Electron Ballistics in High-Frequency Fields*

By A. L. SAMUEL

THIS, the final lecture of a series on Electron Ballistics, is not a summary of the material which has been previously presented but rather it is an attempt to show how the ballistic approach can be extended to the analysis of high-frequency devices. Much that might otherwise be said about ultra-high frequencies cannot be said because of secrecy requirements. However, there is considerable material which can be presented, within the limits of the necessary security regulations, which may be of interest to those who are not already well acquainted with the subject. I will, perforce, not be able to say anything specific about actual devices utilizing the principles to be discussed.

Many of the ultra-high-frequency devices which have come into use during the last few years have employed electron beams of one sort or another. These devices can be analysed in any one of a number of ways. For example, we can write the equation of space-charge flow. This approach considers the electric charge as a continuous fluid subject to Poisson's equation. The small-signal theory of Peterson and Llewellyn is an example of this type of analysis. Or if we wish we can consider the various types of wave motion which can exist in a space-charge region. The space-charge-wave analysis of Hahn and Ramo as applied to velocity-variation tubes is an example of this. In addition there is an electron-ballistic approach to the problem and it is with this method that we will be concerned in the present lecture.

Before we become involved in the details of the analysis, we should perhaps spend a few moments considering the relationship between these various methods. If we have an interaction taking place between electric fields and moving charges, we know at once from Newton's second law that the forces acting on the electrons must of necessity be equal and opposite to those acting on the fields. It is therefore a matter of small concern whether we consider the forces acting on the electrons and the effects of these forces on the electron motion or whether we consider the alteration in fields which the electron motion produces. We can, if we wish, compute the energy transfer to an electric field by the motion of an electric charge or we can compute the change in energy of the electron which accompanies this trans-

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fer. I was tempted to say "which results from this transfer" but this implies a cause and an effect, a notion which has no place in the present discussion. The dual aspect of any energy-transfer problem must always be kept in mind. Much needless discussion frequently arises between proponents of one point of view and those preferring the other when the only difference is one of language and both groups are really saying the same thing. The electron-ballistic approach yields a simple physical picture; it is capable of being applied to widely differing situations, but it is not well suited for a determination of the reactive contributions of an electron stream.

Basic Concepts

There are several concepts which we will find useful in our analysis. These concepts are extremely simple, so simple in fact that one is tempted to assume that they are well known. However, these concepts are so basic to the subject, and their results so far reaching that we must pause to consider them.

The first is the concept of total current, as distinguished from its components. One way of writing Kirchhoff's second law is

$$\text{Div. } J = 0 \tag{1}$$

This simply says that the total current entering or leaving any differential region in space is zero. This expression must of course be generalized by including displacement currents as proposed by Maxwell if applied to alternating currents. The current J is the total current density as here defined. An important consequence of equation (1), actually only an alternate way of stating it, is that the total current always exists in closed paths. Let us take a simple case of a two-element thermionic vacuum tube connected to a battery. Visualize the situation existing if but a single electron leaves the cathode and travels to the plate. The electron takes a finite time to cross from the cathode to the plate. During this time a current exists, the magnitude being given by the relationship

$$I = ev$$

and according to our premise this current is the same in every part of the circuit. The current begins at the instant that the electron leaves the cathode and it ceases when the electron arrives at the plate. In the apparently empty region ahead of the electron there must exist a displacement component, numerically equal to the conduction, or perhaps we should say convection component accounted for by the moving electron. An ammeter, were there one sufficiently sensitive and fast, connected in the external leads would read a current during this same interval of time.

I have chosen to talk about but a single electron to emphasize the electron-ballistic aspect; however, the concept is much broader than this since it is not at all dependent upon a corpuscular concept of the electron. As a result of this property of the total current, the current to any electrode within a vacuum tube does not necessarily bear any relationship to the number of electrons which enter or leave it. Obviously then, currents can exist in the grid circuit of a three-element tube even though none of the electrons are actually intercepted by the grid. This current may have any phase relationship to an impressed voltage on the grid so that the grid may draw power from the external circuit, or it may deliver power to the external circuit, all without actually intercepting any electronic current. The grid-current component resulting from the electronic flow between cathode and plate may equally well bear a quadrature relationship to the impressed voltage, in which case it will either increase or decrease the apparent interelectrode capacitance. If these effects seem queer it is because one is still confusing the electronic component with the total current.

A second basic concept once stated becomes self-evident. This is to the effect that the only one thing which we can do to an electron is to change its velocity, that is, if we are to confine ourselves to the classical concept of an electron. We can change its longitudinal velocity, that is, alter its speed but not its direction other than possibly to reverse it, or we can introduce a transverse component to its velocity, that is, alter its direction as well as its speed. Thought of in this light all electronic devices in which a control is exercised over an electron stream are velocity-modulated devices. It might be argued that one could equally well say that *all we can do is to change the electron's acceleration* (*derivative of velocity*) *or its position* (*integral of velocity*). The singling out of velocity is in a sense arbitrary. It does, however, have some very interesting ramifications.

I might digress for a moment to elaborate on this idea. Since some of the newer devices have been labeled velocity-modulation tubes, there is a perfectly understandable tendency on the part of the uninitiated to assume that these tubes differ from earlier known devices, such as, for example, the space-charge-control tubes, the Barkhausen tube or the magnetron in the fact that they employ velocity modulation. The real difference lies elsewhere as we shall see in a few moments. At the same time that these newer devices were introduced, there was introduced a new way of looking at something which is very old in the art. This newer viewpoint, to my way of thinking, constitutes a far greater fundamental contribution than do the specific devices which have received so much attention. The pioneers in this new approach: Heil and Heil, Bruche and Recknagel, the Varian Brothers, Hahn and Metcalf, to mention a few, and the many other workers who lost in the race to publish their independent contributions in this field—all of these people deserve the greatest of praise for their stimulating contributions to our thinking. My only point in all this discussion is to emphasize that the basic method of acting on the electron stream has not really been changed at all. The entire matter is summarized in the original statement that the only thing which we can do to an electron is to change its velocity.

Before going on to the next aspect of the problem there is a closely related concept which should be mentioned. This concept is that a change in the component of the velocity of an electron along one space coordinate does not introduce components of velocity in directions orthogonal to the first. For example, if an electron beam is deflected by a transverse electric field, there will be no accompanying change in the longitudinal velocity. The difficulty in the way of doing this in a practical case has nothing to do with the concept but only with the problem of producing unidirectional fields. Analyses of deflecting field problems which ignore the longitudinal components of the fringing fields are apt to be wrong. The problem of high-frequency deflecting fields has been treated in great detail in the literature and frequently with more acrimony than accuracy.

One further note should be added at this point. In an earlier lecture it was pointed out that the magnetic effects of an electromagnetic field are in general very much smaller than the electric effects. We will not stop to prove that this is still true at the frequencies which now interest us but will accept it without further discussion.

For our next concept we leave electron flow for a moment and consider the fields within a resonant cavity. You may very properly object that this has nothing to do with electron ballistics, and indeed it does not. However, we will find it necessary to discuss problems involving cavity resonators, and a failure to understand some of the properties of these circuit elements can cause a great deal of trouble. There are two conflicting approaches to this problem which I will attempt to reconcile.

The physicist when first presented with the problem of a resonant cavity is inclined to say: *This is a boundary value problem. The solution consists in writing Maxwell's equations subject to the conditions that the tangential component of E must be zero along the conducting walls. While a scalar and a magnetic vector potential can be defined, the field is not related to the former in the simple manner used in electrostatic problems.*

The engineer, on the other hand, is inclined to say: *This looks like an extension of the usual resonant circuit. A capacitance exists between the top and bottom walls of the cavity; charging currents will flow through the single turn toroidal inductance formed by the side walls. I would like to know what voltage difference exists between the top and bottom walls, and what currents exists in the side walls.*

Now, actually, I am maligning both the physicist and the engineer by my statements; nevertheless, there are these two approaches. Which is correct? Well, they both are. It is not correct to speak of an electrostatic potential within a resonant cavity; nevertheless, we may and do talk about the voltage between the top and bottom of a resonant cavity. What do we mean? Simply the maximum instantaneous line integral of the electric field taken along some specified path. In any practical device utilizing electron beams we are naturally interested in the path taken by the electrons. The fact that the line integral is different for different paths is of no great concern. We are interested in but one of these paths. We shall therefore have occasion to talk about voltages in cavities but we must always remember what is meant, and we must never for one instant forget that this voltage is not unique but that it depends upon some assumed path.

The second peculiarity of this voltage must also be emphasized. The line integral must be taken at a specified instant in time. In effect one takes a photograph of the field at some instant in time and then at one's leisure performs the integration.

