal pair, it has essentially the same components of mutual capacitance. The static shield for the cable pair is not solid and perfectly cylindrical but consists of the other cable pairs or sheath which are immediately adjacent to the pair under consideration. The direct capacitances of the two wires to this ground or shield are very nearly equal.

Rigorous mathematical formulas have been derived by Mrs. S. P. Mead of the Bell Telephone Laboratories for the mutual capacitance (C_{mut}) and the capacitance to ground (C_g) of the ideal pair. These formulas are given in Appendix I. For discussion purposes they can be written as follows:

$$C_{\text{mut}} = \frac{\epsilon}{f_m\left(\frac{d}{2S}, \frac{S}{D}\right)} \tag{2}$$

$$C_g = \frac{\epsilon}{f_g\left(\frac{d}{2S}, \frac{S}{D}\right)} \tag{3}$$

where $\epsilon =$ dielectric constant and f_m and f_g are the functions defined in appendix I.

Dividing equation (3) by equation (2):

$$C_g/C_{\text{mut}} = \frac{f_m\left(\frac{d}{2S}, \frac{S}{D}\right)}{f_g\left(\frac{d}{2S}, \frac{S}{D}\right)} \tag{4}$$

Thus, the C_g/C_{mut} ratio is independent of the dielectric constant, being a function only of the dimensional ratios describing the ideal pair configuration. The ratio $d/2S$ also appears in formula (1). Therefore, knowledge of L, C_g, and C_{mut} for an ideal pair is sufficient to find its dimensional ratios and dielectric constant, using the following procedure:

1. Equation (1) is solved for $d/2S$.
2. Knowing C_g/C_{mut} and $d/2S$, equation (4) is solved implicitly for S/D.
3. Knowing C_{mut}, $d/2S$, and S/D, equation (2) is solved for ϵ.

Curve sheets to facilitate the solution of equations (2) and (4) for the applicable ranges of $d/2S$ and S/D are shown in Figs. 2 and 3.

When this same procedure is applied to a cable pair using its measured 1000 cps L, C_g, and C_{mut}, the dielectric constant value obtained is truly representative of the pair in accordance with our definition of dielectric constant, only if the ideal and actual structures are equivalent. An absence of rigor in this method would be expected due to differences in configuration, and non-homogeneity in the cable dielectric.

The magnitude of the error in the dielectric constant of a cable pair, when determined by this method, was evaluated by comparison of tests on a cable having a known, homogeneous dielectric with tests on an identical cable structure having a non-homogeneous dielectric but a known dielectric constant.

These tests were made on a short length of cable containing twenty-two 19-gauge pairs. The conductors were insulated with solid polyethylene. The core wrap was polyethylene tape under an alpeth* sheath. A low molecular weight polyethylene compound known as DXL-1 has the same dielectric constant as solid polyethylene (2.26). The compound is in a semi-solid state at room temperature and flows easily at 150° Fahrenheit. By filling the interstitial air space in the cable with DXL-1, a cable having a homogeneous dielectric with a dielectric constant of 2.26 is obtained. The dielectric constant of the cable structure before filling with DXL-1 is found from the ratio of mutual capacitance before and after filling, and a check on the accuracy of the ideal pair formulas as applied to cable pairs is available.

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These objectives were not realized with the hoped-for finesse, because of difficulty encountered in filling the interstitial air space with DXL-1. Both vacuum and pressure injection methods were tried, and the best that was

