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Physical Limitations in Electron Ballistics*

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Introduction

THE subject of this talk is "Physical Limitations in Electron Ballistics". It is pleasant to have a chance to talk about such physical limitations, because there is so little we can do about them. And, although these limitations are apt to be discouraging, a knowledge of them is very valuable, for it keeps us from spending time trying, like the inventors of perpetual motion machines, to do the impossible.

As electron ballistics is particularly subject to physical limitations, there are so many that it is impossible to discuss all of them thoroughly at this time. Also, many of the limitations are of a rather complicated nature, and to deduce them from basic principles in a quantitative way requires much thought and patience. I think the best I can do is to try to mention most of the chief limitations, as a warning to the uninitiated that rocks lie ahead in certain directions, but to concentrate attention on only a few of them. I have chosen this evening to devote particular attention to limitations that bear on the production and use of electron beams in which considerable current is required, such as those used in cathode ray tubes and high-frequency oscillators, and to mention only briefly as a sort of introduction problems pertaining more closely to low-current devices such as electron microscopes.

The Wave Nature of the Electron

One of the most important limitations in electron microscopy is the dual nature, wave and corpuscular, of the electron. Without making any attempt to justify or explain the combination of wave and particle concepts which is characteristic of modern physics, we may describe its consequence at once; very small objects don't cast distinct shadows. This cannot be explained merely in terms of the physical size of the electron and the object. When an electron beam is reflected from a surface of regularly

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spaced obstacles (the atoms in a crystal lattice, for instance) diffraction patterns are obtained, similar to those which may be obtained with waves of X-rays or light. It appears that electrons get around sufficiently small objects just as sound waves get around telephone poles, automobiles, and even houses, and if the objects are sufficiently small their effect on the electron flow will either be absent or will consist of a few ripples which are meaningless in disclosing the shape or size of the object.

The electron wave-length, which varies inversely as the momentum of the electron, may be simply expressed in terms of the energy V in electron volts. A simple non-relativistic expression which is only 5% in error at 100,000 volts (a high voltage for electron microscopes), is*

$$\lambda = \sqrt{150/V} \times 10^{-8} \text{ cm} \qquad (1)$$

Thus for 30,000-volt electrons the wave-length is 7×10^{-10} cm or about 1.4×10^{-7} times the diameter of a hair and 1.2×10^{-5} times the length of a wave of yellow light.

In terms of this wave-length λ and the half angle of the cone of rays accepted by the objective, α, we can express the distance d between point objects which can just be distinguished in an electron microscope. This distance is

$$d = .61\lambda/\sin\alpha \qquad (2)$$

For small values of α

$$2\alpha = 1/f \qquad (3)$$

where f is the well known photographic f number, the ratio of the focal length to the lens diameter. We see that, just as with cameras, the smaller the f number the better. In electron microscopes a small f enables us to distinguish smaller objects.

Aberrations

Just as in cameras, the limitation to the f number is imposed by lens aberrations. But in electron lenses the aberrations are much more severe. Why is this so? Because with electron lenses we have less freedom of design than with optical lenses.

Consider an electric lens. The quantity analogous to the index of refraction for light is the square root of the potential with respect to the cathode. Now suppose that with a light lens we know the index of refraction at every point along the axis. Suppose, for instance, that the index of refraction is 1 everywhere along the axis except for a space L long

* The relativistic expression is

$$\lambda = (\sqrt{150/V}/\sqrt{1 + .98 \times 10^{-6} V}) \times 10^{-8} \text{ cm}$$

where it is 2, as in Fig. 1. Our lens may be converging or diverging; strong or weak. In the analogous electric case, however, the potential throughout the lens space must satisfy Laplace's equation, and this means that if it is specified along the axis it is known everywhere. We can easily see this by writing down Laplace's equation for an axially symmetrical field.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{\partial^2 V}{\partial z^2} = 0 \tag{4}$$

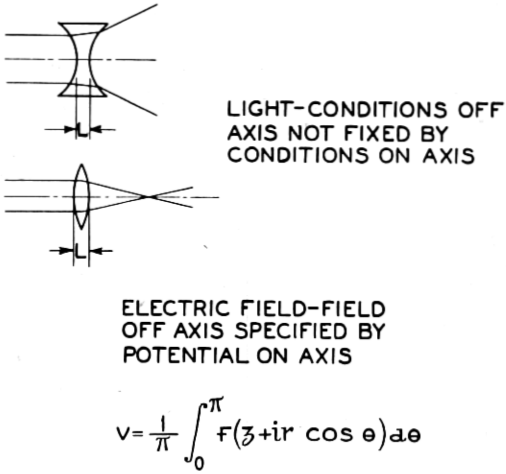

Fig. 1—Contrast between optical and electric focussing conditions.

