Matter, A Mode of Motion

By R. V. L. HARTLEY

(*Manuscript Received Feb. 28, 1950*)

Both the relativistic and wave mechanical properties of particles appear to be consistent with a picture in which particles are represented by localized oscillatory disturbances in a mechanical ether of the MacCullagh-Kelvin type. Gyrostatic forces impart to such a medium an elasticity to rotation, such that, for very small velocities, its approximate equations are identical with those of Maxwell for free space. The important results, however, follow from the inherent non-linearity of the complete equations and the time dependence of the elasticity associated with finite displacements. These lead to reflections which permit of a wave of finite energy remaining localized. Because of the non-linearity, the amplitude and energy of a stable mode, as well as the frequency, are determined by the constants of the medium. Such a stable mode is capable of translational motion and so is suitable to represent a particle. The mass assigned to it is derived from its energy by the relativity relation. While this mass is dimensionally the same as that of the medium it is differently related to the energy and so need not conform to the classical laws which the latter is assumed to obey.

Exchanges of energy between particles and between a particle and radiation involve frequency changes as in the quantum theory. The experimental detection of a uniform velocity relative to the medium is not to be expected. Besides providing a new approach to the problems of particle mechanics, the theory offers the prospect of incorporating the present pictures into a more comprehensive one, with a material reduction in the number and complexity of the independent assumptions.

Introduction

The following quotation states a conclusion which is widely held: "But in view of the more recent development of electrodynamics and optics it became more and more evident that classical mechanics affords an insufficient foundation for the physical description of all natural phenomena."[1] This implies that classical mechanics and classical electromagnetics are so alike that one may be condemned for the shortcomings of the other. Actually, classical electromagnetics is in open disagreement with classical mechanics particularly with respect to those features for which it has been most criticized. According to the mechanical principle of relativity,[2] the equations of any mechanical system are invariant under the Newtonian transformation, $x = x' + Vt', y = y', z = z', t = t'$, where V is a constant velocity in the x direction. Since the classical electromagnetic equations are not invariant under this transformation, they cannot describe the performance of any classical mechanical system. Their failures, therefore, should not stand in the way of a study of the possibilities of such systems.

The system considered here is the so-called rotational ether, suggested

[1] A. Einstein, The Theory of Relativity, Methuen & Co., Ltd., London, 1921, p. 13.
[2] Haas, Introduction to Theoretical Physics, 2nd Ed., Vol. I, p. 46.

by MacCullagh and elaborated by Kelvin, in which the stiffness is associated with gyrostatic forces. Some consideration has been given to an alternative model consisting of a non-viscous liquid in a high state of fine scale turbulence. It is well known that, by virtue of the gyrostatic forces associated with it, a vortex will transmit a wave of transverse displacement along its axis. It would appear, therefore, that a gross wave involving similar displacements would be passed along from vortex to vortex, much as a sound wave is passed from molecule to molecule. However, since this model has not yet been shown to be fully equivalent to Kelvin's, attention will be confined to the latter. While this, as developed by Kelvin, gave a satisfactory description of electromagnetic waves in free space, it had nothing to represent matter. This was assumed to be something different from ether, which might or might not be pervaded by it. A closer study of the model has indicated that the peculiar nature of its stiffness makes possible sustained oscillatory disturbances in which the energy remains localized about a center which may move with any velocity less than that of a free wave. It is proposed to use such quasi-standing wave patterns to describe material particles. Matter, then, has no existence apart from the ether, and the motion of particles is the motion of patterns of mechanical wave motion. While the ether itself conforms to Newtonian mechanics, the mechanics of such a wave pattern, considered as a particle located at its center, is much more complicated than that of the familiar mass point of particle dynamics. This complexity provides a bridge from the older concepts of particle behavior to the new.

The study of this model given below reveals no insuperable obstacles such as were encountered by the electromagnetic theory and the simpler ether model. The properties of the wave-patterns are qualitatively consistent with many of the concepts of modern physics, though in some cases not with the generality of application which is now assigned to them. Among these concepts are: the space-time of special relativity, relativistic mechanics, de Broglie waves, proportionality of energy and frequency, energy thresholds, and transfers of energy according to the quantum frequency formula. The ether model also leads to certain concepts not found in the present theories. It provides, for example, for a possible failure of the mass-energy balance such as has been observed in nuclear reactions. It also suggests the possibility of a new type of particle which, by virtue of its negative inertial mass, is capable of exerting a binding force between other particles.

These results make it more probable that classical mechanics may, after all, afford a sufficient "*foundation* for the physical description of all natural phenomena" even though the super-structure be very different from that contemplated by its originators. The present argument, however, is not that this particular description is necessary, but rather that it offers distinct advantages. On the philosophical side, there is the prospect of greater unification of the basic theory through a reduction in the number of independent assumptions. Matter and radiation appear as wave motions which satisfy the same equations. The apparent conflicts between current concepts appear to be reconcilable through a more exact determination of the conditions under which each applies. On the more practical side, the ether model provides a different approach and technique. It has the advantage inherent in all models that, once one is found which fits one set of conditions, a study of its properties under widely different conditions may bring out relations which it would be difficult to postulate solely on the basis of observations made under the second conditions. The suggested existence of particles having negative inertia, as discussed near the end of the paper, should it lead to anything of value, would be an example of such a relation. Also it makes available the added relationships which are characteristic of non-linear equations, without encountering those difficulties with respect to absolute motion which may arise when non-linearity is introduced arbitrarily. While the working out of the quantitative relations involved is a rather formidable undertaking, any effort in that direction may well throw new light on those problems which have not yielded to other methods.

