Theory of the Negative Impedance Converter

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This paper presents a relatively new approach to the solution of negative impedance problems related to vacuum tube circuits. The approach consists of reducing the vacuum tube circuit of a device for producing negative impedance to an electrically equivalent four-terminal network from which the stability and the operation of the device as an element in a system can be predicted accurately. The theory is of interest at this time because a negative impedance repeater, the E1, has recently been developed for use in the exchange plant. It has been found that such a repeater can be placed in series with a voice frequency telephone line to provide transmission gains which are ample for many purposes.

Introduction

A NEW type of telephone repeater known as the E1 has been developed recently to meet the large demand in the exchange area for an economical means of providing transmission gains of about 10 db in two-wire telephone lines. This repeater costs less than the 22 type which has been the standard two-wire, two-way means of amplifying voice currents in the Bell System. The difference in cost is made possible by a difference in operating principle. The E1 repeater employs a type of feedback amplifier the action of which can be said to have the properties of a negative impedance converter. It is the purpose of this paper to describe the operation of the negative impedance converter, which is a device for transforming positive impedance into negative impedance.

Negative impedance like positive impedance can have two components: reactance and resistance. The reactance component can be either positive or negative. However, for an impedance to be negative the resistance component should be negative at some frequency in the range from zero to infinity.

The idea of negative resistance originated over 30 years ago, and in the beginning was associated with the concept of resistance neutralization. This concept grew from the observation that a two-terminal device could be found which had an unusual property when inserted in series with a single mesh circuit: it could produce the same flow of current as would flow otherwise at some frequency provided a resistance R were removed from this mesh. Apparently, the addition to a circuit of a two-terminal element could neutalize an amount of resistance equal to R. Thus within certain frequency limits this two-terminal device could be treated as a negative resistance equal in magnitude to $-R$.

In the early days of vacuum tube development the negative resistance effect was considered to be an important one. Possibly the regenerative vacuum tube circuits associated with the early radio receivers stimulated interest in the subject. One of the first text books on the theory of vacuum tube circuits[1] devoted about as much space to regenerative means for producing negative resistance as it devoted to the theory relating to any one of the more conventional devices—namely: amplifiers, oscillators, modulators and detectors. In spite of the interest in the subject, little practical use was made of the negative resistance theory.

Negative resistance cannot be completely disassociated from reactance. A vacuum tube circuit arranged to develop negative resistance will present reactance as well at some frequencies, and the effect on an external circuit at these frequencies will be that of taking away resistance and adding or subtracting reactance. At high and low frequencies the circuit may present a positive impedance. Consequently, the term negative impedance is used herein to designate the effect produced by a two-terminal device which has the property of negative resistance at some frequency or frequencies, negative resistance plus reactance at other frequencies and positive impedance at still other frequencies.

The Negative Impedance Converter

Heretofore, many vacuum tube circuits have been devised for converting positive impedance into negative impedance. All known simple circuits employing vacuum tubes for obtaining negative impedance, as distinguished from the combination circuits which can be made up either of vacuum tubes or of negative elements found in nature, have much in common and can be treated as the same type of arrangement. Essentially, this type of arrangement is a feedback amplifier and can be treated as such. Recently, a new method of handling these devices has been developed which has the merit of simplifying computations in many cases. This method is based upon the fact that vacuum tube circuits devised for converting positive impedance into negative impedance can be reduced to an electrically equivalent, four-terminal network consisting of a combination of positive impedance elements together with an ideal negative impedance converter.

This ideal converter, Fig. 1(a), resembles a form of transformer: it has a ratio of transformation of $-k:1$, can have four terminals and is capable of bilateral transmission. Assume that a positive impedance Z_N is connected to terminals 3 and 4 and $-kZ_N$ is seen at terminals 1 and 2, Fig. 1(b). Then it must follow from the theory described herein that if a positive impedance

[1] L. J. Peters—Theory of Thermionic Vacuum Tube Circuits—McGraw-Hill Book Company—1927.

Z_L is connected to terminals 1 and 2, a negative impedance $Z_L/-k$ will be seen at terminals 3 and 4, Fig. 1(c).

George Crisson stated[2] that there are two types of negative impedance which he defined as the series type and the shunt type respectively. His series type is the reversed voltage type of negative impedance $-kZ_N$ equivalent to $-V/I$; and his shunt type is the reversed current type $Z_L/-k$ equivalent to $V/-I$. This is a logical development considering that impedance Z equals V/I, and therefore negative impedance $(-Z)$ equals either $-V/I$ or $V/-I$ where V is the voltage measured across the impedance and I is current flowing through it.

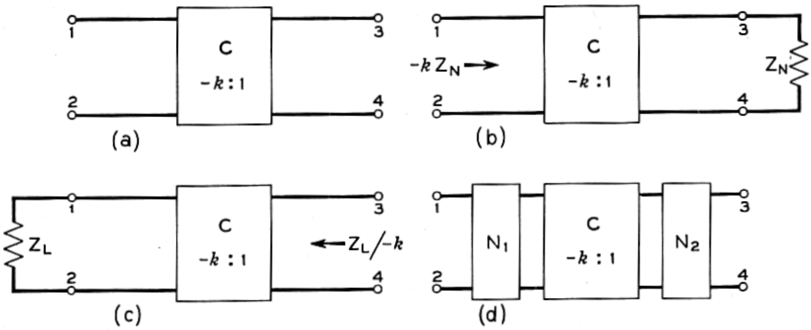

Fig. 1—The negative impedance converter.

It is important to note that k, the ratio of transformation, is of the form $A\underline{/\Theta}$ where both the magnitude A and the angle Θ are changing, however slowly, with frequency. This follows from the fact that, as will be shown, k contains a term $(\mu_1 - 1)$ where μ_1 is a voltage ratio the magnitude and phase of which can change with frequency.

The ratio of transformation k can be made to have a magnitude closely approaching unity, and at some frequency or frequencies the angle can be made zero. If k equals $1\underline{/0}$ the series type negative impedance seen at terminals 1 and 2, Fig. 1(b), will be equal to $-Z_N$, $-V/I$, where the voltage V is reversed by 180 degrees from the voltage which would appear across terminals 1 and 2 were the impedance here the positive impedance Z_N and the voltage to current ratio the conventional V/I. Likewise, the shunt type negative impedance seen at terminals 3 and 4, Fig. 1(c), would be $Z_L/-1$, $V/-I$, where here it is the current which is reversed by 180 degrees from the current which would flow through the positive impedance Z_L. Thus a strange fact is noted: multiplying a positive impedance by -1 does not yield the

[2] George Crisson—Negative Impedance and the Twin 21-Type Repeater—*B.S.T.J.*—July, 1931.

same result from a circuit viewpoint as dividing a positive impedance by -1. A positive impedance multiplied by -1 is a series type negative impedance. A positive impedance divided by -1 is a shunt type of negative impedance.

