The Measurement of Delay Distortion in Microwave Repeaters*

By D. H. RING

Measuring equipment is described which is capable of measuring delay distortion of the order of 10^{-9} seconds in a wide band microwave television relay repeater. Two measuring circuits are discussed. The first is a circuit for measuring the relative phase shift versus frequency from which the delay distortion may be computed. The second circuit gives the delay directly from a single measurement. The measuring equipment is designed to work in the intermediate frequency range from 50 to 80 megacycles, but by applying suitable conversion equipment measurements can be made at microwave frequencies.

THE successful transmission of broadband television and pulse signals over any communication circuit depends upon the preservation of the complex wave shapes of the original transmitted signals. Fourier analysis tells us that a complex signal wave can be resolved into a spectrum of frequencies with certain amplitude and phase relationships. It is well known that the amplitude relationships of all essential frequencies in this spectrum must be substantially preserved. It is equally important that the phase relationships of the essential frequencies should be preserved. The instantaneous value of the received signal is the vector sum of the instantaneous amplitudes of all the component frequencies. Therefore, if the relative phase of some frequency component is changed by 180° the sign of its contribution to the output is reversed, and it is clear that a closer approximation to the original signal could be obtained by suppressing this frequency component rather than permitting it to contribute negatively to the output.

It can be shown[1] that the relative phase relations of the component frequencies in a complex signal wave will be preserved if the phase shift in passing through a circuit is a linear function of the angular frequency. That is

$$\beta = T_0\omega + n\pi \tag{1}$$

where T_0 is a constant and n an integer. Distortion of the transmitted signal will occur if T_0 is not constant over the essential frequency band of the signal. We shall not be interested in distortion due to n not being an integer[2] since this case does not occur in carrier circuits where the phase shift at carrier frequency, rather than the phase shift at zero frequency, is

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[1] Phase Distortion and Phase Distortion Correction, S. P. Mead, *B.S.T.J.*, April 1928. 199–201.

[2] Phase Distortion in Telephone Apparatus, C. E. Lane, *B.S.T.J.*, July 1930, 494–496.

the reference point for phase. Departure of β from the linear relationship given by (1) is the phase distortion in the circuit.

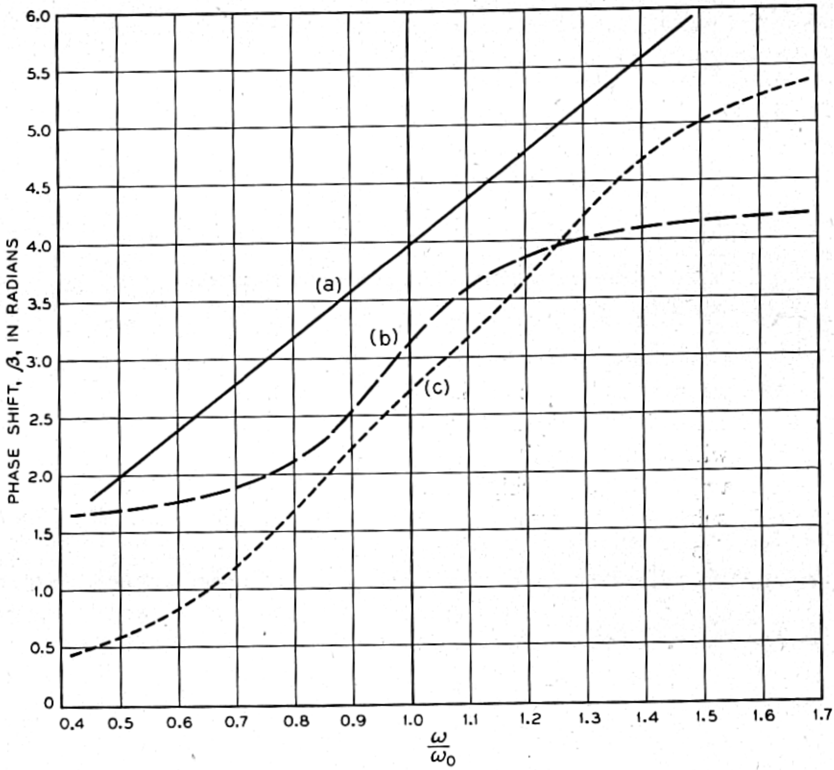

Fig. 1—Typical phase shift curves for various types of circuits
a. A Transmission line terminated in its characteristic impedance.
b. A single tuned circuit.
c. Two tuned circuits with approximately critical coupling.

The time of transmission[3] or the delay in passing through the circuit is obtained by differentiating (1):

$$\text{Delay} = \frac{d\beta}{d\omega} = T_0 \text{ seconds} \tag{2}$$

Variation in T_0 with frequency is the delay distortion in the circuit.

Figure 1 shows some typical phase curves, and Fig. 2 shows the corresponding delay curves. In each figure curve (a) represents an ideal distortionless circuit with linear phase and constant delay such as a simple transmission line. Curves (b) are obtained for single resonant circuits,

[3] T_0 has also been called the group delay and the envelope delay.

and curves (c) for coupled double tuned circuits. It is felt that the delay curves of Fig. 2 are easier to interpret and give a better physical picture of the distortion resulting from phase variations than the phase curves of Fig. 1. Therefore, most of the following discussion will be in terms of delay rather than phase.

