Precision Measurement of Impedance Mismatches in Waveguide

By ALLEN F. POMEROY

A method is described for determining accurately the magnitude of the reflection coefficient caused by an impedance mismatch in waveguide by measuring the ratio between incident and reflected voltages. Reflection coefficients of any value less than 0.05 (0.86 db standing wave ratio) can be measured to an accuracy of ± 2.5%.

LONG waveguide runs installed in microwave systems are usually composed of a number of short sections coupled together. Although the reflection at each coupling may be small, the effect of a large number in tandem may be serious. Therefore, it is desirable to measure accurately the very small reflection coefficients[1] due to the individual couplings.

A commonly adopted method for determining reflection coefficients in phase and magnitude in transmission lines has been to measure the standing wave ratio by means of a traveling detector. Such a system when carefully engineered, calibrated and used is capable of good results, especially for standing waves greater than about 0.3 *db*.

Traveling detectors were in use in the Bell Telephone Laboratories in 1934 to show the reactive nature of an impedance discontinuity in a waveguide. A traveling detector was pictured in a paper[2] in the April 1936 Bell System Technical Journal. Demonstrations and measurements using a traveling detector were included as part of a lecture on waveguides by G. C. Southworth given before the Institute of Radio Engineers in New York on February 1, 1939 and before the American Institute of Electrical Engineers in Philadelphia on March 2, 1939.

Methods for determining the magnitude only of a reflection coefficient by measuring incident and reflected power have been developed by the Bell Telephone Laboratories. A method used during World War II incorporated a directional coupler[3]. The method described in this paper is a refinement of this directional coupler method and is capable of greatly increased accuracy. It uses a hybrid junction[4] to separate the voltage reflected by the mismatch being measured from the voltage incident to the mismatch. Each is measured separately and their ratio is the reflection coefficient.

The problem to be considered is the measurement of the impedance mismatch introduced by a coupling between two pieces of waveguide due to differences in internal dimensions of the two waveguides and to imperfections in the flanges. The basic setup might be considered to be as shown in Fig. 1. The setup comprises a signal oscillator, a hybrid junction, a calibrated detector and indicator, a termination Z', a piece of waveguide EF (the flange E of which is to be part of the coupling BE to be measured) and a termination Z inserted into the waveguide piece EF so that the reflection coefficient of the coupling BE alone will be measured. In addition a fixed shorting plate should be available for attachment to flange B.

Four cases are considered:

I. Termination Z and Z' perfect, only one coupling on hybrid junction.
II. Termination Z imperfect, termination Z' perfect, only one coupling on hybrid junction.
III. Termination Z perfect, four couplings on hybrid junction.
IV. Termination Z imperfect, four couplings on hybrid junction.

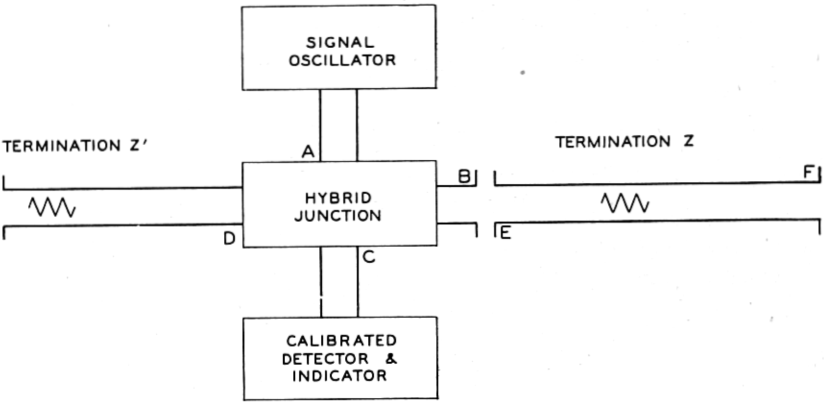

Fig. 1—Block schematic for cases I and II.

It is assumed in all cases that:

1. The hybrid junction has the properties as defined in the discussion of case I.
2. The signal oscillator absorbs all the power reflected through arm A of the hybrid junction.
3. The calibrated detector and indicator absorb all the power transmitted through arm C of the hybrid junction.
4. The oscillator output and frequency are not changed when the hybrid junction arm B is short-circuited.
5. The attenuation of waveguide may be neglected.