A practical (i.e., real or actual) converter circuit can be represented by Fig. 1(d). A vacuum tube circuit contains positive impedance elements. Some of these will show up in the equivalent circuit on the left-hand side of the ideal converter; others will show up on the right-hand side of the ideal converter. This will be clarified in the discussion of the E1 circuit which follows. The equivalent circuit of any practical negative impedance converter of this type can be represented by the equivalent circuit of Fig. 1(d) which shows the positive impedance elements associated with the vacuum tube in

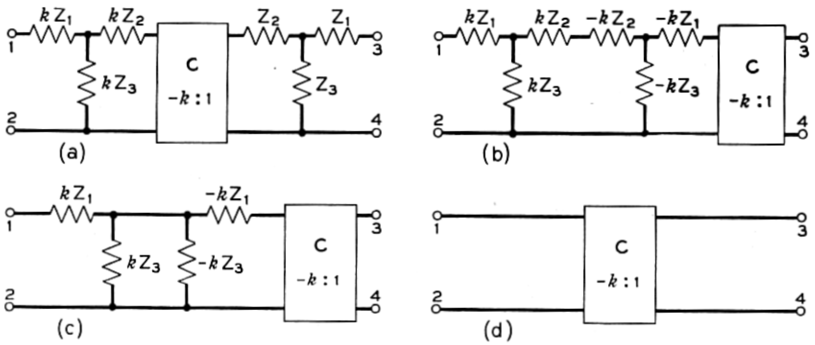

Fig. 2—Equivalent circuits.

the form of two equivalent networks (N_1 and N_2) arranged one on each side of the ideal converter (C) having a transformation ratio of $-k{:}1$.

It should be noted that should the series arms of N_1 equal k times the series arms of N_2 and should the same relationship exist for the shunt arms then the effect of these networks is cancelled, except for power dissipation, and Fig. 1(d) is equivalent to Fig. 1(a). This is illustrated in Fig. 2 where N_1 and N_2 have been represented by equivalent T networks as shown specifically in Fig. 2(a). Network N_2 can be multiplied by $-k$ and transformed to the left-hand side of the ideal converter, Fig. 2(b). The adjacent series arms of these two networks cancel each other as shown in Fig. 2(c). The shunt arms go to infinity and the other series arms also cancel leaving Fig. 2(d).

It is not possible to cancel N_1 and N_2 perfectly in a practical circuit design. But over the frequency range of interest this could be closely approximated by making all impedances shunting the ideal converter as large as possible, and by cancelling all resistances in series in N_1 by a resistance added in N_2.

As a physical concept the idea of negative impedance is difficult to visualize because this type of impedance supplies power to an external circuit rather than dissipates it. This power is supplied either by the application of a reversed voltage or a reversed current. In spite of the difficulty which may be experienced in attempting to visualize the feedback action that takes place inside a negative impedance converter, if its equivalent circuit is known then its stability can be determined readily and its operation as a device for producing negative impedance becomes obvious.

STABILITY

Like any amplifier whose output connects back to its input, the negative impedance converter if not properly terminated can run away with itself and oscillate. Stability can be determined by conventional feedback theory[3]; fortunately, there is a simpler criterion for determining stability. Consider again the ideal converter, Fig. 1(b). Assume that Z_L, not shown on Fig. 1(b), is an impedance connected to terminals 1 and 2. Consider the circuit mesh formed on the left-hand side of the ideal converter by the connection of Z_L to terminals 1 and 2. Here a negative impedance $(-kZ_N)$ is seen looking into terminals 1 and 2, and a positive impedance (Z_L) is seen looking away from them. The total impedance in this mesh is $Z_L - kZ_N$. If $k\,Z_N$ should equal Z_L then the total impedance would be zero; and a voltage inserted in series with this mesh would call for infinite current, a situation obviously impossible. Thus it becomes evident that kZ_N should not equal Z_L; or, what is the same thing, the ratio kZ_N/Z_L should not equal $\underline{1/0}$ if the system is to be stable. Furthermore, it can be shown that for an ideal converter the ratio kZ_N/Z_L contains the characteristics of the feedback factor $(\mu\beta)$ of the amplifier in the converter. In view of this fact, it might be expected that Nyquist's rule[4] for stability in feedback amplifiers could be paraphrased as follows: *To obtain stability in an ideal negative impedance converter the locus of the ratio kZ_N/Z_L over the frequency range from zero to infinity must not enclose the point $\underline{1/0}$.*

The same general rule for stability can be arrived at by connecting an impedance Z_N to terminals 3 and 4 of Fig. 1(c) and by considering the circuit mesh formed by $(Z_L/-k) + Z_N$. It should be noted in this case that $Z_L/-k$ calls for a flow of current 180 degrees out of phase from that which would flow through Z_L/k. This means that where the phase angle of Z_L/k equals that of Z_N the magnitude of Z_L/k must be greater than that of Z_N, which is another way of saying that at this phase angle the magnitude of kZ_N/Z_L must be less than unity.

[3] H. W. Bode—Book—Network Analysis and Feedback Amplifier Design—D. Van Nostrand Company, Inc.—1945.

[4] H. Nyquist—Regeneration Theory—*B.S.T.J.*—Jan., 1932.

From a practical engineering viewpoint there is a simple criterion for judging stability. It can be stated as follows: The ideal negative impedance converter will be unconditionally stable provided that the magnitude of kZ_N/Z_L is less than unity at any frequency where the angle of this ratio is zero.

These same conditions for stability apply to any practical (i.e., real or actual) converter circuit, Fig. 1(d). However, Z_L must be taken as the impedance seen looking into the network N_1 from the position of the ideal converter C, and Z_N must be taken as the impedance seen looking into the network N_2 from the ideal converter C. In other words, the effect of N_1 must be included in Z_L and the effect of N_2 must be included in Z_N.

Negative Impedance Converter Circuits

Negative impedance can be produced by connecting the output of an amplifier back in series or in shunt with the input in the right phase relationship. This type of circuit can be considered as a negative impedance converter similar to Fig. 1(d) where the ratio of transformation $-k$ is of the form $-(\mu_1 - 1)$, in which μ_1 represents a function of the voltage amplification of the amplifier. The disadvantage of a transformation ratio of this kind is that it changes markedly with variations in tube constants and battery supply voltage. Such circuits present a stability problem. One solution to this problem has been described by E. L. Ginzton[5]. He reduced variations in the amplifier gain by stabilizing the amplifier itself with negative feedback and thus reduced variation in μ_1. Note that if μ_1 is set equal to 2, then $-k$ becomes equal to -1.