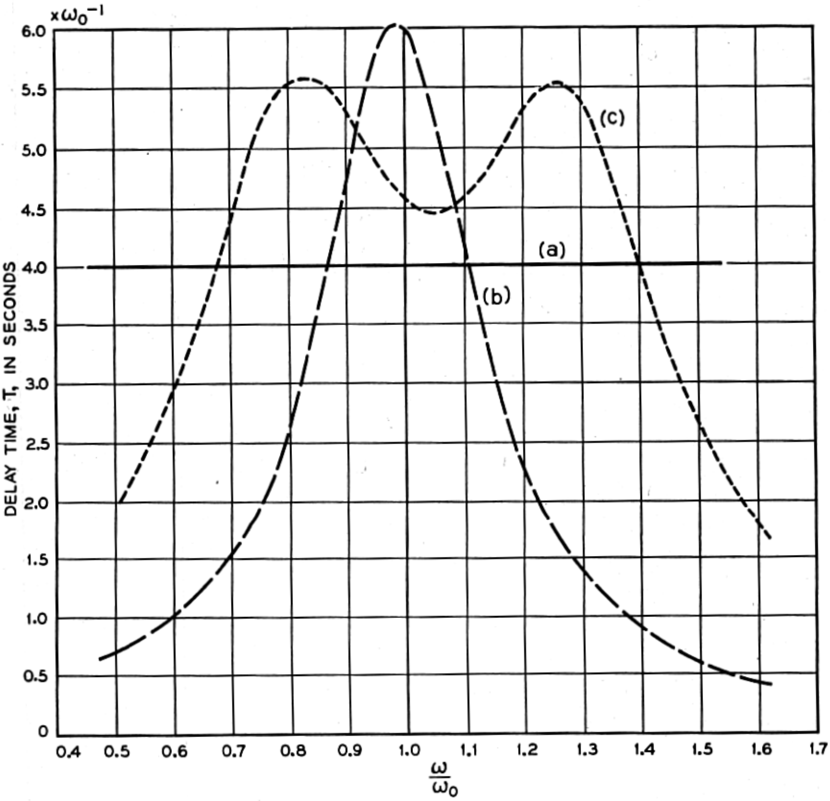

Fig. 2—Typical delay curves for various types of circuits
a. A transmission line terminated in its characteristic impedance.
b. A single tuned circuit.
c. Two tuned circuits with approximately critical coupling.

Thus far we have considered the general case and stated that the delay must be constant over the essential frequency band for distortionless transmission. It should be noted, however, that when delay distortion is present different types of signals may be affected differently. For instance, in the case of an amplitude modulated carrier of angular frequency ω_0, Fig. 2, we note that if the delay curves have arithmetic symmetry about ω_0, then the sidebands at $\omega_0 \pm \Delta\omega$ will suffer the same delay and therefore will add in phase upon demodulation. If the delay distortion is not symmetrical about ω_0 the sidebands will not add in phase upon demodulation, and the demodulated output will suffer both amplitude distortion and some delay distortion which differs from the delay distortion at both $\omega_0 + \Delta\omega$ and $\omega_0 - \Delta\omega$. In the case of frequency modulation dissymmetry introduces harmonics in the demodulated output. A detailed discussion of these effects is beyond the scope of this paper, but we may note that in general a true picture of the delay distortion in carrier circuits is not readily obtained by observing the demodulated output and that an unsymmetrical delay distortion is particularly undesirable in carrier circuits.

Principles of Delay Measurement

Delay cannot be measured directly on a steady state basis with a single test signal in the simple manner in which amplitude response is measured. Instead, the phase shift through the unknown network must be measured at two adjacent frequencies and the delay, or slope of the phase shift, computed from the relation

$$T = \frac{\Delta\beta}{\Delta\omega} \tag{3}$$

Figure 3 illustrates the computation of the average delay in the interval $\Delta\omega$ from two phase measurements.

The steady state phase shift of an unknown network can be measured by using the basic circuit shown schematically in Fig. 4.[4] This is essentially a bridge circuit in which the phase shift in the unknown is balanced by an equivalent calibrated phase shifter. The phase comparator is some kind of device which will give an indication of a known relationship between the phases of the signals arriving over the unknown path and the known reference path.

The exact form of suitable components and circuit arrangements for applying the basic method of Fig. 4 to a particular delay measuring problem is largely determined by the order of magnitude of the delay to be measured and accuracy desired. In the case of microwave television repeaters we are interested in video bands of the order of 5 mc wide. As a rough estimate we might say that the highest frequency in the band should not be shifted more than one quarter period from its normal phase position. In the case of linear delay distortion or a parabolic phase-frequency characteristic, one quarter period for 5 mc is 0.05 microseconds. In a repeater system with 50 repeaters this yields a tolerable systematic delay distortion of 10^{-9} seconds per repeater. Therefore we conclude that in developing

[4] Measurement of Phase Distortion, H. Nyquist and S. Brand, *B.S.T.J.*, July 1930, 526–527.

repeaters for this service an accuracy of better than 10^{-9} microseconds in measuring the relative delay over a band of frequencies will be desirable. The phase shift, $\Delta\beta$ in Fig. 3, which corresponds to a delay of 10^{-9} seconds is a function of the measuring interval $\Delta\omega = 2\pi\Delta f$. If Δf is small $\Delta\beta$ will

