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Principles and Applications of Waveguide Transmission

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Under the above title, D. Van Nostrand Company, Inc. will shortly publish the book from which the following article is excerpted. Dr. Southworth is one of the leading authorities on waveguides and was one of the first to foresee the great usefulness that this form of transmission might offer. The editors of the Bell System Technical Journal are grateful for permission to publish here parts of the preface and the historical introduction and chapter 6 in its entirety.

Preface

Though it has been scarcely fifteen years since the waveguide was proposed as a practicable medium of transmission, rather important applications have already been made. The first, which was initiated several years ago, was in connection with radar. A more recent and possibly more important application has been in television where waveguide methods provide a very special kind of radio for relaying program material cross-country from one tower top to another. Already Boston and New York have been connected by this means and shortly Chicago and intervening cities will be added. Other networks extending as far west as the Pacific may be expected. It is reasonable to expect that these two applications will be but the beginning of a more general use.

Interest in the subject of waveguide transmission is not limited to commercial application alone. A comparable interest, perhaps less readily evaluated but nevertheless extremely important, lies in its usefulness in teaching important physical principles. For example there are many concepts that follow from the electromagnetic theory that, in their native mathematical form, may appear rather abstract. However, when translated to phenomena actually observed in waveguides, they become very real indeed. As a result, these new techniques have already assumed a place of considerable importance in the teaching of electrical engineering and applied physics both in lecture demonstrations and in laboratory exercises. It is to be expected that they will be used even more extensively as their possibilities become better appreciated.

Interest in waveguides has been greatly enhanced by the fact that they brought with them a series of extremely interesting methods of measurement, comparable both in accuracy and scope, with similar measurements previously made only at the lower frequencies. This extension of the range over which electrical measurements may be made has contributed also to neighboring fields of research. One early application led to the discovery of centimeter waves in the sun's spectrum. Another led to important new information about the earth's atmosphere. Still another contributed to the study of absorption bands in gases, particularly bands in the millimeter region. Also of great importance was its contribution to our knowledge of the properties of materials for it led at a fairly early date to measurements at higher frequencies than heretofore of the primary constants, permeability, dielectric constant and conductivity—all for a wide array of substances ranging from the best insulators to the best conductors and including many of the so-called semi-conductors. It is because this new art has already attained considerable stature and is already showing promise as an educational medium that this book has been prepared.

CHAPTER I

INTRODUCTION

1.5 Early History of Waveguides

That it might be possible to transmit electromagnetic waves through hollow metal pipes must have occurred to physicists almost as soon as the nature of electromagnetic waves became fully appreciated. That this might actually be accomplished in practice was probably in considerable doubt, for certain conclusions of the mathematical theory of electricity seemed to indicate that it would not be possible to support inside a hollow conductor the lines of electric force of which waves were assumed to consist. Evidence of this doubt appears in Vol. I (p. 399) of Heaviside's "Electromagnetic Theory" (1893) where, in discussing the case of the coaxial conductor, the statement is made that "it does not seem possible to do without the inner conductor, for when it is taken away we have nothing left on which tubes of displacement can terminate internally, and along which they can run."

Perhaps the first analysis suggesting the possibility of waves in hollow pipes appeared in 1893 in the book "Recent Researches in Electricity and Magnetism" by J. J. Thomson. This book, which was written as a sequel to Maxwell's "Treatise on Electricity and Magnetism," examined mathematically the hypothetical question of what might result if an electric charge should be released on the interior wall of a closed metal cylinder. This problem is even now of considerable interest in connection with resonance in hollow metal chambers. The following year Joseph Larmor examined as a special case of electrical vibrations in condensing systems the particular waves that might be generated by spark-gap oscillators located in hollow metal cylinders. A more complete analysis relating particularly to propagation through dielectrically-filled pipes both of circular and rectangular cross section was published in 1897 by Lord Rayleigh. Later (1905) Kalähne examined mathematically the possibility of oscillations in "ring-shaped" metal tubes. Still later (1910) Hondros and Debye examined mathematically the more complicated problem of propagation through dielectric wires. Transmission through hollow metal pipes was also considered by Dr. L. Silberstein in 1915.

As regards experimental verification, it is of interest that Sir Oliver Lodge as early as 1894 approached but probably did not quite realize actual waveguide transmission. In a demonstration lecture on electric waves given before the Royal Society, he used, as a source of waves, a spark oscillator mounted inside a "hat-shaped" cylinder. An illustration published later suggests that the length of the cylinder was only slightly greater than its diameter. There is no very definite evidence that the short cylinder functioned as a waveguide or that such a function was discussed in the lecture. Perhaps of greater significance were some experiments reported a year later by Viktor von Lang who used pipes of appreciable length and repeated for electric waves the interference experiment that had been performed for acoustic waves by Quincke some years earlier. Other similar experiments were later performed by Drude and by Weber.

About 1913 Professor Zahn of the University of Kiel became interested in this problem and assigned certain of its aspects to two young candidates for the doctorate, Schriever and Reuter by name. They had barely started when World War I broke out, and both left for the front. Zahn continued this work until he was called a year later. It is reported that by this time he had succeeded in propagating waves through cylinders of dielectric, but it is understood that he did little or no quantitative work. Reuter was killed at Champagne in the autumn of 1915, but Schriever survived and returned to complete his thesis in 1920, using for his source the newly available Barkhausen oscillator.

The contributions of Thomson, Rayleigh, Hondros and Debye, and Silberstein were, of course, purely mathematical. Those of von Lang, Weber, Zahn and Schriever were experimental, but they were of rather limited scope. The concept of the hollow pipe as a useful transmission element, for example as a radiator or as a resonant circuit, apparently did not exist at these early dates. Nothing was yet known quantitatively about attenuation, and little or nothing of the present-day experimental technique had yet appeared. At this time, the position of this new art was perhaps comparable with that of radio prior to the time of Marconi.

The history of waveguides changed abruptly about 1933 when it was shown that they could be put to practical use. Several patent applications were filed,[1] and numerous scientific papers were published. More recently a great many papers have appeared, too many in fact for detailed consideration at this time. Three of the earlier papers are mentioned in the footnote below.[2] Others will be referred to in the text that follows.

The writer's interest in guided waves stems from some experiments done in 1920 when such waves were encountered as a troublesome spurious effect while working with Lecher wires in a trough of water. In one case there were found, superimposed on the waves that might normally travel along two parallel conductors, other waves having a velocity that somehow depended on the dimensions of the trough. These may now be identified as being the so-called dominant type. In another case, the depth of water was apparently at or near "cut-off," and conditions were such that water waves in the trough gave rise to depths that were momentarily above cut-off, followed a moment later by depths that were below cut-off. This led not only to variations in power at the receiving end of the trough but also to variations in the plate current of the oscillator supplying the wavepower. Indeed these effects could be noted even when the wires were removed from the trough. These waves were recognized as being roughly like those described the same year by Schriever.[3]

Several years later this work was resumed and since that time a continued effort has been made to develop from fundamental principles of waveguide transmission a useful technique for dealing with microwaves. The earliest of these experiments consisted of transmitting electromagnetic waves through tall cylinders of water. Because of the high dielectric constant of water, waves which were a meter long in air were only eleven centimeters long in water. Thus it became possible to set up in the relatively small space of one of these cylinders many of the wave configurations predicted by theory. In addition it was possible, by producing standing waves, to measure their apparent wavelength and thereby calculate their phase velocity. Also by investigating the surface of the water by means of a probe,

[1] Reference is made particularly to U.S. Patents 2,129,711 (filed 3/16/33, 2,129,712 (filed 12/9/33), 2,206,923 (filed 9/12/34) and 2,106,768 (filed 9/25/34).

[2] Carson, Mead and Schelkunoff, "Hyper-frequency Waveguides—Mathematical Theory," *B.S.T.J.*, Vol. 15, pp 310–333, April 1936. G. C. Southworth, "Hyper-frequency Wave Guides—General Considerations and Experimental Results," *B.S.T.J.*, Vol. 15, pp 284–309, April 1936. Also "Some Fundamental Experiments with Waveguides," *Proc. I.R.E.*, Vol. 25, pp 807–822, July 1937. W. L. Barrow, "Transmission of Electromagnetic Waves in Hollow Tubes of Metal," *Proc. I.R.E.*, Vol. 24, pp 1298–1398, October 1936.

[3] The waves actually observed are now known as TE_{10} waves in a rectangular guide, while those described by Schriever are now recognized as TM_{01} waves in a circular guide.

the directions and also the relative intensities of lines of electric force in the wave front could be mapped. It is probable that certain of these modes were observed and identified for the first time.

Shortly afterwards, sources giving wavelengths in air of fifteen centimeters became available and the experimental work was transferred to air-filled copper pipes only 5 inches in diameter. At this time, a 5-inch hollow-pipe transmission line 875 feet in length was built through which both telegraph and telephone signals were transmitted. Measurements showed that the attenuation was relatively small. This early work, which was done prior to January 1, 1934, was described along with other more advanced work in demonstration-lectures and also in papers published in 1936 and 1937.[4]

It was recognized at an early date that a short waveguide line might, with suitable modification, function as a radiator and also as a reactive element. These properties were likewise investigated experimentally, and numerous useful applications were proposed. Descriptions may be found in the numerous patents that followed. These properties were also the subject of several experimental lectures given before the Institute of Radio Engineers and other similar societies by the writer and his associates during the years 1937 to 1939.[5] Included were demonstrations of the waveguide as a transmission line, the electromagnetic horn as a radiator, and the waveguide cavity as a resonator. An adaptation of the waveguide cavity was used to terminate a waveguide line in its characteristic impedance.

From the first, progress was very substantial and by the autumn of 1941 there were known, both from calculation and experiment, the more important facts about the waveguide. In particular, the reactive nature of discontinuities became the subject of considerable study, and impedance matching devices (transformers), microwave filters, and balancers soon followed. Also a wide variety of antennas was devised. Similarly, amplifiers and oscillators as well as the receiving methods followed.

As might be expected, a great many people have contributed in one way or another to the success of this venture. Particular mention should be made of the very important parts played by the author's colleagues, Messrs. A. E. Bowen and A. P. King, who, during its early and less promising period, contributed much toward transforming rather abstract ideas into practical equipment, much of which found important military uses immediately upon the advent of war. Also of importance were the parts played by the author's colleagues, Dr. S. A. Schelkunoff, J. R. Carson, and Mrs. S. P. Meade, who, in the early days of this work, provided a substantial segment of mathematical theory that previously was missing. During the succeeding years, Dr. Schelkunoff, in particular, made invaluable contributions in the form

[4] A description of one of the earlier lectures appears in the Bell Laboratories Record for March 1940. (Vol. XVIII, No. 7, p. 194.)

of analyses which in some cases indicated the direction toward which experiment should proceed and, in others, merely confirmed experiment, while, in still others, gave answers not readily obtainable by experiment alone. In the chapters that follow, the author has drawn freely on Dr. Schelkunoff, particularly as regards methods of analysis.

Beginning sometime prior to 1936, Dr. W. L. Barrow, then of the Massachusetts Institute of Technology, also became interested in this subject and together with numerous associates made very substantial contributions. No less than eight scientific papers were published covering special features of hollow-pipe transmission lines and electromagnetic horns. For several years the work being done at the Massachusetts Institute of Technology and at the Bell Telephone Laboratories probably represented the major portion, if not indeed the only work of this kind in progress, but with the advent of World War II, hundreds or perhaps thousands of others entered the field. For the most part, the latter were workers on various military projects. Starting with the considerable accumulation of unpublished technique that was made freely available to them at the outset of the war, they, along with others in similar positions elsewhere in this country and in Europe, have helped to bring this technique to its present very satisfactory state of development.

CHAPTER VI

A DESCRIPTIVE ACCOUNT OF ELECTRICAL TRANSMISSION

6.0 General Considerations

The preceding four chapters presented the more important steps in the development of the theory of electrical transmission, particularly as it applies to simple networks, wire lines, and waves in free space and in guides. For the most part, the analysis followed conventional methods and made use of the concise and accurate short-hand notation of mathematics. It had for its principal objective the derivation of a series of equations useful in the practical application of waveguides.

Closely associated with the theory of electricity and almost a necessary consequence of it are the numerous concepts and mental pictures by means of which we may explain rather simply the various phenomena observed in electrical practice Though extremely important, this aspect of the theory was not stressed before. Instead it was deferred to the present chapter where it could be considered by itself and from the purely qualitative point of view. It is hoped that this arrangement of material will be of special use to those who find it necessary to substitute for mathematical analysis, simple models to explain the phenomena which they observe in practice. It is believed that, for these people, this chapter together with a few key formulas taken from the earlier sections will be helpful in gaining a fairly satisfactory understanding of the practical aspects of waveguide transmission.

At the lower frequencies, the current aspect of electricity meets most of the needs and in comparison it is only occasionally that there is a need to discuss lines of electric and magnetic force. In waveguide practice, on the other hand, currents are usually not available for measurement and, although we recognize their reality, they necessarily assume a secondary role. In contrast with currents, we consider the fields present in a waveguide as very real entities and we attach a very great importance to their orientations as well as to their intensities.

6.1 The Nature of Fields of Force

As a suitable introduction to the discussion that follows, we shall review some of the fundamental properties of lines of electric and magnetic force and show pictorially the part that they play in transmission along an ordinary two-wire line.

The Electrostatic Field

As is well known, the concept of the electric field was devised by Faraday to explain the force action between charged bodies. According to his view there exist in the space between the charged bodies, lines or tubes of electric force terminating respectively on positive and negative charges attached to the bodies. These tubes of force are endowed with a tendency to become as short as possible and at the same time to repel, laterally, neighboring lines of force. Their direction at any point is purely arbitrary, but, by subsequent convention, the positive direction is taken from the positively charged body to the negative. This is such that a small positive charge (proton) placed in the field tends to be displaced in the positive sense while an electron tends to move in a negative direction. The force exerted on the unit charge is a measure of the magnitude of the electric intensity **E**. It is measured in volts per meter and, since it has direction as well as magnitude, it is a vector quantity.[1] Figure 6.1-1 illustrates in a general way the arrangement of lines of electrostatic force that are assumed to exist between two oppositely charged spheres. Also shown is a representative vector **E**.