The field near the axis may be expanded in powers of f

$$\frac{\partial V}{\partial r} = ar + \cdots \tag{5}$$

Substituting this into (4),

$$\frac{1}{r}\frac{\partial}{\partial r}(ar^2) = 2a = \frac{-\partial^2 V}{\partial z^2}$$

$$\frac{\partial V}{\partial r} = \frac{-1}{2}\frac{\partial^2 V}{\partial z^2}r \tag{6}$$

As a matter of fact, the potential $V(z,r)$ remote from the axis can be expressed in terms of the potential $V_o(z)$ on the axis as

$$V = \frac{1}{\pi}\int_0^{\pi} V_0(z + ir\cos\theta)\,d\theta \tag{7}$$

If we could introduce charges into our lens, Laplace's equation would no longer hold and we would have more freedom of design. The methods proposed for the introduction of charges comprise the use of free charges (space charge) which are largely uncontrollable, and the use of curved grids, which do more damage than good. In other words, the cures are worse than the disease.

Similar limitations apply to magnetic lenses, and in the end we find that because of the simplest form of aberration, spherical aberration, best definition is achieved in electron microscopes with *f* numbers of 100 or greater, while the *f* number of a light microscope objective corrected for spherical aberration and other defects as well may be around unity. Thus the electron microscope is severely handicapped, and this handicap is overcome only because electron waves are much less than 1/100 the length of light waves.

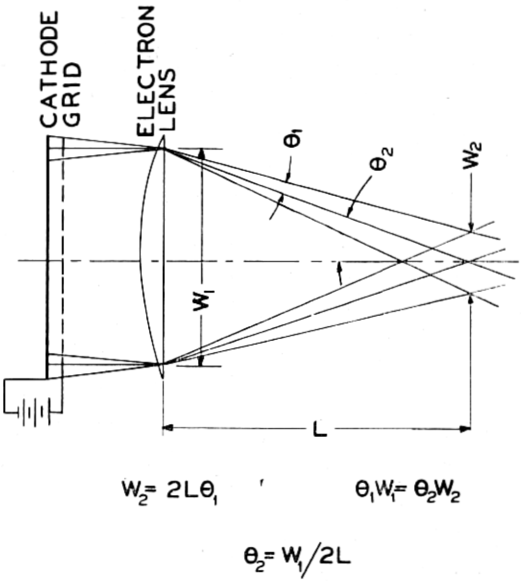

Fig. 2—Approximate relation between beam size and angular spread.

Thermal Velocities of Electrons

In many electron-optical systems, and particularly in such devices as cathode ray tubes, it is desirable to focus an electron beam into a small area, so as to produce a very small spot on a fluorescent screen, or to pass a considerable current through a small aperture. We might think at first that if our focusing system were good enough, that is, if it had very small aberrations, we could focus a current from a cathode of given area into as small a space as we desired. This, unfortunately, is not so. The obstacle is the thermal velocities of the electrons emitted by the cathode.

A simple example will show the sort of thing we should expect to take place. Figure 2 shows a plane cathode and near to it a positive grid so fine as to cause no appreciable deflections of the electrons which pass through it. Farther on we have an aberrationless electron lens designed to focus the electron stream at a spot a distance L beyond it. The electrons will leave the cathode with some slight sidewise velocity components; so, electron paths will pass at several angles through a given point on the lens. The lens will bend these paths approximately equally, and hence we can see that at the point where the beam is narrowest it will still have some appreciable diameter W_2.

Now consider the beam at the lens. Suppose that through a given point all the paths lie within a cone of half angle θ. Then the width W_2 is approximately

$$W_2 = 2L\theta_1 \tag{8}$$

We can also see that the paths at W_2 will lie within an angle approximately

$$\theta_2 = W_1/2L \tag{9}$$

Hence we see that approximately

$$\theta_1 W_1 = \theta_2 W_2 \tag{10}$$

In other words, we can have a small spot through which electrons pass over a wide angular range, or we can have a broad beam in which all paths are nearly parallel, but we can't have a narrow spot and nearly parallel rays.