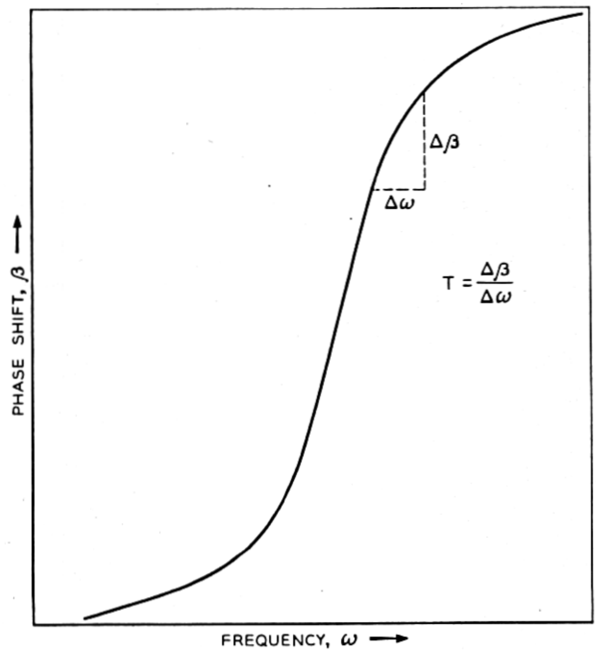

Fig. 3—Factors involved in calculating the delay of an electrical circuit

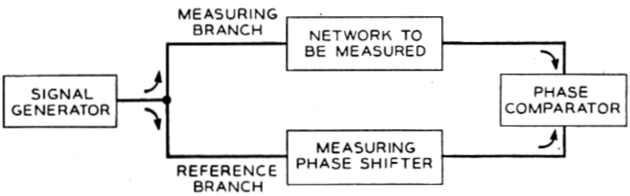

Fig. 4—Basic circuit for measuring the phase shift in a network

be small and difficult to measure. If Δf is large the average slope measured will not be the true slope in the center of the interval. For a 5-megacycle video signal the intermediate and radio frequency bands of interest will be in excess of 10 megacycles wide. These considerations led to a choice of 1 megacycle as a reasonable value for Δf. If $\Delta f \doteq 1$ megacycle a phase shift $\Delta\beta = 0.36°$ will result from a delay of 10^{-9} seconds.

Two circuits for measuring delay distortion will be described. The first is a phase measuring circuit which, in principle, is an adaptation of Fig. 4 to practical operation at intermediate frequencies. The second circuit is a modification in which two frequencies differing by Δf are fed through the circuit under test simultaneously in such a way that the difference in the phase shift at the two frequencies is measured. This arrangement permits the calculation of the delay from a single measurement and is, therefore, a delay measuring circuit.

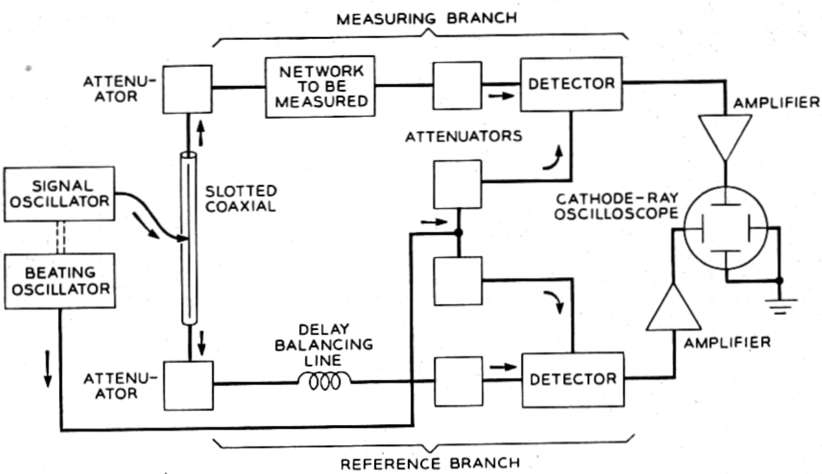

Fig. 5—Schematic circuit for the precise measurement of phase shift in the intermediate frequency range.

Phase Measuring Circuit

Figure 5 shows a schematic diagram of a phase measuring circuit which has been found to be suitable for precision measurements of wide-band circuits in the intermediate frequency range of 50 to 80 megacycles. It is a double detection system with an intermediate frequency of one megacycle. The test signal oscillator and the beating oscillator are ganged to a single control and adjusted to track so that they maintain a difference of approximately one megacycle throughout their tuning range. The test signal is fed to a sliding contact on a section of air dielectric coaxial transmission line. The signal divides at this point to feed the test branch and reference branch.

The sliding tap on the coaxial line provides a high precision measuring phase shifter if the coaxial line is well terminated at each end. The relative change $\Delta\beta$ in phase of the signals at the two ends of the line when the slider is moved a distance Δx is two times the phase shift corresponding to the movement of the slider at the working frequency or

$$\Delta\beta = \frac{720f\Delta x}{c} \text{ degrees}$$

where c is the velocity of light.
For $\Delta x = 0.1$ centimeter and $f = 65$ megacycles, $\Delta\beta$ is 0.156 degrees.

The measuring branch and reference branch feed through the network to be measured and a balancing line respectively to identical detectors and one megacycle amplifiers. The amplifiers are connected to the plates of a cathode ray oscilloscope which is used as a phase comparator. A cathode

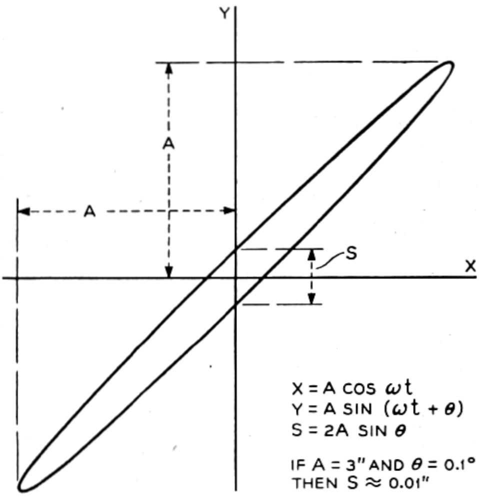

Fig. 6—Calculation of the sensitivity of a cathode ray oscilloscope as a phase comparator.

ray oscilloscope has the advantage that the phase comparison with this instrument is independent of either the relative or absolute amplitudes of the two signals. A straight line on the oscilloscope always indicates that the two voltages are in phase (or phase opposition) regardless of the relative amplitudes, which determine the slope of the line. The sensitivity, of course, is a function of the amplitudes. Figure 6 illustrates how the sensitivity of an oscilloscope phase comparator can be calculated. If each signal alone produces a 6-inch deflection, then 0.1 degree phase difference produces an opening of the pattern of 0.01 inches, which is sufficient to be detected on a "rocking" or in-out basis.

