CHAPTER II

Methods For Specifying Quartz Crystal Orientation and Their Determination by Optical Means

By W. L. BOND

2.1 Quartz and its Axes

The chemist describes quartz as silicon dioxide, SiO_2, crystallized in hard, brittle, glass-like, six sided prisms, often with pyramidal terminations; melting point 1750° Centigrade, density 2.65, hardness on Moh's scale 7. It transforms from alpha to beta quartz at 573°C under atmospheric pressure. Under stress it transforms at lower temperatures. Alpha quartz is insoluble in ordinary acids but soluble in hydrofluoric acid; and in hot alkalis.

At first glance we might say that it had hexagonal symmetry but if we etch two adjacent pyramid faces we find that the microscopic etch pits are of different shape, hence the faces cannot be equivalent. It has three axes of two-fold symmetry and one axis of three-fold symmetry. Let us also remark that it does *not* have a center of symmetry or a six-fold axis. Figure 2.1 shows us that the three two-fold axes are perpendicular to the three-fold axis and are 120° apart. If they were not like this, they would not be self-consistent.

As we examine more and more quartz crystals we find that there is a tendency for pyramid faces to be alternately large and small, the larger faces being brighter than the smaller faces. Also the etch pits of *alternate* faces are similar. (The etch pit study is a powerful tool in determining crystal symmetry.) Further, two other "kinds" of faces are quite commonly found. If we draw such a crystal as though equivalent faces were of equal size we get such a picture as Fig. 2.2. It is an idealized figure used to illustrate the symmetry of quartz. The prism faces are marked *m*, the six faces marked *r* "constitute the primary rhombohedron"—the ones we called the large bright pyramid faces. The crystallographer thinks of these six faces as pieces of the faces of a rhombohedron. (A crystallographer's rhombohedron is like a cube stood on one corner, then the opposite corner pushed in a little towards the other, or pulled away from it. He thinks of it always as standing on this corner, as Fig. 2.3.) The *z* faces constitute a second rhombohedron—the secondary rhombohedron or minor pyramid faces. The *s* and *x* faces illustrate a further property of quartz. Figure 2.3 differs from its mirror image so that we

might expect to find two kinds of quartz that are related to each other as one's right hand is related to his left. We do find them and call them right-hand quartz and left-hand quartz respectively. They are illustrated in Fig. 2.4. These conventional figures are shown in many texts but no one has seen such perfect quartz crystals. They are drawn possessing just these faces and no others merely to illustrate the symmetry of quartz and its occurrence in right-handed and left-handed forms.

These figures are also useful in defining how a blank shall be cut from one kind of quartz. It is found that if a crystal be compressed with forces

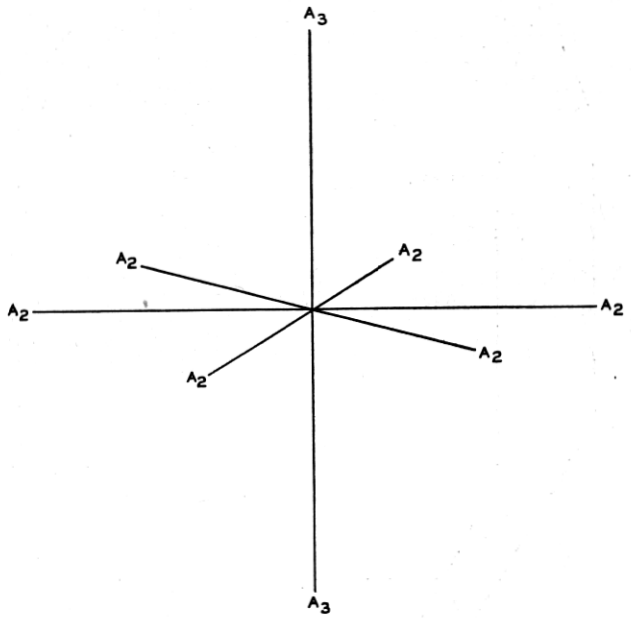

Fig. 2.1—Hexagonal axis system

parallel to a pair of sides of the hexagon an electric polarization takes place in the direction of the forces. The edge "modified" by the presence of s and x faces becomes negative. If we allow these charges to leak off and then suddenly release the mechanical forces the "modified" edge becomes positively charged as the crystal expands. This is true for both right-hand and left-hand crystals.