We see that the actual width of spot will depend on the thermal velocities, which are proportional to the square root of the cathode temperature, and on the forward velocity, which is proportional to the square root of the accelerating voltage. By using more involved arguments we discover that for any point in an electron stream, where the beam is wide, narrow, or intermediate, the current in an arbitrary direction chosen as the x direction can be expressed[4,*]

$$dj = \frac{4kT}{\pi m} j_0 v_x \epsilon^{(1/kT)(eV - mv^2/2)} \, dv_x \, dv_y \, dv_z \tag{11}$$

$$v = v_x^2 + v_y^2 + v_z^2$$

$$\text{when } v > \sqrt{2eV/m}; \tag{12}$$

$$\text{or } dj = 0 \tag{13}$$

$$\text{when } v < \sqrt{2eV/m} \tag{14}$$

Here j_o is the cathode current density, V is voltage with respect to the cathode, T is the absolute temperature of the cathode in degrees Kelvin, and v_x, v_y, and v_z are the three velocity components; dj is the element

* This expression neglects the effects of electron collisions, which may actually make the current density smaller.

of current density carried by electrons which have velocity components about v_x, v_y, v_z, lying in the little range of velocity $dv_x\, dv_y\, dv_z$.

The reason for restriction (12) is that if an electron starts with zero thermal velocity from the cathode, it will attain the velocity given by the right side of (12) by falling through the potential drop V. As electrons cannot have velocities smaller than this, we have (13) and (14).

By integrating (11) with appropriate limits we obtain a more specialized but very useful expression

$$j < j_m = j_o\left(1 + \frac{11600V}{T}\right)\sin^2\theta \tag{15}$$

For usual values of voltage, unity in the parentheses is negligible, and we can say that if all the electron paths approaching a given point in an electron beam lie within a cone of half angle θ, the current density j at that point cannot be greater than a limiting value j_m which is proportional to the

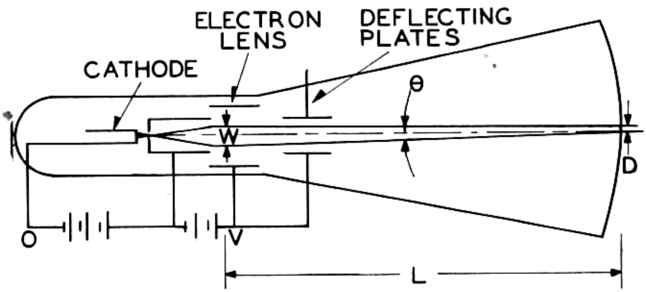

Fig. 3—Parameters important in determining spot size in a cathode ray tube.

cathode current density, to the voltage, to $\sin^2\theta$, and inversely proportional to the cathode temperature.

Let us see what this means in some practical cases. Figure 3 shows a cathode ray tube. The electron stream has a width W at the final electron lens, and is focused on a screen a distance L beyond the lens. The half angle of the cone of rays reaching the screen cannot be greater than

$$\sin\theta = \theta = W/2L \tag{16}$$

Suppose the spot must have a diameter not greater than d. Let the spot current be i. Then from (15),

$$j = \frac{4i}{\pi d^2} < j_0\left(1 + \frac{11600V}{T}\right)(W/2L)^2,$$

$$i < \frac{\pi d^2}{4} j_0\left(1 + \frac{11600V}{T}\right)(W/2L)^2. \tag{17}$$

Thus if for a given spot size we want to increase the spot current, and if we are limited to a given cathode current density because of cathode life, we must make V larger, W larger or L smaller.

Making W larger increases both lens and deflection aberrations. Making L smaller means that for a given linear deflection we must increase the angular deflection, and this too tends to defocus the spot. Because of these limitations, it is necessary to avail ourselves of the remaining variable and raise the operating voltage V.

Another illustration, perhaps a little more subtle, of the effect of thermal velocities, lies in the analysis of the properties of a type of vacuum tube amplifier known as the "deflection tube". In such a device, illustrated in Fig. 4, an electron stream from a cathode is accelerated and focused by a lens and deflected by a pair of deflecting electrodes so as to hit or miss an output electrode. Such a device may be used as an amplifier.

Now it is obvious that as the output electrode on which the beam is focused is moved farther away from the deflecting plates, a given deflecting voltage will produce a greater linear deflection of the beam at the output.

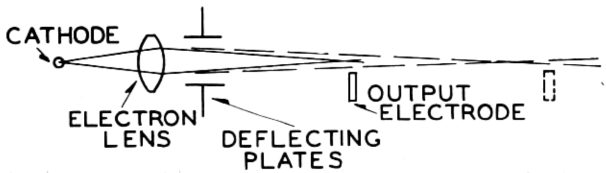

Fig. 4—Amplifying tube making use of electron deflection.