The Magnetostatic Field

In the same way that Faraday provided a satisfactory explanation for the forces between charged bodies, so was he able to explain the forces be-

[1] Black-face type will be used when it seems desirable to emphasize the vector properties of quantities having direction as well as magnitude.

tween magnetized bodies. In the latter case, the two kinds of electrostatic charge are replaced by north-seeking and south-seeking magnetic poles respectively. Similarly the tubes of electric force are replaced by tubes of magnetic force. Roughly speaking, the two kinds of tubes are endowed with analogous properties. Because these magnetic lines are at rest, it is appropriate to speak of them as magnetostatic lines of force and consider them as being comparable but of course not identical with electrostatic lines already discussed. The force exerted on a unit magnetic pole is a measure of magnetic intensity **H**. Like its electric counterpart, it is a vector quantity. In the par-

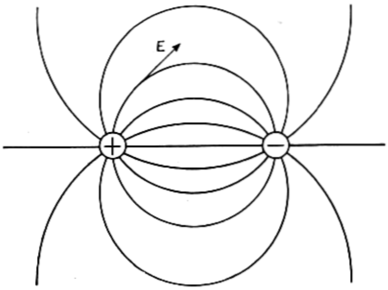


Fig. 6.1-1. Arrangement of lines of electrostatic force in the region between two oppositely charged spheres.

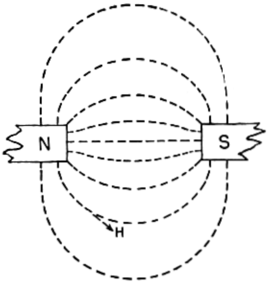


Fig. 6.1-2. Arrangement of lines of magnetostatic force in the region between two oppositely magnetized poles.

ticular system of units used in this text, it is measured in amperes per meter. Figure 6.1-2 illustrates the arrangement of the lines of magnetic force that are assumed to exist between two opposite magnetic poles.

Interrelationship of Electric and Magnetic Fields

As a result of the electromagnetic theory, there are certain properties with which we may endow lines of electric and magnetic force and thereby explain numerous phenomena of electrical transmission. This establishes a relationship between electric and magnetic fields that makes them appear at times as if they were different aspects of the same thing. They are as follows:

1. *Lines of magnetic force, when displaced laterally, induce in the space immediately adjacent, lines of electric force. The direction of the induced electric force is perpendicular to the direction of motion and also perpendicular to the direction of the original magnetic force. The intensity* **E** *of the induced electric*

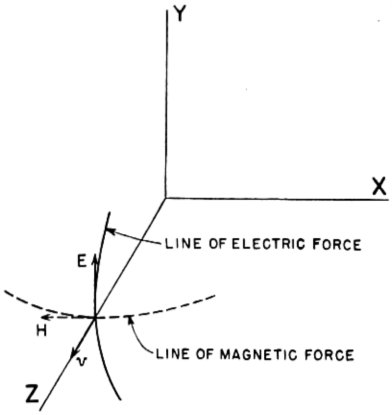


Fig. 6.1-3. Directions of electric vector E and magnetic vector H relative to the velocity v of motion of such lines.

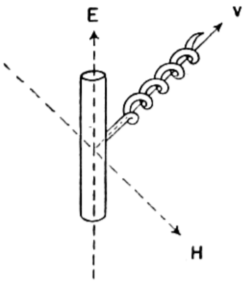


Fig. 6.1-4. Simple corkscrew rule for remembering the directions of E, H and v.

force is proportional to the velocity **v** *of displacement and proportional to the intensity* **H** *of the original lines of magnetic force.*

The directions of the vectors **v**, **E** and **H** are shown in Fig. 6.1-3. They are so related that, when **E** moves clockwise into **H**, it is as though a right-hand screw had progressed in the direction of **v** as shown in Fig. 6.1-4. A convenient short-hand notation used rather generally by mathematicians makes it possible to express these facts by the following vector equation:

$$\mathbf{E} = -\mu(\mathbf{v} \times \mathbf{H}) \qquad (6.1\text{–}1)$$

The quantity μ is the magnetic permeability of the medium under consideration.

2. *Lines of electric force, when displaced laterally, induce in the immediately adjacent space lines of magnetic force. The direction of the induced magnetic force is perpendicular to the direction of motion and also perpendicular to the direction of the original electric force. The intensity* **H** *of the induced magnetic force is proportional to the velocity* **v** *of displacement and proportional to the intensity* **E** *of the original lines of electric force.*

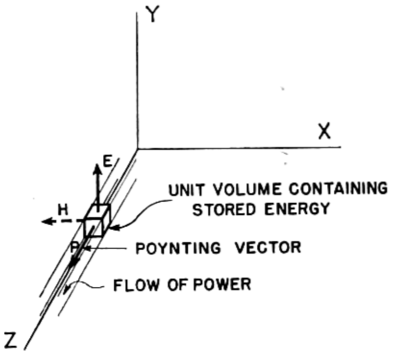


Fig. 6.1-5. Directions of the vectors E and H relative to the Poynting vector P in an advancing wave front.

Again Fig. 6.1-3 and also the right-hand or cork-screw rule apply. In the short-hand notation these facts may be expressed by the following vector equation:

$$\mathbf{H} = \epsilon(\mathbf{v} \times \mathbf{E}) \tag{6.1–2}$$

In this equation, ϵ is the dielectric constant of the medium.[2]

3. *When an electric field of intensity* **E** *is translated laterally, it together with its associated magnetic field* **H** *represents a flow of energy. The direction of the flow of energy is perpendicular to both* **E** *and* **H** *and is therefore in the direction of the velocity* **v.** *The magnitude of the energy flow per unit volume across a unit area measured perpendicular to* **v** *is proportional to the product of the electric intensity* **E** *and the magnetic intensity* **H.** *It may be designated by the vector* **P**.

The relative directions of the vectors **P**, **E**, and **H** are shown in Fig. 6.1-5. The energy per unit volume moves with a velocity expressed by

$$v = \frac{1}{\sqrt{\mu\epsilon}} \tag{6.1–3}$$

[2] The values of permeability μ and dielectric constant ϵ appearing in these equations are not the values found in most tables of the properties of materials. As here given μ is smaller than the usual value μ_r by a factor of 1.257×10^{-6} while ϵ is smaller than ϵ_r by a factor of 8.854×10^{-12}. The use of these special values leads to certain mathematical simplifications.

It therefore corresponds to a flow of power. In the notation just referred to, it may be expressed by the vector equation

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} \qquad (6.1\text{–}4)$$

4. *Lines of force exhibit the properties of inertia. They therefore resist acceleration.*

Other principles not quite so fundamental but nevertheless useful in application are:

5. *Lines of force are under tension and at the same time are under lateral pressure.*

6. *For perfect conductors there can be no tangential component of electric force.* That is to say, lines of electric force when attaching themselves to a perfect conductor must approach perpendicularly. This is substantially true also for common metals such as copper.

In passing it is well to point out that the first principle is really that by which the ordinary dynamo operates. The second is, for practical purposes, Oersted's Principle, if we assume that the lines of electric force are attached to charges flowing in near-by conductors. The third is known as the Poynting Principle. It has a wide field of application contributing very materially to the physical pictures of both radio and waveguide transmission. When applied to the very simple case of low frequencies propagated along a transmission line, it gives a result that is in keeping with the usual view that the power transmitted is equal to the product of the total voltage times the total current. The fourth principle is useful in explaining qualitatively how radiation from an antenna takes place. The usefulness of these four principles will be made more evident by the examples that follow.

6.2 Transmission of Power along a Wire Line

Direct Current

According to the Poynting concept, one may think of an ordinary dry cell as two conductors combined with chemical means for producing a continuous supply of lines of electric force. This need not be counter to the accepted views concerning electrolysis, for we may think of these lines of force as being attached to ionic charges incidental to dissociation. As long as the cell is on open circuit, these lines of electric force remain in a static condition in which many are grouped in the neighborhood of the terminals of the cell as shown in Fig. 6.2-1(a). In this state of equilibrium, the forces of lateral pressure are balanced by the forces of tension. There is no motion and hence no flow of power. For an ordinary dry cell such as used in flashlights, the electric intensity **E** will depend on the spacing of electrodes, but it may be as much as 200 volts per meter. If we attach to the dry cell two parallel wires spaced perhaps a centimeter apart with their remote ends open, electrostatic lines will be communicated to the wires, thereby providing a distribution roughly like that shown in Fig. 6.2-1(b). Except at the moment of contact, there is no motion of the lines of electric force and therefore no magnetic field and, accordingly, there can be no flow of power. The final configuration is to be regarded as the resultant of the forces of tension and lateral pressure. The electric intensity, **E**, measured in volts per meter at any point along the line, may be altered at will, merely by changing the spacing.

If, next, we close the remote end of the line by substituting a conducting wire for the particular line of force shown as a heavy line in Fig. 6.2-1(c), the adjacent lines of electric force will collapse on the terminating conductor,

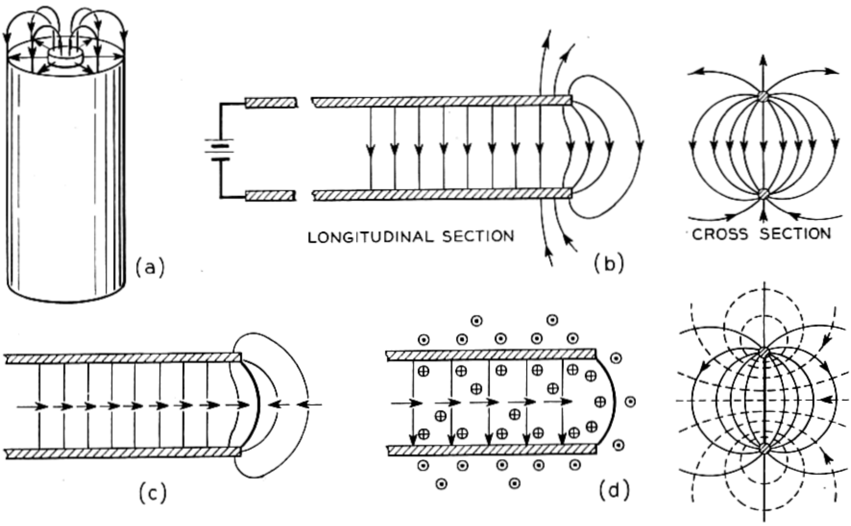


Fig. 6.2-1. Lines-of-force concept applied to the transmission of d-c power along a wire line.

as opposing charges unite. This removes the lateral pressure on the neighboring lines with the result that the whole assemblage starts moving forward. Each line of force meets in its turn the fate of its forerunners, thereby delivering up its energy to the resistance as heat. As soon as the lateral pressure at the cell is relieved, chemical equilibrium is momentarily destroyed and more lines of force are manufactured to fill the gaps of those that have gone before. All of this is, of course, at the expense of chemical action.

According to the electromagnetic theory, as set forth in the second principle, this is but a part of the story of transmission. We must add that the motion of the lines of electric force from the dry cell toward the resistance gives rise in the surrounding space to lines of magnetic force in accordance with Equation 6.1–2 and furthermore the two fields together give rise to component Poynting vectors representing power flow. Each component vector has a magnitude at any point equal to the product of the electric and magnetic intensities there prevailing and a direction at right angles to the two component forces in accordance with Equation 6.1-3. This is illustrated in Fig. 6.2-1(d).

Since the fields reside largely outside the conductors, we conclude that the principal component of power flow is through the space between the wires and not through the wires themselves. If, in the case cited above, there is appreciable resistance in the connecting wires, then we may expect that there will be a small component of energy flowing into the wires to be dissipated as heat. To account for this, we may picture lines of electric force

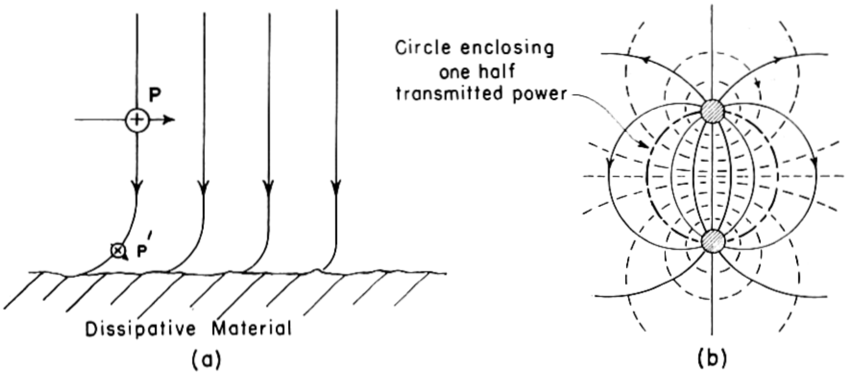


Fig. 6.2-2. Fields of electric and magnetic force and also direction of power flow in the vicinity of conductors. (a) Magnified view showing power flow along a single dissipative wire. (b) Cross-sectional view of parallel-wire line.

which in the immediate vicinity of the conducting wire lag somewhat behind the portions more remote. This is illustrated by Fig. 6.2-2(a) which shows a highly idealized and greatly enlarged section of the field in the immediate vicinity of one of the two dissipative conductors. The very small component of power flowing into the conductor is designated as the vector P' to distinguish it from the much greater power P which we shall assume is being propagated parallel to the conductor.[3]

The magnetic field associated with two cylindrical conductors consists of circles with centers on the line joining the two conductors, whereas the electric field consists of another series of circles orthogonally related to the

[3] For all metals from which conducting lines are ordinarily made, the component of power flowing into the conductor is extremely small compared with the power flowing parallel to its surface. In Fig. 6.2-2(a) therefore, we should regard vector P' as greatly exaggerated in magnitude relative to that of vector P.

first, and having centers on a line at right angles to the first as shown in Fig. 6.2-2(b). The total flow of power through any plane set up perpendicular to the wires is found by adding up the various component products of **E** and **H** from the boundaries of the wires to infinity. The method by which this is carried out is outside of the scope of this chapter, but, as already pointed out, it leads to the same result as obtained by multiplying together the total voltage and the total current. There are two results of this integration that are of special interest. (1) In the case of two parallel cylinders, one-half of the total power flows through the space enclosed by a circle drawn about the wire spacing as a diameter [see Fig. 6.2-2(b)]. The remaining half extends from this circle on out to infinity. (2) Since both the electric and magnetic intensities are greatest in the neighborhood of the wire, most of the total power flow takes place in the immediate vicinity of the wire.