Now, of course, an electron when projected through such a cavity will perform yet another type of integration. The change in squared velocity of the electron as expressed in volts will be given by the line integral of the field encountered by the electron; that is, integrated not instantaneously but with the electron velocity. This is not a simple process, because the electron velocity is continuously being changed by the field interaction and therefore the velocity with which the integration is performed depends upon the integrated value of the field up to the point in question. This has nothing to do with the concept of voltage in a resonant cavity. The cavity voltage can, however, be considered as the maximum change in squared velocity expressed in volts which an electron could receive if its entrance velocity was very large so that the transit time was small compared with the period of the cavity field.

The four basic concepts which I have chosen to recall to your mind are, by way of summary: (1) the total current is the same in all parts of a circuit, that is div. $J = 0$; (2) the only way we can act on an electron is to change its velocity; (3) the changes in the velocity component of an electron along any one rectangular coordinate have no effect on the velocity components along any other coordinate; and (4) for convenience, a voltage can be defined in a resonant circuit as the line integral of the electric field taken along some prescribed path.

Transit Angle

Since we are to deal with the interaction of electrons and high-frequency fields, we frequently find it convenient to measure electron velocity not directly but in terms of the equivalent potential difference through which an electron must fall to obtain the velocity in question, and the unit of measure will be a volt. Instead of measuring the time required for an electron to traverse any given distance in seconds, it is also convenient to use, as a unit of time, one radian of angle at the operating frequency. We frequently refer to the transit angle of an electron rather than the transit time, although both terms are used. In fact, we may on occasion measure distances in terms of transit angle, and this usage is extended to measure dimensions transverse to the direction of travel of the electron beam. When used in this fashion, we mean that the dimension in question is such that were an electron to be projected in this direction with a velocity equal to that of the electrons in the main beam, the high-frequency field would change through the stated number of radians during the transit time.

The Five Functions in an Electronic Device

With this preliminary discussion out of the way we can now answer the question which has probably been troubling quite a few of you. If the only thing we can do to an electron is to change its velocity, then in what basic way does the velocity-modulation tube differ from the conventional negative grid tube or from the magnetron?

Well, this is an involved story. If we are to make any use at all of an electron beam we must in general perform five distinct operations or functions. First we must produce the beam. Then we must impress a signal of some sort onto the beam. From what I have just said this can be done only by varying the velocities of the electrons contained in the beam. The third operation consists in converting this variation into a usable form. It is in this way that the diverse forms of electronic devices differ to the greatest degree. We will go into this matter in more detail shortly. The fourth operation consists in abstracting energy from the beam, and the final operation consists in collecting the spent electrons. While these operations are distinct from an analytical point of view, in many actual devices they are performed more or less simultaneously and more than one operation may be performed by certain portions of the tube structure. In fact, in some devices, for example in the space-charge-control tube, the confusion is so great as to make the separation seem rather forced. This very confusion may partly explain why vacuum-tube engineers who were steeped in the art were so slow to realize the advantages of this new way of looking at things which I will call the velocity-modulation concept.

By way of mental exercise in this new way of thinking let us see how we can analyze a simple space-charge-control triode. Well, first of all we have to identify the electron gun which produces the beam. The electrons most certainly come from the cathode, but where is the first accelerating electrode? Actually there isn't any unless we think of the combined d-c field resulting from the d-c potentials on the grid and plate as assisted by the initial emission velocities as performing this function. The next function, that of varying the electron velocities, is performed by the grid which varies the potential gradient in the vicinity of the cathode and hence the velocity of the electrons as they approach a potential minimum or virtual cathode which is formed a short distance in front of the cathode by the action of space charge. This virtual cathode performs the third function, that of conversion, by sorting out the electrons and allowing only those electrons with emission velocities greater than some specific value to pass. This, then, is one of the conversion mechanisms which we will call virtual-cathode sorting. In this example the virtual cathode occurs very close to the real cathode but this is not always the case. The fourth function, that of utilization, is performed by allowing the sorted electrons to traverse an electromagnetic field between the virtual cathode and the plate. This operation is completed by the time the electrons have reached the plate. Of course in the triode the plate then performs the final operation, that of collecting the spent electrons and dissipating the remaining energy as heat. It should be clearly realized, however, that this last function need not necessarily be performed by the same electrode which provides the output field. Indeed the so-called inductive-output tube proposed by Haeff is a space-charge-control tube in which these two operations are separated.

Conversion Mechanisms

But now to get back to a cataloguing of the different kinds of conversion mechanisms. The first general type involves sorting. The first kind which we have mentioned is by virtual-cathode sorting. A second kind of sorting might involve deflecting the electron beam in proportion to its longitudinal velocity instead of reflecting or transmitting it. Various deflection tubes have been proposed from time to time using this mechanism. We shall be forced to neglect this phase of the problem this evening because of time limitations but those of you who are interested will find the literature filled with detailed discussions. Still a third type of sorting, sometimes called anode sorting, is used in certain Barkhausen tubes when the plate is operated at or near the cathode potential so that fast electrons are collected while slow electrons are reflected and caused to retraverse a high-frequency field. There are still other types of sorting mechanisms but I will not burden you with these.

A second general type of conversion mechanism I will call bunching, to distinguish *sorting* in which electrons are separated according to their velocities from *bunching* in which electrons of differing velocities are brought together. Now it just happens that many of the older devices used sorting, while many of the newer devices use bunching but this is not universally the case. For example, the magnetron as used at high frequencies and the cyclotron both employ a combination of sorting and bunching. A peculiar property of the motion of an electron in a magnetic field lies in the existence of the so called Larmor frequency. You will recall that the angular velocity of an electron in a magnetic field depends only upon the field-strength and not at all upon the electron's linear velocity. This time in seconds is given by

$$t = \frac{0.357 \times 10^{-6}}{H},$$

or in radians

$$\theta = 2\pi \frac{10600}{\lambda H}.$$

Electrons of widely differing velocity can thus revolve together in spoke-like bunches with the faster electrons going around larger circles than the slow ones, but just enough larger to keep them together. This, then, is one kind of bunching, which for simplicity we shall call magnetic bunching. It is used in the magnetron and in the cyclotron. We will have more to say on this subject a little later.

A second kind of bunching was used in some of the early Barkhausen tubes where the plate electrode was operated at a fairly high negative potential so that none of the electrons were able to reach it. Under such conditions a uniformly spaced stream of electrons with varying velocities is reflected as a bunched stream, the slower electrons being reflected almost at once and the faster electrons penetrating the retarding field for a greater distance and hence taking longer to return. This same type of bunching is used in a newer form of oscillator, commonly referred to as a reflex tube which was suggested by Hahn and Metcalf in 1939, and by others at about the same time. The reflex tube differs from the Barkhausen tube, not in the basic mechanisms so much as in the fact that the conversion mechanism occurs in a different region in the tube from the region devoted to velocity modulation and to energy abstraction. A second kind of bunching is then reflex bunching.

A third type of bunching was used in the diode oscillators of Muller and of Llewellyn. The mathematical research done by W. E. Benham may be mentioned as of interest in this connection. In these tubes a uniform stream of electrons becomes bunched simply through the fact that faster moving electrons overtake slower ones which precede them. In these earlier forms of tubes we again have the case where this conversion is performed simultaneously with one or more of the other processes so that it is very difficult to separate them. However, in 1935 Heil and Heil proposed a tube in which the conversion region was separated from the other regions of the tube. This tube, the velocity-modulation tubes of Hahn and Metcalf, and the klystron tubes of the Varian Brothers, are alike in their use of transit-time bunching in a relatively-field-free drift tube. Since this separation of functions renders these devices much easier to analyze and since the structures are quite interesting in any case we will spend most of our time considering them and will, I fear, rather neglect some of the other types of tubes.

We will, of course, keep our analysis as general as possible so that the results may be applied to a variety of different devices.

Input Gap Analysis

Let us begin by a small-signal consideration of a uniform electron stream entering a region in which there is a longitudinal field defined as some function of the distance and of time. This can be the entire Llewellyn diode or it can be the input region of a klystron. We ask ourselves with what velocity will the electrons leave this region and what will be the net exchange of energy between the electrons and the field. At any point within the field a typical electron will experience an acceleration given by

$$\ddot{y} = \frac{1}{\xi} E + \eta f(y) f(t) \tag{1}$$

where η is proportional to the maximum amplitude of the h.f. field, but contains a numerical constant so that $\ddot{y}$ is expressed in centimeters per second per second. Now in the usual case $f(t)$ will be a simple sine function but $f(y)$ may assume a variety of forms. Again, by way of simplifying our work we will assume that it is also a sine function. Let us consider how we can go about solving this apparently simple equation. Unfortunately this expression can not be solved directly because the value of t at any plane (that is, the time of arrival of an electron at this plane) depends upon the interchange of energy between the electron and the field. Here we are forced back to the time-honored mathematical device of assuming a solution in the form of a series and then evaluating these coefficients. There is a large number of ways in which this can be done, and consequently a large number of different solutions which look very different but which all give comparable answers. Usually when such solutions are published, the arithmetical work is omitted leaving one with the feeling that there is something involved that is not within the ken of ordinary mortals. The fact is that the work is usually extremely tedious but actually very simple. It will be instructive to follow through one form of such an analysis in just enough detail to see the amount of work involved.