As this at first sight seems desirable; it has been seriously suggested not only that this be done, but that an elaborate electron optical system be interposed between the deflecting plates and the output electrode to amplify the deflection.

The merit of a deflection tube is roughly measured by the deflecting voltage required to move the beam from entirely missing the output electrode to entirely hitting the output electrode, and, of course, moving the output electrode farther away or putting lenses between the deflecting plates and the output electrode doesn't reduce this voltage at all. As we improve the deflection sensitivity by these means, we simply increase the spot size at the same time. Focusing our attention on the beam between the deflecting plates, we appreciate at once that the electron paths through each point will be spread over some cone of half angle θ, and that to change from a clean miss to a clean hit we must deflect the electrons through an angle of at least 2θ, regardless of what we do to the beam afterwards.

Returning for a moment to equation (15), we see that it says the current density can be less than a certain limiting value depending on θ. Yet expression (15) was obtained by integrating a supposedly exact expression. What does this inequality mean?

The answer is that for the current to have the limiting value, electrons of *all allowable velocities* must approach *each part* of the spot from *all angles* lying within the cone of half angle θ. When the average current density in the spot is less than the limiting current density, the possibilities are

(a) Electrons are approaching each point in the beam from all angles, but along some angles only electrons which left the cathode with greater than zero velocity can reach the spot.

(b) Electrons leaving the cathode with all velocities can reach the spot, but at some portions of the spot electrons don't come in at all angles within the cone angle θ.

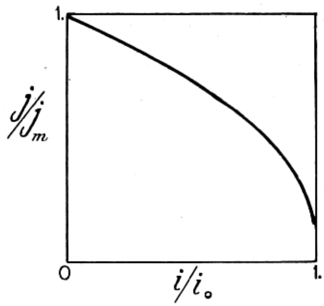

Fig. 5—Relation between nearness of approach to limiting current density and fraction of current utilized.

Thus, we can have less than the limiting current either because electrons do not reach the spot with all allowable velocities or from all allowable angles. Of course both factors may operate.

We can easily see how lens aberrations, which we know are present in all electron-optical systems, can prevent our attaining the limiting current density. There is a more fundamental limitation, however. It can be shown that even with perfect focusing, we must sort out and throw away part of the current in order to approach the limiting current density, and we can even derive a theoretical curve for the case of perfect focusing relating the fraction of the limiting current density which is attained to the fraction of the cathode current which can reach the spot. Figure 5 shows such a curve which applies for voltages higher than, say, 10 volts.

Usually, the failure to approach the limiting current density is chiefly caused by aberrations, and in ordinary cathode ray tubes the current density in the spot may be only a small fraction of the limiting value. A very close approach to the limiting current density has been achieved in a special cathode ray tube designed by Dr. C. J. Davisson of the Bell Telephone Laboratories.

When we become thoroughly convinced that these equations expressing the effects of thermal velocities very much cramp our style in designing electron-optical devices, as good engineers we wonder if there isn't, after all, some way of getting around them. I don't think there is. The suggestion illustrated in Fig. 6 is a typical example of such an attempt. We know that in a strong magnetic field electrons tend to follow the lines of force. Why not use a very strong magnetic field with lines of force approaching the axis at a gentle angle to drag the electron stream toward the axis?

An electron off axis traveling parallel to the axis certainly will be dragged inward by such a field. The catch is that the field pulls the electron in because it makes the electron spiral around the axis. As the beam converges and the field becomes stronger, the pitch of each spiral decreases and the angular speed of each electron increases. Finally, if the field is strong enough, all the kinetic energy of the electron is converted from forward

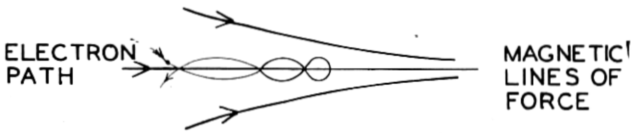

Fig. 6—Reflection of an electron by a magnetic field with strongly converging lines of force.

motion to revolution about the axis; the electron ceases to move into the field and bounces back out. It may be some small consolation to know that very high-current densities can be achieved by this means, but only because in their flat spiralling the electrons approach a spot at much wider angles with the axis than the small inclination of the lines of force.