Transmission of A-c Power

If the simple d-c source mentioned previously is replaced by an alternating electromotive force, a variety of phenomena may take place, the more important of which will depend on the frequency of alternation. If this frequency is low (very long wavelength), the line may be relatively short compared with the wavelength, with the result that changes occurring at the source may appear very soon at the remote end. For this case, the observed phenomena will vary sinusoidally with time everywhere along the line, in substantially the same phase. This is the typical alternating-current power line problem[4] and, except for minor details, which we shall not discuss at this time, it does not differ materially from the simple d-c case already covered.

If, on the other hand, the frequency is high (short wavelength), the line may be regarded as being *electrically long*, with the result that sinusoidal changes occurring at the source may not have traveled very far before the direction of flow at the source has changed. The over-all result in extreme cases may become very complicated indeed; for, wavepower may not only be reflected from the remote end of the line but, if there are sharp bends in the line or abrupt changes in spacing, it may be reflected from these points also. The phenomenon observed is usually referred to as *wave interference* and it often leads to *standing waves*. Though described above as complicated, there are many cases where the results of wave interference may be sufficiently simple to be readily visualized. Practical difficulties of various kinds may arise from these effects, but they may also serve very useful purposes. In fact, a substantial portion of our microwave technique is based on wave

[4] The wavelength corresponding to a frequency of 60 cycles per second is five million meters. A commercial power line having a length as great as 100 miles is therefore but 0.03 wavelength long. It is said to be *electrically short*.

interference. Certain specific examples will be discussed later, but first we shall discuss a somewhat simpler case.

The Infinite Line

Let us take, for discussion, a uniform two-wire line that is infinitely long. Waves launched on such a line are assumed to be propagated to infinity. There are no reflected components and hence no wave interference. If the frequency is very high, the forerunners of the lines of force sent out by the source will not have traveled very far when the emf at the source will have reversed its direction. This gives rise at the source to a second group of lines

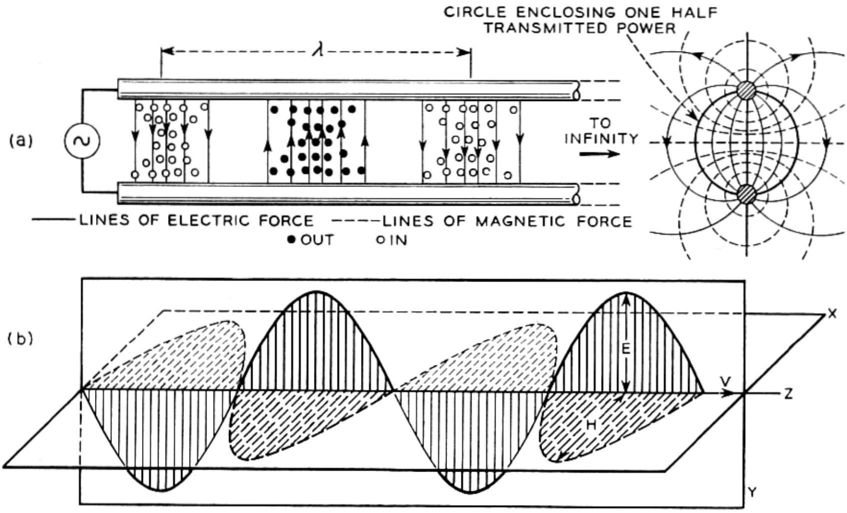


Fig. 6.2-3. (a) Arrangement of lines of electric and magnetic force in both the longitudinal and transverse sections of an infinitely long transmission line. (b) Space relationship between electric vector E and magnetic vector H as observed in a plane containing the two conductors.

of force exactly like the first except oppositely directed. This, in turn, will be followed by a third group identical with the first and a fourth identical with the second and so forth until equilibrium is reached. Because the lines of electric force are in motion, we must expect them to be accompanied by lines of magnetic force. Both are of equal importance. Therefore it is not correct to refer to either alone as a distinguishing feature of the wave. Both components are shown in cross section at the right in Fig. 6.2-3(a).

The distance between successive points of the same electrical phase in a wave is known as the wavelength λ. It depends on the frequency f of alternation and the velocity of propagation v; $\lambda = v/f$. The velocity of propagation in turn depends on the nature of the medium between the two wires. For air, the velocity v_a is substantially 300,000,000 meters per second (186,000 mi per sec). For other media $v = v_a/\sqrt{\mu_r \epsilon_r}$. Thus it will be seen that, by replacing the air normally found between the two wires of a transmission line by another medium such as oil ($\epsilon_r = 2$ and $\mu_r = 1$), the wavelength will be reduced by a factor of $1/\sqrt{2}$.

If A_0 is the maximum amplitude reached by the oscillating source during any cycle, the amplitude at any time t, measured from an arbitrary beginning, may be expressed by the equation

$$A = A_0 \sin (\omega t + \phi) = A_0 \sin \left(\frac{2\pi}{\lambda} vt + \phi \right) \qquad (6.2\text{–}1)$$

where ϕ is the initial phase of the amplitude relative to an arbitrary reference angle.

If the transmission line is free from dissipation and we choose a datum point in a plane at right angles to the direction of propagation and at a distance far enough from the source that the lines of force have had an opportunity to conform to the wire arrangement and if we designate the electric intensity at this point as E_0 and the corresponding magnetic intensity as H_0, then the electric and magnetic intensities at other corresponding points at a distance z further along the line may be represented by

$$E = E_0 \sin \frac{2\pi}{\lambda} (z - vt)$$

and

$$H = H_0 \sin \frac{2\pi}{\lambda} (z - vt) \qquad (6.2\text{–}2)$$

These equations are the trigonometric representations of an unattenuated sinusoidal wave of electric intensity and magnetic intensity traveling in a positive direction along the z axis. They are plotted in the yz and xz planes of Fig. 6.2-3(b). An electromagnetic configuration similar to the above but traveling in the opposite direction is given by

$$E = E_0 \sin \frac{2\pi}{\lambda} (z + vt)$$

and

$$H = H_0 \sin \frac{2\pi}{\lambda} (z + vt) \qquad (6.2\text{–}3)$$

These equations may be further confirmed by plotting arbitrary values on rectangular-coordinate paper. In an infinite line the magnetic intensity **H** and the electric intensity **E** are in the same phase as shown in Fig. 6.2-3.

If the wave is subject to an attenuation of α units per unit distance, possibly due to resistance in the wires, the corresponding components of **E** and **H** are equally attenuated. Either component may be expressed by an equation of the type

$$E = E_0 e^{-\alpha z} \sin \frac{2\pi}{\lambda} (z - vt) \tag{6.2–4}$$

This is a very special form of certain equations appearing in Sections 3.2 and 3.3.

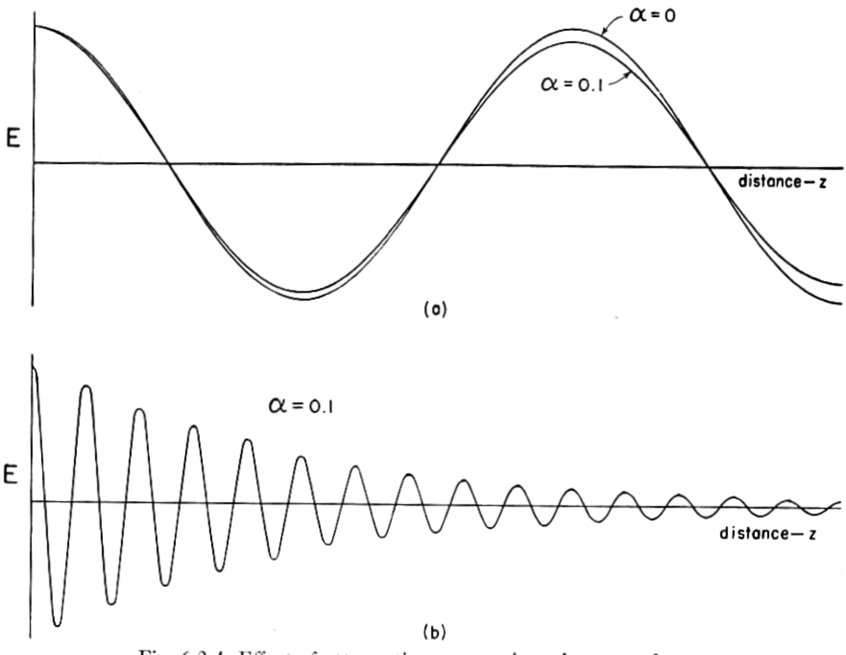


Fig. 6.2-4. Effect of attenuation on an advancing wave front.

If the attenuation is negligible, then $\alpha = 0$ and the term $e^{-\alpha z}$ will be unity. Equation 6.2-4 will then reduce to 6.2-2. If, on the other hand, the attenuation is considerable, the product of α times z will increase rapidly with distance, and the factor $e^{-\alpha z}$ will have the effect of reducing the electric intensity E prevailing at various points along the line. Figure 6.2-4(a) illustrates the variation, with distance, of the electric intensity E for an unattenuated wave $\alpha = 0$. There is included for comparison purposes the case, $\alpha = 0.1$. Figure 6.2-4(b) shows the effect of this rate of attenuation on waves that have traveled for some distance. It is significant that moderate amounts of attenuation have little or no effect on wavelength.

At low frequencies, conductor loss is often the principal cause of attenuation. At high frequency, this loss may be still more important[5] and in addition there may be losses in the medium around the two conductors. The latter is particularly true when the conductors are supported on insulators or are embedded in insulating material. There may also be losses due to lines of force that detach themselves from the wires and float off into the surrounding space (radiation). All three lead to attenuation and may be expressed in terms of an equivalent resistance. They are amenable to calculation for certain special cases.

According to one view of electricity, the individual charges to which lines of force attach themselves are unable to flow through the conductor with the velocity of light. If this is true, lines of force snap along from one charge to the next in a rather mysterious fashion which we will not attempt to picture at this time. This view, like others mentioned previously, tends to relegate the charges and hence the currents to a secondary position.

Although infinitely long transmission lines cannot be constructed in practice, it is possible, by a variety of methods, to approximate this result. In general, a resistance connected across the open end of a short transmission line, of the kind here assumed, absorbs a portion of the arriving wavepower and reflects the remainder. If the resistance is either very large or very small, the reflected power may be very substantial but, by a suitable choice of intermediate values of resistance, the reflected part may be made very small indeed. In the ideal case, the arriving wavepower is completely absorbed. A line connected to this particular value of resistance appears to a generator at the sending end as though it were infinitely long. The particular resistance that can replace an infinite line at any point, without causing reflections, is known as the *characteristic impedance* of the line. This quantity depends on the dimensions and spacings of the two conductors as well as the nature of the medium between. A parallel-wire line, in air, usually has a characteristic impedance of several hundred ohms. A coaxial line filled with rubber often has a characteristic impedance of a few tens of ohms. A line having characteristic impedance connected at its receiving end is said to be *match-terminated.*

Reflections on Transmission Lines

If the transmission line ends in a termination other than characteristic impedance, or if there are discontinuities, due to impedances connected either in series or in shunt with the line, reflections of various kinds will occur.[6] Much of the practical side of microwaves has to do with these reflections.

[5] The losses in most conductors increase with the square root of the frequency.

[6] At the higher frequencies, reflections may also occur at points where the wire spacing changes abruptly. In some instances abrupt changes in wire diameter may be sufficient to cause reflection. These discontinuities may be regarded as changes in characteristic impedance.

A particularly simple form of reflection occurs when the high-frequency transmission line is terminated in a transverse sheet of metal of good conductivity, as for example, copper. An arrangement of this kind is shown in Fig. 6.2-5. As it is difficult to represent a wave front moving toward the reflecting plate, we shall substitute an imaginary thin slice or section of the electromagnetic configuration. A slice of this kind is shown in Fig. 6.2-5(a).

Experiment shows that, at the boundary of the nearly perfect reflector, the transverse electric force E is extremely small. This is consistent with the sixth principle set forth in the previous section which states that there can be no tangential component of electric force at the boundary of a perfect conductor. The result actually observed can be accounted for if it is assumed that the reflecting conductor merely reverses the direction of lines of electric force as they become incident, thereby giving rise to two sets of

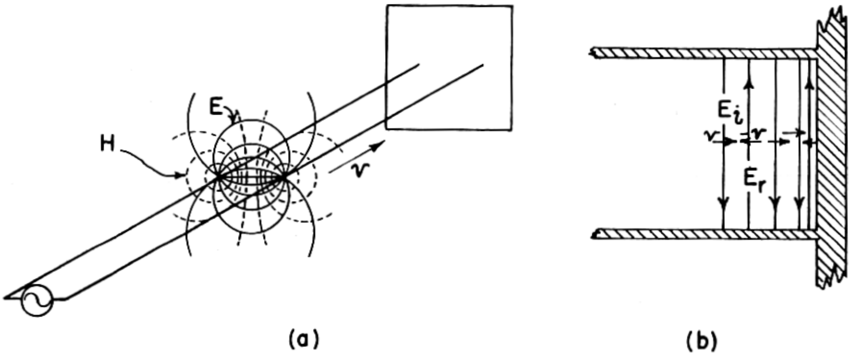


Fig. 6.2-5. (a) Propagation of an electromagnetic wave along a two-wire line terminated by a large conducting plate. (b) Representative lines of force reflected by the conducting plate.

lines of force as shown in Fig. 6.2-5(b), one of intensity $E_i = E$ directed downward in the figure and moving laterally toward the metal sheet (incident wave) and the other of intensity $E_r = -E$ directed upward and moving away from the metal sheet (reflected wave). Accordingly the resultant electric intensity at the surface is zero.