Let us now talk about right-hand quartz. Since expansion is considered as a positive strain (contraction as negative) it is decided to take the positive end of the electric axis as pointing towards the modified edge. This gives us a positive charge at the positive end of the electric axis when a positive stress (tension) is applied along this axis. This positive direc-

tion of an electric axis is taken as the positive x axis of a right-hand xyz rectangular coordinate system. The z axis is taken along the axis of the hexagonal prism, and since the x axis is an axis of two-fold symmetry we

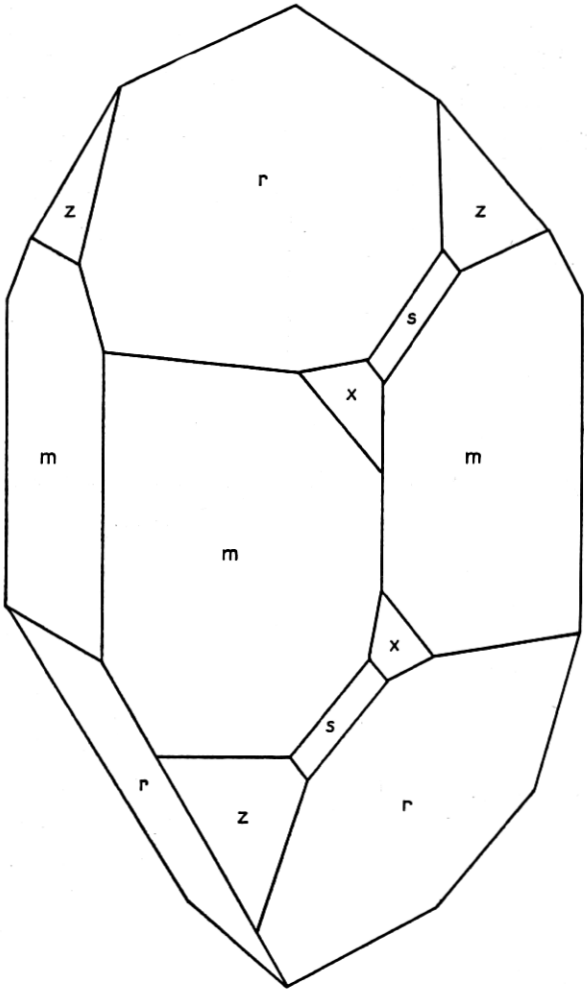

Fig. 2.2—An idealized quartz crystal

can take either end of the prism as the direction of $+z$. We then choose y to form a right-handed coordinate system. (In a right-handed system if a right-handed screw turns about the z axis in the sense x to y it would ad-

vance in the positive z direction.) The y axis will always lie directly under a major rhombohedral face.

We could define the x, y and z axes for a left-hand crystal as forming a left-hand system. Though this is a useful conception in mathematical

