# Electromagnetic Waves

A Textbook* (530 pages, $7.50 net) by S. A. Schelkunoff, published by D. Van Nostrand, Inc., New York City, 1943.

THIS new addition to a well-known series has been awaited with much interest by all those acquainted with Dr. Schelkunoff's contributions to propagation theory, and it will be found that their expectations have been entirely fulfilled. This monumental piece of work is equally remarkable for the originality and consistency of its approach as for the wealth of information contained in its five hundred densely packed pages.

The author's systematic use of the harmonic oscillation, with complex variables and coefficients, is in line with the marvelous development which has occurred in the communication field during the last fifty years. Alternating current theory, then acoustics, then vibrational mechanics successively dropped the differential equations which physics offered as a basis and systematically restricted themselves to harmonic oscillations. This has resulted in the replacement of the differential operator by $i\omega$, leading to a tremendous simplification of steady-state analysis, which has been reduced to the calculation of amplitude ratios and phase differences. The genuinely difficult problems have not disappeared for all that but are now relegated to Fourier or Laplace transform theory, and it has become apparent that an enormous field of application can be covered by purely algebraic processes.

Not the least advantage of this method has been the unification brought into the three chapters of technical science mentioned above. Electrical impedances gave the model after which acoustical and mechanical impedances were fashioned; and mixed mutual impedances, thereafter, made it possible to write the equations of electro-mechanical or acoustico-mechanical transducers. There was an exciting era of intense development in this field during the twenties; and it was amusing to hear at that time, and even a good deal later, irate die-hards denouncing "impedances" with bitter irony or viewing with alarm the spread of "analogies."

Dr. Schelkunoff has set about to carry this point of view into Electromagnetic Theory, and it may well be that his will be the honor of having brought into the fold of harmonic oscillation theory the last chapter of Physics which still had to be incorporated. (One might think of Optics, but of course half of the book is really Optics.) Having given, in the first pages of his fourth

*This review by P. Le Corbeiller is reprinted from *Quarterly of Applied Mathematics*, Vol. 1, No. 2 by permission of the editors. Mr. Le Corbeiller, until coming to the United States where he has joined the faculty of Harvard University, was an engineer with the Administration Française des Postes, Telegraphes et Telephones.

chapter, a short and quite personal derivation of Maxwell's equations (1–15, p. 69), Dr. Schelkunoff without taking breath adds immediately: "Since we are concerned primarily with fields varying harmonically with time, we replace the instantaneous field intensities and current densities by the corresponding complex variables and write Maxwell's equations as follows:

$$\int E_n\, ds = -\iint i\omega\mu H_n\, dS - \iint M_n\, dS,$$
$$\int H_n\, ds = \iint (g + i\omega\epsilon) E_n\, dS + \iint J_n\, dS." \qquad (1\text{–}16)$$

Thus the sacrosanct Maxwell equations are swept away with movie-like swiftness, and instead we have the steady-state equations of a medium characterized by a distributed series impedance $i\omega\mu$ and a distributed shunt admittance $g + i\omega\epsilon$ (p. 81).

The analogy with a transmission line whose series inductance is $\mu$, shunt conductance $g$ and shunt capacitance $\epsilon$, all taken per unit length, is inescapable (p. 243). In particular the above primary constants simply beg to be transformed into the familiar secondary constants of transmission line theory; here the intrinsic propagation constant $\sigma$ and the intrinsic impedance $\eta$ are defined by

$$\sigma = \sqrt{i\omega\mu(g + i\omega\epsilon)}, \qquad \eta = \sqrt{\frac{i\omega\mu}{g + i\omega\epsilon}} \qquad (9\text{–}1)$$

(p. 81) ($\sigma$ is in neper/meter, $\eta$ in ohms; the book is written in MKS—p. 60). For free space we shall have $g = 0$, and the following numerical values of the fundamental constants (p. 82):

impedance of free space $\eta_0 \approx 120\pi$ ohms, (9-4)
characteristic velocity $v_0 \approx 3.10^8$ meters/second.