Since we are interested in the energy which is proportional to $\dot{y}^2$ we will write at once

$$(\dot{y}^2)_{y=a} = K = K_0 + \eta K_1 + \eta^2 K_2 + \eta^3 K_3 + \dots$$

where the K's are a function of the transit time, of the field distribution and of the entrance phase, and we will proceed to evaluate these coefficients. The average energy per unit of change as expressed in volts is then simply

$\frac{\xi\bar{K}}{2}$ at the end of the field while the gain is:

$$V_{av} = (\xi/2)(\bar{K} - \bar{K}_0) = \frac{\eta\xi\bar{K}_1}{2} + \frac{\eta^2\xi\bar{K}_2}{2} + \cdots$$

where the bar means that we are averaging over all values of the entrance phase.

It is of interest to evaluate the value of velocity $\dot{y}^2$ which individual electrons receive as a function of the entrance phase. For small signals it is usually sufficient to evaluate $\dot{y}^2$ maximized with respect to the starting phase, then

$$V_{max} = (\xi/2)(K - K_0)_{max} = \left[\frac{\eta\xi K_1}{2} + \frac{\eta^2\xi K_2}{2} + \cdots\right] \text{max.}$$

We can further define the ratio of V_{max} to the largest value it can have as a coefficient β, sometimes called the modulation coefficient.

But now to evaluate the K's. There are many ways of doing this as I have intimated. We will proceed by writing

$$y = y_0(t) + \eta y_1(t) + \eta^2 y_2(t) + \eta_3 y_3(t) + \ldots$$

where the y's are coefficients depending upon the transit time t which in itself is a function of the applied field thus

$$t = t_0 + \eta t_1 + \eta^2 t_2 + \eta^3 t_3 + \ldots.$$

We can then expand each function of time into a series remembering that

$$f(x + d) = f(x) + \frac{f'(x)\, d}{1!} + \frac{f''(x)\, d^2}{2!} \cdots$$

or for our particular case

$$y_0(t) = y_0(t_0) + \frac{\dot{y}_0(t_0)[\eta t_1 + \eta^2 t_2 + \eta^3 t_3 + \cdots]}{1!} + \frac{\ddot{y}_0(t_0)[\eta t_1 + \eta^2 t_2 + \eta^3 t_3 + \cdots]^2}{2!} + \cdots$$

Now we can expand $y_1(t)$, $y_2(t)$ etc. in exactly the same way. Finally we get a collection of terms which can be grouped in like powers of η thus

$$y = y_0(t_0) + \eta\ [\text{terms in } \dot{y}, \ddot{y}, t_1, t_2, \text{etc.}] + \eta^2\ [\qquad] \ldots$$

The coefficient of the η is in fact $\dot{y}_0(t_0)\, t_1 + y_1(t_0)$. We will not bother to write the rest. This expression can then be differentiated to get $\dot{y}$ and then squared. However, we still have some undetermined coefficients the t_1, t_2 etc. terms. These we can evaluate by noting that we wish these values at $y = a$, where a is a fixed distance in the actual device. At this distance the t coefficients in the expression for y must have such values that the value of y does not change with the value of η. This can only be true if the individual expressions multiplying each power of η are each equal to zero. Equating these expressions to zero one can evaluate all of the t's. For example the first term yields

$$\dot{y}_0(t_0)t_1 + y_1(t_0) = 0$$

or

$$t_1 = -\frac{y_1(t_0)}{\dot{y}_0(t_0)}.$$

Introducing these values, differentiating and squaring, one finally gets an expression for $(\dot{y}^2)y = 0$ as a power series in y, the coefficients all being of a form easily evaluated for any specified field distribution. Since we have by definition called these coefficients K_0, K_1, etc. these values are then

$$K_0 = \dot{y}_0^2$$

$$K_1 = 2(\ddot{y}_0\ddot{y} - y_0\ddot{y}_1)$$

$$K_2 = (\dot{y}_1^2 - 2y_1\ddot{y}_1 + 2\dot{y}_0\dot{y}_2) - 2\ddot{y}_0 y_2 + \frac{\ddot{y}_0 y_1^2}{\dot{y}_0}$$

This then constitutes the formal solution of the problem. We must now particularize our problem to some specific field distribution and evaluate the y coefficients. Suppose, for example, that there is a uniform d.c. field (E of equation 1) and an alternating field which varies as some cosine function of distance. Then the latter is

$$f(y) = \cos\left(\frac{\pi y}{b} + c\right)$$

and

$$\ddot{y} = \frac{1}{\xi} E + \eta \cos(\omega t + \varphi) \cos\left(\frac{\pi y}{b} + c\right)$$

we must eliminate the y which appears in this expression and replace y by its equivalent

$$y = y_0 + \eta y_1 + \eta^2 y_2 + \dots$$

and expanding

$$\cos\left(\frac{\pi y}{b}+c\right)=\cos\left(\frac{\pi y_0}{b}+c\right)+\frac{\frac{\pi}{b}\sin\left(\frac{\pi y_0}{b}+c\right)[\eta y_1+\eta^2 y_2+\cdots]+\cdots}{1!}$$

and as before equating like powers of η with $\ddot{y}$ defined as

$$\ddot{y}=\ddot{y}_0+\eta\,\ddot{y}_1+\eta^2\ddot{y}_2+\ \ldots$$

we finally arrive at

$$\ddot{y}_0=\frac{1}{\xi}\,E$$

$$\ddot{y}_1=\cos(\omega t+\varphi)\cos\left(\frac{\pi y_0}{b}+c\right)$$

$$\ddot{y}_2=-y_1\pi/b\cos(\omega t+\varphi)\sin\left(\frac{\pi y_0}{b}+c\right).$$

Now we need only integrate these expressions to obtain the values of the $\dot{y}$'s and the y's needed to evaluate the K's.

If we average $\dot{y}^2$ over all values of the starting phase we can write the energy contributed by the field to the electron's velocity. When this is done one finds that the odd powers of η are identically zero leaving only the even powers to be considered and for small signal analysis purposes we need only consider $\bar{K}_2$. The energy per electron expressed in volts is

$$V=2.49\times10^{-8}E^2\lambda^2f(\theta)$$

where $f(\theta)=\omega^2\bar{K}_2$, and the power is obtained by multiplying this expression by the beam current in amperes.

The end results can be expressed as curves of $f(\theta)$ against θ as shown in Fig. 1. Three examples are shown: the uniform field case and two different harmonic distributions as indicated by the smaller plot in the lower left-hand corner. You will note that there exist regions of positive $f(\theta)$ where the net transfer of energy is from the field to the electron and regions in which the transfer is in the other direction; the former portions are of considerable interest in connection with the input gaps in velocity modulation tubes, and for that matter in the cathode grid region of the negative grid tube although this is more complicated than is here indicated, as this transfer of energy constitutes a loss to the field which loads the input circuit. The latter portions may be utilized as was done in the Muller and Llewellyn diodes to obtain sustained oscillations.

If, as I have indicated, we maximize $\dot{y}^2$ as a function of the starting phase we can evaluate the modulation coefficient. The value for the uniform field case, as shown in Fig. 2, is simply, $\beta = \frac{\sin \theta/2}{\theta/2}$. For future reference we will write the loss expression for this case as

$$f(\theta) = 2\,(1 - \cos\theta) - \theta \sin\theta.$$

Drift Space Analysis

Now let us consider the conversion region in a typical velocity-variation tube. Figure 3 is a drawing of several such devices with the conversion