Space Charge Limitations

In electron beam devices using reasonably large currents, the space charge of the electrons is a very serious source of trouble both in complicating design of the devices and in limiting their performance.

Let us begin our consideration right at the electron gun, the source of electron flow in many devices such as cathode ray tubes and certain high-frequency tubes. Electron guns are sometimes designed on the basis of radial space charge limited electron flow between a cathode in the form of a spherical cap of radius r_o and a concentric spherical anode a distance d from the cathode. It can be shown that by use of suitable electrodes external to the beam, radial motion can be maintained between cathode and anode along straight lines normal to the cathode surface. A hole in the anode electrode will allow the beam to emerge from the gun. Because of the change in

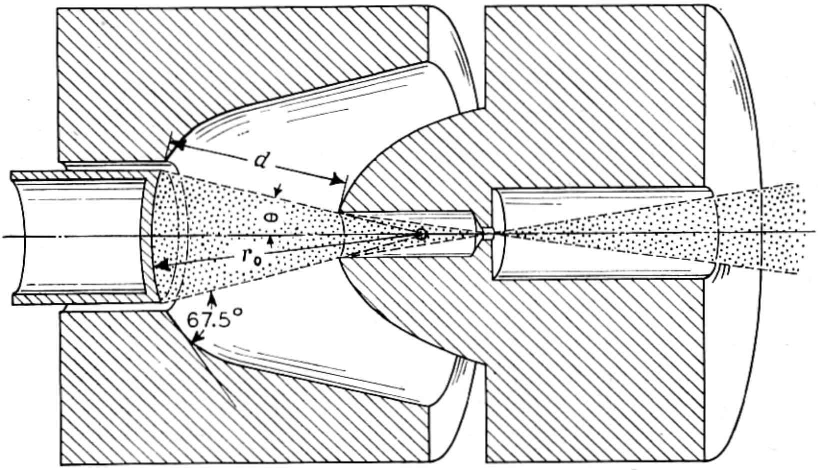

Fig. 7—Electron gun utilizing rectilinear flow.

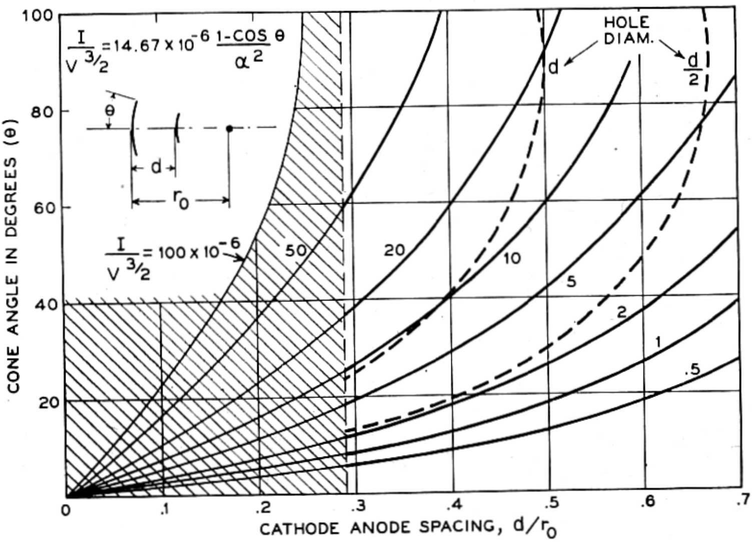

Fig. 8—Relation between perveance, angle of cone of flow, and cathode-anode spacing.

field near the hole, the hole acts as a diverging electron lens.[11] Figure 7 illustrates such a gun.[15] The curves shown in Fig. 8 relate to this sort of electron gun. They are plots of a factor called the perveance, which is defined as

$$P = I/V^{3/2} \tag{18}$$

(that is, current divided by voltage to the 3/2 power) as a function of θ, the half angle of the cone of flow, and d/r_o, the ratio of cathode-anode spacing to cathode radius. In getting an idea of the meaning of the curves, we may note that a perveance of 10^{-6} means a current of 1 milliampere at 100 volts. It is obvious from the curves that to get very high values of perveance, that is, high current at a given voltage, θ must be large and the cathode-anode spacing must be small. Making θ large means that electrons approach the axis at steep angles; aberrations are bad and the beam tends to diverge rapidly beyond crossover. Moving the anode near to the cathode means that the hole which must be cut in the anode to allow the beam to pass through must be large, and cutting such a large hole in the anode defeats our aim of getting higher perveance; we can't pull electrons away from the cathode with an electrode which isn't there. Further, for ratios of spacing to cathode radius less than about .29, the lens action of the hole in the anode causes the emerging beam to diverge, which would make the gun unsuitable for many applications.

