Design of Two-Terminal Balancing Networks

By K. G. VanWYNEN

This paper describes a simple graphical method for designing a two-terminal network, which will simulate a given line impedance to such a degree that return losses of the order of 25 db or better will be readily obtained. The method is particularly useful in those problems in which a reasonably accurate balancing network is adequate, but a high degree of precision is not required.

General

IT IS the purpose of this paper to describe a graphical method which has been found useful in the design of simple two-terminal networks to simulate the impedance of transmission lines or equipment. The discussion which follows is intended to emphasize the simplicity of the method and the rapidity with which it may be employed to arrive at a solution; it will also indicate the analytical background without attempting to develop or establish the rigor of the procedure involved. A solution can frequently be obtained in a fraction of an hour and it is thought that the graphical analysis will appeal to the pragmatist and the engineer who has a job to do, but very little time in which to accomplish his aim, rather than the person interested in the rigor of the solution.

The problem which is considered may be stated as follows: Design a two-terminal network with the minimum number of elements which will give a desired degree of approximation to a given impedance function $Z(\lambda)$, where $Z(\lambda)$ is a fraction whose numerator and denominator are polynomials in frequency in accordance with the customary usage in such problems.

Origin of Problem

This problem has arisen most generally in providing balancing networks which will give satisfactory return losses against various types of telephone facilities. It is obvious that for a given impedance, $(r + jx)$, at a given frequency there are an infinite number of networks which will satisfy the given impedance. It has also been pointed out that the network which simulates a given impedance function is not unique. Hence there are also a large number of networks which will satisfy a given impedance function.

In designing networks for repeater circuits, it is generally satisfactory if the return loss is equal to or greater than some specified number of db. This somewhat simplifies our problem and permits a double infinity of solutions. A method has been given by Brune[1] for designing such networks, in which it is pointed out that there is no unique solution to the problem of designing a finite two-terminal network and also states that any network which satisfies the impedance function may be considered a satisfactory solution to the problem. It is thought that the method which is given below will provide a solution which makes maximum use of the number of elements employed. That is, it will provide a given return loss with the minimum number of parts.

The required degree of approximation and the frequency range to be covered determine the number of elements required in this solution. In one simple case which will be discussed below in the first example, the approximation between the impedance of a transmission line and a network designed to simulate it is the approximation between the curvature of the impedance function and the arc of a circle.

Generating Function

The method discussed here differs from that outlined by Brune in that use is made of known generating functions which are added together in series to approximate the total function, similar to the manner in which sine functions may be added to approximate other functions. This series type network can readily be converted to the ladder type by well known network equivalence theorems and the solution will then have the Stieltjes fraction form pointed out by Fry[2] and Cauer.[3]

The generating function used here is an impedance consisting of a resistance in parallel with a pure reactance or a special case of this. This function plus a real corresponds to a bilinear transformation, the properties of which have frequently been discussed elsewhere. This particular configuration, for instance, has been pointed out both by Brune and by Guillemin[3] at M.I.T. and a discussion of the bilinear transformation has been given by C. W. Carter[4] of the Bell Telephone Laboratories. The series addition of such generating functions is similar to the form given in Foster's reactance theorem except that there only pure reactances are dealt with. The solution can also be worked out with admittances, but will not be discussed here since the average engineer is more accustomed to dealing with impedances.

In many problems, particularly those involving dissipative transmission

[1] *Jour. Math. & Physics*, Vol X, 1930–1931.
[2] *Bull. Am. Math. Soc.*, 35, 1929.
[3] Guillemin—Vol. II.
[4] *B.S.T.J.*, July 1925.

lines, the entire impedance function is found in the fourth quadrant of the complex plane. When this is so, the generating function is reduced to a resistance in parallel with a condenser.

Graphical Representation of Functions

As a first step in utilizing the graphical procedure, it will be advisable to acquire some familiarity with the generating function in its general form