I. TERMINATION Z AND Z' PERFECT, ONLY ONE COUPLING ON HYBRID JUNCTION

In this case the hybrid junction, termination Z' and termination Z, as shown in Fig. 1, are all considered to be perfect. This means for the hybrid junction that its electrical properties are such that the energy from the oscillator splits equally in paths AD and AB. The half in AD is completely absorbed in the perfect termination Z'. The half in AB is partly reflected from the impedance mismatch due to the waveguide coupling BE and the remainder is absorbed in the perfect termination Z. Again due to the properties of the perfect hybrid junction, the impedance presented by the arm B when arms A and C are perfectly terminated is also perfect, and the reflected energy from waveguide coupling BE splits equally in paths BA and BC. The part in BA is absorbed by the oscillator. The part in BC representing the voltage reflected from the coupling BE is measured by the calibrated detector and indicator. The magnitude of the incident voltage may be measured when the waveguide piece EF is replaced by the fixed shorting plate.

It is convenient to measure voltages applied to the calibrated detector and indicator in terms of attenuator settings in *db* for a reference output indicator reading. Then the ratio expressed in *db* between incident and reflected voltages (hereafter called W) is

$$W_2 \text{ (due to the coupling } BE) = A_1 - A_2 \tag{1}$$

where A_1 is attenuator setting for incident voltage and A_2 is attenuator setting for reflected voltage.

Both reflection coefficient and standing wave ratio may be expressed in terms of W. For if

$$X = \text{voltage due to incident power} \tag{2}$$

and

$$Y = \text{voltage due to reflected power,} \tag{3}$$

then reflection coefficient $= \dfrac{Y}{X}$ (4)

and voltage standing wave ratio $= \dfrac{|X| + |Y|}{|X| - |Y|}$ (5)

Since $W(db) = 20 \log_{10} \dfrac{|X|}{|Y|}$ (6)

then in *db*, standing wave ratio $= 20 \log_{10} \dfrac{1 + \text{antilog } \dfrac{W}{20}}{-1 + \text{antilog } \dfrac{W}{20}}$ (7)

Standing wave ratio plotted versus W is shown in Fig. 2. Reflection coefficient versus W can be found in any "voltage ratios to *db*" table.

II. Termination Z Imperfect, Termination Z' Perfect, Only One Coupling on Hybrid Junction

In Fig. 1, if the termination Z is not perfect, there will be two reflected voltages from branch B. The vector diagram of the voltage at C might be represented as in Fig. 3, where vector 0–1 represents the voltage reflected from coupling BE and vector 1–2 represents the voltage reflected from the termination Z. To make measurements, termination Z should be movable and the magnitude of its reflection coefficient be the same at a given position of rest for either direction of approach, and be the same for positions of rest over an interval of a half a wavelength in waveguide.