It is obvious that a circuit of this type measures the difference in the phase shifts of the two branches. The measured phase shift is, therefore, the absolute phase shift in the measuring branch less the phase shift in the reference branch. The balancing line shown in the reference branch of Fig. 5 can be adjusted in length so that it balances out the average delay in the measuring branch. The measured remainder will then be the distortion in the measuring branch, since a good transmission line does not have phase or delay distortion. The use of a balancing line in the reference branch simplifies measurements by reducing the range of movement of the slider, and it greatly decreases the errors in the calculation of the delay distortion due to inaccuracies in the measuring interval Δf.

The simplified diagram of Fig. 7 will be used to show how the delay of the unknown network can be calculated. With the signal oscillator set at frequency f_1 the slider is adjusted until the signals at C and E are in phase

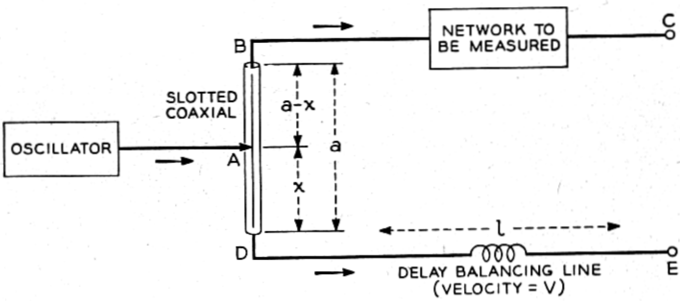

Fig. 7—Simplified circuit electrically equivalent to the circuit of Fig. 5.

as indicated by a straight line on the oscilloscope, and the corresponding distance x_1 is measured. The relationships between the phases of the signals at the various points in the circuit are:

$$\phi_C = \phi_A - \phi_{AB} - \phi_{BC}$$

$$= \phi_A - \frac{2\pi f_1}{c}(a - x) - \beta \tag{4}$$

$$\phi_E = \phi_A - \phi_{AD} - \phi_{DE}$$

$$= \phi_A - \frac{2\pi f_1 x_1}{c} - \frac{2\pi f_1 \ell}{v} \tag{5}$$

$$\phi_C = \phi_E$$

where β is the phase shift in the unknown network at frequency f_1. Solving for β,

$$\beta = \frac{2\pi f_1}{c}\left[\frac{\ell c}{v} + 2x_1 - a\right] \tag{6}$$

The signal oscillator frequency is then changed to $f_2 = f_1 + \Delta f$ and the slider readjusted to the position x_2 that again makes $\phi_C = \phi_E$. Then as above

$$\beta' = \frac{2\pi f_2}{c}\left[\frac{\ell c}{v} + 2x_2 - a\right] \tag{7}$$

where β' is the phase shift in the unknown network at frequency f_2. The delay in the network is, from (3)

$$T = \frac{\beta' - \beta}{2\pi\Delta f} \tag{8}$$

Substituting (6) and (7) in (8) yields

$$T = \frac{1}{c}\left[\left(\frac{\ell c}{v} - a\right) + 2x_2 + \frac{2f_1}{\Delta f}(x_2 - x_1)\right] \tag{9}$$

The first term in T is a constant of the set-up and can be dropped when the delay distortion only is desired. The second term is small and can often be neglected. The third term gives the major part of the difference between the delay of the reference path and the delay of the network.

It has been found that the slider position can be easily reset to $\pm$ 0.05 centimeters for each frequency. This would mean a maximum error of $\pm$ 0.1 centimeter for the difference of two readings which corresponds to an error in T of about $\pm 0.4 \times 10^{-9}$ seconds. However, this reset accuracy will not be realized in overall accuracy unless a number of precautions are observed in setting up the circuit of Fig. 5. In order to avoid stray coupling the two branches of the system must be carefully shielded from each other. At least 60 db net attenuation must be provided between the detectors via the path through the balancing line, phase shifter and unknown network, and 40 db attenuation in the path via the BO supply lines. All attenuators, plugs, etc., must have voltage standing wave ratios of less than about 1.015 and the detectors and amplifiers in each branch must have identical phase shifts over the range of variation of the IF due to imperfect tracking of the oscillators.