When we build guns for small currents at high voltages, such as cathode ray tube guns, space charge causes little trouble; when we try to obtain large currents at lower voltages, we find ourselves seriously embarrassed.

Suppose we now turn our attention to the effect of space charge in beams when the beam travels a distance many times its own width. Consider, for instance, the case of a circular disk forming a space charge limited cathode. Suppose we place opposite this a fine grid, and shoot an electron stream out into a conducting box, as illustrated in Fig. 9a. We immediately realize that there will be a potential gradient away from the charge forming the beam. In this case, the gradient will be toward the nearest conductor; that is outwards, and the electron beam will diverge.

How can we overcome such divergence? One way would be to arrange the boundary conditions in such a fashion that all the field would be directed along the beam instead of outwards; this might be done by surrounding the beam by a series of conducting rings and applying to them successively higher voltages as in 9b, the voltages which would occur in electron flow between infinite parallel planes with the same current density. Another way in which the same effect may be achieved is through use of specially shaped electrodes outside of the beam, as shown in Fig. 9c.[11] In maintaining parallel flow by these means, the electric field due to the electrons acts along the beam, and increases continually in magnitude with distance from the cathode. We can in fact calculate the potential at any distance along the beam by the well known Child's law equation

$$I = 2.33 \times 10^{-6} A V^{3/2}/x^2$$
$$V = 5{,}690 x^{4/3} I^{2/3}/A^{2/3} \tag{19}$$

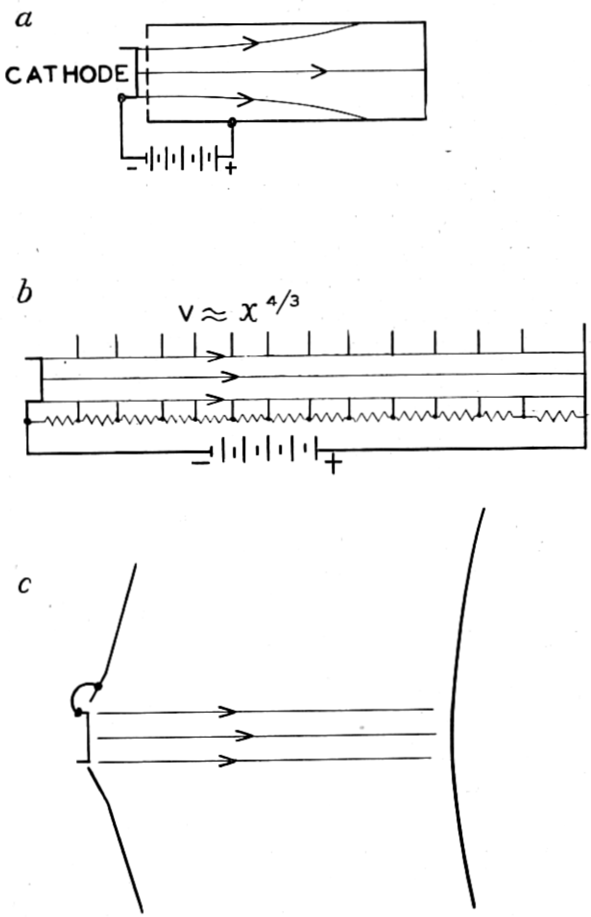

Fig. 9—Avoiding beam divergence by means of a longitudinal electric field.

Here V is the anode voltage, x the cathode-anode spacing, I the current in amperes and A the cathode area.

Suppose we take as an example

$$A = 1 \text{ cm}^2$$
$$I = .01 \text{ amp.}$$
$$x = 10 \text{ cm}$$

Then

$$V = 5{,}700 \text{ volts}$$

Thus to maintain parallel motion of the modest current of 10 milliamperes spread over an area of one square centimeter requires 5,700 volts. Moreover, the requirement of distributing this voltage smoothly along the beam would make it very difficult to put the beam to any use.

One means for mitigating the situation is to use an electron lens and direct the beam inward. Of course, the beam will eventually become parallel and then diverge again, but by this means a fairly large current can be made to travel a considerable distance. Some calculations made by Thompson and Headrick[12] cover this type of motion, with an especial emphasis on the problem in cathode ray tubes, in which the currents are moderate.