If the reflector is non-magnetic, the magnetic intensity **H** will be unaffected by the reflecting material. We find by applying the right-hand rule of Fig. 6.1-4 that the electric intensity $\mathbf{E}_r = -\mathbf{E}$ when combined with **H** constitutes a wave that must travel in a negative direction of **v**. This wave may be represented by Equation 6.2-3. In a similar way the Poynting vector which before reflection is represented by $\mathbf{P} = \mathbf{E} \times \mathbf{H}$ now takes the form $\mathbf{P} = (-\mathbf{E} \times \mathbf{H})$. The negative sign according to the right-hand rule of Fig. 6.1-4 shows that the power approaching the conductor is reflected back upon itself. If **E** and **H** are respectively equal in magnitude before and after incidence, the reflection is perfect, and the coefficient of reflection is said to be unity. Bearing in mind that $\mathbf{H}_i = \epsilon(\mathbf{v} \times \mathbf{E})$ before reflection and $\mathbf{H}_r = \epsilon(-\mathbf{v} \times -\mathbf{E})$ after reflection, it is evident that the direction of the magnetic intensity has been unchanged by the process of reflection and that the resultant magnitude at the surface of the metal is $|H_i| + |H_r| = 2|H|$. Thus we see that, at the moment of reflection from a metallic surface, the resultant electric force vanishes and the resultant magnetic force is doubled.

The reflection of waves at the end of the line naturally gives rise to two oppositely directed wave trains. This is a well-known condition for standing waves. Though a complete discussion of standing waves calls for the mathematical steps taken in Section 3.6, there are certain qualitative results that may be deduced from relatively simple reasoning. Some of these deductions will be made in the paragraphs that follow.

If an observer, endowed with a special kind of vision for individual lines of force, were to be stationed at various points along a lossless transmission line as shown in Fig. 6.2-5, he would observe a variety of phenomena as follows. Near the reflector he would observe a waxing and waning of lines of force, both electric and magnetic, corresponding to the arrival of crests and hollows of waves. Also he would observe a similar waxing and waning corresponding to waves leaving the reflector. The sum of the two waves would give rise at the conducting barrier to a resultant electric intensity of zero and to a corresponding magnetic intensity that would oscillate between limits of plus or minus $2H$. Since it is the magnetic component that is the the more evident near the barrier, this region would appear to the observer much like the interior of a coil carrying alternating current.

If the observer were to pass along the line to a point one-eighth wavelength to the left of the reflector, the distance up to the reflector and back would then be a quarter wave and he would then find that at the moment that a wave crest (maximum intensity) was passing on its way toward the reflector a point on the wave corresponding to zero intensity would be returning from the reflector. Adding the corresponding electric and magnetic intensities at this point, he would observe that the electric intensity would not always be zero but instead it would oscillate between limits of plus or minus $\sqrt{2}\,E$. Similarly the corresponding magnetic intensity would no longer oscillate between limits of plus or minus $2H$, but instead it would never reach limits greater than plus or minus $\sqrt{2}\,H$. Thus at this point the electric and magnetic components would have the same average intensity.

If the observer were to move farther along the line, stopping this time at a distance of one-fourth wavelength to the left of the metal plate, the total electrical distance to the barrier and back again would be a half wavelength and he would now find that at the time a crest passed on its way toward the reflector a hollow (maximum negative intensity) would be passing on its return journey. This time, the resultant electric intensity would oscillate between limits of plus or minus $2E$, and the resultant magnetic intensity would be zero at all times. To this observer then, this quarter-wave point on the line would have many of the characteristics of the interior of a condenser charged by an alternating voltage.

If our observer were to move another one-eighth wave farther along the line, he would note that the resultant electric and magnetic forces would again be equal. Proceeding on to a point one-half wavelength from the metal reflector, he would observe that, at the time crests (maximum positive intensity) were passing on their way toward the reflector, hollows would be returning, and accordingly upon examining the resultant electric intensity he would find it to be zero at all times, whereas the corresponding magnetic intensity would be oscillating between limits of plus or minus $2H$. At this point along the line, he would be unable to distinguish his electrical environment from that prevailing at the metal boundary. The half-wave line, therefore, has had the effect of translating the metal barrier to another point in space a half wave removed.

If the observer were to continue still farther along the line, he would pass, alternately, points where the resultant electric force is zero and other points where the resultant magnetic force is zero. It is important to note that at points in a standing wave where the magnetic force is a maximum, the electric force is a minimum and at points where the electric force is a maximum, the corresponding magnetic force is a minimum. It is customary to call the points of minimum E (or H) "mins," though the term *node* is sometimes substituted. Points of maximum E (or H) are known as "maxs" with the term *loop* as its alternative. If the observer were to measure current and voltage along the line, he would find that points of maximum voltage correspond to maximum E and that points of maximum current correspond to maximum H.

An examination of the energy associated with the incident and reflected waves shows that, except for minor losses not to be considered here, there is as much energy led away from the reflector as is led up to the reflector, and that there is associated with the standing wave a stored or resident energy. The regular arrangement of nodes and loops along a standing wave with minima at half-wave intervals is a very important characteristic, for such points may be located very accurately experimentally, and accordingly wavelength may be measured with considerable precision.

If, instead of terminating the wire line in a large conducting plane assumed previously, it is terminated in a relatively thin cross bar as shown in Fig. 6.2-6, the reflection will assume a somewhat more complicated form. First of all, the thin cross bar will intercept, initially at least, only a portion of the total wave front. The particular lines of force arriving along a plane containing the two wires will be the first to be reflected and they will behave at reflection much like those already discussed, whereas those outside the plane of the two wires will not be intercepted initially by the thin cross bar but instead will advance for a short distance beyond the end of the line before their forces of tension bring them to rest. These outlying lines of force are represented by the lines designated as c in Fig. 6.2-6. After the first lines of force have been reflected, lateral pressure will be removed from those adjacent, with the result that they will close in and collapse on the conductor at a slightly later time than their neighbors. One over-all result of this process is to make the effective length of such a line slightly greater than the true length. Effects of this kind are observed in practice and they are referred to as *fringing*. Discrepancies between the wavelength as measured in the last section of line where fringing may take place and that measured between other minima along the same line are usually small but

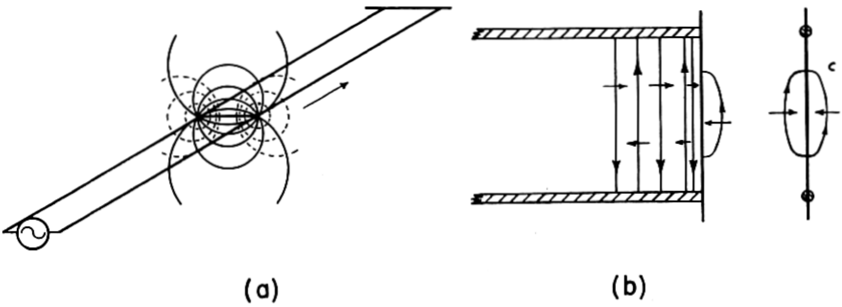


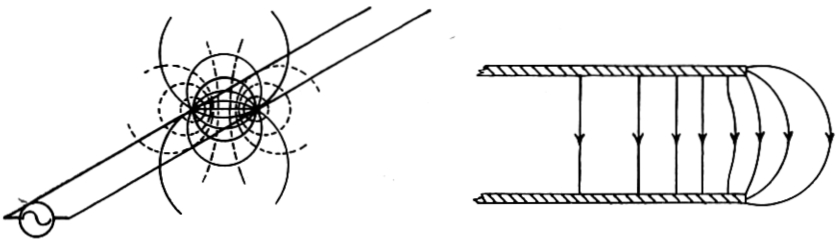
Fig. 6.2-6. (a) Representative transmission line terminated by a conductor of finite dimensions. (b) Nature of reflection by a finite conductor.

they are nevertheless measurable. It is also true that, as the wave front approaches a limited barrier of this kind, some of its energy continues on into the space beyond and is lost as radiation. In general, the smaller the barrier, the larger will be the losses.

Consider next a line open at its remote end, as shown in Fig. 6.2-7. In this case, none of the lines of force of the advancing wave is intercepted by a conductor, with the result that a very considerable number momentarily congregate near the end of the line and, because of inertia, they extend into the space beyond as suggested by Fig. 6.2-7(b). This process continues until forces of tension in the lines, still clinging fast to the ends of the wires, bring the assemblage temporarily to rest. At this moment, there is no magnetic component; for v, in the relation $\mathbf{H} = \epsilon(\mathbf{v} \times \mathbf{E})$, is zero while the corresponding electric intensity is approximately $2E$. The lines of electric force, being momentarily at rest, represent energy stored in the electric form.

This static situation is extremely temporary, for the tension momentarily created in the lines of electric force soon forces the configuration as a whole to move backward. As the wave front gets under way, the magnetic force H increases in magnitude in accordance with the relation $\mathbf{H} = \epsilon(\mathbf{v} \times \mathbf{E})$.

The fact that the wave front extends momentarily for a short distance beyond the physical end of the line and requires time to come to rest and get into motion in the reverse direction implies inertia or momentum in the wave front. This is the inertia referred to in the fourth principle mentioned in Section 6.1. In this form of reflection, fringing is usually very evident, and because of fringing we may have an apparent reflection point that is considerably beyond the end of the wires. Thus the distance from the end of the wires back to the first voltage minimum is much less than the quarter wave that otherwise might be expected.



(a) (b)

Fig. 6.2-7. (a) Transmission along a line open at the remote end. (b) Nature of reflection from open end.

It is generally true that processes of reflection in which fringing takes place are usually attended by considerable amounts of radiation. This suggests that in the process of reflection some of this extended wavepower detaches itself from the parent circuit and is lost. Experience shows that this lost power may be greatly enhanced by separating the two wires or by flaring their open ends. The so-called half-wave dipole, so familiar in ordinary radio, is but a transmission line in which the last quarter-wave length of each wire has been flared to an angle of 90 degrees. If we wish to minimize radiation, we follow a reverse procedure and reduce the spacing between the two parallel wires. This also reduces fringing, for we find that the measured distance from the ends of the wires to the first voltage minimum is now more nearly a quarter wave.

It is of interest to compare reflections taking place at the open end of a transmission line with those at a closed end. When a wave front becomes incident upon a perfect conductor, the electric force vanishes. At the same time, the lines of magnetic force, though effectively brought to rest, are momentarily doubled in intensity. The energy is predominantly magnetic, and the type of reflection may be regarded as inductive. When the wave is reflected from the ideal open-end line, a reverse situation prevails. The lines of magnetic force momentarily vanish while lines of electric force, though brought to rest, are doubled in intensity. At this moment the energy is predominantly electrostatic, and the reflection may be considered as being capacitive.

When a line is terminated in a sheet of metal of good conductivity such as copper or silver, reflection is almost perfect. If the sheet is a poor conductor such as lead or German silver, most of the incident power will still be reflected; but if a semi-conductor, such as carbon, is used as a reflector, a perceptible amount of the incident power will be absorbed. It is interesting also that the penetration into all metals at the time of reflection is very slight, for relatively thin sheets seem to serve almost as well as thick plates. It is therefore possible to use as reflectors extremely simple and inexpensive materials, for example, foils or electrically deposited films fastened to a cheaper material such as wood.[7]

A more general study of reflections on transmission lines shows that the examples cited previously are special cases of a very general subject. Not only may there be reflections from the open and closed ends of a transmission line, but there may be reflections also when the line is terminated in an inductance, in a capacitance, or in a resistance. Details concerning the reflections that may be observed from various combinations of these three impedances are discussed in connection with Fig. 3.6-3. The outstanding results of these discussions may be summarized for the ideal case as follows:

1. A pure inductance (positive reactance) connected at the end of a transmission line always leads to a reflection coefficient having a magnitude of unity. The standing wave resulting from this reflection will be characterized by the following: (a) If the terminating inductance is infinitely large (reactance of positive infinity), the reflection will be identical with that from an ideal open-end line, and the distance to the nearest voltage minimum will be a quarter wave. [See Fig. 3.6-3(a).] (b) If the inductance is finite but very large, the distance to the nearest voltage minimum, as measured toward the generator, will be somewhat greater than a quarter wave. [See Fig. 3.6-3(b).] (c) If the inductance is reduced progressively toward zero (reactance zero), the distance to the same voltage minimum will approach one-half wavelength. In this limiting case, another voltage minimum will appear at the end of the line. [See Fig. 3.6-3(c) and 3.6-3(d).]

2. A pure capacitance (negative reactance) connected at the end of a

[7] One convenient and inexpensive form of reflector is a kind of building paper coated with copper or aluminum foil. Moderately good reflectors can also be made by covering wood with a special paint containing finely divided silver in suspension (Du Pont's 4817). Most aluminum paints are unsatisfactory for this purpose.

transmission line also leads to a reflection coefficient having a magnitude of unity. In this case, the resulting standing wave will be characterized as follows: (a) If the capacitance is zero, (reactance equal to minus infinity), the reflection will correspond to that from the open end of a transmission line, and a voltage minimum will be found at a distance of a quarter wave from the end. [See Fig. 3.6–3(g).] (b) If the capacitance is increased from zero to a small finite value, the distance to the nearest voltage minimum will be somewhat less than a quarter wave. [See Fig. 3.6-3(f).] (c) If the capacitance is increased progressively toward infinity (reactance zero), the distance to the nearest voltage minimum will approach zero. [See Figs. 3.6-3(e) and 3.6-3(d).] The limiting condition, in which the terminating capacitance is zero, is comparable with that in which the termination is an infinitely large inductance.