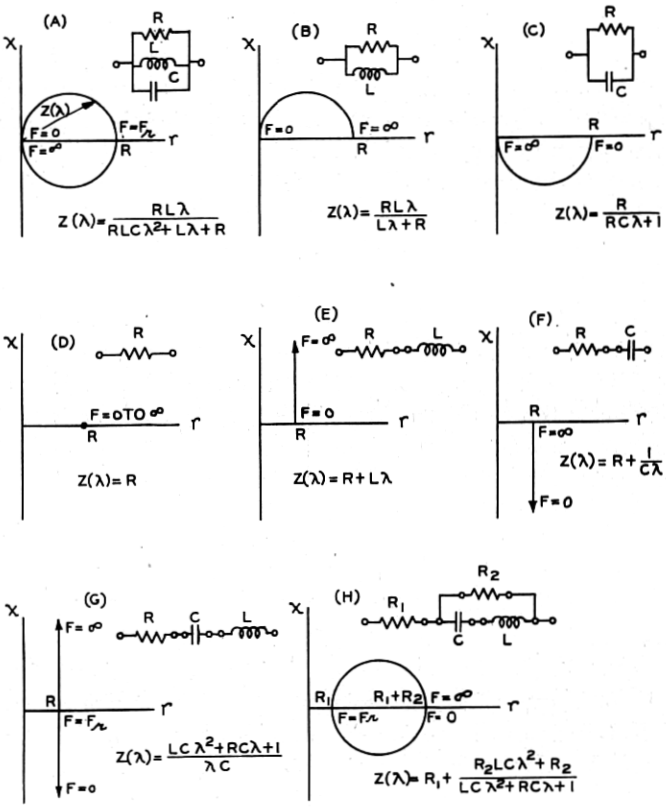

Fig. 1.—The impedance loci, $Z(\lambda)$, for several networks.

and some of its special cases. Plots of various cases are given in Figs. 1(a) through 1(h) together with the network configuration and the impedance function thereof. Obviously the summation of the properly selected generating functions corresponds to the addition of the partial fractions derived by Brune's method. For an accurate solution these partial fractions when combined should approximate the given $Z(\lambda)$.

Figure 1(A) shows the impedance locus of the parallel R, L, C generating function as frequency varies from 0 to ∞. At 0 frequency $Z(\lambda) = 0 + j0$ and at ∞ $Z(\lambda) = 0 - j0$. The locus is a circle and crosses the real axis at R and the frequency at which L and C are anti-resonant. The special cases will be readily apparent and without further discussion attention will be shifted to Fig. 1-c which is the generating function applied to obtain solution of the examples listed below, all of which are located in the fourth quadrant.

The impedance of this function (Fig. 1-c) at zero cycles is a real and has the value R, and at infinite frequency its impedance is $0 - j0$. The locus traced by this function in the fourth quadrant of the complex plane as f varies from zero to infinity is a semicircle of radius $R/2$ whose center is at $R/2$ on the axis of reals. Obviously, the impedance for any given frequency depends only on C when R has been fixed.

One of the most useful networks for voice frequency work is that in which two such functions are added together but the second function is the special case in which $C = 0$. We then have a network which consists of a resistance R_1 in series with the parallel combination R_2 and C_2, and is represented by the semicircle just described but displaced to the right of the origin by the distance R_1. This form corresponds to a special case of the bilinear transformation previously mentioned.

As stated earlier a given impedance function can be obtained from a large number of networks but when the impedance is to be simulated for a limited frequency range, such as the voice band, the selection of the best network is reduced to sorting through a relatively small range of networks to select that one which is the best compromise for the given conditions. This then is a restatement of the problem: *To find the network having the minimum number of circuit elements which will give the desired approximation to a specified impedance function.*

The other sections of Fig. 1 will be evident upon analysis.

Method of Solution

The first step to be followed in finding the solution to a given problem is to plot in the complex plane the locus traced by the given impedance function as the frequency varies over the range which is to be considered and to mark the frequency at those impedances which are essential to the problem. Having done this, the next step is to draw a semicircle with the center on the real axis such that an arc of the semicircle approximates part or all of the locus of the impedance function. In many cases this semicircle is a sufficiently good approximation but where it is not, it will be necessary to add other functions. The examples given below are illustrative of cases requiring three-, four- and five-element networks.

Example 1—104 Mil Open Wire

To demonstrate the method we will now consider the design of a network which simulates a 104-mil copper open-wire line with 12 in. spacing and CS insulators. The impedance function for this particular facility is plotted on Fig. 2. It is perhaps rather obvious that this locus can readily be approximated by a semicircle whose center is on the real axis and whose intercept on the real axis is not at the origin. Such a semicircle has been drawn, but it is recognized that the one shown is not unique, for it would be possible to draw several others which might do equally well. However, they