In order to confine large currents into beams, an axial magnetic field is sometimes used, as shown in Fig. 10 Here a cathode-grid combination shoots a beam of electrons into a long conducting tube. A long coil around the tube produces an axial magnetic field intended to confine the electron

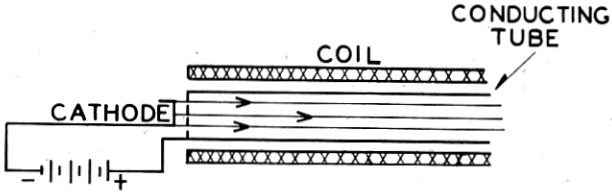

Fig. 10—Avoiding beam divergence by means of a longitudinal magnetic field.

paths in a roughly parallel beam. The radial electric field due to space charge will cause the beam to expand somewhat and to rotate about the axis. As the magnetic field is made stronger and stronger, the electrons will follow paths more and more nearly straight and parallel to the axis. For a given current and voltage, there is one sort of physical limitation in the strength of magnetic field we need to get a satisfactory beam. It is another effect that I wish to discuss.

Suppose we have a very strong magnetic field, in which the electrons travel almost in straight lines. We know, of course, that the radial electric field is still present, and this means that the potential toward the center of the beam is depressed; this in turn means that the center electrons are slowed down. This slowing down of course increases the density of electrons in the center of the beam. The result is that if for some critical voltage or speed of injection we increase current beyond a certain value, the process runs away, the potential at the center of the beam drops to zero, and another type of electron flow with a "virtual cathode" of zero electron velocity at the center of the beam is established. Thus, although the magnetic field has overcome the diverging effect of the space charge, we still have a space charge limitation of the beam current. C. J. Calbick has calculated the value of this limiting current.[13] If the beam completely fills a conducting tube at a potential V with respect to the cathode, the limiting beam current is independent of the diameter of the beam and is

$$I = 29.3 \times 10^{-6} V^{3/2} \qquad (20)$$

If the beam diameter is less than that of the conducting tube, the limiting current is lower.

But perhaps we can completely overcome the effects of space charge. Suppose we put a very little gas in the discharge space. Then positive ions will be formed. Any tendency of the electronic space charge to lower the potential and slow up the electrons will trap positive ions in the potential minimum and so raise the potential. Thus the gas enables us to get rid of the the slowing up effect of the space charge as well as its diverging effect.

Before we congratulate ourselves unduly, it might be well to make sure about the stability of an electron beam in which the electronic space charge is neutralized by heavy positive ions. Langmuir and Tonks, in their work on plasma oscillations, introduced a concept, extended later by Hahn and Ramo, which enables us to investigate this problem. The concept is that of space charge waves. It is found that in a cloud of electrons whose net space charge is neutralized by heavy, relatively immobile positive ions, small disturbances of the electron charge density produce a linear restoring force; and this, together with the mass of the electrons, makes possible a type of space charge wave which may be compared roughly with sound waves, although much of the detailed behavior of space charge waves is quite different from that of sound waves. We may express a disturbance in an electron beam in terms of these space charge waves and then examine the subsequent history of the disturbance as a function of time. This has been done[14] and the perhaps surprising result is that even when the electronic space charge is neutralized by heavy positive ions, the flow tends to collapse if the current is raised above a limiting value

$$I = 190 \times 10^{-6} V^{3/2} \qquad (21)$$

It is true that this current is 6.5 times the limiting current in the absence of ions, but it is a limit nevertheless.

If this limit in the presence of ions seems unnatural, perhaps we should recall a mechanical analogy. Consider a vertical long column subjected to a load F. If we subject it to a sidewise force αF proportional to F, as shown in Fig. 11a, the behavior on increasing F will be a gradual deformation (analogous to the space charge lowering of potential in the absence of ions) ending in collapse. However, even if, as in 11b, there is no sidewise loading and no bending during loading, we know from Euler's formula that beyond a certain loading the column will still collapse. This behavior is analogous to that of an electron beam in which the electronic space charge is neutralized by positive ions and there is no depression of potential in the beam.