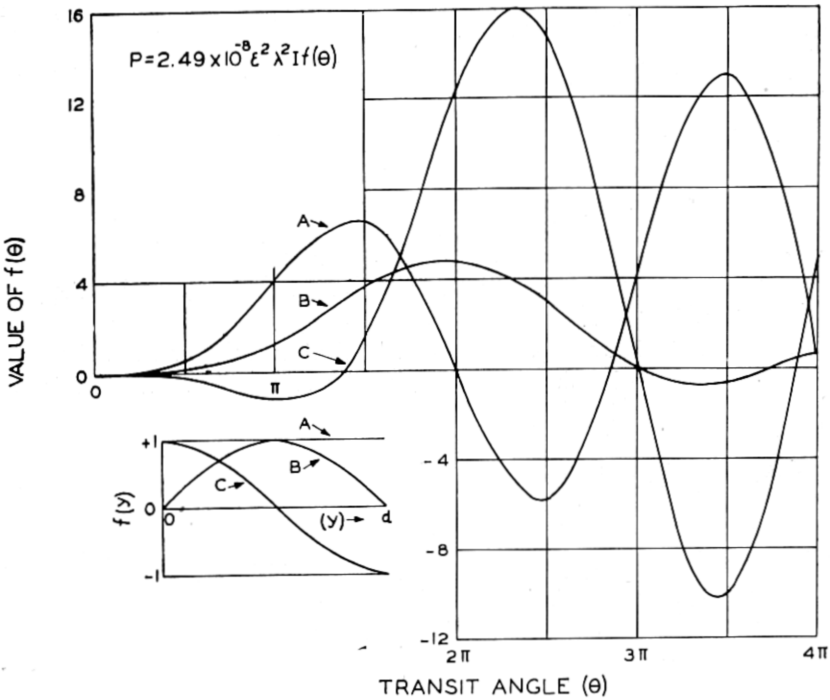

Fig. 1—The energy transfer between an initially uniform electron stream and a longitudinal electromagnetic field as a function of transit angle.

regions indicated. We will assume for the moment that the electrons enter this region with a small variation in velocity and at a perfectly uniform rate. Since the total number of electrons entering the region must be equal to the number of electrons leaving the region we may write

$$i_1\, dt_1 = i_0 dt_0$$

or

$$i_1 = i_0 \frac{dt_0}{dt_1}.$$

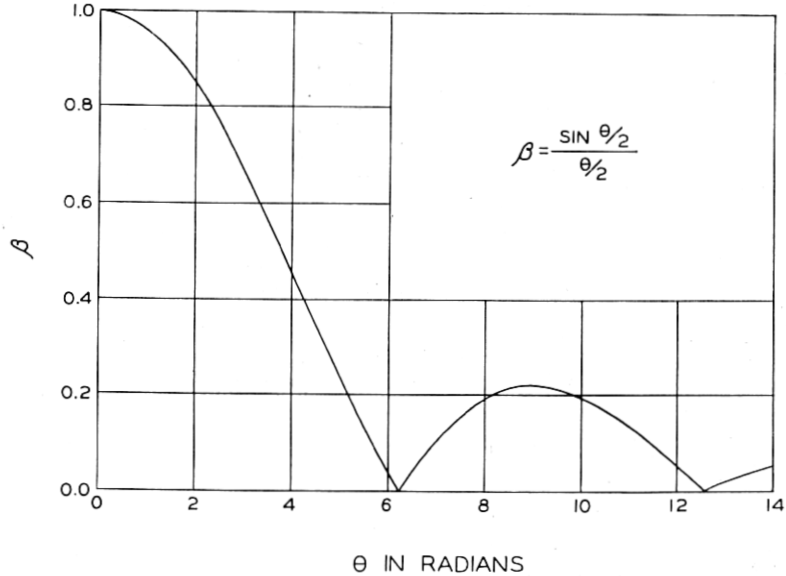

Fig. 2—The (velocity) modulation coefficient between an initially uniform electron stream and a uniform electromagnetic field as a function of transit angle.

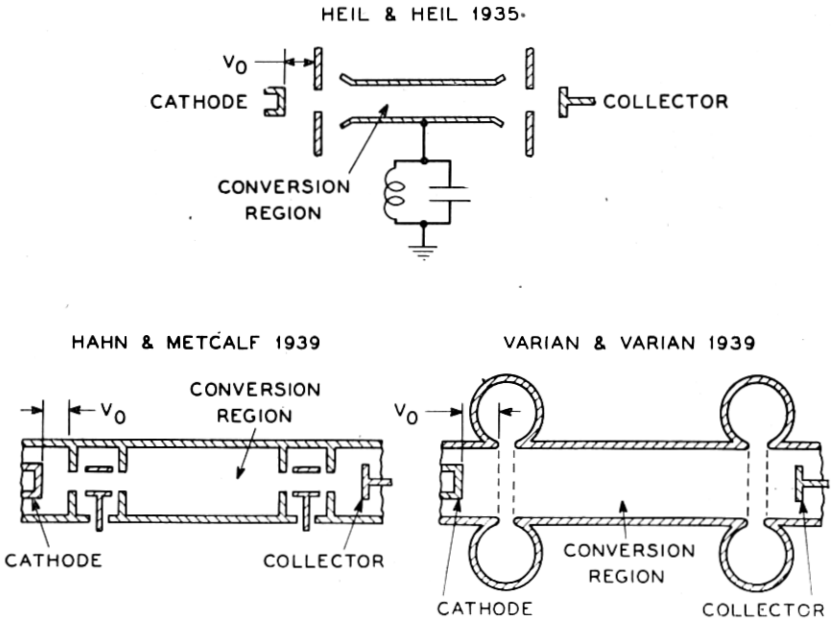

Fig. 3—Typical velocity variation devices employing transit-time bunching.

However, a relationship exists between t_1 and t_0,

$$t_1 = t_0 + \frac{\ell}{v}.$$

Where

$$v = v_0 \sqrt{1 + \alpha \sin \omega t_1},$$

$$t_1 = t_0 + \frac{\ell}{v_0 \sqrt{1 + \alpha \sin \omega t_1}}.$$

Now if $\alpha \ll 1$

$$t_1 = t_0 + \frac{\ell}{v_0}\left(1 - \frac{\alpha}{2} \sin \omega t_1 + \cdots\right)$$

and

$$\frac{dt_0}{dt_1} = 1 + \frac{\ell}{v_0}\frac{\alpha\omega}{2} \cos \omega t_1$$

and finally

$$i_1 = i_0\left(1 + \frac{\ell\alpha\omega}{v_0 \cdot 2} \cos \omega t_1\right)$$

but

$$\frac{\ell\omega}{v_0} = \theta$$

so that finally

$$i_1 = i_0\left(1 + \frac{\alpha\theta}{2} \cos \omega t_1\right).$$

This says that the velocity variation impressed on the beam at the entrance to the drift space or conversion region has resulted in a current variation at the output. For those of you who think in vacuum tube parameters it is of interest to differentiate this expression with respect to the a-c voltage and obtain the transconductance

$$G_m = \left|\frac{di_1}{dV_{a \cdot c \cdot}}\right|$$

rewriting

$$|i| = i_0\left(1 + \frac{V_{ac}\theta}{2V}\right)$$

$$\left|\frac{di_1}{dV_{ac}}\right| = \frac{\theta i_0}{2V}.$$

This result is obtained by neglecting all of the higher order terms and is therefore only a small signal theory of a very restricted sort.

Now let us consider what we have done. Well, we have followed a small interval of time through the drift tube. At the input this time dt_0 had a current i_0 associated with it; at the output the size of this unit of time is different—it is now dt_1 and the current associated with it is i_1. The physical picture corresponding to this phenomenon is that of a uniform distribution of electric charge becoming bunched with time as it traverses the drift space.

The next step in the analysis is to carry our approximation a step further and consider higher-order terms. Expanding the expression for i_1 and using our nomenclature the desired expression is

$$i_1 = i_0\left[1 + 2\left(J_1\left(\frac{\alpha\theta}{2}\right)\cos\omega t + J_2\left(2\frac{\alpha\theta}{2}\right)\cos 2\omega t + J_n\left(n\frac{\alpha\theta}{2}\right)\cos n\omega t\right)\right].$$

This equation is not exact since it neglects space charge effects but it does indicate the presence of harmonics in the beam current and it reveals certain non-linear effects which can also be illustrated by the so-called phase-focusing diagrams of Bruche and Rechnagel.

Phase-Focusing Diagrams

Bruche and Recknagel pointed out that an analogy exists between the focusing in space of a parallel light beam and the focusing in phase of the electrons in a uniform electron beam. In fact a small-signal theory can be developed entirely in terms of optical equations. We will not go into this aspect in detail but we will use their diagram (Fig. 4) to illustrate the bunching effect graphically. A uniform beam of electrons is represented by a series of parallel lines in distance and time coordinates, focus being indicated by a crossing of these lines after they have been deflected by the velocity modulation.

This general type of diagram has been popularized in this country by the Varians, and their associates under the name Applegate diagram, the only difference being an interchange of axis. Figure 5, taken from a recent paper by Dr. A. E. Harrison, illustrates this version of the Bruche and Recknagel diagram.

Now if instead of judging the current density by the density of the lines on the diagram, we make a plot of the current density as a function of time for different fixed distances from the input gap, the pictures are somewhat as shown on Fig. 6. Figure 7 represents a plot presented by Kompfner and combines in one illustration the type of presentation used by Tombs.