There is another method of using negative feedback to stabilize a circuit for producing negative impedance. This method was used in the E1 circuit and will be described in detail in connection with it. Essentially, here negative feedback is arranged together with positive feedback to produce a transformation ratio for the ideal converter of the form $-(\mu_1 - 1)/(\mu_2 + 1)$. The symbol μ_2 represents a voltage ratio. Furthermore, μ_1 equals $\beta_1\mu_2$. If β approximates unity and if μ_2 is very much larger than one, $-k$ approaches -1 in value and is relatively independent of variations in tube constants and battery supply voltage.

In order to illustrate how the equivalent circuit of a negative impedance converter can be derived, consider a circuit credited to Marius Latour about the year 1920, Fig. 3(a). Figure 3(b) is a schematic representation of Fig. 3(a) in the manner originated by G. Crisson. With reference to Fig. 3(b), the polarity of amplifier A is assumed such that, at the instant the a-c current I_1 flows in the direction indicated, the current I_2 will flow in the direction

[5] E. L. Ginzton—Stabilized Negative Impedances—*Electronics*—July, 1945.

shown and the voltages V_2 and $\mu I_1 Z_N$ will have the polarities shown. For the moment it will be assumed that the input impedance of amplifier A is infinitely high, an assumption which will be modified later to agree with actual circuits. The symbols Z_N and Z_1 represent impedances, R_p represents the plate resistance of the output tube (T_2) and $\mu I_1 Z_N$ represents the voltage produced in the plate circuit of the output tube of amplifier A, the tube

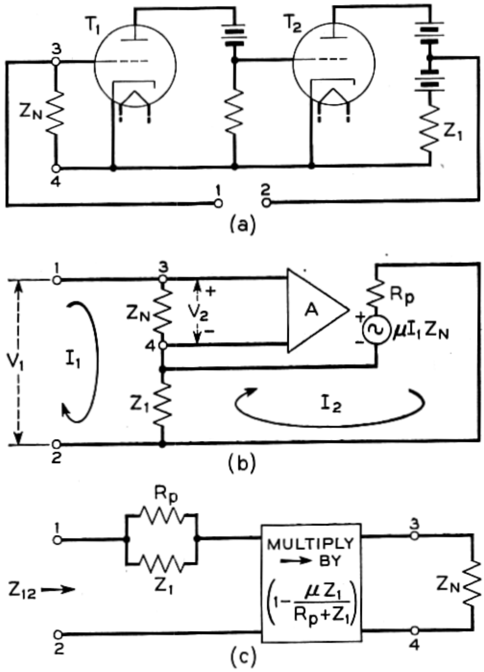

Fig. 3—Latour's circuit.

T_2 of Fig. 3(a), by the voltage drop $I_1 Z_N$ across the input, grid to cathode of T_1, Fig. 3(a). The mesh equations for Fig. 3(b) can be written as follows:

$$\begin{aligned} V_1 &= (Z_N + Z_1)\, I_1 \qquad\qquad - Z_1 I_2 \\ 0 &= -(Z_1 + \mu Z_N)\, I_1 + (R_p + Z_1)\, I_2 \end{aligned}$$

The current I_1 becomes:

$$I_1 = \frac{V_1(R_p + Z_1)}{(Z_N + Z_1)(R_p + Z_1) - (Z_1 + \mu Z_N)Z_1} \qquad \text{Eq. (1)}$$

The impedance seen looking into terminals 1 and 2 can be written:

$$Z_{12} = \frac{V_1}{I_1} = \frac{Z_1 R_p}{Z_1 + R_p} + Z_N\left(1 - \frac{\mu Z_1}{R_p + Z_1}\right) \qquad \text{Eq. (2)}$$

Thus the impedance Z_{12} equals the impedance Z_1 in parallel with R_p which combination is in series with impedance Z_N multiplied by $1 - [\mu Z_1/(R_p + Z_1)]$. An equivalent circuit for Z_{12} is illustrated in Fig. 3(c).

Next, assume that Z_L, not shown on Fig. 3(b), is connected to terminals 1 and 2 of Fig. 3(b) and that Z_N is removed from across terminals 3 and 4. The mesh equations can be written as follows:

$$\begin{aligned} -V_2 &= (Z_1 + Z_L)\, I_1 \quad - Z_1 I_2 \\ \mu V_2 &= -Z_1 I_1 \quad + (R_p + Z_1) I_2 \end{aligned}$$

The current I_1 can be written:

$$I_1 = \frac{-V_2 [R_p + Z_1 - \mu Z_1]}{(Z_1 + Z_L)(R_p + Z_1) - Z_1^2} \qquad \text{Eq. (3)}$$

The impedance looking into terminals 3 and 4, Fig. (3b), with the changes listed above is:

$$Z_{34} = \frac{V_2}{-I_1} = \frac{Z_L + \dfrac{R_p Z_1}{R_p + Z_1}}{\left(1 - \dfrac{\mu Z_1}{R_p + Z_1}\right)} \qquad \text{Eq. (4)}$$

Hence, if the circuit of Fig. 3(b) is redrawn as a four-terminal network as shown in Fig. 4(a) with Z_2 added to represent the input impedance of amplifier A the equivalent circuit of this network can be represented by Fig. 4(b). The equivalent circuit consists of two positive impedance networks, one on each side of an ideal converter. The ratio of transformation of this ideal converter is of the form $-(\mu_1 - 1):1$. Looking into terminals 1 and 2, Fig. 4(b), a series type negative impedance will be seen and looking into terminals 3 and 4 a shunt type negative impedance will be seen. The proof that these impedances are negative and of the reversed voltage or reversed current type has been established by H. W. Dudley, F. H. Graham and R. C. Mathes for similar circuits and will not be taken up here, although the fact could be derived simply from equations (1), (2), (3) and (4). The purpose of this discussion is to illustrate the simplicity with which an equivalent circuit can be derived, and to point out the value of the concept of the ideal negative impedance converter.

A well known circuit which can be used as a negative impedance converter is the circuit of the 21 type repeater. In this circuit the output of the amplifier is connected back to the input through a bridge type of arrangement referred to as a hybrid coil. Negative feedback and positive feedback can be developed across this coil between the amplifier output and the amplifier input. This device was used as a negative impedance converter by Crisson in his twin 21-type repeater.