The Gyrostatic Ether

As stated above the specific form of gyrostatic medium on which the present discussion is based is the ether model proposed by Kelvin. This is discussed in detail in a companion paper.[3] It is there shown that, for infinitesimal displacements, it is characterized by the wave equations:

$$\nabla \times \left(\frac{\overline{T}}{2}\right) = \rho_0 \frac{\partial \bar{q}}{\partial t} \tag{1}$$

$$\nabla \times \bar{q} = -\frac{1}{\eta_0} \frac{\partial}{\partial t}\left(\frac{\overline{T}}{2}\right), \tag{2}$$

where ρ_0 is the density, η_0 is a generalized stiffness determined by the constants of the medium, $\bar{q}$ is the vector velocity, and $\overline{T}$ is a vector torque per unit volume, which has its origin in the torque with which a gyrostat opposes an angular displacement of its axis. For a plane polarized plane wave, the quantity $\frac{\overline{T}}{2}$ can be interpreted as a surface tractive force per unit area, which a layer of the medium normal to the direction of propagation exerts on the layer just ahead. Its direction lies in the surface of separation, and is parallel to that of the velocity $\bar{q}$.

[3] R. V. L. Hartley, "The Reflection of Diverging Waves by a Gyrostatic Medium"—this issue of *The Bell System Technical Journal*.

These equations become identical with those of Maxwell for free space,

$$\nabla \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t},$$

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t},$$

if we replace $\bar{q}$ by $\bar{E}$, $\frac{\bar{T}}{2}$ by $\bar{H}$, ρ_0 by ϵ and $\frac{1}{\eta}$ by μ. Then $\rho_0\bar{q}$ corresponds to $\bar{D}$ and -2φ to $\bar{B}$ where φ is the angular displacement of an element of the medium. Or the roles of the electric and magnetic quantities may be interchanged.

For present purposes, however, we are more interested in finite displacements. The relations which then apply are discussed in detail in the companion proper. It is there shown that changes of two kinds appear in (1) and (2), with corresponding changes in the transmission properties of the medium. The simple linear relations are to be replaced by non-linear ones, which cause distortion of a wave but no reflection. In addition, a qualitative difference appears in the nature of the elasticity, as was pointed out by Kelvin. The restoring torque is no longer proportional to the angular displacement alone. When the axis of a gyrostat is displaced it begins rotating toward the axis of the displacement, thereby decreasing the component of its spin which is normal to that axis. Thus the restoring torque for a constant angular displacement decreases with time. The restoring torque is therefore a function of the time as well as of the displacement. Because of this time dependence, a disturbance of finite amplitude generates waves which propagate both backward and forward.

For a plane progressive sine wave it is found that the reflected waves interfere destructively. However, if a central generator starts sending out a diverging sinusoidal disturbance, a part of the energy is reflected inward as a wave of the same frequency as the generator and another smaller part as waves the frequencies of which are odd multiples of that frequency. This reflection attenuates the outgoing wave. If the incoming wave is reflected rather than absorbed at the generator, it tends to set up a standing wave pattern. As time goes on, the impedance of the medium as seen from the generator becomes more reactive and less power is drawn from the generator. Due to the attenuation, the energy in spherical shells of a given thickness decreases with increasing radius, so that it and the power transmitted at the wave front approach zero as r approaches infinity. This falling off is somewhat similar to that suffered by a wave the frequency of which lies in the stop band of a filter, but with one important difference. There the attenuation is independent of the distance. But here, since the attenuation is a function of the magnitude of the disturbance and of the curvature of the wave-front, the attenuation constant approaches zero as r increases indefinitely.

Whether or not the total energy stored in the wave pattern will approach a finite or infinite value depends on how fast the attenuation decreases with distance, and a more complete solution is needed to give an exact answer. If it does approach infinity it will do so much more slowly than for a medium which does not reflect.

The disagreement between classical electromagnetics and mechanics, referred to above, may now be stated more explicitly. The former says that electromagnetic waves are represented exactly by Maxwell's equations, regardless of the magnitudes of the electromagnetic variables. When these waves are interpreted as existing in a mechanical ether, classical mechanics says that Maxwell's relationship is approached as a limit as the mganitudes approach zero. Waves of finite amplitude are to be represented by the more complicated relations.

The two systems differ in three important respects; their relation to uniform linear motion, the linearity of their equations and the nature of the elasticity involved. Because the classical electromagnetic equations are not invariant under a Newtonian transformation, the set of axes to which the equations refer are uniquely related to other sets which are moving uniformly with respect to them. In special relativity, this condition is avoided by modifying the classical concepts of space and time to conform to the fact that the equations are invariant under the Lorentz transformation. The Newtonian invariance of the ether equations, however, insures that a set of axes at rest with respect to the undisturbed ether is not unique. Hence in the modified model, in which *the motions which constitute matter* conform to the laws of the ether, a uniform linear velocity of the entire system cannot be detected. This is consistent with the accepted principle that absolute velocity is meaningless.