Delay Measuring Circuit

The circuit of Fig. 5 is basically a phase measuring circuit. It can be rearranged as shown in Fig. 8 to yield a circuit that will read delay directly from a single setting of the slider. In Fig. 8 the signals from the two oscillators are both sent through the circuit to be measured, and both are sent through the reference branch. Any delay in either path will alter the relative phases of the two signals in that path and this change in relative phase shift will be passed on to the beat note formed in the detectors. If the delays in the two paths are different a relative phase shift proportional to the difference will appear in the two beat notes. This phase change can be measured with a phase shifter in the beat note circuit or with a phase shifter which varies the relative phase of the two signals fed to one branch

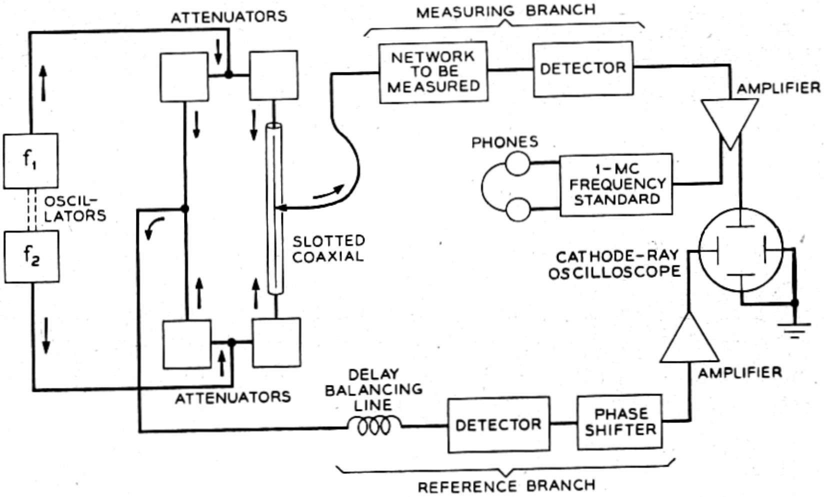

Fig. 8—Schematic circuit for the precise measurement of delay in the intermediate frequency range.

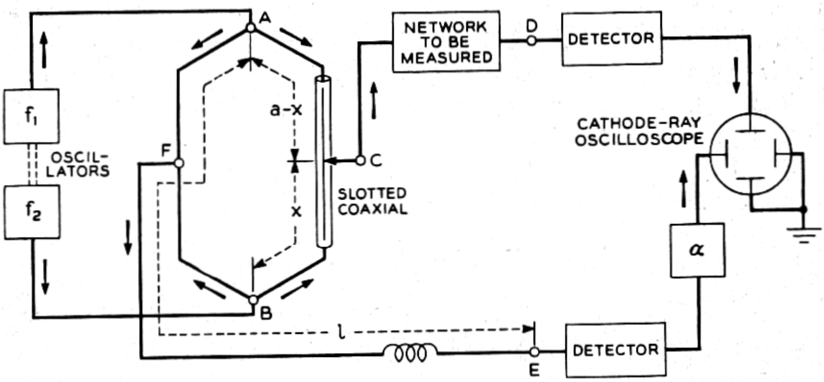

Fig. 9—Simplified circuit electrically equivalent to the circuit of Fig. 8

as compared with the relative phase of the two signals fed to the other branch. Figure 8 shows how the coaxial slider can be used for the latter type of measurement.

The simplified diagram of Fig. 9 will be used to show how the delay of the unknown network can be calculated in terms of this circuit. It will be assumed that the electrical lengths of the paths AF and BF are equal so that this length can be regarded as part of the balancing line ℓ. Then at point E we have

$$\phi_E = \phi_A - \phi_{AE} = \phi_A - \frac{2\pi f_1 \ell c}{v}$$

$$\phi_E' = \phi_B' - \phi_{BE}' = \phi_B' - \frac{2\pi f_2 \ell c}{v}$$

The primes indicate phase shifts at frequency f_2. The phase ϕ''_E of the beat note, $\Delta f = f_2 - f_1$, at E is

$$\phi_E'' = \phi_E' - \phi_E = \phi_B' - \phi_A - \frac{2\pi \ell c}{v} \Delta f \tag{10}$$

Similarly, at point D

$$\phi_D'' = \phi_B' - \phi_A - \frac{2\pi f_2 x}{c} + 2\pi f_1(a - x) - (\beta' - \beta) \tag{11}$$

If x is adjusted so that the Δf's in the two branches are in phase at the oscilloscope then

$$\phi''_D = \phi''_E - \alpha \tag{12}$$

where α is the difference in the phase shifts in the two beat note circuits. Substituting (10) and (11) in (12) and solving for $(\beta' - \beta)$ yields

$$(\beta' - \beta) = \frac{2\pi}{c}\left[\frac{\ell c \Delta f}{v} - x\Delta f - f_1(2x - a) + \frac{\alpha c}{2\pi}\right] \tag{13}$$

The delay in the network is

$$\begin{aligned} T &= \frac{(\beta' - \beta)}{2\pi \Delta f} \\ &= \frac{1}{c}\left[\frac{\ell c}{v} + \frac{\alpha c}{2\pi \Delta f} - f_1 \frac{(2x - a)}{\Delta f} - x\right] \end{aligned} \tag{14}$$

The first two terms in (14) are independent of x and f_1 and therefore are of interest only for absolute measurements. Relative measurements may be made without evaluating these constants. The last two terms are functions of x and yield the change in delay as f is varied. It is usually most convenient to adjust α and ℓ so that the average delay in the measuring branch is given by (14) with

$$(2x - a) = 0 \tag{15}$$

In general this will minimize the variation of the slider and make the delay distortion in the measuring branch roughly proportional to the slider movement. This is helpful in judging the effect of adjustments of the unknown network. The condition (15) corresponds to the optimum condition for the circuit of Fig. 5 which requires that the average delays of the two branches be equal. In the circuit of Fig. 8, the condition (15) can be realized by varying either ℓ or α. If α is made zero and (15) is fulfilled by adjusting ℓ, errors due to variations in Δf are minimized. However, if Δf is held sufficiently constant, the balancing line ℓ can be omitted entirely and a 360° variable phase shifter introduced in the beat note circuit of one branch to vary α.