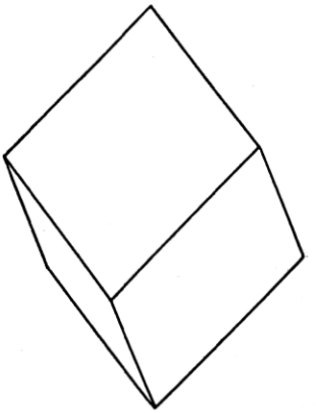

Fig. 2.3—A rhombohedron

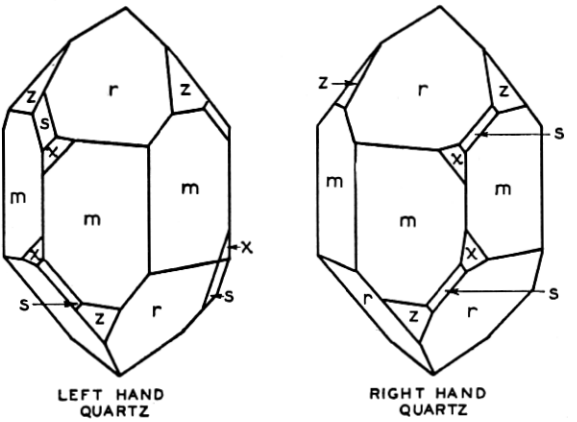

Fig. 2.4

studies, we can dodge this "double standard" by a simple device. For use as a crystal circuit element, left-hand quartz can be used just as well as right-hand quartz. In designing such an element it suffices to think always in terms of right-hand quartz and issue specifications for this kind only, using always right-hand coordinate systems. If now for left-hand

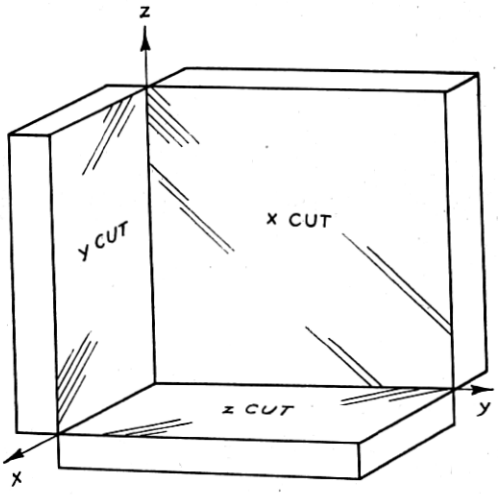

Fig. 2.5—Simple crystal cuts

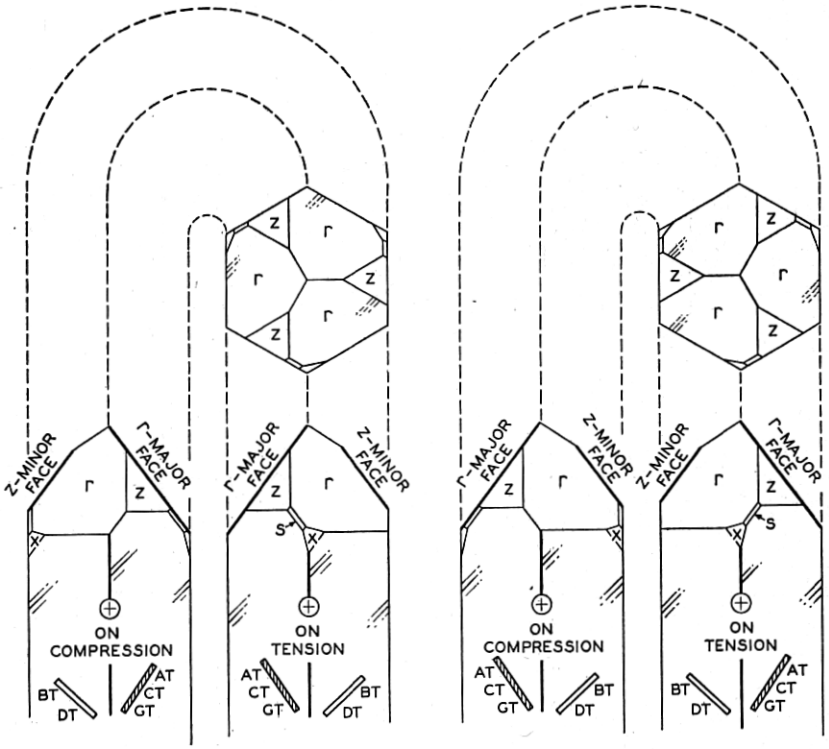

Fig. 2.6—Simply rotated cuts

crystals we mark the negative end of the electric axis as positive we can treat it exactly as though it were a right-handed crystal.