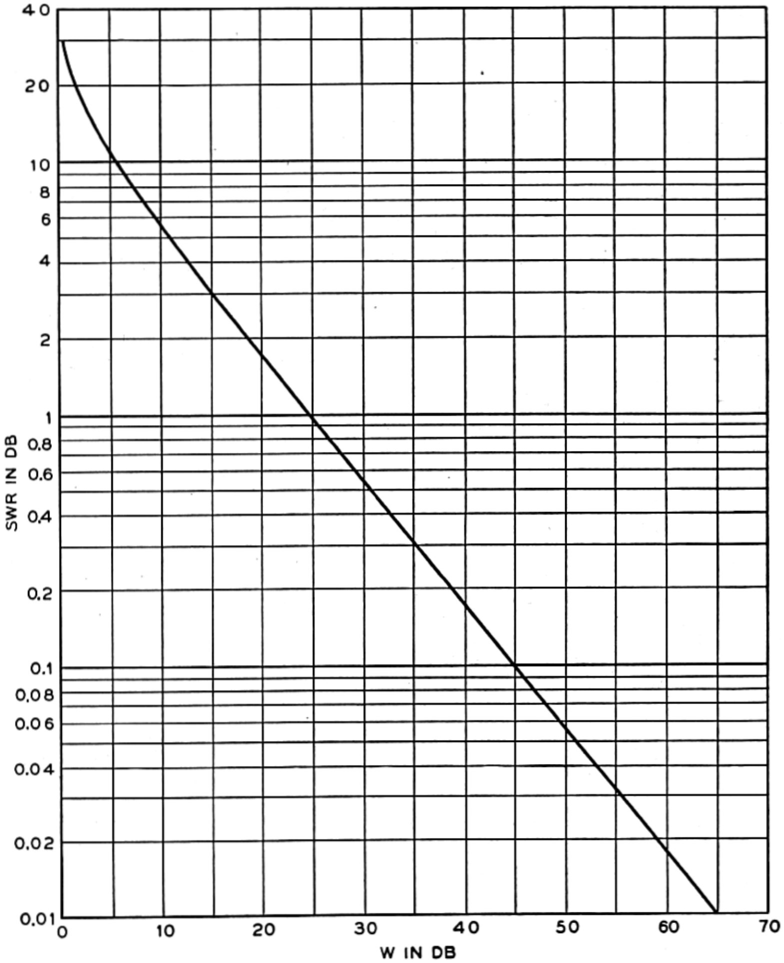

Fig. 2—Standing wave ratio (SWR) versus W.

The reflected voltage is measured twice, once for minimum output as the position of the termination Z is adjusted and again for maximum output. Then

$$V_{\min} = V_b - V_z \text{ and } V_{\max} = V_b + V_z \tag{8}$$

where V_b is voltage reflected from coupling *BE* and V_z is voltage reflected from termination *Z*.

Equations (8) can be solved for V_b and V_z for

$$V_b = \frac{V_{\max} + V_{\min}}{2} \quad \text{and} \quad V_z = \frac{V_{\max} - V_{\min}}{2} \tag{9}$$

The incident voltage is measured as before. Therefore, using equation (6)

$$W' = 20 \log \frac{|V_a|}{|V_b|} \quad \text{and} \quad W'' = 20 \log \frac{|V_a|}{|V_z|} \tag{10}$$

where W' is due to coupling *BE*, W'' is due to termination *Z* and V_a is incident voltage.

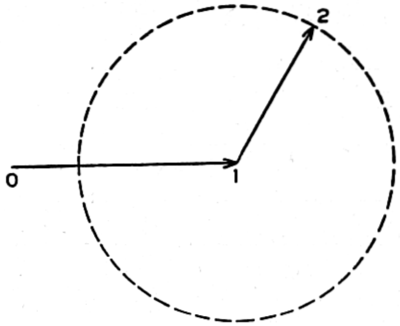

Fig. 3—Vector diagram of voltages reflected from coupling *BE* and termination *Z*.

A more practical solution involving only addition, subtraction and the use of the characteristics in Fig. 4 is now presented. The settings of the detector attenuator for incident voltage, minimum output and maximum output might be A_1, A_3 and A_4.

$$\text{Then } W_3 = A_1 - A_3 \text{ and } W_4 = A_1 - A_4 \tag{11}$$

$$\text{But } W_3 = 20 \log \frac{|V_a|}{|V_b| - |V_z|} \quad \text{and} \quad W_4 = 20 \log \frac{|V_a|}{|V_b| + |V_z|} \tag{12}$$

$$\text{and } W_3 - W_4 = 20 \log \frac{|V_b| + |V_z|}{|V_b| - |V_z|} = 20 \log \frac{1 + \text{antilog} \dfrac{T}{20}}{-1 + \text{antilog} \dfrac{T}{20}} \tag{13}$$