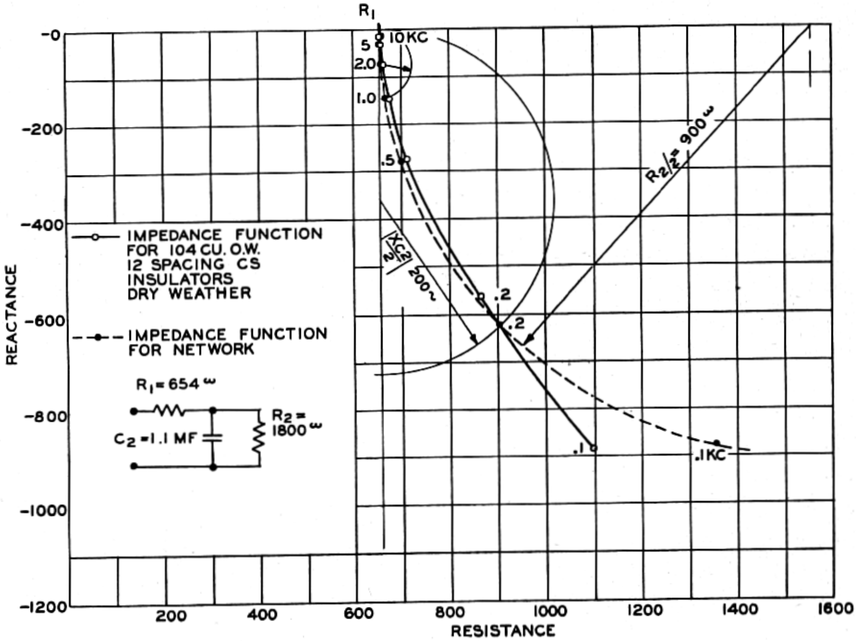

Fig. 2.—Graphical design of two-terminal balancing network for 104-mil. copper open wire.

would in general be fairly close to that shown. Having selected this semicircle, which approximates the impedance function, it is evident that a network consisting of a resistance, R_1, in series with a parallel R_2C_2 combination will provide a reasonable approximation above 200 cycles. The series resistance R_1, is, of course, the left-hand intercept of the semicircle and the R axis and the parallel resistance, R_2, is the diameter of the semicircle. There remains, then, the problem of determining C_2 which obviously governs the distribution of frequencies along the semicircular locus. If C_2 is very small, the 1000 cycle impedance will be near the right-hand end of the locus since

R_2 is controlling and vice versa. The answer as to what value of C should be selected depends on what frequency range we are most interested in approximating closely. Suppose in this case that we say 1000 cycles is the frequency at which we wish to have the best degree of approximation. C_2 will then be determined by drawing a vertical axis passing through R_1 and inscribing a semicircle passing through R_1 and the 1000-cycle impedance of the open-wire line function and having its diameter on the vertical axis. The diameter of this semicircle represents X_C and therefore determines the capacity, C_2, of the parallel combination.

Carrying out the procedure just described it will be seen by reference to Fig. 2, that $X_{C_2} = 145$ ohms at 1000 cycles and therefore $C_2 = 1.1$ mf. The 3-element network thus determined is a resistance of 654 ohms in series with the parallel combination of 1800 ohms and 1.1 mf. By arbitrary choice the 1000-cycle impedance of the line and network are in good agreement. It is now necessary to determine the network impedance at other frequencies in order to compare them against the open-wire line impedance.

As is well known the parallel impedance at any other frequency is the intersection of the corresponding X_C and R_2 semicircles. At 200 cycles $X_C = 725$ ohms. Drawing a semicircle of diameter 725 ohms on the vertical axis through 654 ohms the network impedance is located at the intersection of this semicircle and the R_2 semicircle, i.e., at $900 - j\,620$.

Thus the network impedance locus as a function of frequency may be completely determined over the desired frequency range and compared with the given impedance locus of the open wire.

This may be done visually. If corresponding points on the two loci are close together, the simulation will be a good one and vice versa. If it is found that the simulation is too good at one frequency and not good enough at other frequencies, it will be possible to alter the distribution of frequencies along the locus by changing C_2 or the locus may be shifted by changing R_2 or both C_2 and R_2 may be changed. No specific rule can be stated for this but with a little experience considerable dexterity may be acquired in this sort of juggling and a locus found which will give an approximately constant approximation over a reasonably wide frequency range. As may be seen by referring to Fig. 2, it was found that a network consisting of a 654-ohm resistance in series with the parallel combination of 1800 ohms and 1.10 mf. gives a very good simulation of a 104 mil copper open wire line over the voice range. As is obvious from the graphical method, the simulation rapidly deteriorates below 200 cycles due to departure of the network locus from the impedance locus of the open wire line. If it were necessary to improve this low-frequency simulation, it would be necessary to add further generating functions to the design or compromise at the higher frequencies.

Since this network was intended for use as a balancing network, it was then tested in the laboratory against the open-wire impedances and found to give fairly high return losses as listed in Table I. The corresponding return losses were also computed and tabulated. The impedances are given for both the network and the theoretical line at typical frequencies over the range from 100 cycles to 20,000 cycles.