There is another type of negative impedance converter circuit which should not be confused with the converter circuits mentioned heretofore. This type was disclosed by Charles Bartlett in the March 1927 issue of the

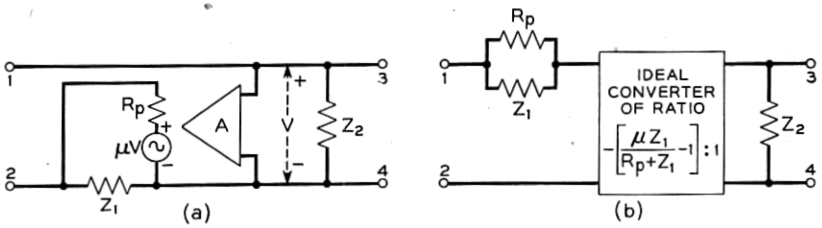

Fig. 4—A negative impedance converter and its equivalent circuit.

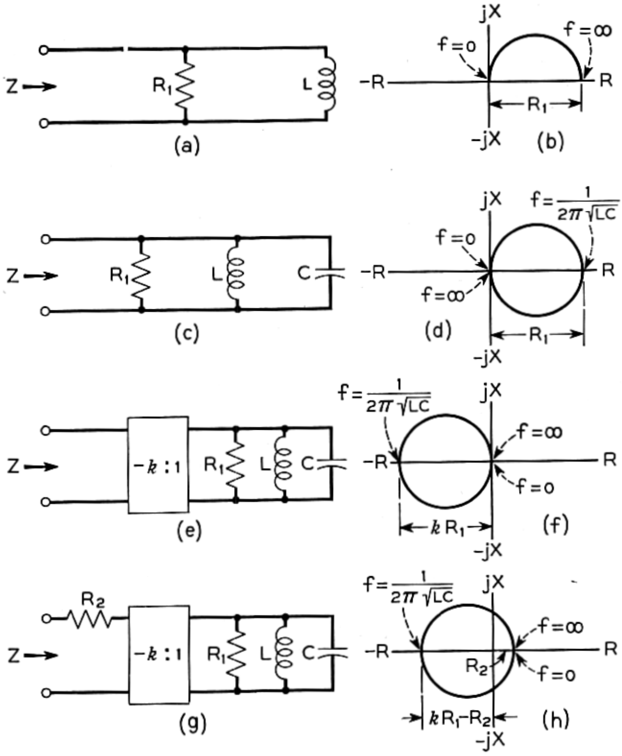

Fig. 5—Impedance loci.

Journal I.E.E. pages 373 to 376. The Bartlett converter consists of a T network of equal resistances, one of them negative. To construct such a converter there must be available a negative resistance element. Over a finite frequency band this negative resistance can be produced by a converter circuit such as that of Fig. 4. However, in this case Bartlett's circuit becomes in a sense, a converter within a converter.

THE NEGATIVE IMPEDANCE LOCUS

Before the E1 type of converter is described, it would be well to consider, in general, the impedance characteristic which can be produced by a negative impedance converter. The shape of the impedance characteristic over the frequency range, zero to infinity, looking into terminals 1 and 2 or into terminals 3 and 4 of a negative impedance converter will be called the negative impedance locus. It is convenient to plot this locus in the polar form with frequency as a "running" parameter. The locus can be derived for any circuit by means of the theory outlined by K. G. Van Wynen for positive two-terminal impedances.[6]

For example, consider the converter of Fig. 4. Assume that an impedance such as that shown in Fig. 5(a) is connected to terminals 3 and 4 of Fig. 4; assume Z_2 of Fig. 4 is a capacitance and that both R_p and Z_1 are resistances. Now Fig. 5(a) represents a two-terminal network made up of a resistance shunted by an inductance. The locus of this positive impedance plotted on the R and jX plane over the frequency range from zero to infinity is shown in Fig. 5(b). At zero frequency the impedance is zero; at infinite frequency the impedance is R_1. If to the network of Fig. 5(a) a capacitance C is added, to represent Z_2 of Fig. 4, the impedance of the network so formed, Fig. 5(c), follows the circle of Fig. 5(d). At zero frequency the impedance is zero, at the resonant frequency the impedance is R_1, and at infinite frequency the impedance is zero again. If the impedance of this network, Fig. 5(c), were viewed through an ideal negative impedance converter, Fig. 5(e), having a ratio of transformation of $-k{:}1$ where, for the moment, k is assumed to be a numeric over the entire frequency range, the impedance locus can be represented by Fig. 5(f). Of course, k will always have an angle at high and low frequencies but, to a first approximation, at least, it can be assumed that over most of the frequency range shown in the impedance diagrams of Fig. 5(f) and Fig. 5(h) k approaches a numeric. If the circuit configuration of Fig. 5(g) is created by adding resistance R_2 in series with Fig. 5(e) the impedance locus looks like that of Fig. 5(h). This is a series type of negative impedance and simulates the impedance seen over a large portion of the frequency range looking into terminals 1 and 2 of Fig. 4 with the two-terminal network of Fig. 5(a) connected to terminals 3 and 4.

[6] Design of Two-Terminal Balancing Networks—K. G. Van Wynen—*B.S.T.J.*—Oct., 1943.

By a similar analysis, the locus of the shunt type of negative impedance can be derived. With the proper choice of an impedance connected to terminals 1 and 2 of the negative impedance converter the locus of the impedance seen at terminals 3 and 4 can be made to follow a circle in the clockwise direction (at least as long as k approximates a numeric). Thus, over a portion of the frequency range the series and shunt type of negative impedance can be made to have very similar impedance characteristics. At very high or low frequencies where k has an appreciable angle there will be a distinct difference between the locus of the series and the corresponding shunt type of negative impedance. This would be expected because in one case the network impedance is multiplied by $-k$; and in the other it is divided by $-k$.

The E1 Converter

The circuit of the negative impedance converter used in the E1 telephone repeater is shown in Fig. 6(a). It consists of a transformer, two triode tubes, an RC network and an inductor. The transformer T couples the cathodes of the two tubes to terminals 1 and 2. The tubes while apparently in push-pull are biased for Class A operation. The RC network couples the plate of each tube to the grid of the other. The inductor L supplies plate current.

The equivalent circuit of Fig. 6(a) is shown in Fig. 6(b). In obtaining this equivalent circuit the two tubes have been assumed to be identical. Thus the converter used with the E1 repeater can be reduced to a four-terminal network consisting of (reading from left to right): the equivalent circuit of the line transformer T; the two biasing resistances R_2; the two plate resistances R_p divided by $(1 + \mu_2)$; the ideal negative impedance converter C of ratio $-(\mu_1 - 1)/(\mu_2 + 1)$ to 1; the elements of the RC coupling arrangement which appear in shunt across terminals 3 and 4; the inductor L also shunted across these terminals; and the capacitor C_X which has been added to represent both the distributed capacitance of the windings of L and the capacitance between vacuum tube plates.