This space charge limitation either in the presence or absence of ions allows the passage of quite a large current through a tube, as the table below will show:

Voltage	Current, amperes, no ions	Current, amperes, ions
1000	.927	6.01
100	.029	.190
10	.009	.060

We might therefore feel that the space charge is disposed of in a practical sense, and so it is in many cases.*

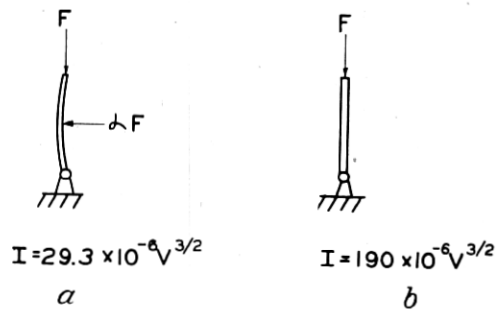

Fig. 11—Comparison of limiting stable beam currents with and without positive ions.

Power Dissipation Limitations

Having talked about various limitations imposed by wave effects, aberrations, thermal velocities and space charge on the electron flow in the beam itself, I want to close by discussing briefly a topic which seems hardly included in electron ballistics but yet is vital to any application in that field. I refer to the problems associated with power dissipation when electrons strike something and stop. This is a good deal like the problem imposed by suddenly coming down to earth while studying the sensations of a free fall. It is inevitable and may be fatal unless satisfactory provision is made for the dissipation of kinetic energy.

What I want chiefly to bring out are the consequences of scaling a given electronic device down in size. If we change the size of each part of an

* It appears that in many gas discharges, including those in which plasma oscillations are observed, the current is too high to allow persistence of the homogeneous flow upon which the plasma oscillation equations are based.

electron device in the ratio R, if we keep all voltages the same, and if we change all magnetic fields in the ratio $1/R$, electron current will remain the same (provided the cathode is still capable of giving space charge limited emission). Electron paths will remain exactly similar, though smaller; the power into the electron beam will remain the same, but what will happen to the power dissipation capabilities of the device and what will happen to the temperature?

In a device cooled by radiation alone and with cool surroundings, the radiating area varies as R^2, and since the radiation per unit area varies as T^4, the temperature will vary as $R^{-\frac{1}{2}}$.

In considering a case of cooling by conduction alone, think of a rod carrying a certain amount of power away. If all the dimensions of a rod are changed by a factor R, the length will be changed by a factor R, the cross sectional area will change by a factor R^2, and if the thermal conductivity

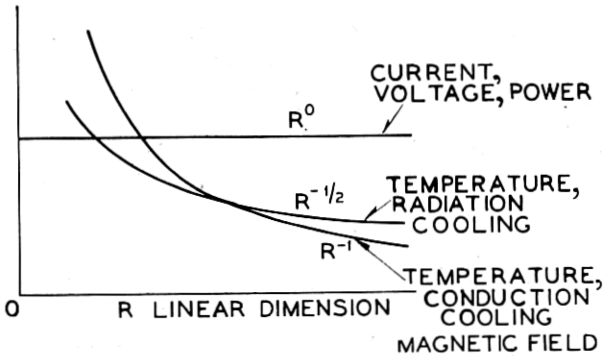

Fig. 12—Variation of magnetic field and temperature in scaling an electronic device.

remains constant the temperature will vary as R^{-1}. This is a faster rate of variation than in the case of cooling by radiation, and hence as the system is scaled to a smaller and smaller size, cooling by conduction will become negligible and radiation cooling only will remain effective and will determine the temperature.

Figure 12 gives an idea of the variation of various quantities discussed.

We want to make electronic devices smaller for a number of reasons; perhaps chiefly to reduce transit time and so to secure operation at higher frequencies. In doing this, we encounter the fundamental limitation of reduced power dissipation capabilities and increased temperature. What is the trouble? We have scaled everything. Or have we? The answer is, we have not. The electrons, atoms, and quanta are still the same size. Had we been able to scale these, we should have increased the heat conductivity and the radiating power of our device, and all would have been well. As it is, if we make a tube for given power smaller and smaller, using the most refractory materials available we eventually reach a size of tube which will, despite our best efforts, melt, thaw, and resolve itself into a dew.

CONCLUSION

Perhaps after these somewhat gloomy words concerning physical limitations in electron ballistics, you may wonder how it is at all possible to surmount the difficulties mentioned. It certainly is not easy; all electronic devices represent compromises of one sort or another between fundamental physical limitations of electron flow on the one hand and structural complications on the other. In working with vacuum tubes one is perhaps troubled more by physical limitations, difficulties of construction, inadequacy of materials and the lack of quantitative agreement between complicated phenomena and relatively simple theories than in any other part of the electric art. It is for this reason that a friend of mine twisted an old aphorism into a new one and said, "Nature abhors a vacuum tube".

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