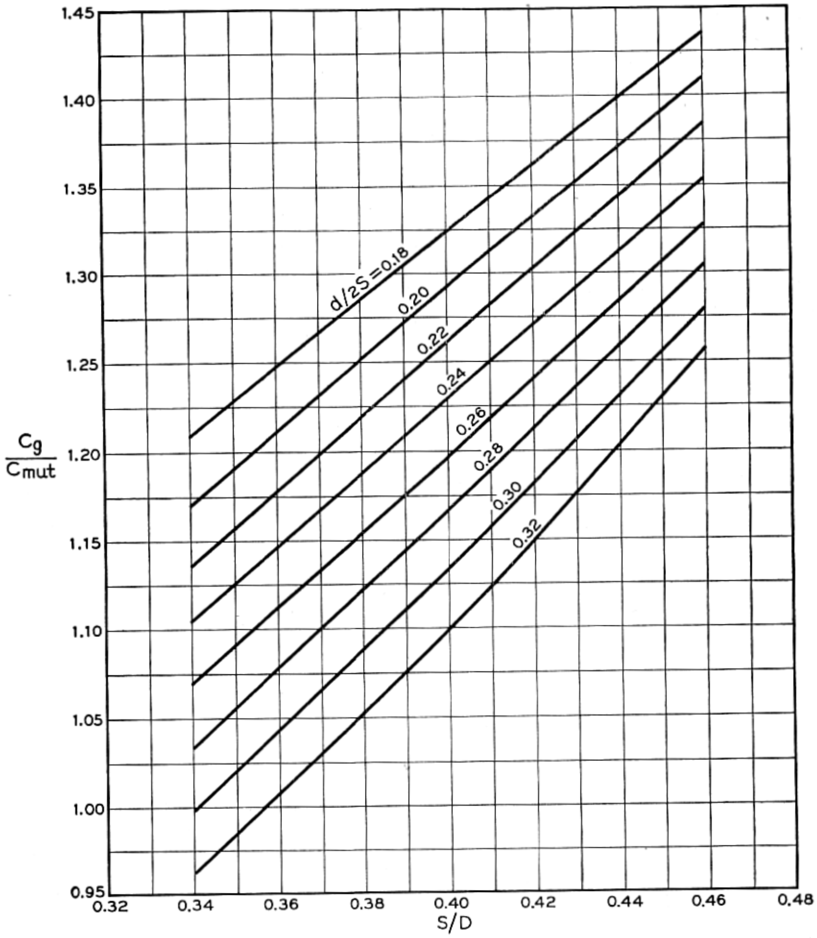

Fig. 2—Cg/C_{mut} as a function of d/2S, S/D.

obtained was a cable with from 85 to 90% of the interstitial space filled with DXL-1, or about 93 to 95% of the total dielectric space filled with polyethylene. These percentages are based on the measured increase in cable weight as a result of filling with DXL-1, and the air volume before filling calculated from nominal insulated conductor and core diameters. Evidently the cable did not fill completely, because small bubbles of either air or ethylene gas formed during the injection process and would not "wash out" by continued flow. While these results are short of the objective, they are close enough to be useful.