It should be noted that μ_2 is the amplification factor of each tube; and that μ_1 equals $\beta_1\mu_2$ where β_1 is a proportionality factor representing the fraction of the voltage, between the plate of one tube and ground, which is fed back to the grid of the other tube. The value of β_1 depends upon the values of C_1, R_3, R_5 and C_2 of the RC coupling circuit. If β_1 approaches unity in value then μ_1 approximates μ_2. If this is so and if both μ_1 and μ_2 are relatively large in magnitude compared to unity then the ratio of transformation, $-(\mu_1 - 1)/(\mu_2 + 1)$ to 1, approaches although it cannot equal $-1{:}1$.

As an illustration of how the elements in the E1 circuit may be proportioned, consider the practical design of the E1 converter. Here the ratio $-(\mu_1 - 1)/(\mu_2 + 1)$ to 1 is $-0.9{:}1$ over most of the voice frequency range. This is not the over-all ratio of transformation of the device, but only the ratio of the ideal converter C, Fig. 6(b). The ratio of transformer T and the effect of the other circuit elements must be considered in determining the over-all effect of the converter from terminals 1 and 2 to terminals 3 and 4. The transformer ratio is 1:9 from terminals 1 and 2 to the tube

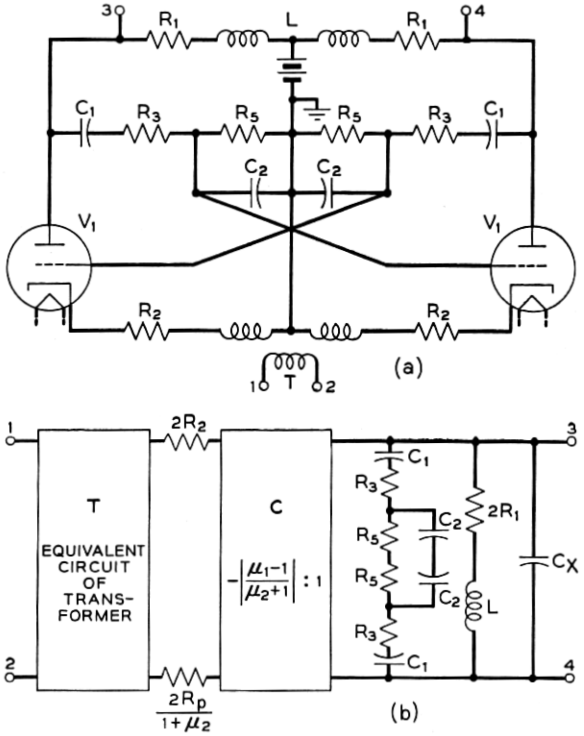

Fig. 6—The E1 converter circuit.

cathodes. The shunt arms of the networks on both sides of the ideal converter C are relatively high compared to the impedances between which this converter has been designed to operate at voice frequencies. Therefore, these shunt arms can be disregarded at voice frequencies although at frequencies above and below the voice band these shunt arms represent a problem for the circuit designer from the viewpoint of stability. In the actual E1 circuit the series arms such as $2R_p/(1 + \mu_2)$ and $2R_2$ could be cancelled out by adding in series on the right-hand side of the ideal converter a resistance of 1800 ohms (not shown in Fig. 6). The final result is that the impedance seen, looking into terminals 1 and 2 of the E1 converter when 1800 ohms plus a network Z_N is connected to terminals 3 and 4, equals $-0.1Z_N$ within a reasonable percentage of error over the frequency range from about 300 to 3500 c.p.s. for values of negative impedance from about 100 to 2000 ohms.

In the practical design two line windings instead of the one shown connected between terminals 1 and 2 in Fig. 6(a) are provided on transformer T. In practice one of these windings is inserted in each side of the telephone line in a balanced arrangement. Terminals 1 and 2 are thus effectively connected in series with the line, and the E1 repeater presents to the telephone line a reversed voltage type of negative impedance $(-V/I)$, which is the means of introducing additional power in the line thereby providing a transmission gain. The value of the negative impedance is controlled by a

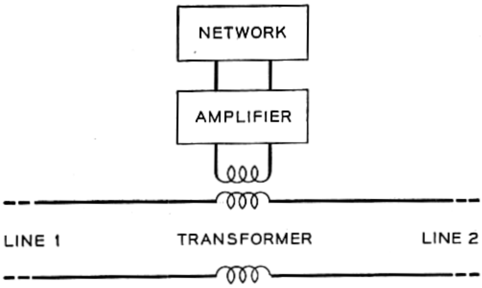

Fig. 7—E1 telephone repeater.

network connected to terminals 3 and 4. Thus the E1 repeater consists of a transformer, an amplifier unit and a gain adjusting network (Fig. 7). The transformer, the amplifier unit and part of the network make up the negative impedance converter under discussion.

For overload conditions the action of a repeater of this kind differs markedly from what might be expected from a conventional amplifier. What can be expected of a conventional amplifier is well known. An idea as to the performance of a negative impedance converter under overload conditions can be had from the following example: Assume that terminals 1 and 2 of the converter are connected in series with a telephone line and a network is connected to terminals 3 and 4 so that a reversed voltage type of negative impedance of a value less than the line impedance is inserted in the line. The combination described will be stable, and some transmission gain will be provided by this negative impedance. If now the volume of speech on the line is increased beyond the overload point the result will be a noticeable reduction in the amount of negative impedance, and a consequent reduction in gain. Under excessive overload conditions the negative impedance becomes small; and the effect of the converter is scarcely discernible on the transmission of speech as far as gain is concerned. The harmonic distortion introduced on the line by the vacuum tubes overloading will increase to a maximum and then decrease with further increase in volume. The constants of the circuit coupling the plate to the grid of the vacuum tube or tubes in the converter will determine the maximum amount of distortion. A push-pull circuit is better from this standpoint than a single sided one. In the E1 repeater harmonics are not particularly objectionable under any condition of overload.

The E1 repeater employs a single 407A vacuum tube which is a twin triode of the 9-pin miniature type. When operated on a plate voltage of 130 volts this repeater will pass speech volumes of +10 *vu* before compression begins because of overloading in the vacuum tube.