Phase Focusing in a Reflex Tube

It might be well to pause for a moment in our discussion of transit time bunching to consider how the phase focusing diagrams can be applied to a reflex tube. The elements of a modern reflex tube are shown in Fig. 8

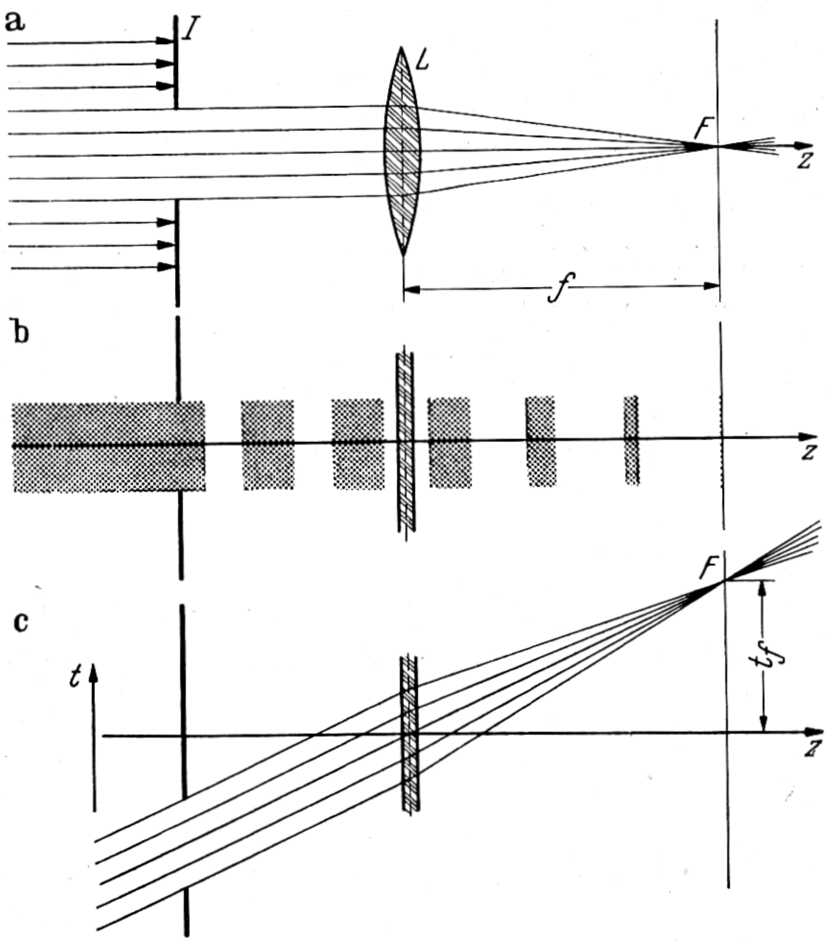

Fig. 4—The phase-focusing diagram of Bruche and Recknagel showing the analogy to optical focusing.

which was taken from a recent I.R.E. paper by Dr. J. R. Pierce. Electrons from the cathode pass through an input gap defined by two grids where they are modulated in velocity. In traveling in the retarding field produced by the repeller those electrons which passed the gap when the field was becoming progressively less accelerated, become bunched; the faster electrons

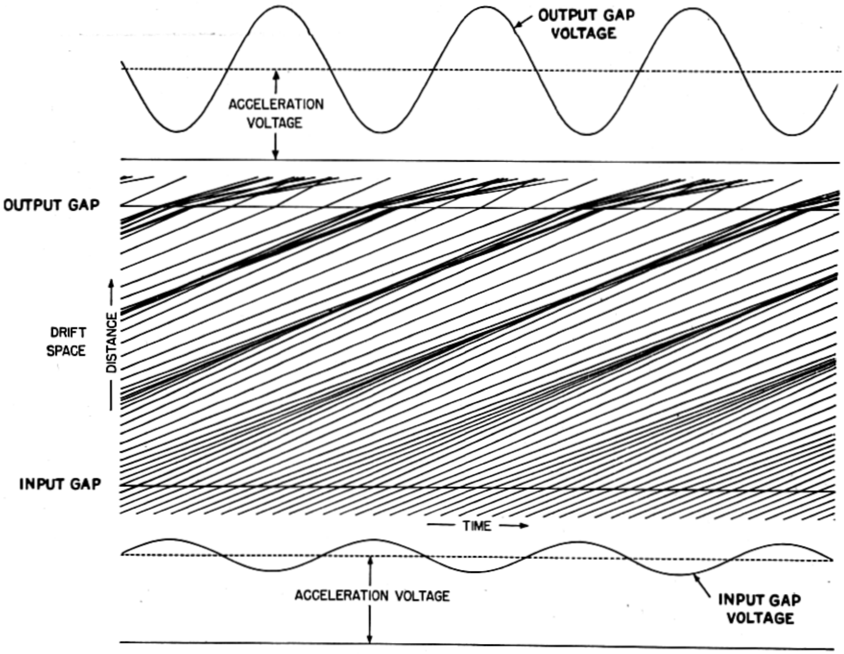

Fig. 5—Applegate's version of the phase-focusing diagram (Harrison).

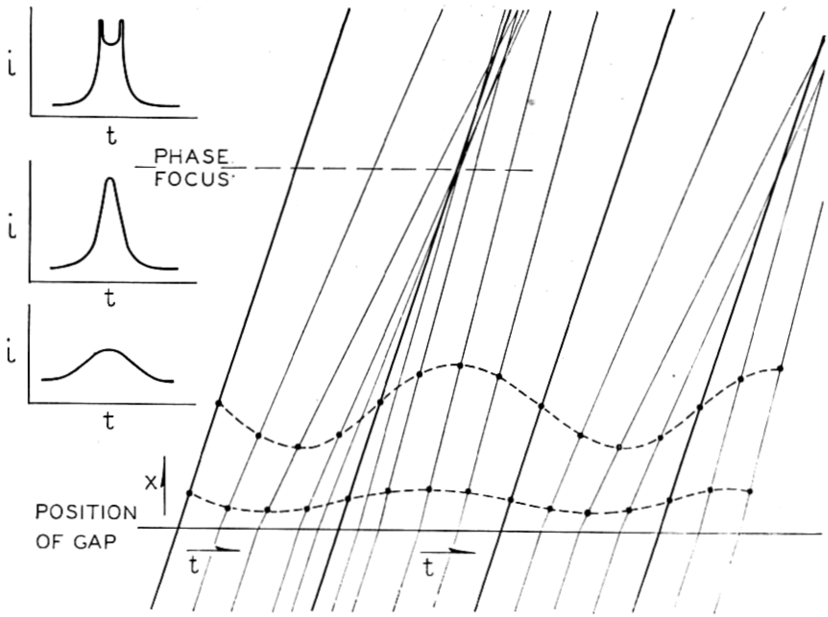

Fig. 6—The conduction-current distribution at different distances along the beam as predicted by the phase-focusing diagram.

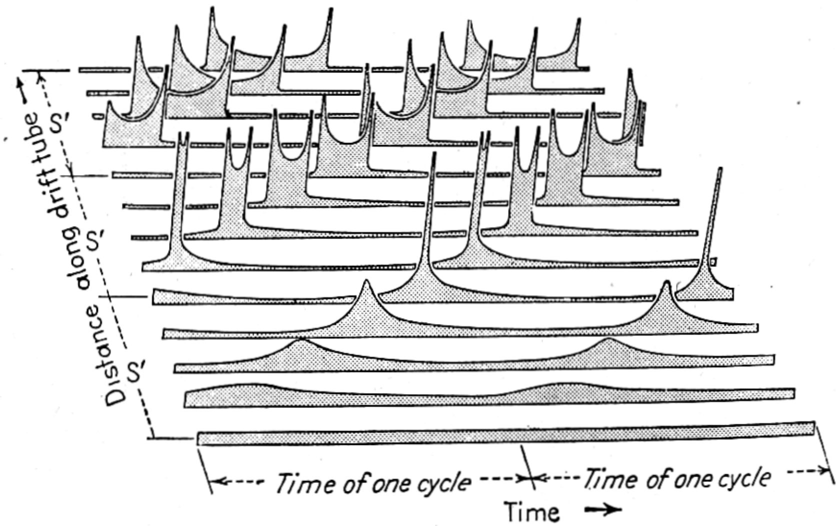

Fig. 7—Kompfner's presentation of the bunching effect.