The measuring interval Δf is determined by the difference in the two signal oscillators. Since Δf has an important influence on the measurement, oscillator tracking cannot be relied upon to maintain Δf with sufficient accuracy. A one-megacycle crystal frequency checker has therefore been included in the equipment as shown in Fig. 8. A sample of the signal from one of the amplifiers is compared with the crystal oscillator and a trimmer on one of the oscillators adjusted for zero beat before measuring each point. When T is so large that a balancing line is not practicable, as in the case of loop circuits including radio paths or long transmission lines, Δf must be held constant to about 1 part in 10^6. This has been accomplished in a modification of this equipment built by Messrs. W. J. Albersheim and J. P. Shafer of these laboratories, by deriving the two measuring frequencies from a crystal oscillator.

In order to obtain the absolute value of T, the constants ℓ, v, and α must be known. The value of ℓ may be found from the physical length of the line; α can be measured by measuring the movement of the slider required to rebalance the circuit when the connections to the two detectors are reversed. The absolute delay is particularly sensitive with respect to the difference between $2x$ and a. If a is measured as accurately as possible, then the exact setting of the slider corresponding to condition (15) can be found by reversing the connections of the slider at points A and B in Fig. 9. In this way a reference value of x may be found which will yield accurate results in spite of a small error in the value of a.

Successful operation of this circuit depends on low standing waves throughout and upon sufficient padding for satisfactory isolation of the oscillators. It will be noted that in the analysis it was assumed that f_1 reached the slider via the path AC, Fig. 9. There must be an attenuation for f_1 in the path AFBC sufficient to render the signal traversing this path negligible. Similar unwanted paths exist for f_2 from B to C and also for f_1 and f_2 to point F. Attenuation inserted as shown in Fig. 8 can be arranged to make these unwanted signals negligible.

Radio Frequency Measurements

The frequency range of a particular measuring equipment using the circuits of Fig. 5 or Fig. 8 is determined only by the range of the oscillators and the range over which the plugs and jacks and attenuators operate with sufficiently low reflection coefficients. While this range is greater than the IF range encountered in microwave repeaters, it does not include the actual microwave frequencies. However, it has been found that microwave components can be measured satisfactorily by using the circuit shown in Fig. 10. In this circuit the measuring equipment is operated at

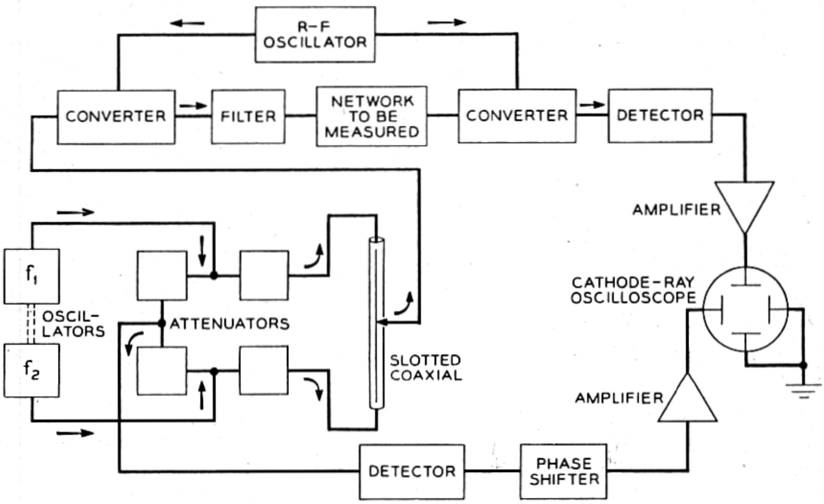

Fig. 10—Method of using the intermediate frequency measuring circuit of Fig. 8 for the measurement of radio frequency networks.

IF, and the reference branch is unaltered. The measuring branch signal is fed first to a converter where it is beat up to the desired microwave frequency. The converter is followed by a filter which eliminates the beating oscillator frequency and one of the beat frequencies. The filter output is then applied to the microwave component under test. The output of this is fed to another converter and converted back to IF by beating with the same oscillator that was used in the first converter. This process removes any variations in phase due to variations in the beating oscillator if the connections from the beating oscillator to each converter are of equal electrical length.

Since considerable extra equipment is included in the measuring branch in Fig. 10, it will usually be necessary to make a calibration run with the device to be measured removed in order to eliminate any possible delay distortion in the converters and filter. Figure 10 uses the measuring