We are, however, still faced with the question of the detection of uniform motion of matter relative to the ether. This is discussed at length below, where it is shown that the properties of the ether lead directly to an auxiliary space-time, which applies very closely under the experimental conditions and accounts for the failure to detect the motion. This "experimental" space-time is formally identical with that of special relativity. Thus the modification of the space-time of classical electromagnetics which appears in special relativity might be said to bring it into closer formal agreement with the classical mechanics of ether wave patterns. At any rate the establishing of this theoretical connection between the space-time of special relativity and a classical mechanical model is a step toward unification.

On the matter of linearity, proposals have been made to add arbitrary non-linear terms to Maxwell's equations. While this also makes the electromagnetic equations more like those of the ether, an important difference still remains. An equation obtained in this way is not necessarily invariant under either a Newtonian or a Lorentz transformation. If, then, the axes with respect to which it is expressed are not to be unique, it must be shown that some transformation exists under which it is invariant. Not only is the form of the equation important here but also the interpretation of the dependent variables. For example, since the complete equations of the ether contain $q \cdot \nabla$, if the mechanical variables be replaced by the analogous electromagnetic ones, the equations will be Newtonian invariant only if $\bar{E}$, which replaces $\bar{q}$, is interpreted as a velocity. It is evident, therefore, that the fact that we are dealing with a mechanical model is an important point in the argument. Also, unless the added terms make the effective constants depend on the time as well as the dependent variables, there will be no reflection of the energy in a finite disturbance and the medium will not have the energy trapping property which is essential to the present argument.

Stationary Wave Patterns

The first question to be considered is the possibility of setting up a sustained wave pattern suitable to represent a particle at rest with respect to the ether. The simplest procedure might seem to be to look for it as a solution of the approximate linear equations in the form of a pair of spherical waves propagating radially, one outward and one inward, so as to form together a standing wave pattern. However, certain difficulties are encountered. There is nothing in the free linear ether which can serve as boundary conditions to fix the position or size of the pattern. Even if these were determined, there would be nothing to fix the amplitude, and so the energy. Most patterns, particularly those which involve a single frequency, have one or more of the following features. Some of the variables become infinite at the center; the total energy is infinite, energy is propagated away radially.

These difficulties disappear, however, when we take account of the properties of the ether for disturbances of finite amplitude. Let us suppose that the energy which is to constitute the pattern is supplied by a central generator, the impedance of which is mainly reactive, so that reflected waves which reach it are reflected outward again. Once a standing wave pattern has been established as described above, let the force of the generator be reduced to zero without changing its impedance. The pattern will then persist except for a small and decreasing damping due to the outward radiation at its periphery. However, in the region near the center the displacements will be very large, and the incoming reflected waves will suffer reflections which increase with decreasing radius. These reflections will effectively take the place of the assumed reactive impedance of the generator, and so the latter may be discarded. The fact that the reflections take place from a somewhat diffuse inner boundary prevents the amplitude from building up to an infinite value at the center as it would with a linear medium.

However, the reflected wave includes components of triple and higher frequencies and, due to the non-linearity, other frequency components will be generated. If the entire pattern is to be stable, all of these must satisfy the boundary conditions. Their magnitudes relative to the fundamental, for a particular mode of oscillation, will depend on the amplitude and frequency of the fundamental, as well as on the constants of the medium. Hence the amplitude as well as the frequency of a stable pattern of a particular mode should be uniquely determined. Particles of different properties would then be expected to consist of patterns involving different modes of oscillation.

Returning to the lack of complete reflection at the outer boundary and the change it might be expected to make in the pattern with time, this might be an important factor for a single particle alone in the universe. Actually, however, a very large number of particles are present. If we consider a point at a considerable distance from any one particle, a point in a vacuum, the resultant of the disturbances produced there by all the patterns will be very large compared with that due to any one. But the effect on a particular pattern of its own loss by radiation will be determined by this small component, and so will be small compared with the effect exerted on it by the combined small fields of its neighbors. This combined field due to a large number of patterns, randomly placed, and moving at random, will constitute a randomly varying electromagnetic field in a vacuum, such as has recently been postulated for other reasons. If, now, the center of a pattern be placed at the point in question, this random field may occasionally take on so large a value as to disturb the equilibrium conditions of the pattern.

It may be argued that, in spite of the merging of a given pattern in that of the random group, the group as a whole will suffer a progressive loss of energy through incomplete reflection. Were this to occur the total loss of energy would not be evenly distributed among the particles. As discussed below the particles would exchange energy through the mechanism of the non-linearities, continually forming less stable group patterns of greater energy, which in turn suffer transitions to more stable patterns of lower energy. A small continuous decrease in total energy would manifest itself as an increase in the rate of transitions downward in energy compared to those upward.