3. If a pure resistance is connected at the end of a transmission line, the magnitude of the reflection coefficient varies with the resistance chosen. The relations are such that: (a) If the terminating resistance is infinite, the magnitude of the reflection coefficient will be unity and its sign will be positive. [See Fig. 3.6-3(h).] (b) If the terminating resistance approaches the characteristic impedance of the line, the distance to the nearest voltage minimum will remain constant, but the magnitude of the reflection coefficient will approach zero. [See Figs. 3.6-3(i) and 3.6-3(j).] (c) If the terminating resistance is made less than characteristic impedance, the sign of the reflection coefficient will be reversed, and, as the terminating resistance approaches zero, its magnitude will approach unity. [See Figs. 3.6-3(k) and 3.6-3(l).]

When the terminating resistance is infinite, the reflection is comparable with that in an ideal open-end line, and the nearest voltage minimum will be found at a distance of a quarter wave. When the terminating resistance is zero, the reflection is comparable with that in a closed-end line, and the voltage minimum will appear at the end of the line and also at a point one-half wave closer to the generator. If the line is terminated in a pure resistance of intermediate value, the voltage minima of such standing waves as may be present will be found at the end of the line for all values of the resistance that are less than characteristic impedance and a quarter wave removed from the end of the line for all values greater than characteristic impedance. When the terminating resistance equals characteristic impedance, there is no standing wave.

If, instead of terminating the line considered above in an inductance coil or in a capacitance or a resistance, we assume that it continues indefinitely into a mass of material having either a conductivity or a dielectric constant different from that of air, similar reflections may take place at the surface. A particular example is shown in Fig. 6.2-8. In general, a part of the wave-power arriving at the surface will be reflected and a part will be transmitted.

One may picture a portion of the Faraday tubes of force turned back at the interface while the remainder continue into the second medium. If one were to reverse the direction of transmission and consider wavepower transmitted from the second medium back into the first, a similar partial reflection would be noted. In both cases the part turned back and returned to the source may be regarded as a reactive component since no energy is really lost. In a similar way, the transmitted component, since it is not returned to the source, may be regarded as a resistive or dissipative component.

If the medium into which wavepower is transmitted is a perfect insulator, the transmitted wave will continue indefinitely except as attenuated by the

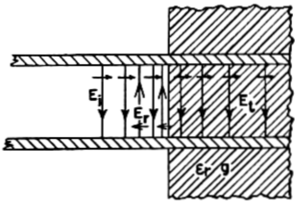


Fig. 6.2-8. Reflection and transmission of lines of force incidental to a change of medium along a transmission line.

wires along which it is guided. Its wavelength, λ, in the dielectric will be less than the wavelength, λ_0, in air as expressed by the relation

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}}$$

If the second medium is somewhat conducting, the wave will be further attenuated, the rate of attenuation being related in a rather complicated way not only to the conductivity of the second medium but to its dielectric constant and permeability as well. Thus far in microwave practice, little practical use has been made of materials having permeabilities very different from unity. However, considerable use has been made of materials having various dielectric constants, ϵ_r, and conductivities, g. Sometimes these take the form of plates placed across a waveguide transmission line. Examples will appear in Section 9.8.

If a thin sheet of insulating material having a dielectric constant, ϵ_r, and conductivity of zero is placed across a two-wire transmission line, the percentage of power reflected is given approximately by

$$q_W = \frac{\pi t}{\lambda_0} (\epsilon_r - 1) \qquad (6.2\text{–}5)$$

A thin sheet of this kind is approximated when wires carrying very high frequencies pass through the glass walls of a vacuum tube. If the glass thickness, t, is small compared with the wavelength in air, λ_0, the power reflected by the glass envelope will likewise be small.

Sometimes it is not feasible to reduce the wall thickness sufficiently to avoid serious reflections. In these instances it may be possible to make the thickness one-half wavelength as measured in glass whereupon the wave reflected from one face of the plate will be approximately equal in amplitude to that from the other face and, since they are separated by one-half wavelength, they tend to cancel.

Another case of practical interest is that in which the line is terminated in a plate of very special dielectric constant ϵ_r, conductivity g_1, and thickness t. This is followed by a second plate of nearly infinite conductivity. This arrangement is shown in longitudinal section in Fig. 6.2-9. By a proper choice of constants, the combination may be made a good absorber of wave-

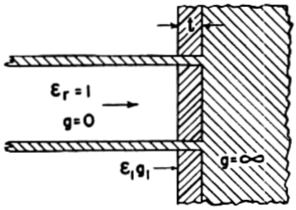


Fig. 6.2-9. A transmission line terminated in a conductor coated with a special materia such that all of the incident wave power is absorbed.

power. It will therefore be substantially reflectionless It may be shown that to satisfy this requirement

$$\lambda_0 = \frac{\sqrt{\epsilon_r}}{15\pi g_1} \tag{6.2–6}$$

and

$$t = \frac{1}{60\pi g_1(2n - 1)} = \frac{\lambda_0}{4\sqrt{\epsilon_r}\,(2n - 1)} \tag{6.2–7}$$

where n is any integer. One common example is that in which $n = 0$. The plate is then a quarter wave thick as measured in the medium.[8] A reflectionless plate of this kind when placed at the end of a transmission line appears to the source as though the line were terminated in its characteristic impedance. Devices incorporating this principle are sometimes used as match terminators for waveguides.[9]

[8] A more complete discussion of this problem was published in 1938 by G. W. O. Howe, "Reflection and Absorption of Electromagnetic Waves by Dielectric Strata," *Wireless Engr.*, Vol. 15, pp 593–595, November 1938.

[9] Plates of this kind may be made very simply by mixing carbon with plaster in varying proportions until the right combination is reached.

When a two-wire transmission line assumes the coaxial form, the lines of electric force are radial and lines of magnetic force are coaxial circles. The directions of these two components obey the right-hand rule. (See Fig. 6.2-10.) Since the wave configuration is completely enclosed except for a small exposure at each end, radiation from this type of line can be made very small.

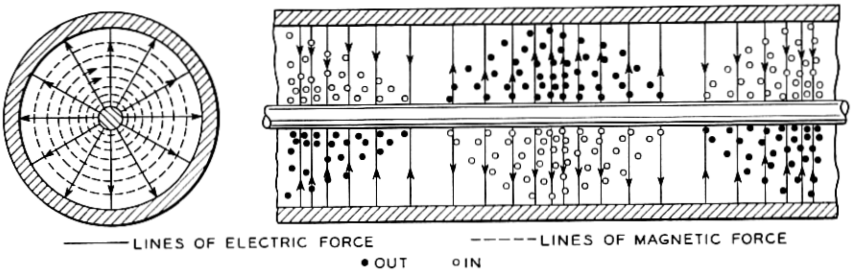


Fig. 6.2-10. Arrangement of lines of electric and magnetic force associated with transmission along a coaxial arrangement of conductors.

6.3 Radiation

Electromagnetic waves, including both light and radio waves, are not unlike the waves that are guided along wire lines. Their difference is largely a matter of environment. In one case they are attached to wires while in the other they have presumably detached themselves from some configuration of conductors and are spreading indefinitely into surrounding space. We shall present in this section one of several possible pictures of the launching of radio waves from a transmission line. Like other verbal pictures drawn in this chapter, it should be regarded as highly qualitative.

Assume a two-wire line with one end flared as shown in Fig. 6.3-1. If at some point to the left there is a source of wavepower, there will flow from left to right along the line a sinusoidal distribution of lines of electric and magnetic force not unlike that shown in Fig. 6.2-7. In order to simplify our illustration, we shall single out for examination two representative lines of electric force *a–b* and *c–d* located a half wave apart. It is understood, of course, that there are present many other lines both before and behind those represented. Also there are lines of magnetic force at right angles to the electric force. As time progresses each element of length of the line of force *a–b* moves laterally with the velocity of light. In the region where the wires are parallel, it remains straight but, upon reaching the flared section, its two ends fall behind the central section, thereby forming a curve as shown in Fig. 6.3-1(c). As this line of force moves to the end of the flared section [Fig. 6.3-1(d)], its successor *c–d* follows one-half wavelength behind.

Because of the property of inertia with which all lines of force are assumed to be endowed, the central section of a–b, which is already greatly extended due to curvature, continues in motion for some time after the two ends, attached to the conductors, have come to rest. The result is shown approximately by Fig. 6.3-1(e). An instant later and perhaps after the two ends of line of force a–b have started on their return journey, the line of force c–d approaches sufficiently close to a–b that a coalescence ensues [Fig. 6.3-1(f)]. An instant later fission takes place as illustrated in Fig. 6.3-1(g), leaving a portion of the energy of each a–b and c–d now shared by a radiated com-

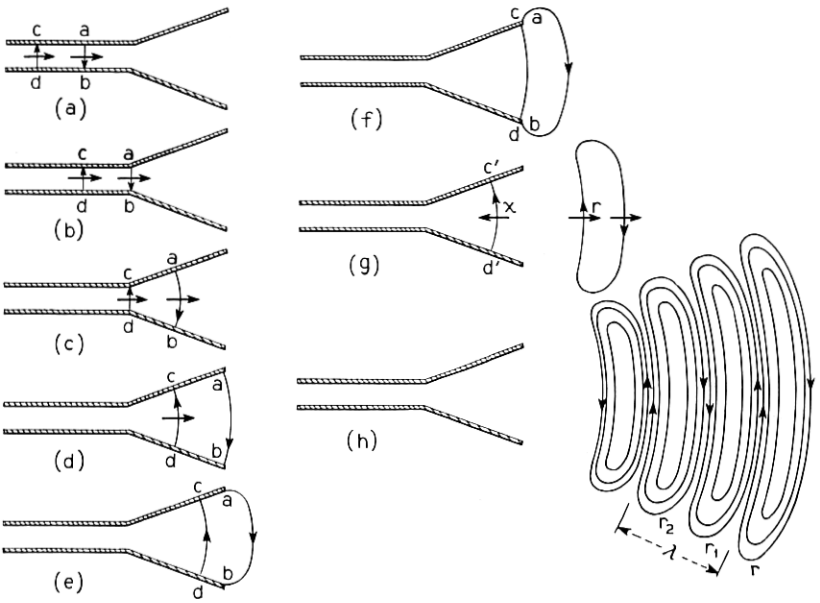


Fig. 6.3-1. Successive epochs in a highly idealized representation of radiation from the flared end of a transmission line.

ponent, r, and a reflected component, x. That the two components r and x should travel in opposite directions seems reasonable when it is noted that lines of electric force in x are in the same direction as in the adjacent portion of r. They may therefore be expected to repel. The first of these components, r, appears to the transmitter as though it were a resistance since it represents lost energy. The second, x, appears as a reactance since it represents energy returned to the transmitter. The radiated component, r, will be followed by other components r_1, r_2, etc., as represented in Fig. 6.3-1(h).

In the radiated wave front, the two components E and H are everywhere mutually perpendicular and in the same phase. Because the wave front is curved, as shown in cross section in Fig. 6.3-2, the component Poynting vectors which specify the directions in which energy is flowing will be slightly divergent. As a result, only a portion of the total wavepower will proceed in the preferred direction. It follows that, for best directivity, the emitted wave front should be substantially plane, and the lines of force should be as nearly straight as possible. There is shown in Fig. 6.3-3 a series of configura-

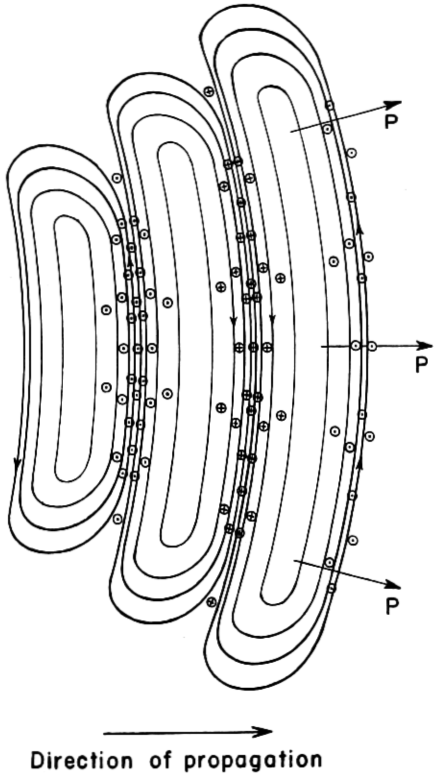


Fig. 6.3-2. Cross section of electromagnetic waves radiated from the flared end of a transmission line. Lines of electric force lie in the plane of the illustration; lines of magnetic force are perpendicular to the illustration while the flow of power is along the divergent arrows P.

tions based partly on speculation and partly on deductions from Huygens' principle. They illustrate in a rough way how, by increasing the aperture between the two wires of the elementary radiator, we may make the individual component Poynting vectors more nearly parallel.[10]

[10] Figure 6.3-3 has been greatly oversimplified. Experiment shows that, to achieve the result desired, the angle between the two wires of Fig. 6.3-3 must be smaller for larger apertures than for small apertures.

Thus far, we have restricted our considerations to directivity in the plane of the two conductors (vertical plane as here assumed). Experiment shows that, in the plane perpendicular to that illustrated, the directivity from a single pair of wires is slight. However, we may obtain additional directivity by increasing the horizontal aperture. One method of accomplishing this result is to array, at rather closely spaced intervals, identical elementary radiators each of the kind just described. [See Fig. 6.3-4(a).] An infinite number of these elements infinitesimally spaced become two parallel plates as shown in Fig. 6.3-4(b). If metal plates are now attached at the right and left

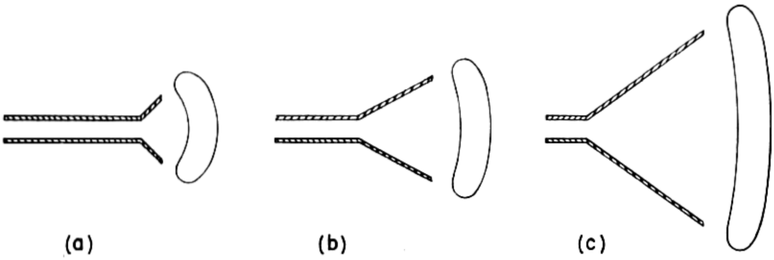


Fig. 6.3-3. Illustrating how radiating systems of large aperture may give rise to wave fronts of large radius of curvature and hence lead to increased directivity.