The first plates used were x and y piezoids (squeezing solids). For these simply described cuts one does not need to know the quartz "handedness." These crystals had large frequency temperature coefficients. But when Lack, Willard and Fair brought out the low temperature coefficient AT plate, its more complicated orientation required the right-left differentiation. The AT, the subsequent BT, CT, DT, etc., were thought of as y-cuts rotated through various angles about the edge that coincided with x. For example, the AT was a $+35\frac{1}{4}°$ cut, or was a y plate rotated $35\frac{1}{4}°$ about x; the BT was a $-49°$ cut. Their orientations are illustrated in Fig. 2.6.

As more complicated orientations were designed to give even better temperature coefficients at extreme frequencies the description became more difficult, requiring the specification of two or three angles. Many schemes for specification have been devised but the Institute of Radio Engineers is recommending the adoption of a system we shall call the I.R.E. system.

The crystal designer has the problem: "How shall I orient the length, width and thickness of a piezoid with respect to the x, y and z axes so as to give the required electrical properties? He thinks in terms of fixed x, y and z axes, variable piezoid edge directions. The crystal cutter has the converse problem. "How shall I orient the x, y and z axes of the crystal so that fixed saws will give the required surfaces?" For this reason the most convenient orientation angles from the designer's viewpoint may not be the simplest from the cutter's viewpoint. Also the translation from one set to the other may not be simple.

The early methods of orientation specification were somewhat chaotic. There was no overall plan of what angles were to be specified and from what axes they were to be measured. Each group of crystals was a law unto itself.

The I.R.E. Orientation Angles

The relations between the x, y and z axes of the crystal and the length, width and thickness of the piezoid are given in Fig. 2.7.

The position of Fig. 2.7 may be considered as a result of turning the piezoid through the successive angles ϕ, θ, ψ starting from an initial position length parallel to x, width parallel to y and thickness parallel to z as in Fig. 2.8. First the crystal is turned through angle ϕ about z in the direction shown in Fig. 2.7. Then it is lowered through angle θ about an axis parallel to the width direction, again in the direction shown in Fig. 2.7. Finally it is skewed through an angle ψ about an axis parallel with the thickness direction in the sense shown in Fig. 2.7.