Development of the E1 Equivalent Circuit

One purpose of this paper is to prove that Fig. 6(b) is equivalent to Fig. 6(a)

Assume that an impedance Z_N is connected across terminals 3 and 4. Assume, furthermore, that the elements C_1, C_2, R_3 and R_5, which also connect across terminals 3 and 4, are all included in impedance Z_N. Assume an impedance Z_L is connected to terminals 1 and 2. The circuit of Fig. 6(a) then can be represented by Fig. 8 where the vacuum tubes have been replaced by their plate resistances (R_p) and their equivalent circuit voltages $\mu_1 e_1$ and $\mu_2 e_2$. The voltage $\mu_1 e_1$ is that voltage which appears in the plate circuit of each tube as a result of the action on the grid of all voltages between the tube plates and ground. The voltage $\mu_2 e_2$ is that which appears in the plate circuit of each tube as a result of the voltages between the cathode and ground, Fig. 6(a). The resistors R_1, R_2 and R_6 designate the resistance in the various coil windings plus other circuit resistance which might be inserted at the points indicated. The reactances X_1, X_2 and X_3 represent the effect of the self-inductance in the coil windings. The reactances M_1, M_2 and M_3 represent the effect of the mutual inductances between coil windings. The numbers on a coil winding determine the polarity of the winding with respect to other windings on the same core. As mentioned previously the vacuum tubes are operated Class A.

The basic circuit equations can be written as follows for Fig. 8:

$$
\begin{aligned}
0 &= PI_1 + QI_2 - M_3(1 + \mu_2)I_3 - S(1 - \mu_1)I_4 \\
0 &= QI_1 + PI_2 - M_3(1 + \mu_2)I_3 - S(1 - \mu_1)I_4 \\
E_3 &= -M_3I_1 - M_3I_2 + (Z_L + R_6 + X_3)I_3 + 0 \\
E_4 &= -SI_1 - SI_2 + 0 + (Z_N + 2S)I_4
\end{aligned}
$$

Where

$$
\begin{aligned}
P &= R_0 + R_1 + X_1(1 - \mu_1 k_1) + X_2\,(1 + \mu_2) \\
Q &= -\mu_1 R_1 + X_1(k_1 - \mu_1) + X_2(1 + \mu_2)k_2 \\
S &= R_1 + X_1(1 + k_1) \\
R_0 &= R_p + R_2(1 + \mu_2) \\
k_1 &= \text{coupling factor} = M_1/X_1 \\
k_2 &= \quad \text{“} \qquad \text{“} \quad = M_2/X_2 \\
k_3 &= \quad \text{“} \qquad \text{“} \quad = M_3/\sqrt{X_2 X_3} \\
E_3 \text{ and } E_4 &= \text{applied voltages}
\end{aligned}
$$

While the derivation of most of the coefficients in these mesh equations is obvious, the derivation of P and Q may require further explanation. Coefficient P can be considered as follows:

1—The term R_0 equals $R_p + R_2(1 + \mu_2)$. The plate resistance R_p is in the plate circuit and hence stands alone. The resistance R_2 being between cathode and ground, Fig. 6(a), produces negative feedback and must be multiplied by $(1 + \mu_2)$.

2—The resistance R_1 is in the plate circuit, and here as in the case of R_p the flow of current in R_1 does not produce a voltage across the grid of the same tube through which this current flows.

3—The reactance X_1 is in the plate circuit of each tube, and by means of the mutual reactance (M_1) is coupled to the respective grid of the other tube. This coupling provides positive feedback for current flowing through X_1 which can be expressed as $-\mu_1 M_1$ or $-\mu_1 k_1 X_1$. Thus the term $X_1(1 - \mu_1 k_1)$ is derived.

4—The reactance X_2 being between cathode and ground, Fig. 6(a), provides negative feedback. Hence X_2 must be multiplied by $(1 + \mu_2)$.

Coefficient Q can be explained in similar fashion:

1—Although the flow of current through R_1 does not produce a voltage across the grid of the same tube through which this current flows, a voltage drop is produced across the grid of the other tube because of the cross coupling of these grids, Fig. 6(a). Thus voltage drop in one plate circuit appears between grid and ground of the other tube in the direction to aid the flow of current in this other mesh; hence the term $-\mu_1 R_1$.

2—Likewise, the reactance X_1 acts in the same manner as R_1 in aiding the flow of current in the other tube circuit. Furthermore, X_1 is coupled by the mutual reactance to this other mesh. These effects can be expressed as $X_1(k_1 - \mu_1)$.

3—The reactance X_2 is coupled by mutual reactance to both tube circuits. It appears in each circuit between cathode and ground in the polarity to produce negative feedback equal to $X_2(1 + \mu_2)k_2$.

In order to establish the fact that Fig. 6(a) can be represented by the equivalent circuit of Fig. 6(b) the basic mesh equations will be developed in the following manner:

First—The impedance seen looking into the converter from terminals 1 and 2, Z_{12}, will be found.

Second—An equivalent circuit which will provide this impedance will be drawn.

Third—The impedance seen looking into the converter from terminals 3 and 4, Z_{34}, will be obtained.

Fourth—The equivalent circuit for Fig. 6(a) should be the logical result.

$$Z_{12} = \frac{E_3}{I_3} - Z_L \qquad \text{Eq. (5)}$$

$$I_3 = \frac{E_3 \begin{vmatrix} P & Q & -S(1-\mu_1) \\ Q & P & -S(1-\mu_1) \\ -S & -S & (Z_N + 2S) \end{vmatrix}}{\Delta} \qquad \text{Eq. (6)}$$

where:

$$\Delta = -M_3 \begin{vmatrix} Q & -M_3(1+\mu_2) & -S(1-\mu_1) \\ P & -M_3(1+\mu_2) & -S(1-\mu_1) \\ -S & 0 & Z_N + 2S \end{vmatrix} + M_3 \begin{vmatrix} P & -M_3(1+\mu_2) & -S(1-\mu_1) \\ Q & -M_3(1+\mu_2) & -S(1-\mu_1) \\ -S & 0 & Z_N + 2S \end{vmatrix} + (Z_L + R_6 + X_3) \begin{vmatrix} P & Q & -S(1-\mu_1) \\ Q & P & -S(1-\mu_1) \\ -S & -S & Z_N + 2S \end{vmatrix}$$

Solving for I_3

$$I_3 = \frac{E_3(P-Q)\left[(P+Q)(Z_N+2S) - 2(1-\mu_1)S^2\right]}{(P-Q)\left[[Z_L + R_6 + X_3][(P+Q)(Z_N+2S) - 2S^2(1-\mu_1)] - 2M_3^2(1+\mu_2)(Z_N+2S)\right]} \qquad \text{Eq. (7)}$$

$$\frac{E_3}{I_3} = (Z_L + R_6 + X_3) - \frac{2M_3^2(1+\mu_2)(Z_N+2S)}{(P+Q)(Z_N+2S) - 2(1-\mu_1)S^2} \qquad \text{Eq. (8)}$$

$$Z_{12} = R_6 + X_3 - \frac{2M_3^2(1+\mu_2)(Z_N+2S)}{(P+Q)(Z_N+2S) - 2(1-\mu_1)S^2} \qquad \text{Eq. (9)}$$

where

$$P + Q = R_0 + (1 - \mu_1)S + X_2(1 + \mu_2)(1 + k_2) \qquad \text{Eq. (10)}$$

Substituting for $P + Q$ and rearranging

$$Z_{12} = R_6 + X_3 \left| \frac{R_0 + X_2(1 + \mu_2)(1 + k_2 - 2k_3^2) + \dfrac{(1 - \mu_1)SZ_N}{Z_N + 2S}}{R_0 + X_2(1 + \mu_2)(1 + k_2) + \dfrac{(1 - \mu_1)SZ_N}{Z_N + 2S}} \right| \qquad \text{Eq. (11)}$$