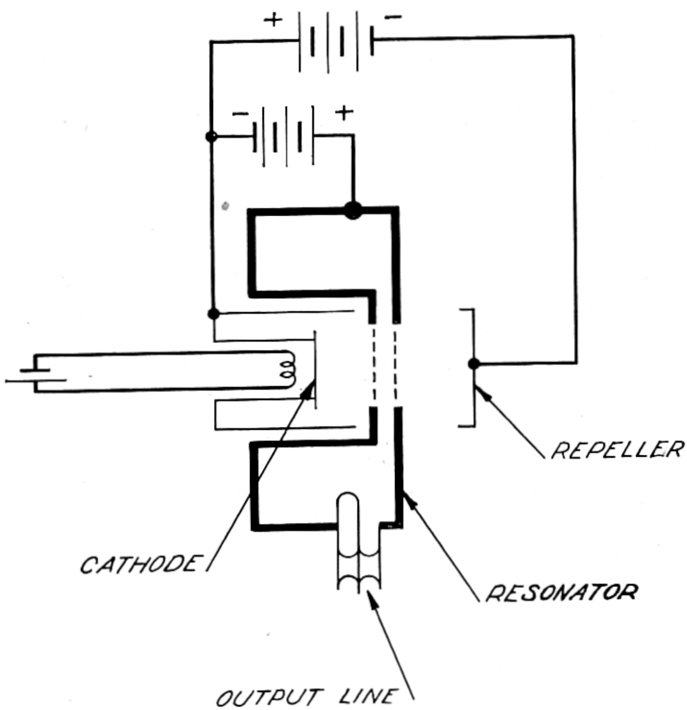

Fig. 8—The elements of a modern reflex tube (Pierce).

penetrating the field to a greater extent and waiting, as it were, for the slower electrons which follow to catch up. The electrons which pass across the gap while the field is becoming progressively more accelerating are spread out. If the retarding field is uniform it can be likened to the earth's gravitational field and the phase-focusing paths on our time-distance plot are parabolas. Figure 9, taken from Pierce's paper, illustrates this while Fig. 10 is such a plot taken from the paper by Harrison. One interesting and,

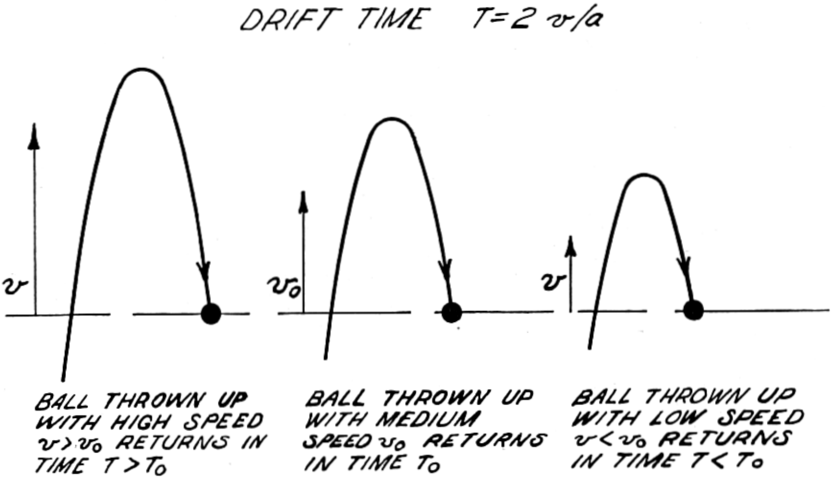

Fig. 9—The gravitational-field analogy to reflex bunching (Pierce).

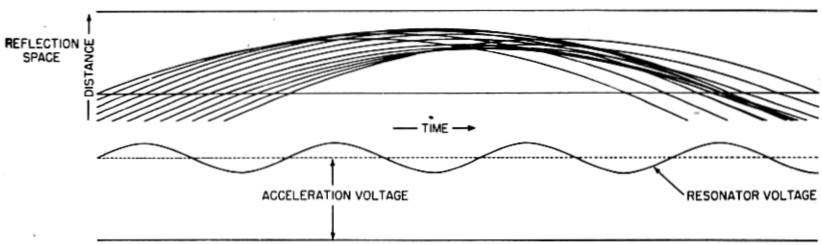

Fig. 10—The phase-focusing diagram for a reflex oscillator (Harrison).

in a way, unfortunate difference between reflection bunching and direct transit-time bunching is the fact that for reflection bunching the slow electrons catch up with the fast ones while the reverse is true for the other type. This means that if both types of bunching are present as shown in Fig. 11, (also taken from Harrison's paper) one will tend to undo the effect of the other.

Another way of combining effects of separate bunching actions is to build a cascade transit-time-bunching amplifier in which a series of three gaps is used together with two drift spaces. The first gap velocity modulates the beam; this modulation is converted into a current modulation in the first drift space. The beam then excites the second cavity, which again velocity modulates the beam in quadrature with the original modulation. This action of course occurs in the output gap of a two-gap tube but it is not there used. Here this second and larger velocity modulation is converted to current modulation in the second drift space. The output is finally taken off the beam by the third gap. A phase-focusing diagram of this sort (again taken from Harrison's paper) is shown in Fig. 12.

Space-Charge-Wave Analysis

This phase-focusing approach is rather intriguing as one feels that one has a physical picture of what is going on. The picture is, however, very inexact except under certain highly specialized cases, as it completely ignores

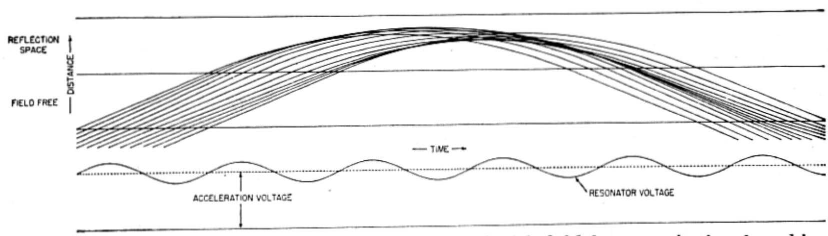

Fig. 11—Diagram showing reflex bunching combined with field-free transit-time bunching (Harrison).

space-charge effects. These space-charge effects are of two sorts: a d-c effect, if you will, and an r-f effect; that is, the presence of the electrons of the beam will alter the average velocity of the electrons at different parts of the beam, and will tend to undo the bunching action. Because of this second effect, the electrons are effectively prevented from passing each other as the graphical solution suggests. Instead, as the density of the electrons in the bunch becomes greater, the mutual repulsion forces tend to prevent a further concentration of charge. The electron bunch then tends to disperse. The action could be likened to the propagation of a sound wave in a moving column of air. While there are several approximate ways to handle this problem, Hahn was the first to propose a really satisfactory theory. Incidentally it should be noted that the Benham, Muller, Llewellyn and Peterson type of theory is capable of treating this aspect of the problem in a rigorous way and including all space-charge effects, but unfortunately these theories are limited in that they have been applied only to the parallel-plane case, and of course they are only small-signal theories.

Hahn's analysis starts by treating an infinitely long electron beam, using cylindrical co-ordinates and is limited to a small signal theory where the a-c motions are small compared to the d-c but it does not ignore the r-f effects of the space charge forces. The electron beam is thought of as a moving dielectric rod which is capable of propagating axial waves much as a dielec-

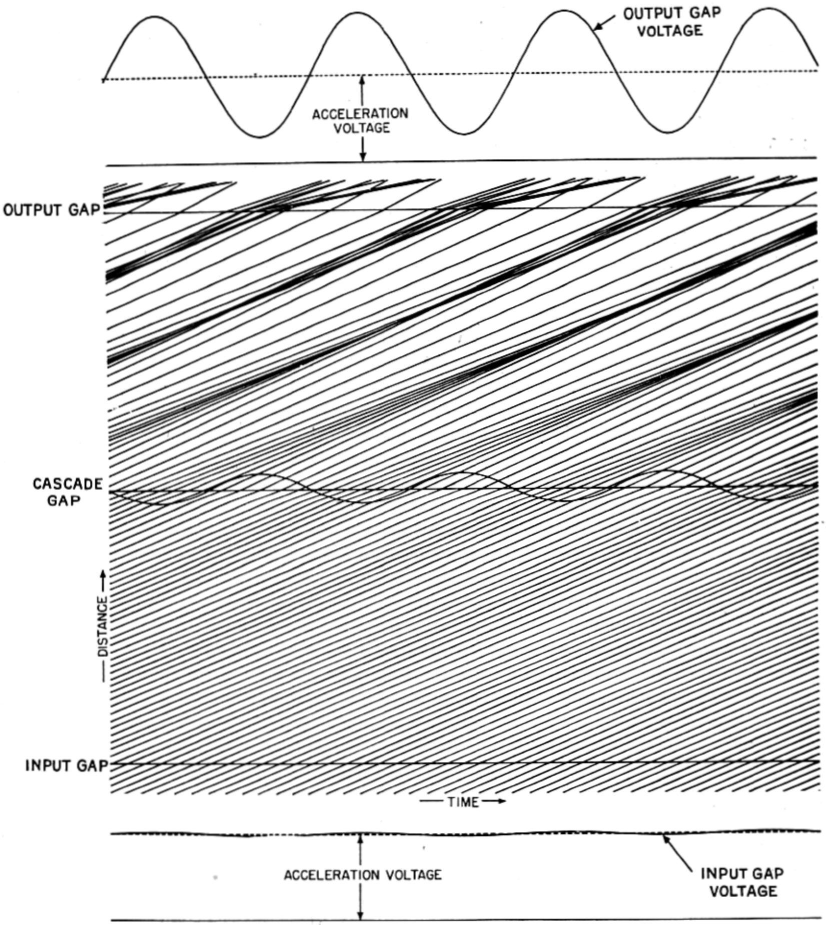

Fig. 12—Diagram for a cascade amplifier (Harrison).

tric wave guide will do. He assumes an axial magnetic field and a stream of positive ions having the same velocity axially and the same charge density. These ions are assumed to have infinite mass. The solution is much too complicated and involved to present here even in abstract. It involves the complete solution of Maxwell's equations subjected to the stated assumptions as restricted by the assumed boundary conditions at the edge of the beam.