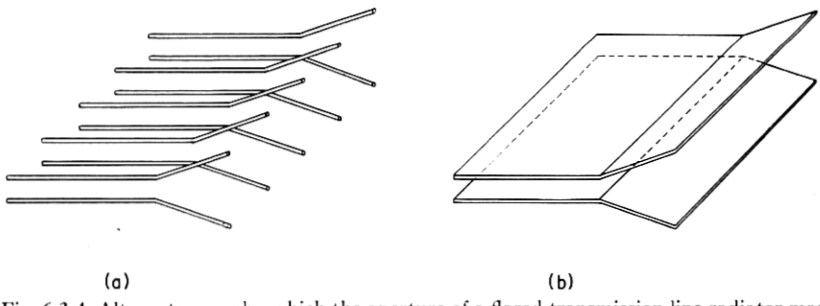


Fig. 6.3-4. Alternate ways by which the aperture of a flared transmission line radiator may be increased.

sides, the resulting configuration will become a waveguide horn. As a general rule, the larger the area of aperture, the more directive will be the antenna. The highly schematic array shown in Fig. 6.3-4(a) is introduced for illustrative purposes only. It is not one of the preferred forms used in microwave work. More practicable forms will be found in Chapter X.

The wave model shown in Fig. 6.3-2 conveys but a portion of the known facts about a radiated wave. A more accurate model is shown in skeleton form in Fig. 6.3-5. It is assumed that the transmitted wave has been launched with about equal directivity in the two principal planes and that the observer is looking into one-half of a cut-away section of the total configuration. In the complete configuration, the individual lines of electric force (solid lines) and magnetic force (dotted lines) form closed loops, thereby producing in each half-wave interval a packet of energy. The stream of projected energy from an antenna is, according to this view, a series of these packets one behind the other moving along the major axis of transmission. At the transmitter each packet may have lateral dimensions that are only slightly greater than the corresponding dimensions of the radiating antenna; but, since the packet has curvature and since propagation is radial, the packet spreads as it progresses so that at the distant receiver it may be very large indeed.

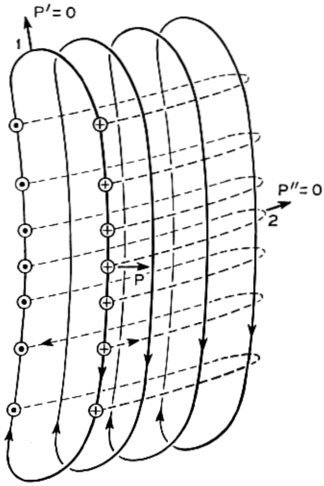


Fig. 6.3-5. Highly idealized representation of a wave-packet radiated by a typical microwave source. One half of the total packet is assumed to be cut away.

Around the edge of each packet there is a region where the relationship between the vectors E, H, and v is rather involved. For example, in the vicinity of point 1 in Fig. 6.3-5, there is a substantial component of E but at this point the vector H is zero and accordingly the Poynting vector P' at that point is also zero. (See Equation 6.1–4.) In a similar way there may be in the vicinity of point 2 a substantial component of magnetic force $\mathbf{H}$; but, since at this point the electric force is substantially zero, we conclude that the Poynting vector P'' is again zero and again no power is propagated.[11]

[11] The peculiar edge effects noted may be regarded as a result of a kind of wave interference not unlike that prevailing in the regions of minimum E and H in the case of standing waves as discussed in Section 6.3. A similar kind of wave interference is cited in Section 6.5 to account for regions of low E and H in transmission along a waveguide.

The sharpest radio beams now in general use are only a few tenths of a degree across. We conclude that for these sharp beams a small but nevertheless appreciable curvature remains in the radiated wave packet. This means that, when the wave front has arrived at a distant receiver, it is still many times larger than any receiving antenna it may be practicable to construct, and accordingly the latter can intercept but a small portion of the total advancing wavepower. This implies a considerable loss of power, which is indeed the case.

In the process of radio reception, one may think of the antenna structure as a device that cuts from the advancing wave front a segment of wavepower which it subsequently guides, preferably without reflection, to the first stages of a nearby receiver. To be efficient, the wavepower intercepted should be large. This, in turn, calls for a receiving antenna of considerable area. It will be remembered that a large aperture was also a necessary feature for high directivity at the transmitter. This is consistent with the accepted view that the processes of reception and transmission through an antenna are entirely correlative and that a good transmitting antenna is a good receiving antenna and vice versa. The directive properties of an antenna are sometimes specified in terms of its *effective area*. (See Section 10.0.)

The term *uniform plane wave* is a highly idealized entity assumed in many problems for purposes of simplicity but never quite attained in practice. In an idealized wave front, the electric and magnetic components E and H are not only everywhere mutually perpendicular but both components are exclusively transverse. That is, there is no component of either E or H in the direction of propagation. Such a wave belongs to a class known as *transverse electromagnetic waves* (TEM). These may be compared with others, to be described later, known as *transverse electric waves* (TE) and *transverse magnetic* (TM) *waves*. Waves guided along parallel conductors are also TEM waves, but except in the case of infinitely large conductors they are not *uniform plane waves*.

6.4 Reflection of Space Waves from a Metal Surface

One of the early triumphs of the electromagnetic theory was its ability to account satisfactorily for the reflection and refraction of light. This theory was so general as to include not only a wide range of wavelengths but also a wide range of surfaces as well. According to this theory, reflections may occur whenever electromagnetic waves encounter a discontinuity. This may happen, for example, when waves fall on a sheet of metal, in which case the discontinuity is due to the sudden change in conductivity. Reflection may also occur when waves are incident on a thick slab of glass or hard rubber, in which case reflection is due to a sudden change in dielectric constant.[12] Similar reflections may theoretically take place also at an interface where the permeability of the medium changes suddenly. The case in which there is a change of conductivity has an important bearing on waveguide transmission. It will therefore be discussed in considerable detail.

Assume a plane wave incident obliquely upon a conducting surface as shown in Fig. 6.4–1. The line along which the wave is progressing (wave-normal) is referred to as the *incident ray*. It intersects the conducting surface or interface at a point O and makes an angle θ with the perpendicular OZ. After reflection, the normal to the new wave wave front makes an angle θ' with the perpendicular OZ. This second wave-normal is known as the

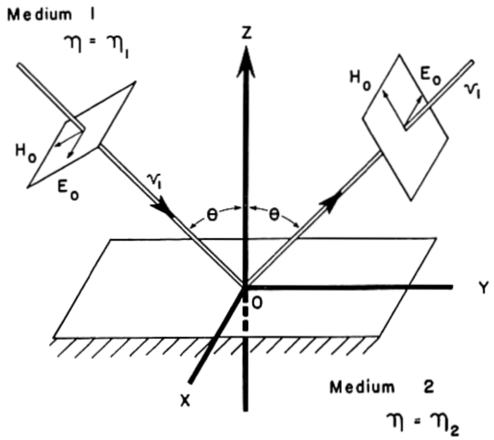


Fig. 6.4-1. Reflection at oblique incidence from a metal plate for the particular case where the electric vector is perpendicular to the plane of incidence.

reflected ray, and its angle with the perpendicular OZ is known as the *angle of reflection*. The plane containing the incident ray and the perpendicular OZ is known as the *plane of incidence*. The incident and reflected rays lie in the same plane, and their corresponding angles of incidence and reflection are numerically equal.

In problems of oblique incidence there are two cases of interest, depending on whether the electric or the magnetic component lies in the plane of incidence. For our particular purpose, the second of these two cases is of special interest and it will therefore be discussed in considerable detail. The vector relations corresponding to this case are shown in Fig. 6.4-1.

[12] For a more general discussion of the electromagnetic theory of reflection: L. Page and N. I. Adams, "Principles of Electricity," D. Van Nostrand Co., Inc., pp 569–575, New York 1931. R. I. Sarbacher and W. A. Edson, "Hyper and Ultra-high Frequency Engineering," John Wiley & Sons, Inc., pp 105–116, New York 1943.

Included are the relative directions of E and H both before and afte reflection.

In Fig. 6.4-2 there are shown in cross section representative lines of electric force in an advancing plane wave front. They are numbered respectively 1, 2, 3, 4, 5, 6, and 7. Each individual figure [(a), (b), (c), etc.] represents a succeeding period of time. We shall assume that the particular wave front singled out for illustration represents the crest of a wave Both ahead and behind this crest there are located alternately at half-wave intervals other crests and hollows, and their respective lines of force alternate in direction. Each line of force in the wave front is assumed to be moving in a direction indicated by the vector v. It is furthermore assumed that there is also present a magnetic component, indicated by the dotted vector H that is perpendicular to E and also to v. The vectors v and H must of course be so directed as to be in keeping with the right-hand or cork-screw rule, both before reflection and after reflection. Also at the point of incidence the tangential electric force must be zero. To account for this, we assume that as each line of electric force moves up to the conducting plane it is reversed in direction, thereby making on the average as many lines of electric force at the surface directed toward the observer as directed away from the observer. Consider, for example, lines of force 3 and 5, 2 and 6, and 1 and 7, in Fig. 6.4-2(c).

Associated with these two components of electric force which, let us say, are E and E', there are two components of magnetic force H and H'. These may be specified by $\mathbf{H} = \epsilon(\mathbf{v} \times \mathbf{E})$, each of which at the interface may be resolved into two components shown in Fig. 6.4-3 as $H = H_\perp + H_\parallel$ at the left and $H_\perp' = -H_\parallel'$ at the right. Combining these four vectors, assuming reflection to be perfect, we find that at the interface $H_\perp - H_\perp' = 0$ and $H_\parallel - (-H_\parallel') = 2H$, giving as an over-all result: (1) the electric force at the interface is everywhere zero; (2) the vertical component of the magnetic force at this point is also zero; and (3) the tangential component of the magnetic force at the interface is $2H$.

The peculiar configuration that resides close to the metal boundary is propagated to the right as a kind of magnetic wave. It has rather interesting properties which will become more evident by referring again to Fig. 6.4-2. Two conclusions may be drawn from this figure, depending on the point of view assumed. To a myopic observer located at the interface and unable to see far beyond the point p and unable to distinguish one line of force from another, the advancing wave front would look like a configuration of amplitude $H_\parallel = 2H$ and $E_\parallel = 0$ moving parallel to the interface with velocity $v_z = v/\sin\theta$. To this observer the apparent velocity would increase as θ becomes progressively smaller until, at perpendicular incidence, v_z would approach infinity. These results follow from the geo-

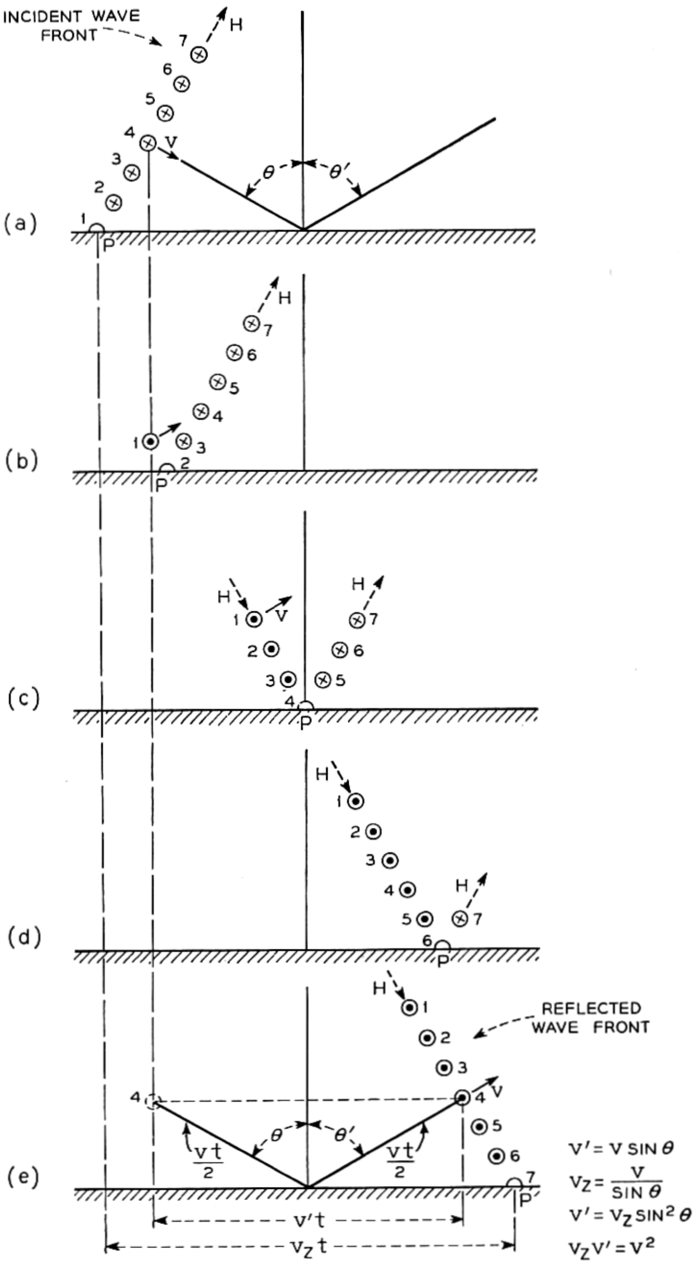


Fig. 6.4-2. Successive steps in the reflection of a single plane wave front by a metal plate.

metrical relations shown in the lower part of Fig. 6.4-2. Phenomena similar to this are sometimes observed when water waves, coming in from the ocean, break upon the beach. If the approach is nearly perpendicular, the point at which the wave breaks may proceed along the beach at a phenomenal speed. A similar effect may be produced by holding at arm's length a pair of scissors and observing the point of intersection as the blades are showly closed. A relatively slow motion of the blades leads to a rather rapid motion of the point of intersection.