Equation (11) can be rewritten in a form from which the equivalent circuit can readily be constructed. This form is given below in Equation (12).

$$Z_{12} = R_6 + \left| \frac{1}{\dfrac{1}{X_3} + \dfrac{1}{\left[\dfrac{X_3}{2X_2}\right]\left[\dfrac{R_0}{k_3^2(1 + \mu_2)}\right] + \dfrac{X_3(1 + k_2 - 2k_3^2)}{2k_3^2} + \left[\dfrac{1 - \mu_1}{1 + \mu_2}\right]\left[\dfrac{X_3}{4k_3^2 X_2}\right]\left[\dfrac{2SZ_N}{Z_N + 2S}\right]}} \right| \qquad \text{Eq. (12)}$$

From Equation (12) the equivalent circuit of Fig. 9(a) can be developed. Starting at the lower right-hand side of Equation (12) it will be observed that $2SZ_N/(Z_N + 2S)$ represents Z_N and $2S$ in parallel. This parallel combination is multiplied by $(1 - \mu_1)X_3/(1 + \mu_2)(4k_3^2\, X_2)$. In series with the parallel combination is the leakage inductance of transformer T equal to $X_3(1 + k_2 - 2k_3^2)/2k_3^2$; and the term containing R_0, which stands for $R_p + R_2(1 + \mu_2)$. In parallel with all of this is X_3. The resistance R_6 is simply a series resistance as evident from Fig. 8.

It can be seen from Fig. 9(a) that $X_3{:}4X_2k_3^2$ is the impedance ratio of transformer T, and that $-(\mu_1 - 1)/(\mu_2 + 1)$ is the transformation ratio of an ideal converter. The resulting circuit can be represented by Fig. 9(b).

Next consider the impedance Z_{34}, which is seen looking into terminals 3 and 4 of Fig. 8.

$$Z_{34} = \frac{E_4}{I_4} - Z_N \qquad \text{Eq. (13)}$$

$$I_4 = \frac{\begin{vmatrix} P & Q & -M_3(1 + \mu_2) \\ Q & P & -M_3(1 + \mu_2) \\ -M_3 & -M_3 & Z_L + R_6 + X_3 \end{vmatrix} E_4}{\Delta} \qquad \text{Eq. (14)}$$

$$\frac{E_4}{I_4} = \frac{(Z_N + 2S)[(Z_L + R_6 + X_3)(P + Q) - 2M_3^2(1 + \mu_2)] - 2(1 - \mu_1)S^2(Z_L + R_6 + X_3)}{(P + Q)(Z_L + R_6 + X_3) - 2M_3^2(1 + \mu_2)} \qquad \text{Eq. (15)}$$

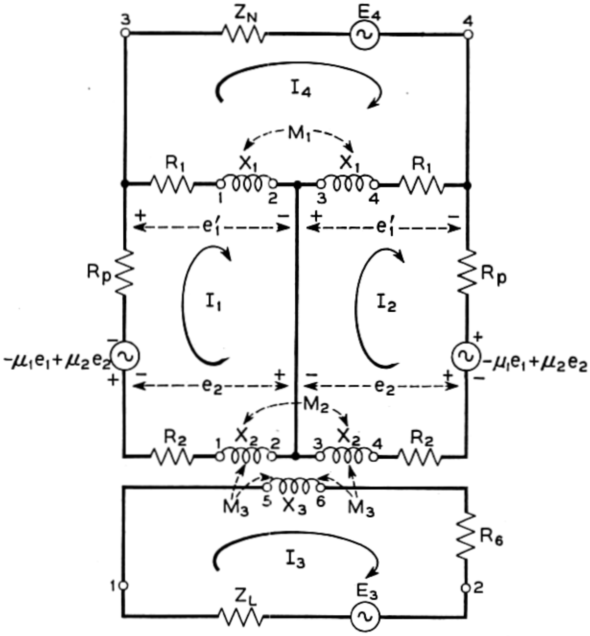

Fig. 8—Schematic of E1 repeater.

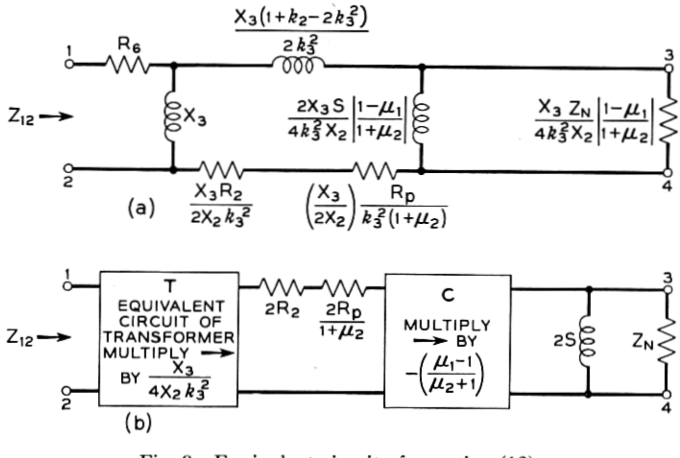

Fig. 9—Equivalent circuit of equation (12).

$$Z_{34} = 2S - \frac{2(1-\mu_1)S^2(Z_L+R_6+X_3)}{(P+Q)(Z_L+R_6+X_3) - 2k_3^2X_2X_3(1+\mu_2)} \quad \text{Eq. (16)}$$

Substituting for $P + Q$ and rearranging

$$Z_{34} = \left| \frac{1}{\dfrac{1}{2S} + \dfrac{1}{\dfrac{2R_p}{(1-\mu_1)} + \dfrac{2X_2(1+\mu_2)}{(1-\mu_1)}[1 + k_2 - 2k_3^2] + \dfrac{4k_3^2X_2}{X_3}\dfrac{(1+\mu_2)}{(1-\mu_1)}\dfrac{X_3(Z_L+R_6)}{(Z_L+R_6+X_3)}}} \right| \qquad \text{Eq. (17)}$$

From Equation (17) the circuit of Fig. 10(a) can be developed in a manner similar to the way Fig. 9(a) was developed from Equation (12). Furthermore, Fig. 10(a) can be rearranged in the form shown in Fig. 10(b).