The impedance function for an open-wire line is given by the equation

$$Z(\lambda) = \left(\frac{R + L\lambda}{G + C\lambda}\right)^{\frac{1}{2}} \tag{1}$$

Expanding this function by the binomial theorem and taking the first approximation and further letting $G = 0$, the impedance function becomes

TABLE I

104 Mil Cu Open Wire, Dry Weather, 12" Spacing, CS Insulators

Freq. Cycles	Impedance				Return Loss of Network vs Line-db	
	Network		Line			
	Rect.	Polar	Rect.	Polar	Measured	Computed
100	1360—j878	$1620/\overline{32.9}$	1101—j883	$1410/\overline{38.8}$	20.9	21.3
200	904—j623	$1097/\overline{34.7}$	865—j562	$1032/\overline{33.0}$	27.4	29.4
300					34.4	
500	699—j281	$754/\overline{21.9}$	712—j273	$764/\overline{21.0}$	37.8	39.6
1000	665—j143	$681/\overline{12.2}$	674—j144	$689/\overline{12.0}$	42.3	43.6
2000	656— j72	$660/\overline{6.3}$	662— j74	$666/\overline{6.4}$	45.2	47.8
5000	654— j28	$654/\overline{2.4}$	658— j32	$659/\overline{2.8}$	43.1	48.4
10000	654— j14.5	$654/\overline{1.0}$	653— j12	$653/\overline{1.1}$	39.5	55.5
20000	654— j7.2	$654/\overline{0.5}$	652— j10	$652/\overline{0.9}$		51.2

$$Z(\lambda) = \left(\frac{L}{C}\right)^{\frac{1}{2}} + \frac{1}{(LC)^{\frac{1}{2}}(2/R)}\cdot\frac{1}{\lambda} \tag{2}$$

Applying the method of Brune, this equation yields a network consisting of 646.4 ohms in series with a condenser of 1.09 mf. It will also be apparent that eq. (2) has the same form as that of Fig. 1(f), i.e.,

$$Z(\lambda) = R_1 + \frac{1}{C_1}\cdot\frac{1}{\lambda}$$

and by a 1 to 1 comparison of terms it is evident that

$$R_1 = \left(\frac{L}{C}\right)^{\frac{1}{2}} \tag{3-a}$$

and

$$C_1 = (LC)^{\frac{1}{2}}\left(\frac{2}{R}\right) \tag{3-b}$$

Including G, the expression for the first approximation of the impedance function may be written in the form

$$Z(\lambda) = \left(\frac{L}{C}\right)^{\frac{1}{2}} \left[1 + \frac{\left(\frac{R}{2L} - \frac{G}{2C}\right)}{\lambda + \frac{G}{2C}} \right] \tag{4}$$

Following Brune's method or noting the correspondence with the impedance function given in Fig. 1(c), it is apparent that the network is a resistance, R_1, of 646.4 ohms in series with a parallel R_2C_2 combination. C_2 is 1.09 mf as before but R_2 computes as 312,000 ohms which is so large compared to 1.09 mf. that the additional resistance provides negligible improvement over the previous network for the voice frequency range.

Obviously then, the analytical method requires at least a second order approximation entailing considerable additional analytical work and computation which will not be carried out here. This points out the advantages of the graphical method; namely, it is rapid, requires no special skill, and gives a reasonably accurate answer.

Example 2—Siral Four Cable—1320 Foot Spacing—6 Milhenry Loading (SP4–1320–6)

In order to indicate the procedure when two complete *RC* regenerating functions are required, another example is given which covers an impedance simulation of a SP4–1320–6 line. A plot of this impedance function is shown on Fig. 3, and it is at once obvious that two semicircular generating functions should give a reasonably good approximation to the given impedance function.

The method of selecting these functions may be somewhat as follows: Consider first the simulation in the low-frequency range, i.e., 200 cycles to 500 cycles. For this region a semicircle may be selected much as in the first example and the one chosen yields a network consisting of $R_1 = 480$ ohms in series with the R_2C_2 parallel combination in which $R_2 = 1460$ ohms. C_2 was found by choosing an X_{C_2} at 500 cycles close to that of the line and from which C_2 was found to be 1.38 mf.

It is evident that to provide high-frequency simulation a condenser must be placed in parallel with $R_1 = 480$ ohms. Its value is determined by the intersection of the R_1 and X_{C_1} semicircles at 10,000 cycles and C_1 is found to be .0161 mf. The construction lines involved in these determinations are shown as light weight solid lines.