$$\text{where } 20 \log \frac{|V_b|}{|V_z|} = T = W'' - W' \tag{14}$$

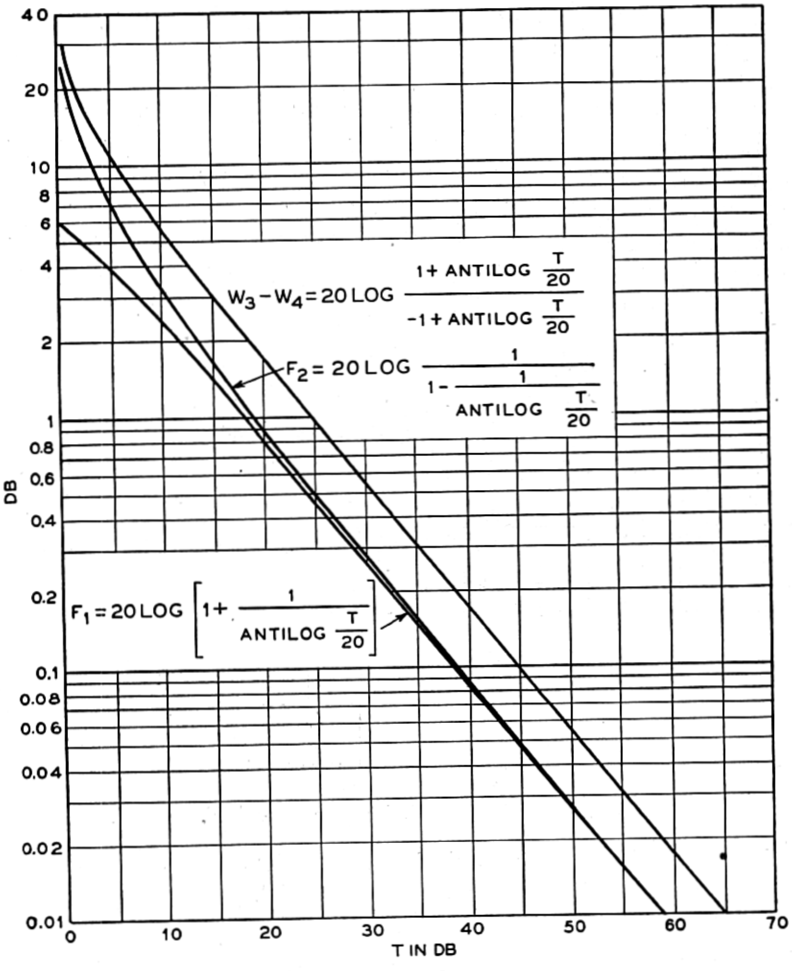

Fig. 4—F_1, F_2 and $W_3 - W_4$

There is an $F_1(T) = 20 \log \left(1 + \dfrac{1}{\text{antilog} \dfrac{T}{20}} \right)$

$$\text{and an } F_2(T) = 20 \log \frac{1}{1 - \dfrac{1}{\text{antilog } \dfrac{T}{20}}} \tag{15}$$

$$\begin{aligned} \text{such that } W' &= W_4 + F_1 = W_3 - F_2 \\ W'' &= T + W_4 + F_1 = T + W_3 - F_2 \\ \text{and } \quad F_1 + F_2 &= W_3 - W_4 \end{aligned} \tag{16}$$

Figure 4 shows F_1, F_2 and their sum $W_3 - W_4$ plotted versus T. It may be noted that $W_3 - W_4$ versus T has the same values as SWR versus W in Fig. 2.

Using equations (16) and Fig. 4, W' and W'' may be evaluated for the particular values of W_3 and W_4 in equation (11). In the evaluation, if there is uncertainty as to which reflection coefficient belongs to the waveguide coupling BE and which belongs to the termination Z, a termination with a different magnitude of reflection coefficient should be used and the technique repeated. The reflection coefficient which is the same in the two cases is of course that due to the waveguide coupling BE.