Since, in the case of incident waves, the apparent velocity is $v_z = v/\sin\theta$, the corresponding wavelength is $\lambda_z = \lambda/\sin\theta$. Both quantities play an important part in the picture of waveguide transmission to be drawn later. In particular, the apparent velocity v_z will prove to be identical with a quantity known as *phase velocity*.

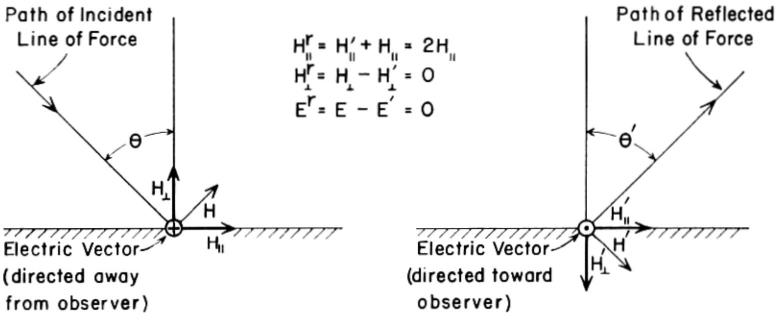


Fig. 6.4-3. Relationship between various components of E and H before and after reflection by a metal plate.

A second observer located at the interface, shown in Fig. 6.4-2, endowed with better vision and able to single out particular lines of force may obtain a somewhat different view of reflection. If he observes a particular line of force such as (4) in Fig. 6.4-2 for the considerable period of time, t, required for it to approach the conducting interface [Figs. (a) to (c)] and recede to a comparable distance [Figs. (c) to (e)], he will note that, whereas the line of force has really traveled a total distance vt, its effective progress parallel to the interface has been $v't = vt \sin\theta$. (See geometrical relations in lower part of Fig. 6.4-2.) This provides another kind of velocity ($v' = v \sin\theta$) known as *group velocity*. It is the effective velocity with which energy is propagated parallel to the metal surface. It approaches zero at perpendicular incidence. It will be observed that

$$v' = v_z \sin^2\theta$$

and

$$v'v_z = v^2 \tag{6.4-1}$$

Group velocity also plays an important part in waveguide transmission.

6.5 Waveguide Transmission

It was pointed out in an earlier chapter that each of the various configurations observed in waveguides may be considered as the resultant of a series of plane waves each traveling with a velocity characteristic of the medium inside, all multiply reflected between opposite walls. In the case of certain of these waves, this equivalence may not be readily obvious, but for the dominant mode in a rectangular guide, which is one of the more important practical cases, it is relatively simple. It also happens that the analysis of such waves throws considerable light on the nature of guided waves, and furthermore it enables us to deduce many of the useful relations used in waveguide practice—relations that might otherwise call for rather complicated mathematical analysis.

It is assumed in Fig. 6.5-1 that we are viewing, in longitudinal section and at successive intervals of time, a hollow rectangular pipe having transverse dimensions of a and b measured along the x and y axes respectively. In this case the illustration is in the xz plane. It is further assumed that the electric force lies perpendicular to the larger dimension a and is consequently perpendicular to the plane of the illustrations. We assume in Fig. 6.5-1 (a) a particular plane wave front 1, perhaps a crest, that has recently entered the guide from below. Let us say that its velocity is $v = v_a/\sqrt{\mu_r\epsilon_r}$ and that is it so directed as to make an angle θ with the left-hand wall as shown.[13] Reflection at the left-hand wall will therefore be identical with that already shown in Fig. 6.4-2. A portion of the wave front that has just previously undergone reflection is shown immediately below at 2 in Fig. 6.5-1(a). We assume further that this front is made up of lines of electric force perpendicular to the illustration together with associated lines of magnetic force lying in the plane of the illustration. It will be obvious presently that, like the case of reflection from a single conducting sheet discussed in the previous section, we may obtain two rather different pictures of what takes place within the guide, depending on whether we fix our attention on the configuration as a whole or on some particular line of force which we may identify and follow through a considerable interval of time. We shall first consider the configuration as a whole.

[13] It is to be noted that the angle ϕ which the wave front makes with the metal wall is equal to the angle which the wave-normal (ray) makes with the perpendicular to the metal wall.

We show in Fig. 6.5-1(b) the same wave front shown in Fig. 6.5-1(a) but at an epoch later—after it has progressed a considerable distance along the guide. We now find the reflected portion 2 complete and a new portion

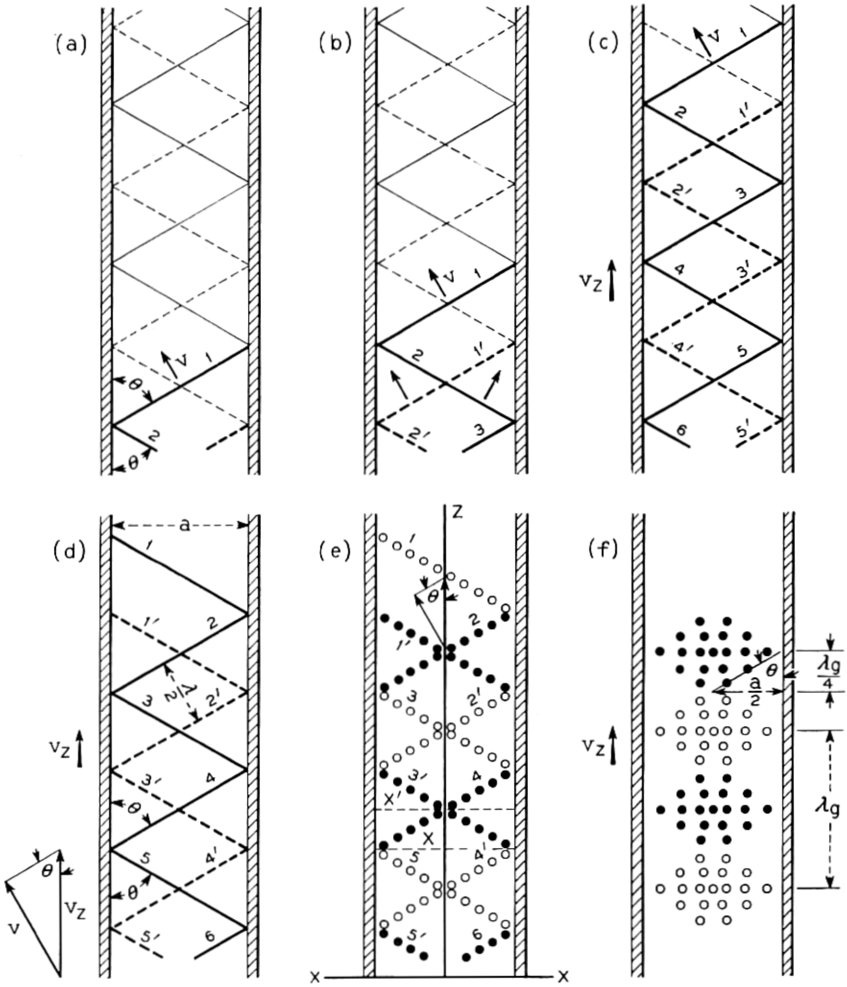


Fig. 6.5-1. The propagation of a multiply reflected wave front between two metal plates [Figs. (a)–(d)] is equivalent to the transmission of a TE wave parallel to the two plates. [Fig. (f)].

3 about to enter the guide. Following wave front 1 and at a distance of one-half wave behind, we find, shown dotted, the "hollow" of the wave. This we shall designate by the numeral 1′. We find here also a new portion of the "hollow" 2′ that has just undergone reflection.

In Fig. 6.5-1(c) and again in Fig. 6.5-1(d) we find successive positions of these same wave fronts as they have moved forward in the guide. We may, if we like, think of these fronts as discrete waves moving zig-zag through the guide or as a single large wave front folded repeatedly back upon itself. Fixing our attention for the moment on Fig. 6.5-1(d), we observe that the velocity v at which any point of incidence of the wave front (say at point 5) moves along the guide is given by the relation

$$v_z = \frac{v}{\sin\theta}$$

This particular velocity v_z is the phase velocity of the wave as seen by a myopic observer located near a lateral wall of the guide.

Referring again to Fig. 6.5-1(d) and fixing our attention on the geometrical relation between the wavelength λ and the width of the guide a, we may construct a right triangle with $\lambda/2$ and a as sides and show that

$$\cos\theta = \frac{\lambda}{2a} \tag{6.5–1}$$

and since

$$\sin\theta = \sqrt{1 - \cos^2\theta} \tag{6.5–2}$$

$$\sin\theta = \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2} \tag{6.5–3}$$

and

$$v_z = \frac{v}{\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}} \tag{6.5–4}$$

This says that for very large guides, that is, $\lambda < 2a$, $v_z \doteq v$, but as λ approaches $2a$, v_z approaches infinity. The particular case where $\lambda = 2a$ and $v_z = \infty$ is referred to as the *cut-off condition*. At cut-off, it would appear that the individual waves approach the wall at perpendicular incidence and a kind of resonance between opposite walls prevails. At wavelengths greater than cut-off no appreciable amount of power is propagated through the guide.

The particular value of wavelength measured in air, corresponding to cut-off, is referred to as the *critical* or *cut-off wavelength* and is designated thus: $\lambda_c = 2a$. The corresponding frequency is similarly known as the *critical* or *cut-off frequency* and it is designated thus: $f_c = v/\lambda_c$. It is sometimes convenient to designate the ratio of the operating wavelength to the critical wavelength by the symbol ν. From Equation 6.5–4 it follows that

$$\frac{v_z}{v} = \frac{1}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} = \frac{1}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{1}{\sqrt{1 - \nu^2}} \qquad (6.5\text{–}5)$$

Referring to Fig. 6.5-1(a) we have indicated that the wave front 1 is made up of lines of electric force directed through the plane of the illustration and hence away from the observer. There are, of course, lines of magnetic force and also other lines of electric force both ahead and behind the wave front drawn, but these have purposely been omitted in order to simplify the illustration. If we were to take the magnetic force into consideration we would find as in Fig. 6.4-2 that, at the reflecting surface, a tangential component only is present and its magnitude is twice that of the magnetic component of the incident wave.

In the discussion of reflection of plane waves in the previous section, it was also pointed out that the act of reflecting a wave reverses the direction of the electric force. Applying this principle to the case at hand, we see that if the electric force is directed downward in the section of wavefront 1 of Fig. 6.5-1(a), it will be directed upward in 2. Carrying this idea forward to Fig. 6.5-1(e) we find that in fronts 1, 2, 3, etc., which we rather arbitrarily called crests, the electric vector alternates in direction as shown by the open and solid circles. Likewise the direction of the electric vector alternates in the fronts designated as 1′, 2′, and 3′, but in this case they are respectively opposite in direction to 1, 2, and 3. Continuing to fix our attention on Fig. 6.5-1(e), it will be observed that the direction of lines of force is the same in 1′ and 2, in 2′ and 3, and in 3′ and 4, indefinitely along the entire length of the guide. Thus there are regularly spaced regions along the length of the guide where the electric vector is directed toward the observer alternating with other regions where the electric vector is directed away from the observer. Between the two are still other regions where the respective component vectors are oppositely directed and hence their sum may be zero.

Adding the foregoing effects, bearing in mind that there are lines of force both ahead and behind the highly simplified wave fronts shown, we have a new wave configuration moving parallel to the main axis of the guide with a phase velocity v_z as suggested by Fig. 6.5-1(f). Examining more carefully the wave interference that is here taking place, it becomes evident that if we pass laterally across the guide along the line x in Fig. 6.5-1(e) the instantaneous value of the resultant electric vector as shown is everywhere zero. On the other hand, if we cross the guide along a parallel line x', the electric vector varies sinusoidally beginning at zero at either wall and reaching a maximum in the middle of the guide. It will be observed that if we pass along the major axis z of the guide the electric vector at any instant again varies sinusoidally with distance. However, at the boundary of the guide the resultant electric vector is everywhere zero. Since there was no component of the electric force lying along the axis z of the guide in the component waves that gave rise to this configuration, there can be no such component in the resultant. Waves in which the electric vector is exclusively transverse are known as *transverse electric*, or TE, *waves*.

A complete account of transmission of this kind should include, of course, a consideration of the lines of magnetic force. From Fig. 6.4-3 it is evident that, at the point of reflection of the component plane wave on the guide wall, there are two components of magnetic force $H_{\perp}$ and $H_{\parallel}$ in both the incident and reflected waves. When these are added, the resultant of the transverse magnetic force, like that of the electric force, differs at different points in the guides. Following alone the line x', it is found that for the particular condition here assumed, the magnetic force is zero at each wall increasing sinusoidally to a maximum midway between. At this point the magnetic component is entirely transverse. Following along the line x, it will be found that the magnetic vector is a maximum near each wall decreasing cosinusoidally to zero in the middle. It is of particular interest that, at the wall of the guide, the magnetic component lies parallel to the axis. Magnetic lines of force are, in this type of wave, closed loops, whereas lines of electric force merely extend from the upper to the lower walls of the guide. The arrangement of lines of electric and magnetic force in this type of wave is shown in Fig. 5.2-1. The quantitative relationships between the various components of E and H are specified more definitely by Equation 5.2–1. The significance of the wavelength λ_g of this new configuration will be obvious from Fig. 6.5-1(f).