Since there are now two impedance functions to be added in series the locus will depart somewhat from the two semicircles. However, the departure will not be great since the effect of C_1 is small at low frequencies,

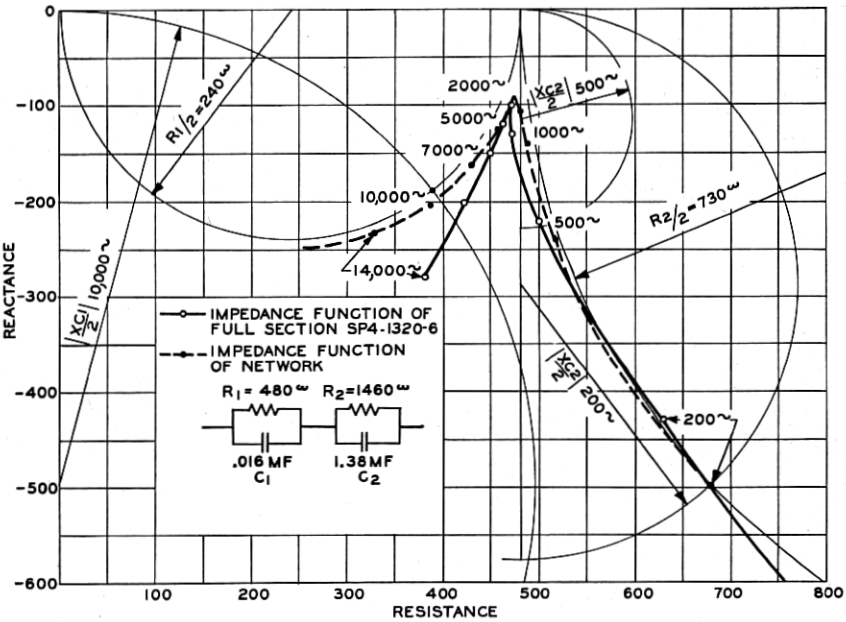

Fig. 3—Graphical design of two-terminal balancing network for spiral four cable.

TABLE II

Spiral Four Cable—1320 Foot Spacing—6 Milhenry Loading—Full Section Termination

Freq. Cycles	Impedance				Return Loss vs Theoretical Line-db		Measured Return Loss vs Artificial* Line-db
	Network		Line				
	Rect.	Polar	Rect.	Polar	Measured	Computed	
100	1046—j713	$1266/\overline{34.3}$	810—j670	$1051/\overline{39.6}$	14.8	19.8	21.8
200	701—j528	$878/\overline{37.0}$	630—j430	$763/\overline{34.3}$	22.8	22.6	22.9
500	516—j239	$569/\overline{24.9}$	500—j220	$546/\overline{23.8}$	33.2	33.0	25.2
1000	488—j139	$507/\overline{15.9}$	470—j130	$488/\overline{15.5}$	37.1	33.9	26.2
2000	479—j108	$491/\overline{12.7}$	470—j100	$481/\overline{12.0}$	41.2	38.1	26.6
3000							26.2
5000	456—j125	$473/\overline{15.3}$	460—j120	$475/\overline{14.6}$	39.9	43.4	26.3
7000	430—j163	$460/\overline{20.8}$	450—j150	$475/\overline{18.4}$	36.7	31.8	25.1
10000	389—j201	$438/\overline{27.3}$	420—j200	$465/\overline{25.5}$	32.4	29.3	23.5
12000							23.6
15000	328—j231	$401/\overline{35.1}$	380—j280	$472/\overline{36.4}$		21.8	

* 120 sections terminated in 450 ohms.

and that of R_2 is small at high frequencies. In this case the two functions may be thought of as virtually independent.

Table II gives the theoretical impedance of this facility and the computed impedance of the network at frequencies from 100 cycles to 15,000 cycles and, as may be seen by the comparison, a fairly good simulation exists throughout the range. This fact has been verified by making return loss measurements in the laboratory against the theoretical line with the results indicated in the table. Return loss measurements have also been made between the network and an artificial line consisting of 120 sections of this facility terminated in 450 ohms. These results show a fairly constant return loss of about 25 db throughout the frequency range. This seems to indicate that the simulating network is a fairly close approximation to the artificial line so far as frequency is concerned and differs from it by a constant multiplying factor which is of the order of 1.12. It is therefore apparent that whenever it is necessary only to simulate the impedance of this particular facility, this four-element network will provide a fairly adequate simulation. The analytical derivation of this network will be omitted.