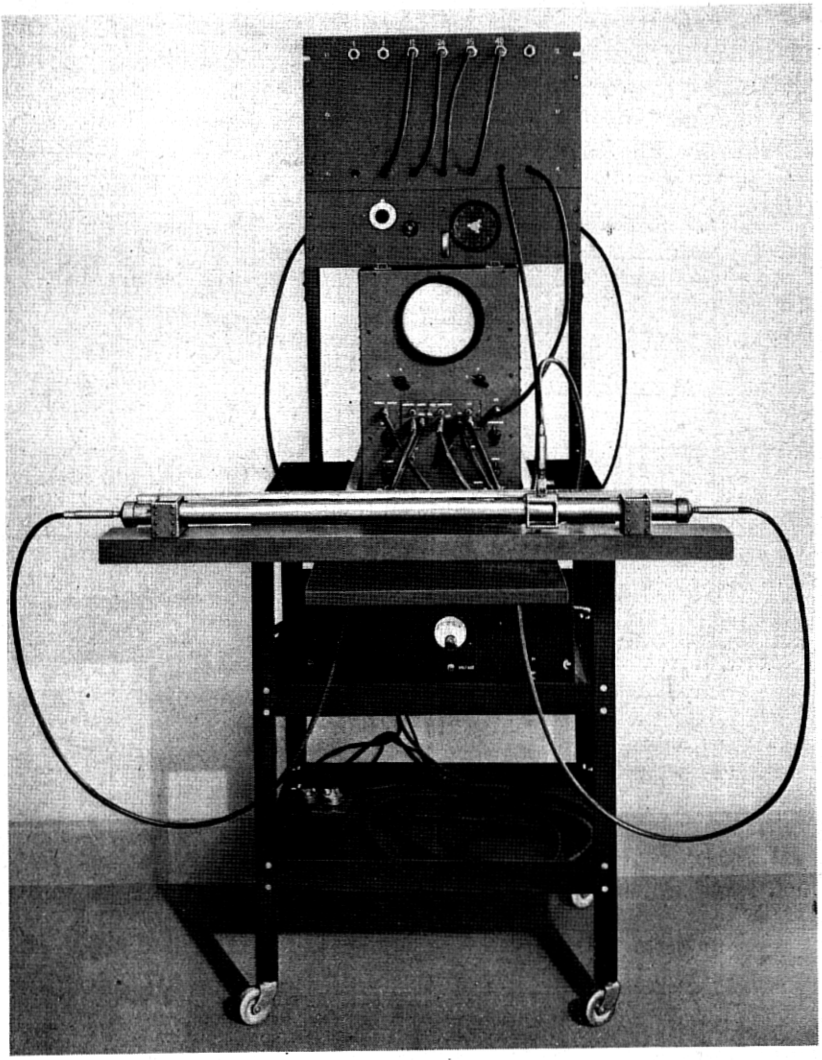

Fig. 11—Photograph of apparatus used in the measuring circuits shown in Figs. 5 and 8.

circuit of Fig. 8 rather than that of Fig. 5 because it was found that small variations in the transit time in microwave amplifiers cause small variations in the phase shift which are the same at all frequencies in the band. These variations cause changes in the relative phase of the two successive measurements required when using the circuit of Fig. 5 which do not represent changes in the delay. In effect the circuit of Fig. 8 makes the two phase measurements simultaneously, and thus eliminates effects due to variations of phase with time.

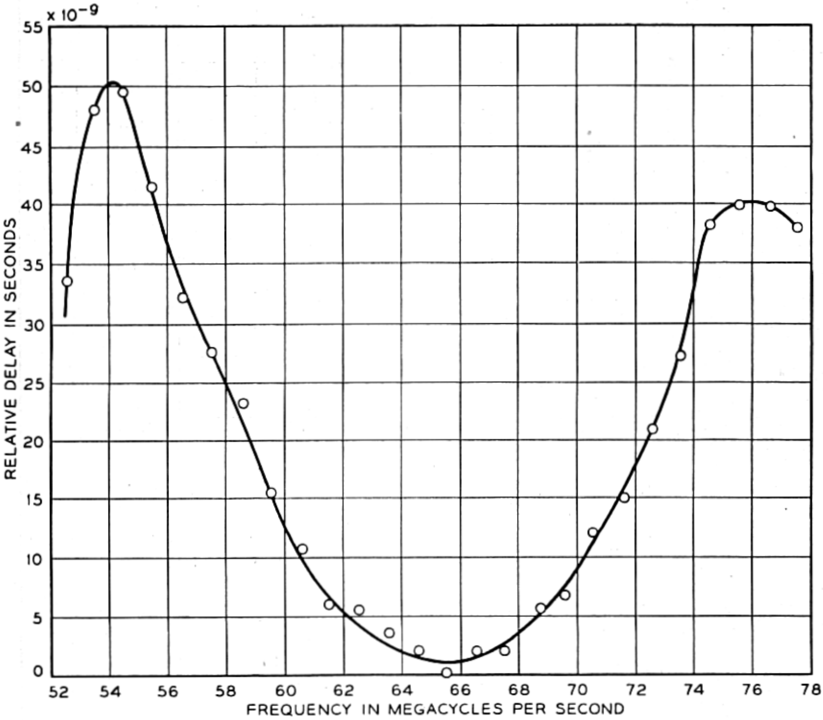

Fig. 12—Measured curve of the relative delay of an intermediate frequency amplifier

Results

Figure 11 shows a photograph of the delay measuring equipment which has been described. With the aid of patch cords and switches that have been included in the equipment this apparatus can be set up according to either Fig. 5 or Fig. 8. The ganged oscillators are on the panel above the oscilloscope box. The box contains the dividing attenuators, detectors, amplifiers and oscilloscope. A separate power supply is required for the oscillators and the output stages of the amplifiers. A number of different lengths of flexible coaxial cable are mounted on the panel above the oscillators and arranged so that various lengths of balancing line can be conveniently obtained by patching. The coaxial phase changer can be seen on the shelf in front of the oscilloscope.