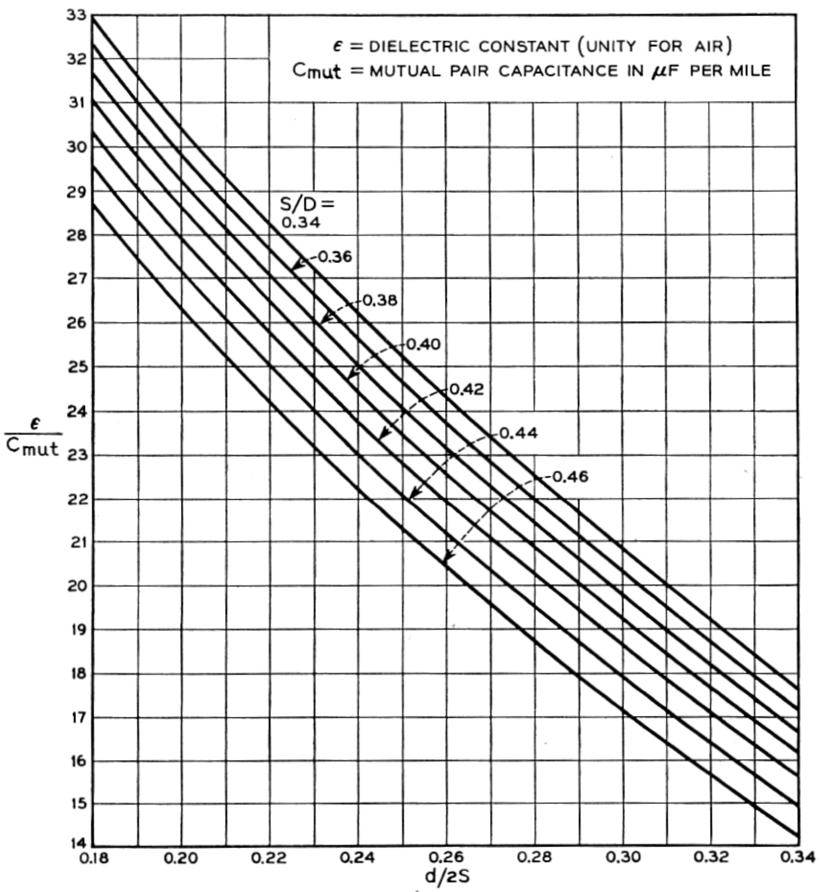

Fig. 3—ϵ/C_{mut} as a function of d/2S, SD.

The remaining air in the cable, occupying interstitial space, should not reduce the dielectric constant below that of polyethylene (2.26) in proportion to the percentage of air volume. This is because the electric field is weaker in the interstitial space than in the space immediately surrounding the conductors. The dielectric constant of the experimental cable should, therefore, be somewhere between 2.26 and 1 + (93% of 1.26) = 2.17.

Table II shows the results of application of the ideal pair formulas to 1000 cps inductance and capacitance measurements on the experimental cable both before and after injection of DXL-1 compound. The dielectric constant so obtained for the cable pairs after injection of DXL-1 is 2.20. This figure is within the limits set forth previously, and it seems probable that it is accurate to ±1%, although there is evidently no way to make an independent check. If we accept 2.20 for the dielectric constant of the cable after filling with DXL-1, the dielectric constant for the unfilled or normal condition can be evaluated. Since only the dielectric was altered in filling the cable, the dielectric constant before filling is given by:

$$\epsilon_{\text{mut}} = 2.20 \times \frac{0.0881}{0.1070} = 1.81,$$

Where: $0.0881\ \mu f/\text{mi.} = C_{\text{mut}}$ before filling.

$0.1070\ \mu f/\text{mi.} = C_{\text{mut}}$ after filling.

Table II shows, however, that application of the ideal pair formulas gives 1.88 for the normal dielectric constant. This figure is 4% higher than the correct 1.81 value. Examining the changes in mutual capacitance and capacitance to ground which occurred when the cable was filled with DXL-1, it is seen that C_g increased by 27.9% whereas C_{mut} increased by only 21.4%. The direct capacitance between wires of a pair, C_{12}, increased by only 12.5%. Since C_{12} is a component of C_{mut} but not of C_g, this accounts for the lower increase in C_{mut} as compared with C_g. The amount of air in the cable after filling is so small that it can be assumed that the dielectric is substantially homogeneous. Had the dielectric been homogeneous before filling, the percentage changes in C_g, C_{mut}, and C_{12} would all have been equal. Thus it is evident that in the normal condition there is a dielectric constant (ϵ_g) applicable for C_g which is different from ϵ_{mut}. We find that ϵ_g is:

$$\epsilon_g = 2.20 \times \frac{0.0983}{0.1256} = 1.72,$$

Where $0.0983\ \mu f/\text{mi.} = C_g$ before filling.

$0.1256\ \mu f/\text{mi.} = C_g$ after filling.