Example 3a—Non-loaded Exchange Area Cable

Another case will be cited to show the application of the graphical method. This is the simulation of non-loaded cable of which the local plant is largely composed in urban areas. A first approximation of the analytical method does not yield a useful network but the graphical method provides a three-element network of the type discussed above which gives a return of about 20 db in the 300 cycles to 3000 cycles range. The graphical derivation of the three-element network is shown on Fig. 4 which also gives the impedance function for 22 ga. BSA non-loaded cable. This latter function is virtually a straight line in the voice range whereas the network is the arc of a circle. Hence it would be impossible to obtain an appreciably closer approximation throughout the range with a three-element network. However, the addition of elements will improve the match as will be shown in example 3b.

The network just derived can be expressed in terms of the 1000-cycle impedance and applied for any gauge of non-loaded cable as follows:

$$R_1 = .42\text{ K} \tag{5-a}$$

$$R_2 = 2.8\text{ K} \tag{5-b}$$

$$X_{C_2} = .9\text{ K} \tag{5-c}$$

where K is the magnitude of the 1000-cycle impedance and

$$X_{C_2} = \frac{1}{2\pi f C_2} \tag{5-d}$$

Table IIIa gives a comparison of the network and line impedances and the computed return loss for frequencies through the 200 to 3000 cycle range.

Example 3b—19-Gauge Quadded Non-Loaded Toll Cable

Two complete *RC* functions plus a resistance are required to give a good simulation for non-loaded toll cable when the simulation is carried through the voice and carrier frequency ranges. The impedance function for 19 ga.

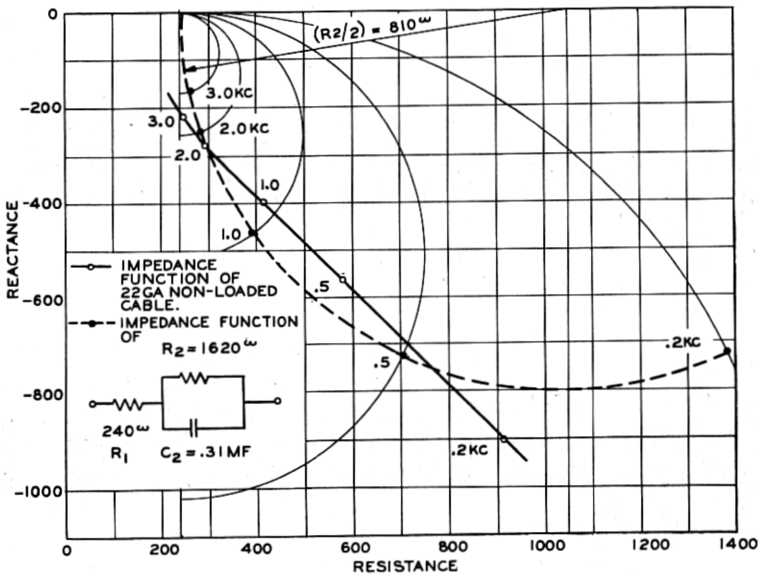

Fig. 4—Graphical design of two-terminal balancing network for 22-ga. non-loaded exchange area cable.

Table III-a

22 Gauge Non-Quadded Non-Loaded BSA Exchange Area Cable

Freq. kc	Line Impedance		Network Impedance		Computed Return Loss—db
	Rect.	Polar	Rect.	Polar	
0.2	915—j905	$1287/\overline{44.7}$	1380—j725	$1555/\overline{27.8}$	15.0
0.5	580—j565	$808/\overline{44.2}$	705—j725	$1010/\overline{45.8}$	19.1
1.0	415—j400	$576/\overline{44.0}$	390—j460	$603/\overline{49.7}$	25.2
2.0	295—j280	$407/\overline{43.5}$	285—j245	$376/\overline{40.7}$	26.6
3.0	250—j220	$333/\overline{41.3}$	260—j165	$308/\overline{32.4}$	21.2

toll cable is plotted on Fig. 5. The method followed in determining the elements is somewhat as follows: R_1 will be given by the intercept of the function on the R axis and is 130 ohms. Next look at the low-frequency range determined by R_3C_3 and draw a semicircle which approximates the given function in the range of 200–500 cycles. The diameter of this semicircle determines R_3 as 2100 ohms and R_2 is then automatically determined as the difference between the R-intercept of the R_3 semicircle and R_1, hence $R_2 = 420 - 130 = 290$ ohms. To determine C_3, choose the X_{C_3} semicircle at 500 cycles to intersect the R_3 semicircle at a point near the 500-cycle impedance of the cable impedance function, but make some allowance for the added negative reactance of the R_2C_2 generating function. The determination of C_2 can be made in either of two ways. First an X_{C_2} semicircle can be drawn at 5000 cycles which intersects the R_2 semicircle at an impedance near the 5000-cycle impedance of the cable. The impedance at 1000 cycles can then be found graphically for R_2C_2 and R_3C_3 and added together to R_1. This