Figure 12 shows the measured relative delay of a typical intermediate frequency amplifier. This amplifier has a substantially flat amplitude response over a band of about 12 mc centered on 65 mc. The delay distortion over the 10 megacycle band from 60 to 70 mc is about 10×10^{-9}

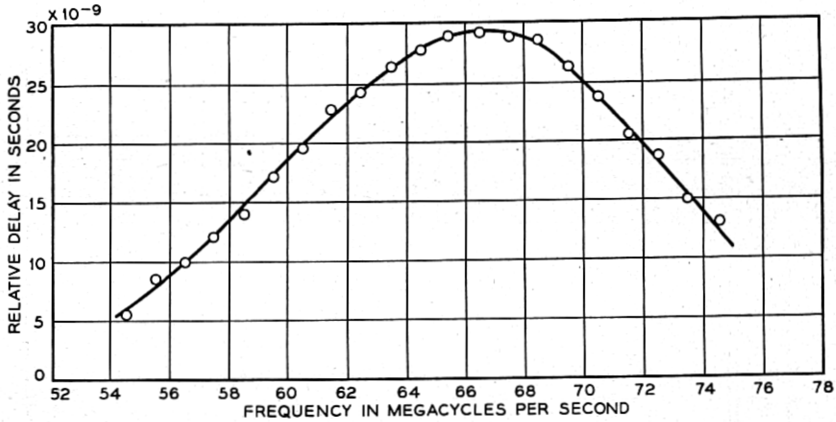

Fig. 13—Measured relative delay of an experimental delay equalizer

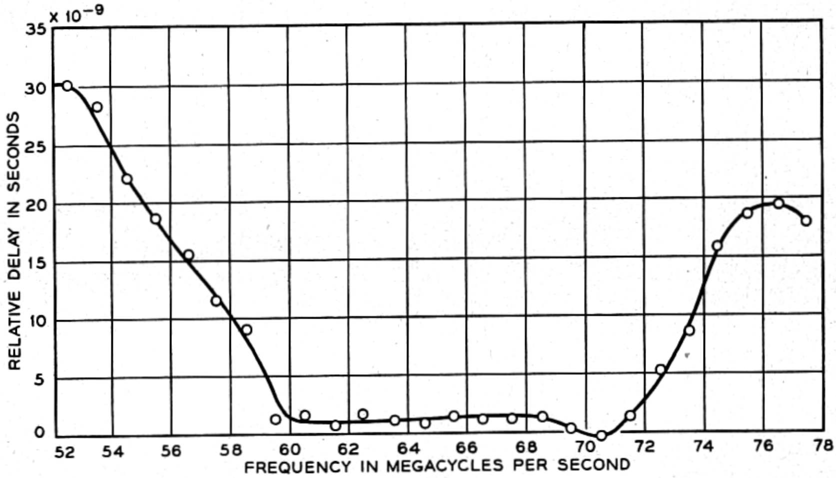

Fig. 14—Measured relative delay of the amplifier of Fig. 12 plus an equalizer

seconds. Figure 13 shows the measured relative delay of a bridged T phase equalizer which can be used to compensate for the distortion in the amplifier. Figure 14 shows the measured relative delay for the amplifier of Fig. 12 and an equalizer measured together. The equalizer has reduced the delay distortion over the 10 mc band from about 10^{-8} to 10^{-9} seconds. These measurements were made with an early model of the measuring equipment. Smoother curves are obtained with the apparatus shown in Fig. 11.

In Fig. 15 the top row shows the distortion of a square top pulse by the amplifier of Fig. 12 for 1, 10, and 30 trips through the amplifier without the equalizer. The lower row shows a similar set of pictures of the pulse when the distortion was reduced by a phase equalizer as shown in Fig. 14. The improvement due to the elimination of phase distortion is clearly illustrated. These pictures were obtained by the circulated pulse[5] technique which

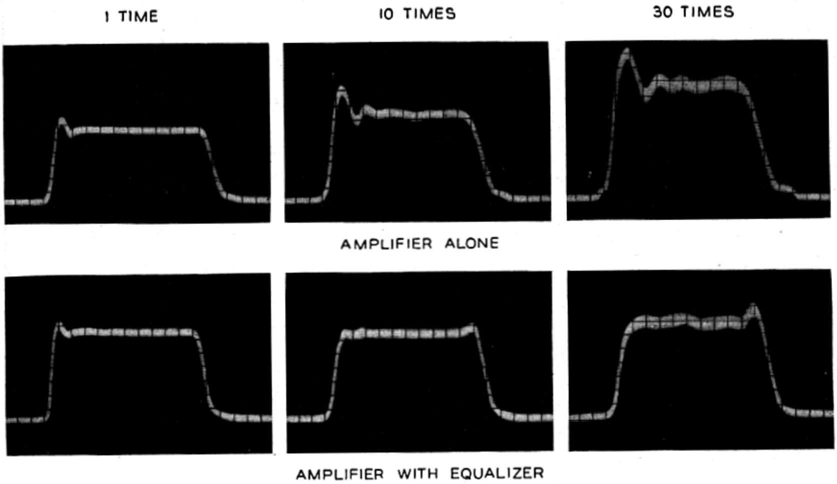

Fig. 15—Oscillograms showing the improvement in the square wave response of the amplifier of Fig. 12 obtained by delay equalization. The numbers above the traces indicate the number of times the test pulse has passed through the amplifier and equalizer.

permits the observation of a pulse after it has passed a number of times through the same amplifier.

Conclusions

Two measuring circuits have been described which are suitable for measuring the small variations in relative transmission time which are present in wide band microwave repeaters. If sufficient care is exercised in setting up these circuits an accuracy of better than $\pm 10^{-9}$ seconds can be realized in relative delay measurements. The circuit of Fig. 5 measures the relative phase shift as a function of frequency. It has the advantage of requiring less signal power and fewer important parameters for absolute

[5] Testing Repeaters with Recirculated Pulses, A. C. Beck and D. H. Ring. Proc. *I.R.E.* Nov. 1947, 1,226–1,230.

delay measurements. The circuit of Fig. 8 measures delay directly with a single measurement and has the advantage of ignoring uniform phase variations with time. It is useful for making relative measurements on circuits with long constant delay times. A significant improvement in the square wave response of carrier amplifiers has been obtained by applying delay equalizers based on measurements made with this equipment.