It is found that two waves are possible, one traveling slightly faster than the electron beam and the second traveling slower. A point where the velocity components are in phase will correspond to the input to the beam, while points where the current components are in phase correspond to the desired positions for the output. The propagation constants for these two waves in a simplified special case where the magnetic field strength is infinite are given by Hahn, as well as expressions for the optimum drift tube length. He goes on to consider the case where the magnetic field is zero and finds that for this case the density of the charge does not vary much but instead the beam swells in and out so that instead of being lumps of charge with spaces between, the lumps appear in the outer boundary. Hahn has extended his general method of analysis to consider the modulation coefficient of gaps through which the beam must pass. His results are a great deal more general than those we have presented.

Ramo has reformulated Hahn's theory by means of retarded potentials for the most important case. This results in some simplification of the theory. He computes the more important design constants for a velocity modulated tube, such as the optimum drift tube length and the amount and phase of the transconductance. Those of you who are particularly interested are referred to the original paper. An interesting aspect brought out rather forcibly by Ramo's analysis is the existence of higher-order waves on the beam, always occurring in pairs, one faster and the other slower than the beam velocity.

The Magnetron

In what time remains I want to say just a very few words about the magnetron. This is a very complicated subject and one which cannot be adequately dealt with in an entire evening, and certainly not in the time remaining.

As you all know, the magnetron was invented and named by Dr. A. W. Hull. Habann, Zacek, Okabe and others pioneered in the use of the magnetron as an ultra-high-frequency oscillator. As envisioned today a magnetron is a two-element device, usually cylindrical with a centrally located cathode and a surrounding anode. The anode may be continuous or it may be split into a number of segments as suggested by Okabe, and these segments joined together either externally or internally by resonant circuits.

The basic ballistic problems of the magnetron, and hence the only problems which directly concern us at this time are (1) that of determining the electron paths within the magnetron and having determined these paths (2) that of getting an understanding of the mechanism whereby electrons in traversing these paths are able to deliver energy to the connected high-frequency circuits. One might think that the first problem would be a relatively easy job. As a matter of fact the literature is surfeited with papers purporting to give the answer. Unfortunately almost all of the published work ignores the effect of space charge. A few moments' thought will suggest that space charge may be a controlling factor because of the long electron paths which are sure to result in crossed electric and magnetic fields, and indeed more detailed computations bear this out. Nevertheless the neglect of space charge greatly simplifies the problem. There are those who believe that the no-space-charge theories have no bearing on the way actual magnetrons work and that any correspondence between the predictions of such theories and the actual behavior of magnetrons is simply the result of an unfortunate coincidence. In fact Brillouin points out that the simplified form in which the Larmor theorem is applied by many, is in itself an approximation which was perfectly valid as originally applied by Larmor to the electronic orbits within the atom but which does not apply to conditions as they exist in the magnetron.

A number of recent workers have attempted to include the effects of space charge but have unfortunately largely restricted themselves to small signal theories while the magnetron is seldom operated under small signal conditions, at least not intentionally. Most theories are further restricted to a consideration either of the coaxial case where the cathode radius is small compared to the anode radius or of the plane case. Most practical structures are intermediate between these extremes.

As an example of the difficulties involved, Fig. 13, reproduced from a paper by Kilgore, shows the electron paths as computed neglecting space charge and also shows experimental proof that these paths actually exist. This illustration has been frequently reproduced and widely accepted. The experimental picture was obtained in the presence of gas, to make the electron beam path visible, and unfortunately the ionization which makes the beam visible also tends to neutralize space charge effects. The experimental arrangement departs still further from reality in that the electron emission from the cathode was restricted to a limited region so that the space charge forces were still further reduced. Now it is probably true that some magnetrons operate with electron paths as shown; still it is not true that all magnetrons operate in this way.

Contrasting with this picture which was until recently commonly accepted, Brillouin, Blewett and Ramo, and others have shown that stable distributions are possible in which a space charge of almost uniform density rotates with a uniform angular velocity about the axis. Brillouin goes so far as to label the curves due to Kilgore as wrong, and pictures the possible electron trajectories as shown in Fig. 14.

One of the earliest papers to consider this newer picture of the electron paths in the magnetron was published by Posthumus in 1935. This was definitely a ballistic approach and hence suitable for discussing tonight.

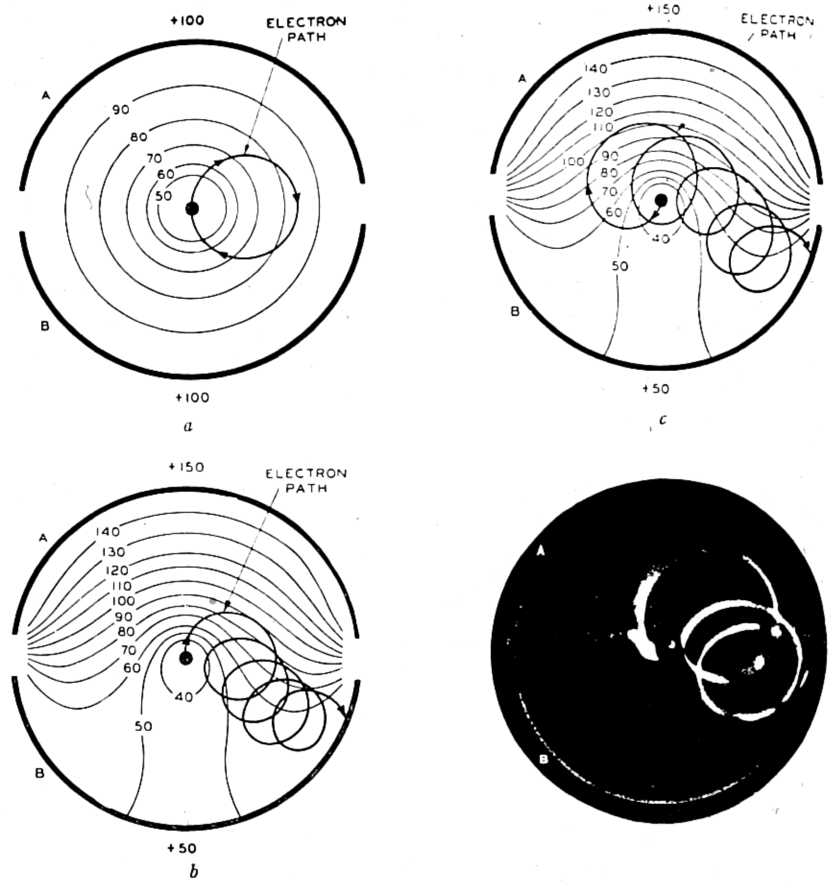

Fig. 13—Typical electron paths in a two-segment magnetron showing how electrons arrive at the plate-half of lower potential (Kilgore).

Posthumus limits his discussion to but one type of oscillation which can be obtained in the split-anode magnetron. Those of you who are familiar with the early literature on the magnetron will recall that two distinct types of oscillations were frequently described. One type usually called "electronic" was found to occur under conditions when the magnetic field was just high enough to cut off the anode current under static conditions. This field has the value computed by Hull:

$$H = \frac{6.72\sqrt{V}}{R}$$

Hull's first computation, by the way, was made neglecting space charge, but, strangely enough, the result is not changed by space charge. These electronic oscillations were assumed to be related in frequency to the time of transit of an electron from the cathode to the anode, and at cutoff this is inversely proportional to the field strength, as expressed by the empirical relationship

$$\lambda H = 13{,}100.$$

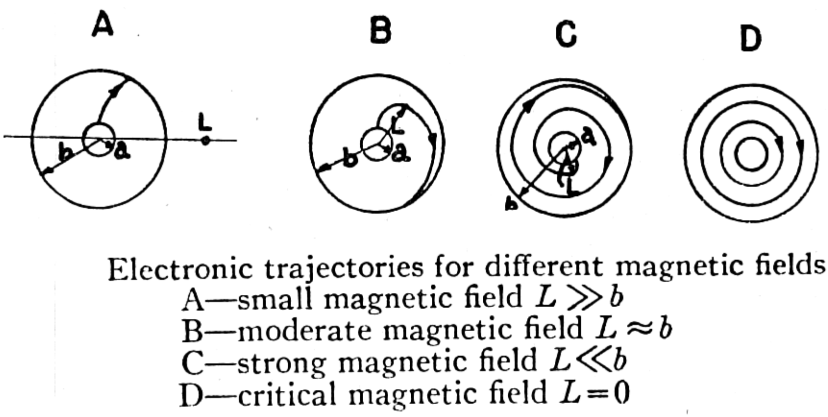

Fig. 14—Electronic trajectories for different magnetic fields varying from weak fields to the critical field shown to the right (Brillouin).