There are certain useful results that follow from Fig. 6.5-1(f). It may be seen from the triangle there shown that

$$\frac{\lambda_g}{4} = \frac{a}{2} \cot \theta \tag{6.5–6}$$

From Equations 6.5–1 and 6.5–3, it will also be seen that

$$\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{\lambda}{2a\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}} \tag{6.5–7}$$

Therefore

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}} = \frac{\lambda}{\sqrt{1 - \nu^2}} \tag{6.5–8}$$

Since $1/\sqrt{1 - \nu^2}$ is the ratio of the apparent wavelength in the guide to that in free space and since for hollow pipes it is greater than unity, it is sometimes referred to as the *stretching factor*. It appears frequently in quantitative expressions relating to waveguides. Since velocity is equal to the number of waves passing per second times the length of each wave, we have

$$v_z = \frac{v}{\sqrt{1 - \nu^2}} \tag{6.5–9}$$

This is equivalent to the relation shown as Equation 6.5–5.

A matter of special interest is the rate at which energy is propagated along the guide. For present purposes, it is convenient to regard a moving line of force and its associated magnetic force as a unit of propagated energy. A knowledge of the path followed by such a line of force will therefore shed light on the rate at which energy is propagated along a waveguide.

It was pointed out in connection with Equation 6.4–2 that, when a wave is incident obliquely upon a metal surface, the apparent phase of the wave progresses at a velocity v_z greater than the velocity of light v, but that the energy actually progresses parallel to the interface at a velocity v' less than the velocity of light. It was pointed out, too, that $v' = v \sin \theta = v_z \sin^2 \theta$. Because of multiple reflections between opposite walls of a waveguide, its *phase velocity* is identical with v_z. Also, because of these multiple reflections, energy being carried by these component plane waves follows a rather devious zig-zag path and will therefore progress along the axis of the guide at a relatively slow rate. This velocity which is known as the *group velocity* is idential with v' above. From relations already given, it will be seen that

$$v' = v\sqrt{1 - \nu^2} \tag{6.5–10}$$

also

$$v' = v_z(1 - \nu^2) \tag{6.5–11}$$

It will be apparent from this relation that, at cut-off, where $\nu = 1$, energy is propagated along the guide with zero velocity. This is consistent with the idea already set forth that, at cut-off, energy oscillates back and forth between opposite faces of the guide. As we leave cut-off and progress toward higher frequencies (shorter waves), the group velocity v' increases as the phase velocity v_z decreases, until, at extremely high frequencies, both approach the velocity v characteristic of the medium. This relationship is made more evident by Fig. 6.5-2.

Reviewing again the simple analysis just made, we find that the wave configuration that actually progresses along a conventional rectangular waveguide may be regarded as the result of interference of ordinary uniform plane waves multiply reflected between opposite walls of the guide. This viewpoint accounts for not only the distribution of the lines of force in the wave front but also for the velocity at which the phase progresses and the velocity at which energy is propagated. As we shall soon see, it accounts also for the rate of attenuation.

In the particular configuration just described the electric component is everywhere transverse, whereas the magnetic component may be either longitudinal or transverse, depending on the point in a guide at which observations are made. These waves are plane waves, but, since the elec-

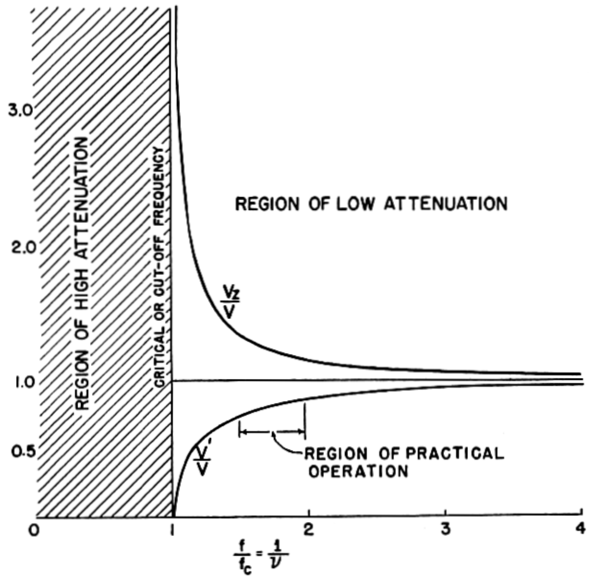


Fig. 6.5-2. Relative phase velocity v_z and group velocity v' for various conditions of operation of a waveguide.

tric intensity is not uniformly distributed over the wave front, they are not uniform plane waves.

The concept of multiply reflected waves provides a basis for calculating the attenuation in rectangular guides as was shown by John Kemp several years ago.[14] The procedure is outlined briefly below. The reader is referred to the published article for details.

There is shown in Fig. 6.5-3 a short section of hollow waveguide in which we imagine multiply reflected plane waves are propagated. We fix our attention on a zig-zag section cut from the guide and so directed that it

[14] John Kemp, "Electromagnetic Waves in Metal Tubes of Rectangular Cross-section," *Jour. I.E.E.*, Part III, Vol. 88, No. 3, pp 213–218, September 1941.

lies parallel to the direction of propagation of the elemental wave fronts. The top and bottom conductors so formed may be regarded as a uniform flat-conductor transmission line with oblique reflecting plates (sections of the side walls) spaced at regular intervals. Other transmission lines adjacent to that under consideration behave in exactly the same way as that singled out for examination and at the same time act as guard plates to insure that the lines of force so propagated remain straight.

It is clear that the attenuation in each elemental transmission line will be that incidental to losses in the upper and lower conductors plus the losses incidental to reflection at oblique incidence from the several reflecting

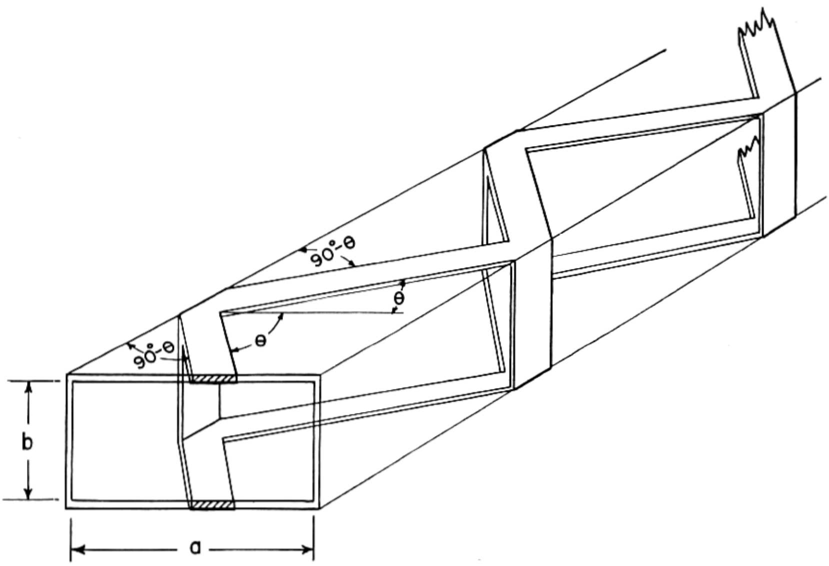


Fig. 6.5-3. Elementary transmission lines terminated periodically by reflecting plates which go to make up a rectangular waveguide.

plates. The total attenuation of the rectangular guide may then be found by summing up over a unit length of waveguide all of the elemental lines. This has been done with results that are equivalent to the corresponding equations given in Chapter V. The results are plotted in Fig. 6.5-4.

Certain characteristics of these curves may be readily accounted for. For instance, at cut-off ($\theta = 0$), both the number of unit reflection plates and the number of flat-plate transmission lines in a given length of waveguide will be infinite. As a result, the component attenuations arising in each of these two sources will likewise be infinite. As the frequency is increased above cut-off the angle θ will increase accordingly, leading thereby to fewer side-wall reflections and to a shorter over-all length of zig-zag transmission line. Thus, in this frequency range, the attenuations contributed both by the side walls and by the top and bottom plates decrease with increasing frequency. Proceeding to frequencies far above cut-off, where θ approaches 90 degrees, there will not only be very few reflections but the over-all length of zig-zag line will approach as its limit a single, straight two-conductor line made up of the top and bottom plates alone. Thus the attenuation due to the side walls will approach zero and that due to the top and bottom plates will increase as the square root of the

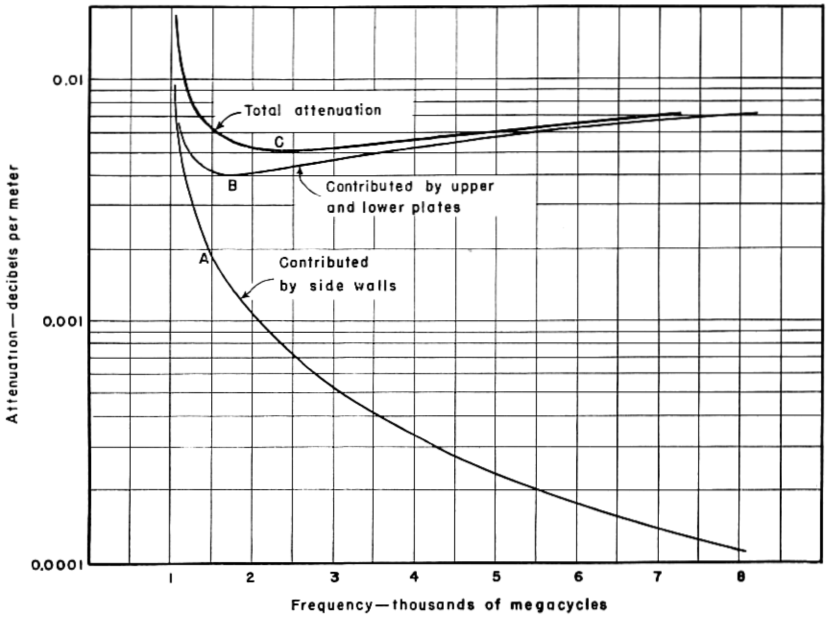


Fig. 6.5-4. Component attentuations contributed by the top and bottom plates and also the two side walls of a rectangular waveguide.

frequency. Since the attenuation contributed by the top and bottom plates first decreases but later increases with frequency, we may expect, between these two ranges, a region of minimum attenuation. The attenuations contributed by the upper and lower plates and also by the side walls of a 7.5 cm × 15 cm copper guide carrying the dominant mode have been calculated. The results have been plotted as curves A and B in Fig. 6.5-4. They follow the courses predicted by the preceding qualitative reasoning.

The fact that the reflection type of attenuation, such as is evident in the side walls above, decreases with frequency, suggests that, if a kind of waveguide could be devised where this type of attenuation alone exists, we could then operate the guide at extremely high frequencies and thereby obtain relatively low attenuations. This can, in effect, be done. It calls for a guide of circular cross section and a special configuration, known as

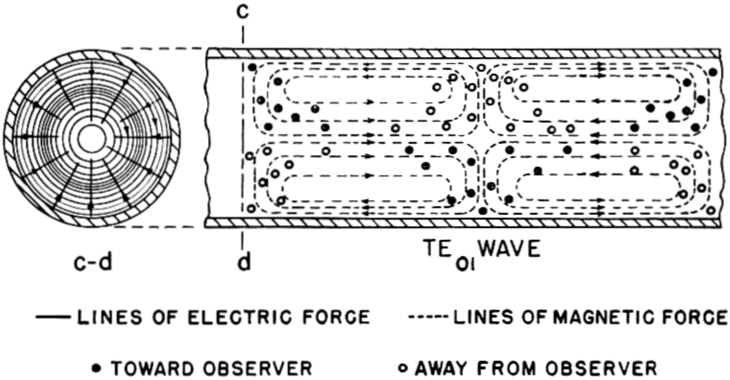


Fig. 6.5-5. The circular electric or TE_{01} configuration in a circular waveguide.

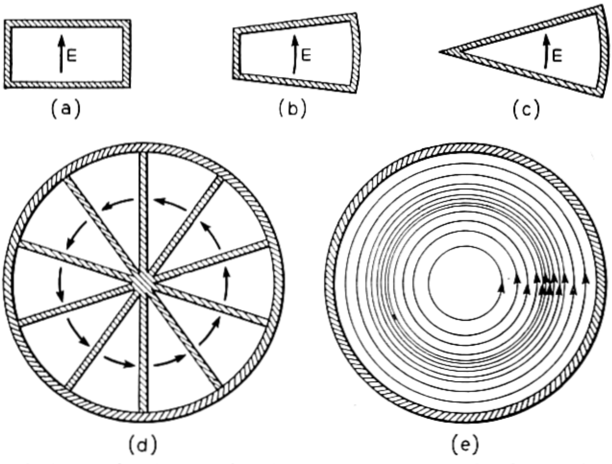


Fig. 6.5-6. Evolution of the circular-electric wave in a circular pipe from a dominant wave in a rectangular pipe.

the *circular-electric wave*. In this configuration, the resultant electric force is everywhere parallel to the conducting boundary as shown in Fig. 6.5-5.

That such a wave will lead to the interesting frequency characteristic noted is made more plausible by referring to Fig. 6.5-6 and its associated discussion. Figure 6.5-6(a) shows a conventional form of rectangular guide in which plane waves are multiply reflected from the two short sides.

In Fig. 6.5-6(b) the proportions of the guide have been altered somewhat, but since the lines of electric force are still perpendicular to the top and bottom plates, the guide may be expected to function substantially as before. At the most, some attenuation that previously originated in the left-hand side wall may now be transferred to the top and bottom walls. As a second step, we may extend the width of the top and bottom walls as shown in Fig. 6.5-6(c) until they intersect, thereby forming an arc-shaped guide. The attenuation now prevailing is evidently confined to the top and bottom walls and the right-hand wall. It is reasonable to assume that the side wall attenuation still decreases with frequency since incident lines of force are everywhere parallel to this wall. As a third step, we assemble as in Fig. 6.5-6(d) a number of identical arc-shaped guides to form a composite circular guide with radial partitions. If, finally, we imagine the radial partitions removed as in Fig. 6.5-6(e), the resulting configuration will not be altered and we shall have removed the component of attenuation attributable to the top and bottom walls leaving only the component of attenuation attributable to the one side wall, which, as we have pointed out, becomes progressively smaller as the frequency is indefinitely increased.