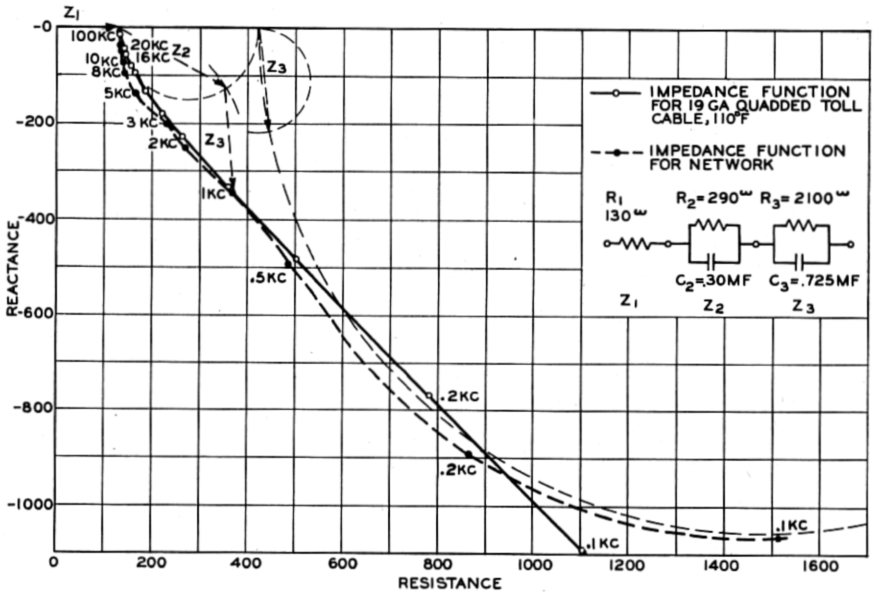

Fig. 5—Graphical design of two-terminal balancing network for 19-ga. quadded non-loaded toll cable.

total impedance at 1000 cycles should provide a good simulation of the 1000-cycle impedance of the cable. A second procedure for finding C_2 would be to follow a somewhat reverse process: Determine the 1000 cycle $\boldsymbol{Z}$ for the R_3C_3 function and subtract it from the 1000 cycle $\boldsymbol{Z}$ of the cable. Choose C_2 such that the intersection of the R_2C_2 semicircles is near the point determined by the subtraction of R_3C_3 from the cable.

To avoid confusion of lines the construction circles have been omitted from this last drawing except to show the addition of the 1000-cycle impedances. As may be seen this network shown in Fig. 5 provides a rather good simulation throughout the frequency range above 200 cycles.

As pointed out earlier, if the first guess is not a sufficiently good approximation a second try can be made based on the evident shortcomings of the first try. In this case if a closer approximation is required up to 20 kc the next step might be to change C_3 to .8 mf which would make the Z_3 contributions above .1 kc somewhat less negative and would therefore raise the network locus. Then changing R_1 to 140 ohms would shift the locus 10 ohms to the right. The resulting locus would be somewhat closer at the upper frequencies but the change would not be necessary unless a rather high degree of balance is required.

TABLE III-b

19 Ga. Quadded Non-Loaded Toll Cable

Freq. kc.	Network Impedance		Line Impedance		Computed Return Loss vs Theoretical Line-db
	Rect.	Polar	Rect.	Polar	
.1	1515 —j1064	1852 /35.9	1103—j1093	1554 /44.7	18.3
.2	867 — j893	1244 /45.8	783— j770	1097 /44.5	24.8
.5	488 — j493	693 /45.3	501— j482	696 /43.9	38.2
1.0	376 — j340	507 /42.1	361— j335	492 /42.8	36.0
2.0	271 — j254	371.6/43.2	265— j229	350 /40.8	29.0
3.0	217 — j203	297.2/43.2	223— j180	287 /38.9	27.8
5.0	166.5— j139	211.3/38.0	187— j131	228 /35.1	26.1
8.0	145.5— j96.8	172.0/32.2	164— j94	189 /29.8	25.8
10.0	140.0— j74.4	158.0/28.0	155— j79.2	174 /27.1	26.5
16.0	134 — j47.2	142.1/19.5	145— j55.0	155 /20.8	27.9
20.0	132 — j37.9	137.9/16.0	141— j45.1	148 /17.7	27.9
100	131 — j7.3	131.0/ 3.2	130— j14.0	131 / 6.2	31.7

In general the success of a trial of the graphical construction may be determined immediately by comparing about three frequencies of the line and network.