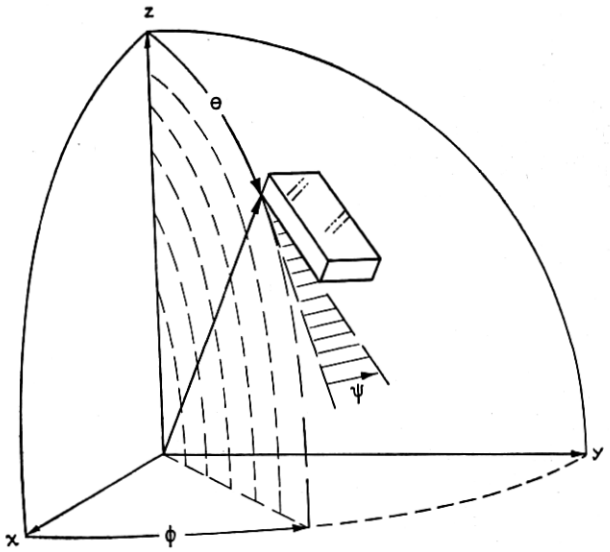

Fig. 2.7—The I.R.E. orientation angles

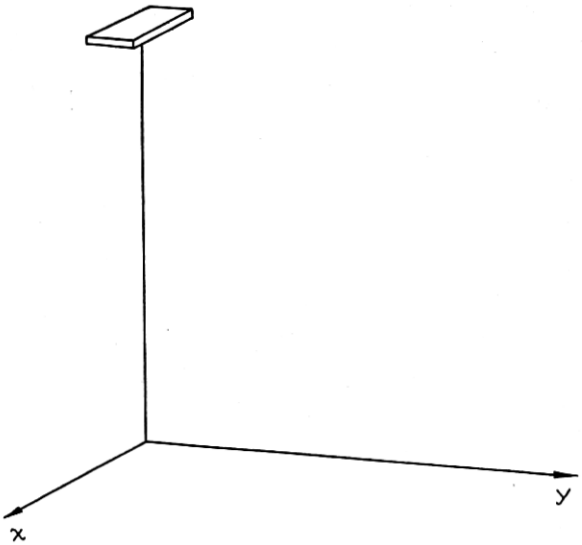

Fig. 2.8—The initial position 0, 0, 0 for the I.R.E. angles

THE I.R.E. ANGLES FOR A FEW STANDARD PIEZOIDS

Name	ϕ	θ	ψ
z	0	0	0
x	0	90	90
−18° Filter	0	90	108
+5° "	0	90	85
y	90	90	90
AT	−90	$54\frac{3}{4}$	90
BT	−90	−41	90
CT	−90	52	90
GT	−90	38° 52′	±45°
MT	6° 40′	50° 28′	79° 36′
NT	9° 25′	40° 40′	77° 40′

2.2 ORIENTATION BY NATURAL FACES

With well faced material one can do an accurate job of orienting without X-rays if he knows the faces of quartz thoroughly.

The quartz rhombohedral faces are highly perfect and polished, the major often more so than the minor. With two such faces a device like that illustrated in Fig. 2.9 could be used to give an orientation accurate to a minute or two. An adjustable base, symbolized here as a ball and socket, is adjusted so that the eye centers the lamp filament image on the cross hairs, first for one face, and then, turning the base about on the reference table it is adjusted for another face. When the images all pass through center as the base is turned on the table the optic axis is perpendicular to the table. When any one image is centered, the electric axis is perpendicular to the plane of the paper. This with the already mentioned fact that AT plates are cut near a minor face and BT's near a major, allows us to cut the crystal accurately.

Although the rhombohedral faces are highly perfect the prism faces never are. On the prism face, true prism faces that are very short in the z direction alternate with short rhombohedral faces to give the general contour a slant. These "steps" give the face a striped look. The stripes are known as growth lines or striations. They are parallel to x and can be used to find x to a degree or so. If we sight on striations on two adjacent faces we can locate the optic axis to nearly the same accuracy since the optic axis is perpendicular to the striations on all faces.

There are several indications that help us find, from the prism, where the major rhombohedron would be in the absence of such faces. Some crystals grow in the form shown in Fig. 2.10. They are symmetrically doubly terminated and a very narrow prism is found under the major rhombohedron, a wide face under the minor. Hence given a portion like that enclosed in the dotted line we could deduce the complete orientation.