In general, it was found that best operation occurred when the magnetic field was not quite perpendicular to the electric field. The efficiency and outputs as reported for this type of oscillator were always low, in spite of the large amount of effort devoted to it by an equally large number of workers. A second type of oscillation, usually referred to as negative resistance oscillations, has also been the subject of considerable study and some practical use has been made of it at relatively low frequencies.

Contrasting with this, Posthumus described a third kind of oscillation which he called rotating field oscillations. As in the electronic oscillations the preferred frequency is determined by the magnetic field-strength and the anode potential, the frequency being inversely proportional to the magnetic field-strength. Contrasting with the electronic oscillations, the rotating field oscillations occur with the magnetic field-strength very much above the critical cutoff value and the efficiency on occasion reached as much as 70%. While a careful reading of the literature will reveal that some of the earlier experimenters were occasionally dealing with these oscillations, Posthumus' observations represent a new departure in magnetron theory and practice and one which we might do well to investigate.

Posthumus' approach consisted in studying the electron paths in a magnetron in detail in order to find the conditions under which electrons may reach the plate with considerably less energy than that corresponding to the plate potential. He assumed a magnetron having k pairs of plates and based his calculations on the supposition of a rotating electric field with k pairs of poles. In reality there exists a simple alternating field but this can be resolved into two rotating fields rotating in opposite directions. Power engineers will recognize this as identical with the procedure used in analyzing single-phase rotating machinery. Posthumus neglected the field opposite to the static angular velocity and considered only one component. This is an approximation but a fairly plausible one which can be partially justified.

In the absence of oscillations there is a radial electric field independent of the angular position and inversely proportional to radius (for the coaxial cylindrical case). When oscillations are present there is an additional radial field which varies as some periodic function of the angle and with a period 2π, and a tangential component of the same general type. For simplicity these functions are taken to be simple harmonic functions and can therefore be split into two circular rotating fields.

Posthumus writes the two simultaneous differential equations determining the path of an electron, neglecting space charge, and inquires if a solution is possible for an elecron path which travels at approximately the same angular velocity as the rotating field but lags it by an angle α. An equally satisfactory way of looking at this is to say that we transform our coordinates from a fixed system to one rotating with the field and inquire if a solution is possible where α the angular motion is always small. He finds that such a solution is indeed possible and that for the electron motion to be stable the value of α must be such that the electrons are somewhat behind the line for which the field has its maximum retarding value. The electrons are thus in a position to lose energy to the field and to spiral out toward the anode.

Posthumus defined the value of the electron's radial velocity squared at the anode as P and the total velocity squared at the anode as Q. Normalized plots of these two parameters are shown in Fig. 15 as a function of frequency. The upper plot shows the radial velocity. Obviously for electrons to reach the plate at all they must have a positive velocity at the plate. Electrons can therefore reach the plate with any given field value, say $Z = 2$,

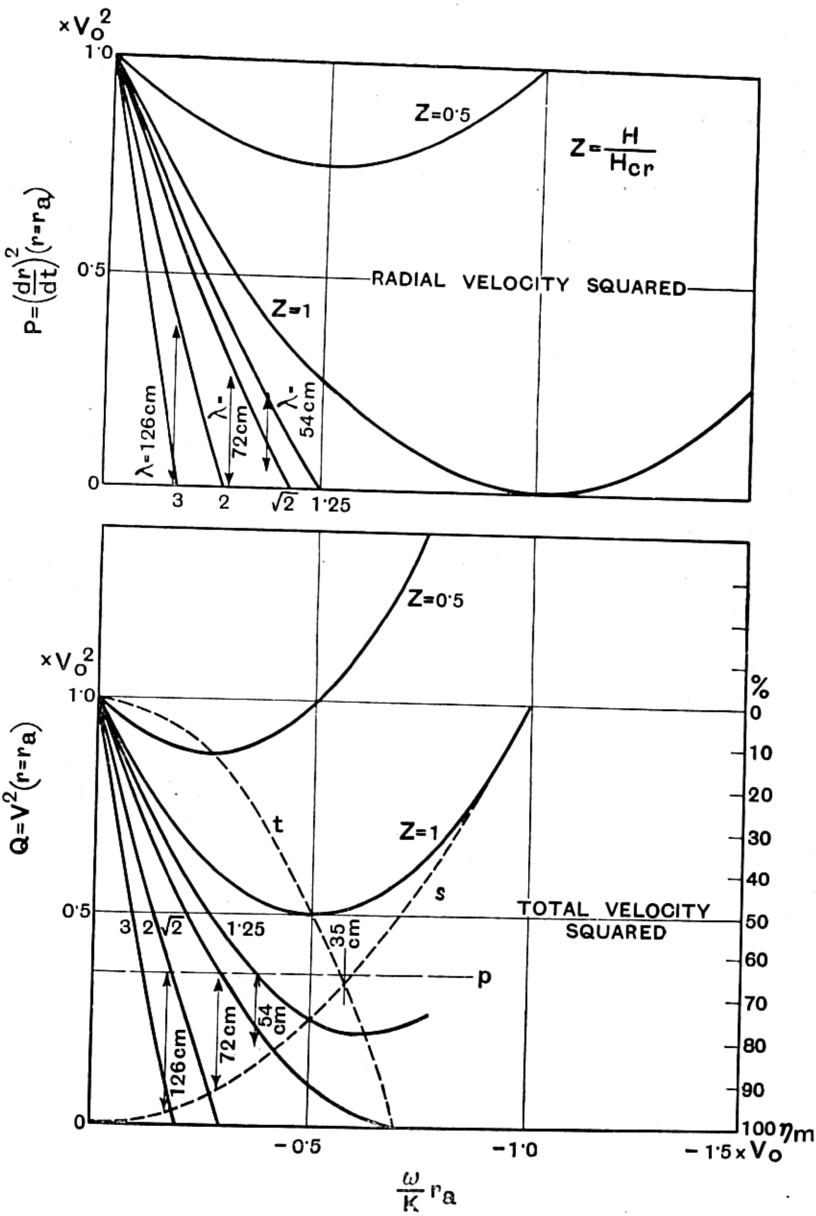

Fig. 15—Electron velocities in the magnetron according to Posthumus.

that is with a field equal to twice the cutoff value, for all frequencies less than the equivalent value defined by the intercept of the $Z = 2$ line with the abscissa axis. The line for $P = 0$ appears on the lower curve as the dotted line s. Here the ordinate is the total velocity squared, normalized with respect to the value without oscillation. Efficiencies can therefore be put on the plot directly as shown by the right-hand scale in per cent. The line s is therefore a plot of the maximum possible efficiency. This refers to what we might call the electronic efficiency since no account is taken of circuit losses. Now in any physical device there are some circuit losses and hence a lower value of electronic efficiency for which sustained oscillations are not possible. The dotted line p is Posthumus' experimental value for this lower limit. Between the lines p and s, then, oscillations are possible at frequencies given by the abscissae and with field values shown on the solid lines. Actual data for an experimental tube are shown on the plot, oscillations occurring at the wavelengths indicated and over the ranges in field shown by the lines terminating in arrows.

One additional line t is shown on the plot connecting points on the different Z lines for which the efficiency is a maximum. The optimum design would be one based on the intersection of this line with the p line. Still other facts will appear from a detailed study of these results but we shall not be able to devote any more time to this interesting subject.

Conclusion

In concluding a talk of this sort and particularly in concluding a series of talks, it is usually appropriate to look ahead to the future and predict the trend of affairs, or perhaps to point out certain fruitful fields of research. I find this a singularly difficult thing to do. However, it is not revealing any military secrets to say that much of the progress of the last few years has been in the direction of making things work and not toward getting a clearer understanding of the underlying theory. If, for example, an illuminating approach could be devised which would make the problems associated with transverse fields, both electric and magnetic, appear as simple and straightforward as do longitudinal-electric-field problems, as a result of the velocity-modulation concept, then I believe even more striking advances could be made in the ultra-high-frequency field than those which the war years have brought forth.

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