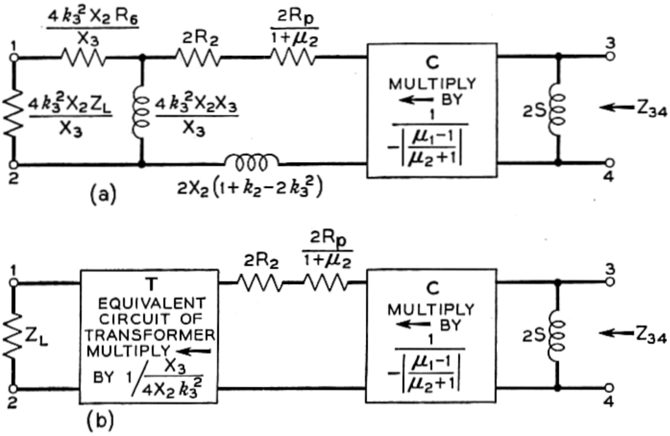

Fig. 10—Equivalent circuit of equation (17).

A comparison of Fig. 9(b) with Fig. 10(b) shows these two circuits to be essentially the same except for the point of view. If it is remembered that Z_N in Figs. 8, 9 and 10 has been chosen to include the elements in the RC network which couples the plate of one vacuum tube to the grid of the other, then from consideration of Figs. 9(b) and 10(b) it is obvious that Fig. 6(b) is the equivalent circuit of Fig. 6(a).

If Equation (6) or Equation (7) were studied it would be found that for all conditions of circuit stability the current I_3 would flow in the same general direction in which it would be expected to flow in any circuit mesh where all impedances were positive. But Z_{12} can equal a negative impedance. Hence, Z_{12} must be a reversed voltage type of negative impedance equal to $-V/I$. For an impedance to be negative either the voltage V at some frequency must be the reverse in polarity of that measured across a like positive impedance or the current must flow in the reverse direction. If the current does not reverse direction the voltage must, to produce a negative impedance.

If Equation (14) for the current I_4 were studied it would be found that all conditions which meet the requirement for circuit stability when the impedance Z_{34} is negative would produce a flow of current through this mesh with a phase impossible to realize were Z_{34} positive. Hence, if impedance Z_{34} is to be negative the current must be reversed and the impedance must be of the reversed current type equal to $V/-I$, where here V is the voltage across Z_{34} and I is the current flowing through it.

Design Considerations

If a resistance were connected to the network terminals 3 and 4 of Fig. 6(a) the impedance seen at the line terminals 1 and 2 would follow a locus

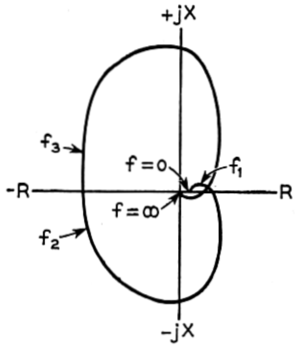

Fig. 11—Impedance locus of E1 repeater.

similar to that shown in Fig. 11 for the frequency range from zero to infinity. At zero frequency this impedance would be a small positive resistance equal to the d-c resistance of the primary windings of transformer T. At some low frequency f_1 (Fig. 11) the locus would show a positive impedance. In the E1 repeater it is this portion of the impedance characteristic which is used for the passage of low frequencies such as ringing, dialing and the like. Between the frequencies of f_2 and f_3 (Fig. 11) is seen an impedance which approximates a negative resistance. At high frequencies the locus would approach the origin again. In Fig. 11 this approach is shown through the first and then the fourth quadrant. At frequencies above the speech band passed by the telephone line a negative impedance is not wanted for the E1 repeater because it is of little value for voice transmission and may be detrimental in adding to the difficulty of obtaining stable operation.

In the design of the E1 repeater it is desirable for the network to have control of the impedance presented at the line terminals over the voice frequency range (f_2 and f_3 of Fig. 11). To accomplish this all impedances shunting the converter circuit are made as large as possible and all impedances in series are made as small as possible at these frequencies. Furthermore, μ_1 is made as close to μ_2 as practical, and large enough so that the factor $(\mu_1 - 1)/(\mu_2 + 1)$ is made as close to unity as possible. If this factor approximates unity and if the series term $R_p/(1 + \mu_2)$ is relatively small with respect to impedances between which the converter is operating, it can be seen from the equations that battery supply variations and tube changes should have little effect upon the negative impedance presented by the converter at these frequencies.

The retard coil and transformer inductances should be considered at low frequencies as a possible source of instability. At low frequencies the inductance of the retard coil shunting terminals 3 and 4, and the inductance of the transformer shunting terminals 1 and 2 materially affect the impedances facing the ideal converter.

Another important consideration in the design is the ratio of the line transformer. The ratio must be such that the tubes will operate efficiently with the normal network, line load, and plate supply voltages. This part of the design follows conventional methods.

The distributed capacitances of the line transformer windings should be taken into account. In general, these are large compared to the tube capacitances between cathodes and between ground and each cathode. These interelectrode capacitances can be neglected at voice frequencies, but at the higher frequencies all of them should be considered, and conventional methods of suppression of parasitic oscillations should be applied. Also, the distributed capacitance of the retard coil must be considered from the standpoint of stability at the higher frequencies.

Conclusion

A vacuum tube arrangement for producing negative impedance can be represented by an equivalent four-terminal network consisting of an ideal converter having a ratio of transformation of $-k{:}1$ and two positive impedance networks located one on each side of the ideal converter. In these two networks some of the elements can be cancelled in effect by balancing those in one network against corresponding elements in the other. Elements which cannot be balanced can be made either relatively large or relatively small compared to the impedances between which the converter is designed to operate. Across one pair of the four terminals of a practical converter can be seen a reverse voltage type of negative impedance; across the other pair can be seen a reversed current type. This device can be made to approach the ideal only over a finite frequency range. At some frequencies k will not be a numeric. Likewise, at some frequencies the two internal networks will produce appreciable effect. Stability can be ascertained readily from a working knowledge of the components of the equivalent circuit. Designs are practical wherein variations in battery voltages and vacuum tube constants are second-order effects.

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In recent years there has been a growing need for an inexpensive two-wire repeater having a cost lower than that of the 22 type for use in the exchange plant. Mr. G. C. Reier encouraged the study of negative impedance devices as a possible way of meeting this need. Had it not been for his encouragement the E1 repeater development might not have been undertaken.