For a given set of capacitance and inductance data, there is, of course, only one value of dielectric constant which will satisfy the ideal pair formulas. This value will not be truly representative of actual cable conditions if non-homogeneity exists. Non-homogeneity, as evidenced by the experimentally determined inequality in ϵ_g and ϵ_{mut}, accounts for the 4%

TABLE II

DIELECTRIC CONSTANT STUDIES ON LENGTH OF 19-GAUGE 22-PAIR POLYETHYLENE INSULATED CABLE

	C_g μf/mi.	C_{mut} μf/mi.	C_{12} μf/mi.	C_g/C_{mut}	L mh/mi.	d/2S	S mils	Effective		ϵ/C_{mut}	ϵ
								S/D	D mils		
Before injection of DXL-1	0.0983	0.0881	0.0390	1.115	0.9783	0.281	63.0	0.378	167	21.4	1.88
After injection	0.1256	0.1070	0.0438	1.175	0.9783	0.281	63.0	0.404	156	20.6	2.20
% Increase	27.9	21.4	12.5								

error in the 1.88 figure. This follows from the fact that if the C_g/C_{mut} ratio obtained in the filled or homogeneous condition is used in the determination of the normal dielectric constant, then the correct 1.81 figure is obtained. Of course, these statements regarding the accuracy of the normal dielectric constant are predicated on the absolute accuracy of the 2.20 figure for the dielectric constant of the filled cable. However, as pointed out previously, there is but little room for uncertainty in this connection.

To summarize, it appears that the principal error involved in the use of ideal pair formulas to determine the dielectric constant of cable pairs is due to non-homogeneity of the dielectric. For polyethylene insulated 19-gauge pairs the error is about +4%. The magnitude of error expected for paper and pulp insulated pairs is less, as is explained below.

a. *Effective Diameter of Shield*

It has been shown that solution of equation (1) provides a value for S which does in fact equal the average interaxial spacing between wires of a cable pair. In the process of finding ϵ using the ideal pair formulas, a value for S/D is found from which a value for D can be determined. The relation between the D value and the physical configuration of the cable pair structure is not precise. It is necessary to consider that D represents an effective diameter of shield; it is the diameter of the cylindrical shield of an ideal pair structure having the same $d/2S$ and C_g/C_{mut} ratios and the same d as the cable pair.

The value of effective D is larger if the dielectric is non-homogeneous, as shown by comparing the figures for D in Table II obtained before and after filling with DXL-1. The D value obtained under homogeneous conditions is more representative of the average physical placement of conductors comprising the shield.

b. *Paper Ribbon and Paper Pulp Insulated Cable*

The distribution of solid insulating material in paper ribbon or pulp insulated cable is different from that in polyethylene insulated cable. Whereas in the latter there is a solid sheath of insulation around the conductor with air occupying interstitial space only, the paper ribbon or pulp insulated cable has some air dispersed among the insulating material in all portions of the dielectric space. "Interstitial space" is not well defined since the paper insulation tends to deform during cabling operations. This sort of dielectric would seem to be more homogeneous than that of the polyethylene insulated cable.

A method for approximating the $\epsilon_g/\epsilon_{mut}$ ratio in terms of power factor measurements on the balanced and grounded circuits is discussed in Appendix II. $\epsilon_g/\epsilon_{mut}$ ratios obtained for three pulp insulated cables do not differ from unity by more than $\pm 2\%$. These results, although not conclusive, tend to substantiate the above qualitative comparisons. It is believed, therefore, that ϵ values obtained for paper ribbon or pulp insulated cables are probably accurate to about 1 or 2%.

Table III shows results of the use of the methods herein described to obtain the average S, ϵ and effective D for examples of some cable designs now in use or under experimental investigation. These data are included primarily to illustrate the method for a variety of cases, and are not comprehensive enough to serve as the basis for a study of the various cable designs and insulations.

c. *Dielectric Constant from L × C at High Frequencies*

For the ideal pair structure the mutual capacitance and inductance are inversely related when the frequency is so high that current does not penetrate either conductors or shield. The relation between inductance and capacitance, in practical units, is then:

$$\epsilon = 34.70\, C L_\infty \tag{5}$$

L_∞ = limiting value approached by inductance as the frequency increases indefinitely—mh/mi.