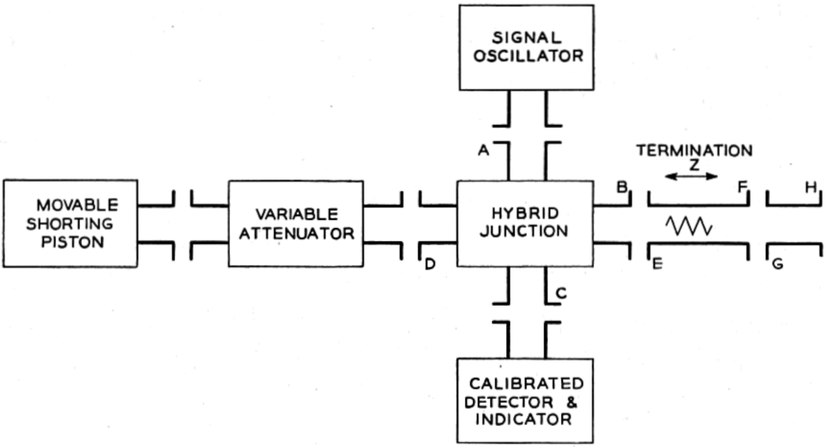

Fig. 5—Block schematic for cases III and IV.

It is assumed in the above solution that multiple reflections between the two impedance mismatches are inconsequential. Appendix A outlines a procedure for evaluating the maximum probable error due to multiple reflections.

III. TERMINATION Z PERFECT, FOUR COUPLINGS ON HYBRID JUNCTION

In this case the setup might be as shown in Fig. 5. This setup differs from that shown in Fig. 1 in that the hybrid junction has four couplings shown, termination Z' has been replaced by a variable attenuator and a movable shorting piston, and the waveguide coupling FG is to be measured instead of coupling BE. The hybrid junction and the termination Z are assumed to be perfect as defined for case I.

Since it is the object of the measuring method to measure impedance mismatches in branch B, it is desirable to make the voltage at C depend only on power reflected from branch B. This is accomplished by adjusting branch D so that the voltages due to the flanges of the hybrid junction are cancelled.

The vector diagram of the voltage at C might be represented as in Fig. 6. Vector 0–1 represents the voltage at C when input is applied to A, due to the impedance mismatch at the coupling BE. Vector 1–2 represents that due to the mismatch at coupling D. Vector 2–3 represents that due to the mismatch at the variable attenuator, (which will usually change in magnitude and probably in phase for different settings). Vector 3–0 represents the voltage at C due to the cancelling voltage from the branch D. Its phase can be varied by changing the position of the movable shorting piston. Its magnitude can be varied by changing the setting of the variable attenuator. When the adjustment is accomplished effectively no power reaches the detector. It is necessary that the reflection coefficients of couplings A, B, and C be small so that multiple reflections caused by them will not affect the accuracy of measurement.

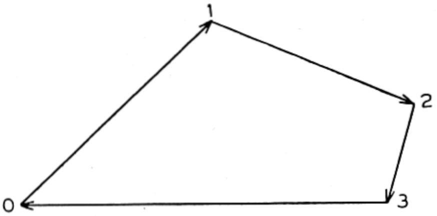

Fig. 6—Vector diagram of voltages at terminal C.

The reflected power from coupling FG may be measured when waveguide GH is connected to waveguide EF as shown in Fig. 5 and termination Z is located within waveguide GH. The detector attenuator setting might be A_5. The incident power may be measured as before when termination Z is withdrawn from the waveguide EF and the piece of waveguide GH is replaced by a fixed shorting plate.

$$W_5 \text{ (due to reflection coefficient of the coupling } FG) = A_1 - A_5 \quad (17)$$

IV. TERMINATION Z IMPERFECT, FOUR COUPLINGS ON HYBRID JUNCTION

In Fig. 5 if the movable termination Z is not perfect, there will be two reflected voltages in branch B when the adjustment is being made. The vector diagram of the voltage at C might be as in Fig. 7. This is the same as Fig. 6 except that a new vector 0–5 represents the voltage due to the mismatch of the movable termination Z. The adjustment is accomplished the same as in the last section except that the criterion is to have no change in detector output as the movable termination Z is moved axially over a range of a half a wavelength in waveguide. As for the last case it is necessary that the reflection coefficients of the couplings A, B and C be small if good accuracy is desired.

When measuring the coupling FG the procedure and evaluation are the same as for case II.

Part of a laboratory setup as used at about 4 kilomegacycles is shown in Fig. 8. It includes a hybrid junction, a variable attenuator, a movable shorting piston, a straight section of waveguide and a movable termination which consists of a cylinder of phenol resin and carbon with a tapered section at one end. It is mounted in a phenolic block so that it may be moved axially in the wave guide.