Associated with a standing wave pattern such as that described above would be three regions. Near the center would be a relatively small core in which the non-linear effects predominate and linear theory is totally inapplicable. Farther out the departure from linearity is only moderate, and the variation of the constants with distance is slow enough that the reflections are small. It should be possible to treat wave propagation in this region by the methods developed for a string of variable density, which are sometimes cited as analogous with those employed in wave mechanics. The analogy is made closer by the fact that the variations in impedance which correspond to the varying density are determined by the energy density of the pattern itself. Still farther out the amplitudes become still smaller, the ether constants become very nearly but not quite uniform, and the pattern approaches very closely to that in a linear medium.

While the nature of the pattern is determined largely by the non-linear inner region, because of the small volume of this region most of the energy will be located in the nearly linear region. So we might expect some at least of the macroscopic properties of the pattern to differ very little from those deduced from a consideration of the corresponding pattern in a linear medium. We will therefore begin by examining such a pattern. For the linear case, when the axes are at rest with respect to the undisturbed ether, (1) and (2) lead to the wave equation for the vector displacement $\bar{s}$,

$$\frac{\partial^2 \bar{s}}{\partial t^2} = c^2 \nabla^2 \bar{s}. \tag{3}$$

As is well known, this is satisfied by any function of the form

$$\bar{s} = f(\omega t \pm k_x x \pm k_y y \pm k_z z),$$

where

$$\frac{\omega^2}{c^2} = k_x^2 + k_y^2 + k_z^2, \tag{4}$$

and the constants ω, k_x, k_y and k_z, are real or complex. Since an imaginary frequency is interpreted as an exponential change with time, it is not suitable for representing a permanent pattern, so ω will be taken to be real. Imaginary values of k are interpreted as exponential variations with distance. But, since $\bar{s}$ is always real, we may, by a four-dimensional Fourier analysis, represent f as the summation of components of the form

$$\bar{s} = \bar{A} \cos (\omega t \pm k_x x \pm k_y y \pm k_z z), \tag{5}$$

where $\bar{A}$ is a complex vector representing the amplitude and phase of the component, and k_x, k_y and k_z are real. Since each component must satisfy (3), the new constants must satisfy (4). Each such component constitutes a plane progressive wave traveling, with velocity c in a direction, the cosines of which are proportional to the wave numbers k_x, etc.

As a first step in building up a stationary pattern, in which there is no steady propagation of energy in any direction, we combine two progressive wave components (5) which are identical, except that their directions of phase propagation along, say, the z axis are opposite. The signs of the last terms are then opposite and the sum can be written

$$\bar{s} = 2\bar{A} \cos (\omega t \pm k_x x \pm k_y y) \cos k_z z.$$

Proceeding in the same way for x and y, we arrive at the standing wave pattern,

$$\bar{s} = 8\bar{A} \cos \omega t \cos k_x x \cos k_y y \cos k_z z. \tag{6}$$

Components of this sort, each with its own amplitude and phase, may be combined to build up possible stationary patterns. However, we shall not attempt here to build such patterns, but rather to deduce what information we can from a study of a single component.

Moving Wave Patterns

In order to represent approximately a particle in uniform linear motion, we are to look for a solution of (3) which represents a moving wave pattern. For this we make use of two functions which may readily be shown to be such solutions,

$$\bar{s} = g_+ \left(\beta(\omega + Vk_x)t - \beta \left(k_x + \frac{V\omega}{c^2} \right) x \pm k_y y \pm k_z z \right),$$

$$\bar{s} = g_- \left(\beta(\omega - Vk_x)t + \beta \left(k_x - \frac{V\omega}{c^2} \right) x \pm k_y y \pm k_z z \right),$$

where ω, k_x, k_y and k_z are real and satisfy (4), V is a real constant, and

$$\beta^2 = \frac{1}{1 - \dfrac{V^2}{c^2}}.$$

g_+ represents a plane progressive wave the propagation of which along the x axis is in the positive direction. g_- represents one of lower frequency, propagating in the negative x direction. Their wave numbers in the x direction differ in such a way that those in the y and z direction are the same for the two. In the plane wave case, where $k_y = k_z = 0$ and $\omega = ck_x$, they reduce to

$$\bar{s} = g_{\pm} \left(\beta \left(1 \pm \frac{V}{c} \right) \omega \left(t \mp \frac{x}{c} \right) \right).$$

The two waves then travel in the x direction with velocities c and $-c$, and their frequencies are in the ratio $\dfrac{c + V}{c - V}$.

In order to derive a quasi stationary pattern we replace the functions $g_+(\ \ )$ and $g_-(\ \ )$ by $\bar{B} \cos \alpha(\ \ )$ and combine components in a manner similar to that used in deriving (6). The result is

$$\bar{s} = 8\bar{B} \cos \alpha\beta\omega \left(t - \frac{V}{c^2} x\right) \cos \alpha\beta k_x(x - Vt) \cos \alpha k_y y \cos \alpha k_z z, \quad (7)$$

where $\bar{B}$ is a complex vector, and α may be any real scalar function of V. When we compare this with (6) we find that the last three factors, which in (6) describe a fixed envelope, in (7) describe an envelope which moves in the x direction with velocity V. For the same values of k_x, k_y and k_z, the moving pattern has its dimensions in the x direction reduced relative to those in the y and z in the ratio $\frac{1}{\beta}$. The first factor in (6) describes a sinusoidal variation with time which is everywhere in the same phase. In (7) it describes one, the phase of which varies linearly with x. This factor also describes a wave which progresses in the x direction with a velocity $\frac{c^2}{V}$. The existence of such a wave as a factor in the expression for a moving wave pattern was commented on by Larmor.[4] Aside from the constant α in (7) it will be recognized as the Lorentz transform of (6), as it should be since the approximate equations of which it is a solution are invariant under this transformation.