* * *

Surprising as it may appear to transmission engineers and sound engineers, who daily handle their respective characteristic impedances $Z_0$ or $\rho c$, there still are very competent physicists who balk at the idea of free space having a characteristic impedance of about 377 ohms. Yet, in the words of Professor Ronold W. P. King:[1] "The existence of such a characteristic resistance for electromagnetic effects is just as mysterious, but not more so, than the existence of the finite velocity $v_0$." Dr. Schelkunoff explains very well how this constant could have been overlooked by the builders of the classical theory: "The physicist concentrates his attention on one particular wave: a wave of force or a wave of velocity or a wave of displacement. His original differential equations may be of the first order and may involve both force and

[1] Mimeographed "Notes on Antennas" for the course of Electronics and Cathode Ray Tubes (Eng. 270), Harvard University.

velocity; but by tradition he eliminates one of these variables, obtains a second order differential equation in the other and calls it the 'wave equation.' Thus he loses sight of the interdependence of force and velocity waves . . ." (p. vii). Still, it is surprising to see that one has started with two constants $\epsilon_0$ and $\mu_0$, and recognizing the fundamental importance of their product, yet has not enquired about their ratio.

Then, the reader will ask, how can the Theory of Relativity give a leading role to the velocity of light and not mention the impedance of free space. Has Einstein no use for $\eta_0$? Well, he has, and he has not. First, an essential point in Special Relativity is the merging of the magnetic and the electric fields into one skew-symmetrical tensor. When doing this in the MKS system, homogeneity requires the use of the components of $E$ and of $\eta_0 H$; but the factor $\eta_0$ is not apparent, for instance, in the equations on p. 44 of "The Meaning of Relativity" (by A. Einstein, Princeton Univ. Press, 1923) which uses a system of units in which $\eta_0 = 1$. Secondly, if we try to connect the universal constant $\eta_0$ with other members of this interesting family, we find that $\eta_0$ times a (charge)[2] has the dimensions of "action," and more precisely that

$$\eta_0 e^2 = \frac{2h}{137}$$

($e$ = charge of the electron, $h$ = Planck's constant). We see from this that there is more to $\eta_0$ than appears in Special Relativity, the first step in the successive Einsteinian extensions of Maxwell's theory.

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We have dealt at length with this question of the "impedance of free space" because it exemplifies the spirit of the whole work. It occurs in the course of a short but apt presentation of the "Fundamental Electromagnetic Equations" (Chapter IV), immediately applied to harmonic oscillations. The book as a whole is devoted not to Electromagnetism in general but, as specified in the title, to *Electromagnetic Waves*.

Three preliminary chapters introduce the more advanced mathematical tools which will be used, but sparingly, in what follows: such topics as contour integration, Bessel and Legendre functions. Chapter V is a short and original presentation of Network Theory.

The central part of the book begins with Chapter VI, "About Waves in General," a sort of preview of the questions which will be treated in detail later, during which we are introduced to radiation from given currents, propagation along wave guides, and to such general tools as electric and magnetic current sheets, the method of images and conformal representation.

In the following four chapters, we meet the most thorough treatment

[2] See *Quarterly of Applied Mathematics*, 1, 78 (1943).

available of the propagation of waves, guided or bounded, in one, two and three dimensions. It is impossible to do justice here to the richness of the material, which must have cost tremendous labor and which is in great part taken from the author's own publications. We find in Chapter IX, however, classical problems of Fresnel optics, adroitly adapted to contemporary radio needs. Chapter XI is a relatively short treatment of antenna theory, principally of conical antennas, and in the last chapter we return to wave guides and solve various problems involving discontinuities, even to an iris or a transversal wire. This subject is still under development by the author, and the readers of the *Quarterly* have had the benefit of one of its recent extensions.[2]

The specialist in wave propagation has no need to be told of the value of this book; but the reviewer would like to explain to his fellow non-specialists why it is particularly important that they should not miss it. When the results of much present-day research will suddenly be made available, it will be a hard task to catch up, not only with the new knowledge, but still more with the new modes of attack. The borderland between radio and optics is one of the fields from which great things can confidently be expected. Dr. Schelkunoff's book is a great opportunity for those not at present engaged in research to get familiar with methods which they will want to use tomorrow.

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