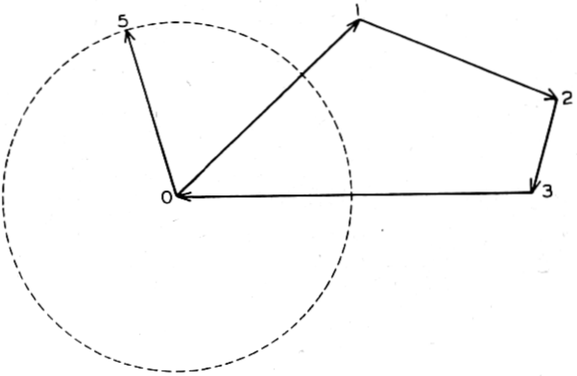

Fig. 7—Part of a laboratory setup as used at 4 kilomegacycles.

In cases III and IV if the hybrid junction has "poor balance" so that voltage appears at C when input is applied to arm A even though B and D are perfectly terminated, the adjusting procedure will cancel this voltage as well. Measuring accuracy will not be impaired provided the other assumptions are fulfilled.

MEASURING W—A FITTING WHICH DOES NOT ADMIT OF MEASURING EACH END SEPARATELY

A piece with a configuration unsuited to the preceding technique may be measured by connecting it between two straight pieces of waveguide such as between flanges F and G in Fig. 5. The W due to the vector sum of the reflection coefficients of the coupling at one end, any irregularities and the coupling at the other end, is measured. Due to the distance between the mismatches, the vector sum will vary over the frequency band of interest.

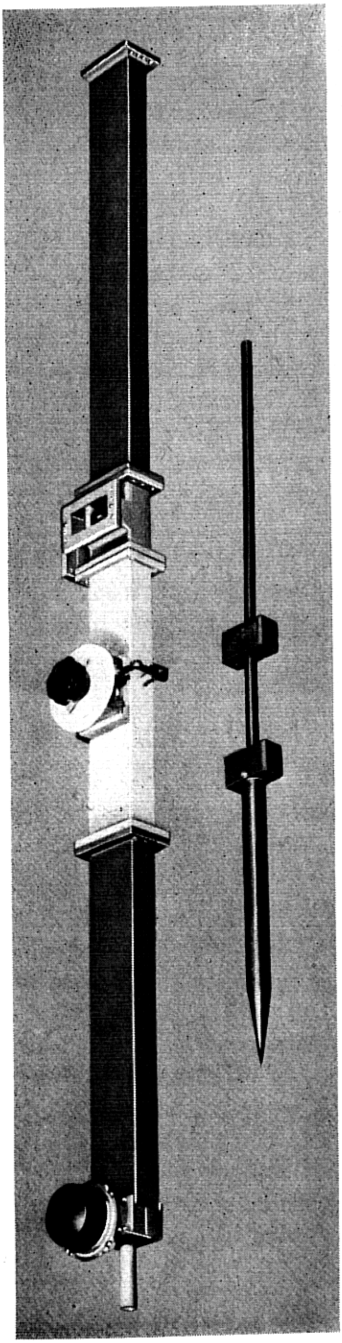

Fig. 8—Vector diagram of voltages at terminal *C* when termination *Z* is not perfect.

ACCURACY

There are three important sources of error. The first is lack of proper adjustment. The second is that due to the detector attenuator calibration. The third is that due to multiple reflections.

Experience and care can almost eliminate the first source. The second source may have a magnitude of twice the detector attenuator calibration error. In equations (1) and (17) this is readily apparent. The evaluation of W using equations (16) introduces negligibly more error provided $W_3 - W_4$ is made large by proper choice of the magnitude of the reflection coefficient of the termination Z. The possible errors due to multiple reflections between the waveguide impedance discontinuity being measured and an imperfect termination are discussed in Appendix A. If the impedance presented by the arm B of the hybrid junction is not perfect, energy reflected from the hybrid junction will be partly absorbed in the termination and cause an error in the measurement. If the magnitude of this reflection coefficient is known, the maximum error may be computed.