We shall take (7) to represent one component of a moving wave pattern which represents a moving particle. If we transform this to axes moving with the pattern by a Newtonian transformation it becomes

$$\bar{s} = 8\bar{B} \cos \alpha \left(\frac{\omega}{\beta} t' - \frac{\beta\omega V}{c^2} x'\right) \cos \alpha\beta k_x x' \cos \alpha k_y y' \cos \alpha k_z z', \quad (8)$$

in which the envelope is at rest. This may be thought of as a stationary wave in an ether which is moving relative to the axes with a velocity $-V$. It is a solution of the wave equation for such an ether, as obtained by transforming (3) to the moving axes, or

$$\frac{\partial^2 \bar{s}}{\partial t'^2} = c^2 \nabla'^2 \bar{s} + 2V \frac{\partial^2 \bar{s}}{\partial x' \partial t'} - V^2 \frac{\partial^2 \bar{s}}{\partial x'^2}.$$

The one dimensional form of this equation is identical with that given by Trimmer[5] for compressional waves in moving air, except that in one case $\bar{s}$ is solenoidal and in the other divergent.

So far we have found no reason to associate any particular moving pattern with the assumed stationary one, in the sense that the moving pat-

[4] Larmor, Ency. Brit. 11th Ed., 1910; 13th Ed., 1926, Vol. 22, p. 787.
[5] J. D. Trimmer, *Jour. Acous. Soc. Am.*, 9, p. 162, 1937.

tern describes the result of setting in motion the particle which is described by the stationary pattern. Without further knowledge or assumptions regarding the factors which control the form of the pattern, we can go no farther in this direction by theory alone. Rather than try to guess at these factors, it seems preferable to investigate what properties the wave patterns must have in order to conform to the known results of experiment.

Let us start with the Michelson-Morley experiment to which the earlier ether theory did not conform. The entire apparatus involved in the experiment is now to be considered as made up of particles each of which consists of a wave pattern in the ether. The apparatus as a whole may be regarded as a more complicated wave pattern. The interference pattern formed by the light beams may, if we wish, be included in the over-all pattern. The results to be expected in the experiment do not depend on the oscillatory nature of the wave, nor on its amplitude or phase, but only on its spatial distribution, which is determined by the envelope factors. It is obvious from (8) that, for any uniform velocity $-V$ of the ether relative to the apparatus, the ratios of the dimensions of the envelope along the motion to those across it are reduced, relative to their values when V is zero, in the ratio $\frac{1}{\beta}$. That is to say the apparatus like the fringes undergo this change in relative dimensions. But, as is well known, this is exactly what is required in order that there shall be no apparent motion of the fringes. Hence any one of the stationary patterns in a moving ether, as represented by (8), is consistent with the experiment. This experiment therefore furnishes no basis for selecting any particular pattern.

More generally, in any experiment, the distances and time intervals which are available as standards of comparison are associated with the wave patterns and change with their motion. Thus we may, following the special theory of relativity, define an auxiliary space and time, the units of which are associated with the dimensions and cyclic interval of a particular periodic wave pattern. This pattern then plays the roles of the "practically rigid body" and the "clock" which determine space and time in relativity theory. An examination of (8) shows that the dimensions of the pattern, its frequency, and its phase change with the velocity of the ether relative to the pattern in just the way that the corresponding quantities associated with the rigid body and clock change with velocity in the relativity theory. But there these changes are known to be such that no experiment can detect the velocity involved. It follows, therefore, that no experiment in which the apparatus consists of wave patterns of small amplitude is capable of detecting the velocity V, in (8), which in this case is the velocity of the ether relative to the apparatus. Hence any of the above patterns are consistent with the failure of all experiments designed to detect motion relative to the ether. When account is taken of the non-linearity of the ether the result to be expected should differ from that just found for the linear case only by the small difference between the linear and non-linear patterns, which may easily be too small to measure. Thus the principal obstacle to the older ether theory is removed.

While the special theory of relativity is usually written in the form which corresponds to α being unity in (8), it has long been recognized that there is no theoretical basis for this particular value. The ether patterns are consistent with the more general formulation. In order to pin down the value of α for the ether patterns we resort to another experiment. Ives and Stillwell[6] found that a molecule which emits radiation of frequency ω when at rest emits a frequency $\frac{\omega}{\beta}$ when in motion. This moving frequency is taken relative to axes moving with the molecule, and so is to be compared with the frequency of oscillation $\frac{\alpha}{\beta}\omega$ in (8). This indicates that in order to represent a component of the pattern which results when the fixed pattern is set in motion, we are to put α equal to unity.