Some crystals grew out at right angles to a wall and because they grew

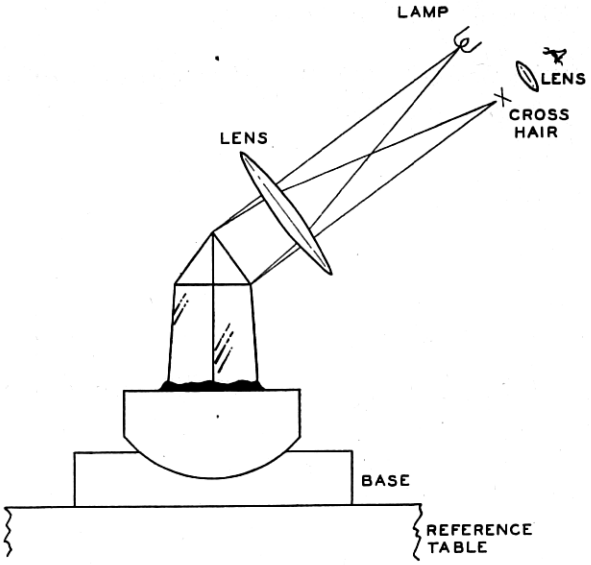

Fig. 2.9—Optical orientation by reflection of light from faces

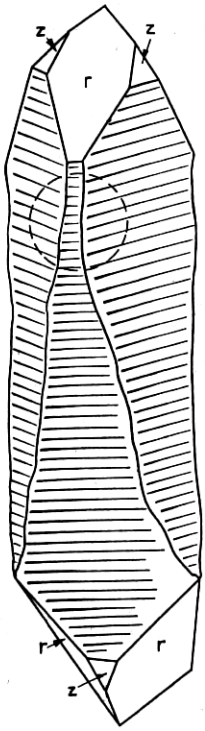

Fig. 2.10—A type of quartz growth

along a z axis in one direction only, the x axis does not appear to be one of two-fold symmetry. Such a crystal is illustrated in Fig. 2.11. Here the prism faces under a major rhombohedron are tapered and bright, the prism faces under a minor are relatively parallel sided and very dull. The bright prism faces are much more nearly parallel to the optic axis than the dull ones. Again, given a portion of the prism we can deduce the orientation.

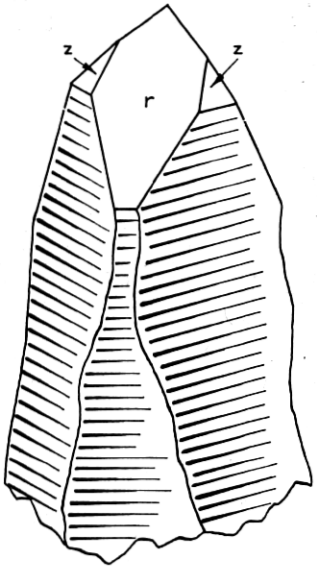

Fig. 2.11—Another type of quartz growth

2.3 Flat Lay Cutting

Flat lay cutting takes advantage of the fact that, although tapering quartz prisms have their faces non-parallel to the z axis the prism faces are parallel to the x axis. A crystal is cemented prism face down, to a mounting plate. The mounting plate is tilted and turned on a base plate to render the optic axis parallel to the long edge of the base plate. This is done in a conoscope. Now the edges of the base plate are the x, y, z axes of the crystal.

The crystal can now be cut directly into wafers for dicing into AT's, BT's etc. by mounting on an angle bracket as shown in Fig. 2.13 or cut into X sections from which AT or BT bars shall be made by merely sawing down the length. Again z sections can be cut by cross cutting. Good z sections can be made in this way in the total absence of faces. These sections can

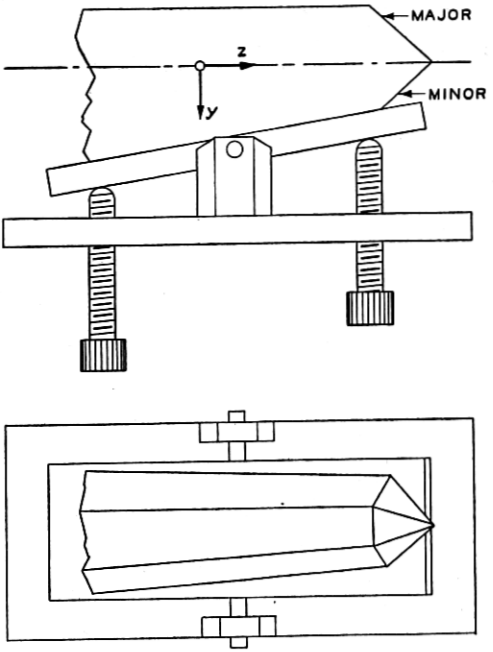

Fig. 2.12—Optical adjustment for the sawing of Z sections or direct crystal blank slabs