If a detector attenuator calibration error of ±0.1 *db* is assumed to be the only contributing error, it is possible to measure the W due to an impedance mismatch to an accuracy of ±0.2 *db* provided the W is greater than 26 *db*. These numbers correspond to measuring a standing wave ratio of any value less than 0.86 *db* to an accuracy of ±0.02 *db* or reflection coefficients of any value less than 0.05 to an accuracy of ±2.5%.

APPENDIX A

MAXIMUM PROBABLE ERROR DUE TO MAGNITUDE OF REFLECTION COEFFICIENT BEING MEASURED WHEN MEASURING A WAVEGUIDE COUPLING

The purpose of this appendix is to derive equations so that the maximum probable error due to multiple reflections may be calculated. The assumptions may not be rigorous but the mathematical treatment appears to represent a reasonable approximation. It is assumed that there is no dissipation in waveguide *EF*, waveguide *GH* and in coupling *FG*.

The electrical relations of the coupling *FG* and the movable termination Z might be represented as in Fig. 9, where K_a = characteristic impedance of waveguide *EF* and K_b = characteristic impedance of waveguide *GH*. The first few multiple reflections from the two discontinuities, coupling *FG* and termination Z, can be illustrated as in Fig. 10.

Evaluation of the magnitudes of the reflections can be accomplished as outlined in paragraph 7.13, page 210 in the book "Electromagnetic Waves"* by S. A. Schelkunoff.

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* Published by D. Van Nostrand, Inc., New York City, 1943.

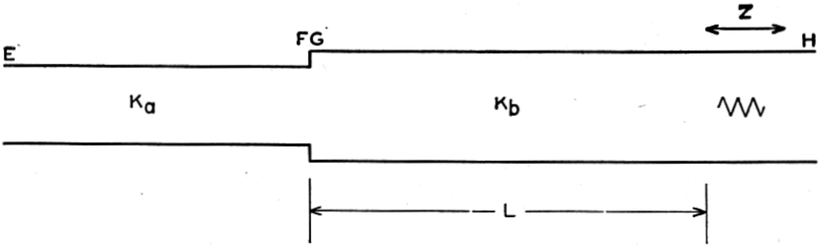

Fig. 9—Relation between coupling *FG* and termination *Z*.

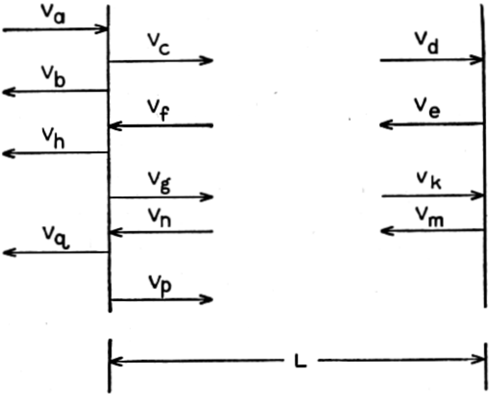

Fig. 10—Multiple reflections from two planes of discontinuity.