Another observed relation is that the energy of a moving particle is β times that of the same particle at rest. This information should be useful in checking any theory of the mechanism by which the non-linearity of the medium determines the energy of the pattern. All we shall do here is to point out one relation, the significance of which from the standpoint of mechanism will be discussed below. In (7), where the frequency is expressed relative to the same axes as the energy of the moving pattern, if we put α equal to unity, the frequency also varies as β. Hence if the pattern conforms to experiment with respect to its energy, the energy must be proportional to the frequency.

Obviously, if we define the mass of the particle-pattern as its energy over c^2, the particle will conform to relativistic mechanics. The mass of a particle as so defined, while dimensionally the same as that of the ether, is in other respects quite different. Since it is derived from the energy associated with a disturbance of the ether, it would be zero in the undisturbed ether, while the ether mass would be finite. The momentum of a particle would be determined by the flow of energy associated with it. Also within a particle, if the mode of oscillation were such that the wave propagated continuously around the axis in one direction, the resulting rotation of the energy would be interpreted as an angular momentum or spin. This concept of spin was suggested by Japolsky[7] in connection with cylindrical waves in a linear medium. There is, therefore, no *a priori* reason to expect that the motion

[6] H. E. Ives and C. R. Stillwell, *Jour. Opt. Soc. Am.*, 28, 215, 1938 and 31, 369, 1941.
[7] N. S. Japolsky, *Phil. Mag.* 20, 417, 1935.

of particles should conform to the laws of classical mechanics. As just noted, it should conform much more closely to those of relativistic mechanics. Also, to the extent that the flow of energy follows the laws of wave mechanics, as suggested below, the behavior of the particles will also conform to those laws. Similar considerations apply to the mass of radiation as derived from its energy.

Another experiment which helps to fix the required properties of the patterns is that of Davisson and Germer, in which it is shown that a particle moving with velocity V is diffracted as if it had a wave length λ such that

$$\lambda = \frac{h}{\beta m_0 V},$$

where h is Planck's constant and m_0 is the rest mass.

If, in (7) with α unity, we assume the energy frequency ratio to be equal to h, the wavelength associated with the first factor reduces to the value given by experiment. This does not mean that an ordinary physical wave of this length is present in the pattern. It does mean that, at any instant, the amplitude of the sinusoidal variation of displacement with distance, as given by the remaining factors, varies sinusoidally with the wave length λ, and is zero as points separated by $\frac{\lambda}{2}$. Hence, when the presence of equally spaced obstacles calls for zero values of displacement at equally spaced intervals, the distorted wave should be capable of forming a stable diffraction pattern when the translational velocity of the pattern is such that the interval between points of zero displacement has the value required by the spacing of the obstacles.

Thus the wave pattern will conform to this experiment provided, first, that it is characterized by a particular wave length, and second, that the factor of proportionality between its energy and frequency is equal to h. The first requirement implies that the wave pattern when at rest has practically all of its energy associated with components which are all of the same frequency, or else are confined to a narrow band near the characteristic frequency.

At this point let us pause for a short review and discussion. Briefly, we have replaced the "rigid body" of special relativity by an oscillatory motion of the ether, the envelope of which is analogous with the configuration of the rigid body. We have found that when in motion this envelope behaves as does the rigid body, and the time relations conform to those of a moving clock. These latter may also be interpreted as a multiplying factor which has the form of a plane wave of the DeBroglie type. In wave mechanics, this is treated as a wave of a single frequency and of a variable phase velocity greater than that of light. In the ether theory this wave is interpreted as one factor in the description of an interference pattern which results from the superposition of component progressive waves of different frequencies, each of which travels with velocity c. This difference in viewpoint leads to other differences.

One of these has to do with the possibility of describing accurately both the position and velocity of a particle, which is ruled out from the wave mechanics viewpoint. An ether wave pattern, however, may have its position accurately described by its envelope, while at the same time the pattern moves with a definite velocity. The particle velocity may here be regarded as a group velocity derived from two waves progressing in opposite directions, but does not depend on the presence of dispersion as does that for waves in the same direction. It is not to be concluded from this that the position and velocity can be *measured* with this accuracy, for we have still to deal with the disturbing effect of the measurement.

From the ether viewpoint, one of the limitations of wave mechanics is to be expected, its inability to calculate directly the position of a particle. The information regarding this position is contained in the expression for the envelope, while the wave factor depends only on its state of motion. A calculation based on a solution which involves the wave factor without the envelope would be expected to be indefinite regarding position. We should expect, however, that it would give information as to the probability of the presence of the particle in a given region, since this is derivable from its state of motion.

Returning to the comparison with experiment, while wave patterns based on the linear equations have shown close agreement so far, the next experiment upsets the applecart. It has been observed that the motion of one particle is modified by the presence of other particles in its neighborhood. So long as the assumed equations are linear, the law of superposition holds, and every solution is independent of every other one. So any wave pattern, when once set up, will continue in its state of rest or of uniform motion indefinitely, and will not be influenced by the presence of other patterns or of free progressive waves. But these together comprise all other matter and radiation. Hence, while we have provided for the property of inertia, there is nothing which tends to alter the state of motion of a body, that is, there are no forces. In this respect the present linear treatment is similar to the special theory of relativity. So, in order to represent the interactions between particles, account must be taken of those between patterns which result from the non-linearity and time dependence of the ether.