C = capacitance—μf/mi.

It is known that L_∞ of individually shielded pairs, that is, pairs having metallic tape shields applied with an overlapped longitudinal or helical seam, can be accurately approximated from a series of inductance tests using the relation:

$$L_F = L_\infty + \frac{M}{\sqrt{F}} \tag{6}$$

Where L_F = distributed inductance at frequency F.

M = a constant

F = frequency

The tests are usually made in the range from 2 to 5 mc. Application of formulas (5) and (6) to individually shielded pairs is known to be an accurate technique for evaluating the dielectric constant of non-homogeneous combinations of air and solid materials and has been in use for many years.

This same method has sometimes been used with cable pairs. However, for the filled cable having a dielectric constant of about 2.20, these techniques gave a dielectric constant value of 2.58, which is about 17% too high. The reason for this large discrepancy is apparent when it is considered that the inverse relationship of L and C obtains only when the static and magnetic

TABLE III

DIELECTRIC CONSTANT, INTERAXIAL SPACING, AND EFFECTIVE DIAMETER OF SHIELD FOR SOME TYPICAL MULTI-PAIRED CABLES

Description			Cable Average at 1000 cps			Space per pr. KCM*	d/2S	S mils	C_g/C_{mut}	S/D	Effective D mils	ϵ
AWG	No. pairs	Type of insulation	C_{mut} μf/mi.	C_g μf/mi.	L mh/mi.							
19	61	Paper ribbon	0.0610	0.0706	1.071	18.3	0.243	73.8	1.157	0.369	200	1.53
19	51	Paper ribbon	0.0826	0.0869	0.910	12.2	0.313	57.5	1.052	0.373	154	1.57
19	51	Paper pulp	0.0811	0.0810	0.892	11.75	0.320	56.0	0.999	0.357	157	1.53
22	51	Paper pulp	0.0773	0.0779	0.907	6.5	0.314	40.3	1.008	0.354	114	1.50
19	26	Polyethylene	0.0831	0.0915	1.003	14.3	0.275	64.7	1.101	0.366	177	1.85

* KCM = 1000 circular mils.

Note: The dimensional ratios and dielectric constant values listed above are based on arithmetic averages of capacitance and inductance for all pairs of a given cable. If these parameters were determined for each pair individually and averaged, the results would not precisely match the above figures, due to the non-linear functions involved. However, the distinction is slight since the RMS deviation of capacitance or inductance for the pairs of a cable is usually about 3%.

fields are completely terminated on the same shielding surface. The shield around a cable pair resembles a Faraday "cage" of wires which is inherently transparent to the magnetic field at low frequencies. Furthermore, at the highest frequency investigated (about 5 mc), the magnetic shielding of a cable pair by the adjacent surrounding conductors is still much less effective than the essentially perfect static shielding. Thus, it is concluded that the $L \times C$ method should not be used for cable pairs when good accuracy is desired.

Acknowledgments

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APPENDIX I

The following formulas were developed by Mrs. S. P. Mead:

I. Formula for the mutual capacitance of a balanced shielded pair:

$$C_{\text{mut}} = \frac{0.01944\ \epsilon}{\log\left(\dfrac{1}{u}\dfrac{1 - v^2}{1 + v^2}\right) - 0.1086\ \delta_{12}}$$

C_{mut} is in μf/mi.
$u = d/2S$
$v = S/D$
d = conductor diameter
S = interaxial separation
D = inside diameter of shield
ϵ = dielectric constant (unity for air)
δ_{12} is a complicated function of u and v. It increases for larger values of u and/or smaller values of v. The term 0.1086 δ_{12} amounts to from $\frac{1}{2}\%$ to about 7% over the ranges of u and v values plotted in Fig. 3.