$$V_a = \text{Incident voltage} \tag{18}$$

$$V_b = rV_a \tag{19}$$

$$\text{where } r = \frac{K_b - K_a}{K_b + K_a} \tag{20}$$

$$V_c = V_a + V_b = V_a(1 + r) \tag{21}$$

$$V_d = e^{-i\beta L} V_c = e^{-i\beta L} V_a(1 + r) \tag{22}$$

$$V_e = zV_d = ze^{-i\beta L} V_a(1 + r) \tag{23}$$

$$\text{where } z = \frac{Z - Z_b}{Z + Z_b} \tag{24}$$

$$V_f = e^{-i\beta L} V_e = ze^{-i2\beta L} V_a(1 + r) \tag{25}$$

$$V_g = -rV_f = ze^{-i2\beta L} V_a(1 + r)(-r) \tag{26}$$

$$\text{where } -r = \frac{K_a - K_b}{K_a + K_b} \tag{27}$$

$$V_h = V_f + V_g = ze^{-i2\beta L}V_a(1 + r)(1 - r) \tag{28}$$

$$V_k = e^{-i\beta L}V_g = ze^{-i3\beta L}V_a(1 + r)(-r) \tag{29}$$

$$V_m = zV_k = z^2e^{-i3\beta L}V_a(1 + r)(-r) \tag{30}$$

$$V_n = e^{-i\beta L}V_m = z^2e^{-i4\beta L}V_a(1 + r)(-r) \tag{31}$$

$$V_p = -rV_n = z^2e^{-i4\beta L}V_a(1 + r)(-r)^2 \tag{32}$$

$$V_q = V_n + V_p = z^2e^{-i4\beta L}V_a(1 - r^2)(-r) \tag{33}$$

For purposes of analysis it is now assumed that further multiple reflections are negligible.

Fig. 11—Vector voltage diagram for maximum vector sum.

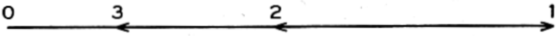

Fig. 12—Vector voltage diagram for minimum vector sum.

Equations (19), (28) and (33) are the reflected voltages that combine vectorially to be measured. If $\beta L = 0, \pi, 2\pi, \cdots n\pi$ then the vector voltage diagram might appear as in Fig. 11. If $BL = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \cdots \frac{(2n - 1)\pi}{2}$ then the vector voltage diagram might appear as in Fig. 12.

The following example illustrates the calculations involved in computing the errors due to the magnitude of the reflection coefficient being measured. The assumptions are such that an appreciable error is computed. If one assumes $r = 0.316$ and $z = 0.282$, then from equation (6) $W_r = 10\ db$ and $W_z = 11\ db$. In Figs. 11 and 12,

$$\text{vector } 0\text{–}1 = r,\ \text{vector } 1\text{–}2 = z(1 - r^2),\ \text{vector } 2\text{–}3 = rz^2(1 - r^2) \tag{34}$$

then

$$W_{0-1} = 10\ db,\ W_{1-2} = 11.00 + 0.92 = 11.92\ db,$$

$$\text{and } W_{2-3} = 10.00 + 22.00 + 0.92 = 32.92\ db \tag{35}$$

In order to evaluate vector 0–2 in Fig. 11 (the vector sum of vectors 0–1 and 1–2), one calculates their difference T.

$$T = 11.92 - 10.00 = 1.92\ db \tag{36}$$

$$\text{For } T = 1.92\ db,\ F_1 = 5.10\ db \tag{37}$$

$$\text{therefore } W_{0-2} = 10.00 - 5.10 = 4.90\ db \tag{38}$$

In order to evaluate vector 0–3 in Fig. 11 (the vector difference of vectors 0–2 and 2–3) one calculates their difference T.

$$T = 32.92 - 4.90 = 28.02\ db \tag{39}$$

$$\text{For } T = 28.02\ db,\ F_2 = 0.36\ db \tag{40}$$

$$\text{therefore } W_{0-3} = 4.90 \pm 0.36 = 5.26\ db = W_4 \tag{41}$$

In order to evaluate vector 0–2 in Fig. 12 (the vector difference between vectors 0–1 and 1–2), one uses T from equation (36).

$$\text{For } T = 1.92\ db,\ F_2 = 14.10\ db \tag{42}$$

$$\text{therefore } W_{0-2} = 10.00 + 14.10 = 24.10\ db \tag{43}$$

In order to evaluate vector 0–3 in Fig. 12 (the vector difference between vectors 0–2 and 2–3), one calculates their difference T.

$$T = 32.92 - 24.10 = 8.82\ db \tag{44}$$

$$\text{For } T = 8.82\ db,\ F_2 = 3.93\ db \tag{45}$$

$$\text{therefore } W_{0-3} = 24.10 + 3.93 = 28.03\ db = W_3 \tag{46}$$

Using equation (16)

$$W_3 - W_4 = 22.77\ db,\ T = 1.24\ db,\ F_1 = 5.40 \text{ and therefore } W = 9.66\ db.$$

Since we started by assuming $W_r = 10\ db$, the error amounts to 0.34 db.

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