Reactions between Patterns

The general problem of the effect of one pattern on another is even more intricate than that of the stable state of a single pattern, which it includes, and its solution will not be attempted here. Some conclusions may, however, be drawn. Since the amount of reflected energy generated by an element of the medium depends on powers of the instantaneous disturbance higher than the first, the superposition of a second pattern will alter the standing wave pattern of the first, and vice versa. Also, as pointed out in the companion paper, the propagation of both the main and reflected waves also depends on higher powers of the instantaneous disturbance there. The resulting variations in the propagation will also affect the conditions for a stable pattern. Neither pattern, then, can satisfy its stability conditions independently of the other; but if the combined patterns are to be stable they must together satisfy a new set of conditions common to both. How much each is altered by such a union will depend on the degree of coupling between them, that is, on the amount of energy which must be regarded as mutual to the two.

The effect of this coupling will be very different, depending on whether the frequencies of the two patterns are the same or different. When they are different the non-linear terms give rise to frequencies related to the first two by the quatum formula. The transfer of energy to these frequencies may, under favorable conditions, set up a new mode of oscillation the stability conditions of which are better satisfied than those of the original frequencies. The new mode might be that of an excited atom. Or the frequency of one or both of the patterns may be changed to that corresponding to the particle in motion with a particular velocity. In either of these processes some of the energy may be released as radiation at one of the difference frequencies.

If, however, the frequencies of the two patterns are identical, no new frequencies will result from their superposition. If the combined pattern is to persist there must be a stable mode for the combination, the frequency of which is identical with that of the separate patterns. This is hardly to be expected. Also the oscillations of the second pattern, being of the same frequency as those of the first, would have a much greater disturbing effect on its conditions for stability. It would appear, then, that if it were possible to bring two patterns of identical frequency into superposition, they would mutually disintegrate. This does not mean that two particles of the same type cannot exist in the same neighborhood. If they have different velocities, for example, their frequencies will be different. The similarity of these considerations to Pauli's exclusion principle is obvious.

If the second pattern has much greater energy than the first, as it will if it represents a much heavier particle, its stability conditions may be little affected by the presence of the first. The behavior of the first, an electron, may then be discussed on the assumption that it exists in a medium, the properties of which vary with position in accordance with the fixed pattern of the second particle, the nucleus. Since the stability conditions for the electron pattern particle are most strongly influenced by the effective constants of the medium near its center, we would expect its energy and frequency to be controlled largely by that part of the nuclear pattern which is near its center. Let us assume that, through some external agency, the center of the electron pattern is transferred from one position of rest to another which is differently placed relative to the nucleus. Owing to the different effect of the nuclear pattern on the effective constants of the medium as viewed by the electron pattern, the stable energy of the latter would be different at the second position. This change in rest energy with position may be interpreted as a measure of the change in a field of static potential associated with the massive nucleus. The similarity between this relationship and that which exists between the electron and the nuclear potential in wave mechanics is obvious.

In speaking of a change in the effective constants of the medium, we refer to an average value taken over a number of cycles and wave lengths of the oscillations which make up the second pattern, or nucleus. Calculations based on this concept should not therefore be expected to give valid results when the time intervals involved in the averages are comparable to the period $\frac{h}{m_0c^2}$ of the second particle at rest, or the distances are comparable to the corresponding wave length $\frac{h}{m_0c}$ of the pattern. For a proton this period is 4.38×10^{-24} seconds and the wave length is 1.31×10^{-13} cms. If, then, an electron is to be subject to the kind of nuclear potential field just described, the linear dimensions of that part of it which is controlled by the potential field of the proton must be at least of the order of 10^{-13} cm. This is consistent with Gamow's[8] observation that "It seems, in fact, that a length of the order of magnitude of 10^{-13} centimeters plays a fundamental role in the problem of elementary particles, popping out wherever we try to estimate their physical dimensions."

The variations in the medium due to the nucleus might be treated in terms of their effect on the progressive wave components, the interference of which gives rise to the wave pattern of the electron. The component waves as so influenced should combine to form an interference pattern which represents the behavior of the electron in the field of the nucleus. It is also possible that a technique may be found for treating their effect on that factor of the electron wave which is similar to the DeBroglie wave. This should be more nearly like the techniques now used in wave mechanics.

If two particles are brought so close together that the central cores of their patterns overlap, the departure from linearity becomes so great that

[8] G. Gamow, Physics Today, 2, p. 17, Jan., 1949.

a procedure which may be successful at intermediate separations becomes inadequate. Relativistic mechanics breaks down and Lorentz invariance may lose its significance. This is in agreement with the experimental result that, in some nuclear reactions, the energy balance, as calculated from the relativistic relations, is not satisfied. Also the difficulty which has been encountered in calculating nuclear phenomena by the techniques of wave mechanics suggests that the extremely non-linear condition is approached for the separation of the particles within a nucleus. This viewpoint suggests that an understanding of the nucleus might make possible an experimental determination of velocity relative to the ether.