Table IIIb gives the computed network impedance and the line impedance. The computed return loss is also given and equals or exceeds 25 db at all frequencies above 200 cycles.

It is apparent that the resistance and condenser elements of the generating functions are in descending order of magnitude with increasing frequency for the non-loaded cable the impedance locus of which is essentially a straight 45° line. As pointed out earlier, the series addition of such generating functions may be converted to a ladder structure[5], whose sections will have a tapered characteristic rather than repetitive.

[5] Appendix D of Transmission Circuits by K. S. Johnson.

Return Loss

When designing such networks for balancing purposes, it has been found convenient to plot the function on a sheet such as Fig. 6 which divides the right half of the complex plane into circular regions such that all points on or within the boundary of a given region have a return loss against the network $1 + j0$ equal to or greater than that corresponding to the boundary. These circles are determined by the return loss voltage ratio k and the ratio

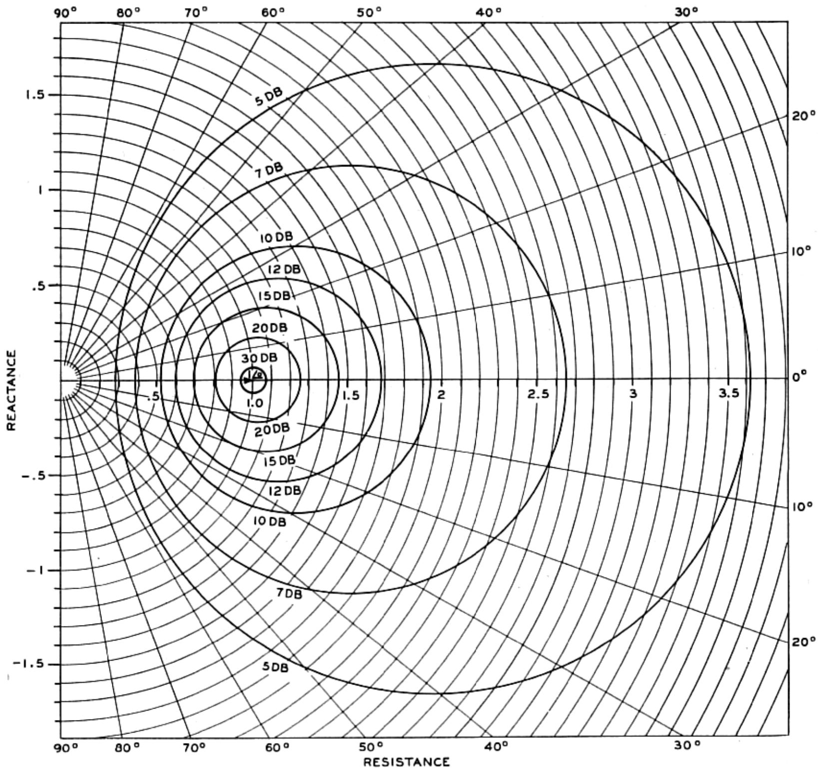

Fig. 6—Curves of constant return loss for the network $1 \angle 0 \equiv 1 + j0$.

of the line and network impedances. They may be computed from the equation

$$(1 + L/N)/(1 - L/N) = k \tag{6}$$

By plotting the line and simulating network loci on such a sheet it is generally possible to observe visually whether or not a given network meets the specified return loss requirement. If visual accuracy is not adequate, it is always possible to measure off N and L and the angle between them, spot the complex ratio L/N on the complex plane and read immediately the approximate return loss.

Conclusion

The examples of the foregoing discussion have been confined to the fourth quadrant. It was shown that by graphical means a number of parallel resistance-condenser functions could be determined which when added together would yield a close approximation to the given function. In the most general case these functions would involve the generating function of R, L and C in parallel, the locus of which is a circle having impedance $+j0$ at zero frequency and $-j0$ at infinite frequency, and crossing the axis of reals at R and the frequency at which L and C are anti-resonant. A case which has been found useful in simulating such things as telephone sets and other inductive elements is the parallel combination of R and L which, of course, is the special case for $C = 0$ and occurs in the first quadrant.

The foregoing has been discussed with the thought that it may be useful where there is limited time and where the required degree of simulation is consistent with a graphical method. At some future time it may be possible to pursue the problem further and devise the analytic counterpart to the somewhat heuristic graphical method.