II. Formula for the capacitance to ground of a balanced shielded pair:

$$C_g = \frac{0.03889\ \epsilon}{\log\left(\dfrac{[3 - \sqrt{1 + 4u^2}]\,[1 - v^4(1 + 4u^2)]}{8uv^2}\right) + 0.4343\Delta_1}$$

C_g is in μf/mi.
ϵ, u and v are defined in section I.
Δ_1 is also a complicated function of u and v. The term 0.4343 Δ_1 amounts to from less than $\frac{1}{2}\%$ to about 3% over the ranges of u and v plotted in Fig. 2. It increases for larger values of u and/or larger values of v.

APPENDIX II

It has been shown that if there is a dielectric constant for capacitance to ground (ϵ_g) which is different from ϵ_{mut}, then values of ϵ_{mut} as found from the ideal pair formulas will be in error. This appendix describes results of an attempt to evaluate the $\epsilon_{mut}/\epsilon_g$ ratio for paper ribbon and pulp insulated cable using a method suggested by Mr. M. C. Biskeborn. It was reasoned that whatever non-homogeneity exists in the cable dielectric would be caused by variations in the insulation density in various portions of the dielectric space. Compressive forces encountered in twisting and stranding the pairs tend to make the density for the space between wires of a pair high as compared with an average density for the total dielectric space. Since ($\epsilon - 1$) and the power factor, or simply G/C, go to zero approximately in direct proportion to the amount of solid dielectric material, the following equation may be used to find an approximate value for $\epsilon_g/\epsilon_{mut}$:

TABLE A

EVALUATION OF $\epsilon_g/\epsilon_{mut}$ FOR PULP INSULATED CABLE

AWG	Description	Cable Averages 1000 cps		$A = \frac{G_{mut}}{C_{mut}}$	Cable Averages 1000 cps		$B = \frac{G_g}{C_g}$	F = B/A Power Factor Ratio	$\frac{\epsilon_g}{\epsilon_{mut}}$
		C_{mut} µf/mi	G_{mut} µmhos/mi.		C_g µf/mi.	G_g µmhos/mi.			
19	51 Prs. Pulp	0.0858	2.00	23.3	0.0850	1.86	21.9	0.94	0.980
19	51 Prs. Pulp	0.0811	1.12	13.8	0.0810	1.19	14.7	1.065	1.022
22	51 Prs. Pulp	0.0773	1.09	14.1	0.0779	1.08	13.9	0.985	0.995

$$\frac{\epsilon_g - 1}{\epsilon_{mut} - 1} = \frac{\text{Power Factor of Grounded Circuit}}{\text{Power Factor of Balanced Circuit}}$$

$$\frac{\epsilon_g - 1}{\epsilon_{mut} - 1} = \frac{G_g/C_g}{G_{mut}/C_{mut}} = F \text{ (power factor ratio)}$$

Dividing by ϵ_{mut} and rearranging:

$$\frac{\epsilon_g}{\epsilon_{mut}} = F + \left(\frac{1 - F}{\epsilon_{mut}}\right)$$

Substituting 1.50 for ϵ_{mut} on the right hand side introduces but small error in the eventual result since F is known to be closely equal to unity and permits solution for $\epsilon_g/\epsilon_{mut}$ as follows:

$$\frac{\epsilon_g}{\epsilon_{mut}} = 1 - \left(\frac{1 - F}{3}\right)$$

This relation was used to evaluate $\epsilon_g/\epsilon_{mut}$ for 3 pulp insulated cables for which the necessary data were available. The results are shown in Table A. None of the 3 cables has a $\epsilon_g/\epsilon_{mut}$ ratio different from unity by more than 2%. The average for the 3 cables is 1.0 and it is felt that the $\pm 2\%$ deviation could be due to inaccuracies inherent to conductance measurements. These data represent the only attempt to quantitatively evaluate non-homogeneity in pulp or paper insulated cable dielectrics and, while not by any means conclusive, they tend to substantiate the belief that the effects of any such non-homogeneity on the $\epsilon_g/\epsilon_{mut}$ ratio is small.