The reactions between wave patterns of appreciable amplitude may also be viewed from a somewhat different angle. We may think of the various wave patterns as being the analogs of the various modes of motion of, say, an elastic plate. For very small amplitudes they have negligible effect on one another. For larger amplitudes, where Hooke's law does not hold, the force may be represented as a power series of the displacement. The first power term represents the linear stiffness. If the frequencies of two modes which are in oscillation are ω_1 and ω_2, the higher power terms represent forces of frequencies $m\omega_1 \pm n\omega_2$ where m and n are integers or zero. These forces set all the modes into forced oscillation at the frequencies of the various forces, in amounts which depend on the impedance of the particular mode for the particular frequency. When the frequency of the force coincides with the resonant frequency of one of the natural modes, the forced oscillations may be large. Thus the variation in stiffness with displacement provides a coupling whereby energy may be transferred from one or more modes, that is wave patterns, to other modes. But in this transfer the energy always appears associated with a new frequency which is related to those of the modes from which it came in accorance with the familiar formula of quantum theory.

The theory of such energy transformations with change of frequency has been worked out in considerable detail for vacuum tube and other variable resistance modulators, and the results show little in common with the quantum theory beyond the relations connecting the frequencies. When, however, the variation is not in a resistance but in a stiffness, as occurs in the ether case, the situation is quite different. This problem has been explored both theoretically[9] and experimentally.[10] It is found that an oscillation of one frequency in one mode may provide the energy to support sustained oscillations of two other lower frequencies in two other dissipative modes. For this to occur the frequencies involved must be related through the quantum formula. Also the amplitude of the generating oscillation must exceed a

[9] R. V. L. Hartley, *Bell Sys. Tech. Jour.*, 15, 424, 1936.
[10] L. W. Hussey and L. R. Wrathall, *Bell Sys. Tech. Jour.*, 15, 441, 1936.

threshold value which depends on the frequencies, the impedance involved, and the constant of non-linearity. The transformed energy divides itself between the generated modes in the ratio of their frequencies. In a non-dissipative system, the frequencies of possible combinations of sustained oscillations are determined by the energy of the system. Here also they are connected by the quantum formula.

The particle wave pattern discussed above would approximate very closely to such a non-dissipative non-linear system. We should therefore expect its frequency to be related to its energy through the constants of the ether. In the more complex wave patterns associated with more than one particle, it is unlikely that the pattern representing, say, an electron could maintain its identity as part of some arbitrarily chosen pattern, the magnitudes of which are not commensurable with its own. This suggests that the stable states of the complex pattern would be confined to a sequence of discreet patterns which are related to one another through some property of the electron. These possible non-dissipative combinations of energy and frequency would represent the stable quantum states of the atom. The radiation process would then be similar to that referred to above in which energy from a source of higher frequency distributes itself between two lower frequencies in the ratio of the frequencies. The energy in the pattern of an excited atom would serve as the source. One of the two lower frequencies would be that of a pattern corresponding to a lower energy state to which the transition occurs. The other would be that of the radiating wave which carries off the energy lost in the transition.

A Suggested New Particle

We saw above that the observed variation of the energy of a particle with its velocity calls for a mechanism in which the energy varies directly as the frequency. The fact that a system, in which the stiffness varies with the displacement, is characterized by this relation suggests that the energy of a particle pattern depends mainly on variations in the stiffness of the ether. However, the non-linearities of the ether equations cannot all be interpretated as variable stiffnesses. The non-linearity which appears in (1) when the displacements are finite is equivalent to a variable inertia. It is in order, therefore, to inquire into the properties of a pattern in which the energy is determined by this kind of non-linearity. The variable inductance of an iron-core coil constitutes such a variable inertia. Theoretical and experimental studies of circuits involving these coils have shown that they behave very much as do systems having variable stiffness, with one important exception. The energy distributes itself in the inverse ratio of the frequencies.

If, then, we assume that the energy of a moving pattern is determined by a mechanism which conforms to this relation, it follows from (7) that its energy will vary as $\frac{1}{\beta}$. Expanding in the usual manner we then have

$$W = m_0c^2 - \tfrac{1}{2}\, m_0V^2 + \cdots$$

This says that a particle represented by such a wave pattern would have a positive rest mass and a negative inertial mass. Its momentum is directed oppositely to its velocity, and energy must be taken from it to set it in motion and given to it to stop it. Such a particle, when bouncing back and forth between two rigid walls or rotating about two centers of force, would exert a force tending to draw them together, instead of the usual repulsion. It is interesting to speculate that if, in an atomic nucleus, the positive charges which are passed back and forth between other nuclear particles were associated with particles of this type their motion would exert a binding force on the other particles.

Conclusion

It appears, then, that the ether model is capable of sustaining wave patterns the behavior of which is qualitatively in agreement with the results of experiment. In order to establish fully the sufficiency of classical mechanics for the physical description of natural phenomena, it will be necessary to work out the complicated quantitative relations whereby the constants of the ether may be deduced from experimental measurements. However, until a serious attempt to do this has failed for some reason other than sheer mathematical complexity, the insufficiency of classical mechanics can scarcely be argued.

In conclusion, I wish to acknowledge the contributions of those of my colleagues who, through discussions over the years, have helped in developing the concepts which have been put together in the above picture.