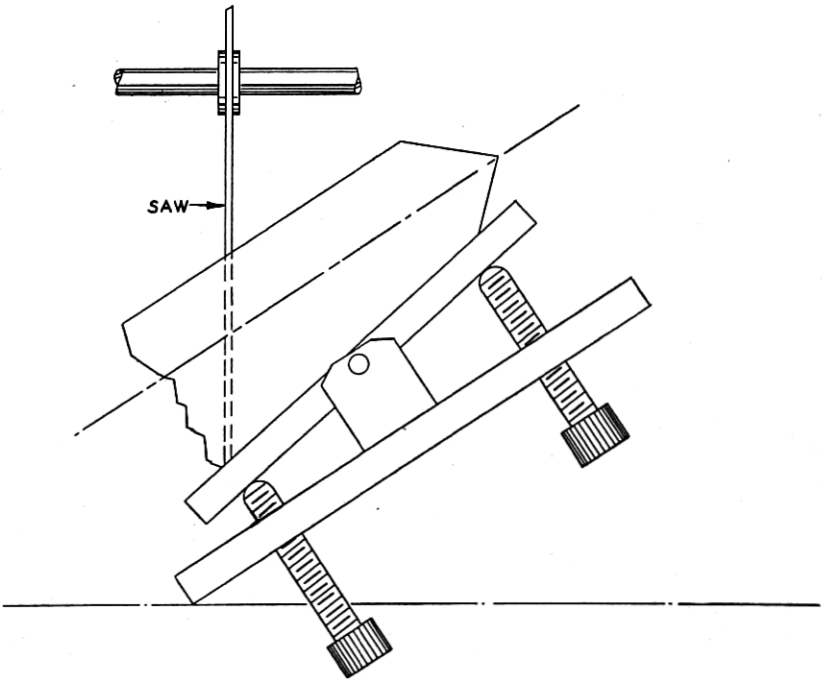

Fig. 2.13—Direct sawing of the slabs

then have their x axes determined by etching and X-rays and cut up by the Z section cutting method.

By turning the base plate on the angle bracket and dicing the wafers at an angle any orientation can be obtained.

2.4 *Z* Section or Vertical Cutting

Having a true Z basal section it is first marked for the $+x$ axis. The simplest procedure is to use the star mark; for right-hand quartz (R.H.Q.)

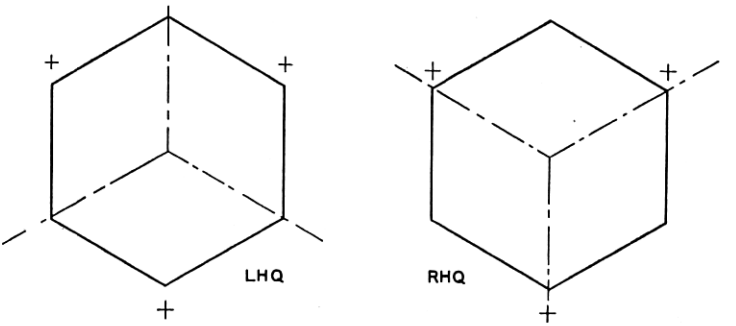

Fig. 2.14—Marking the "sense" of righthand and lefthand sections

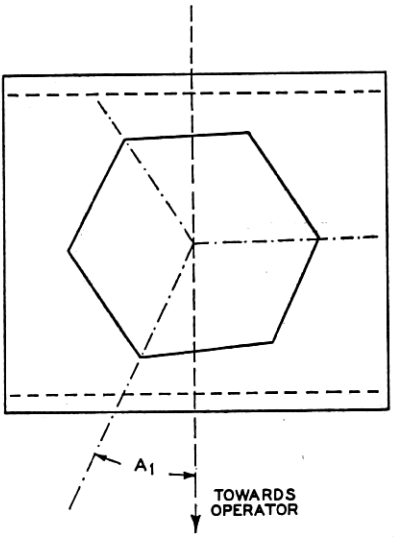

Fig. 2.15—Making the rotations A_1

the rays should point toward the plus electric axis, for left-hand quartz the rays should point towards the negative electric axis.

The section is now placed on the carriage plate, one ray pointing towards the operator (which ray is decided on the basis of the economy of quartz).

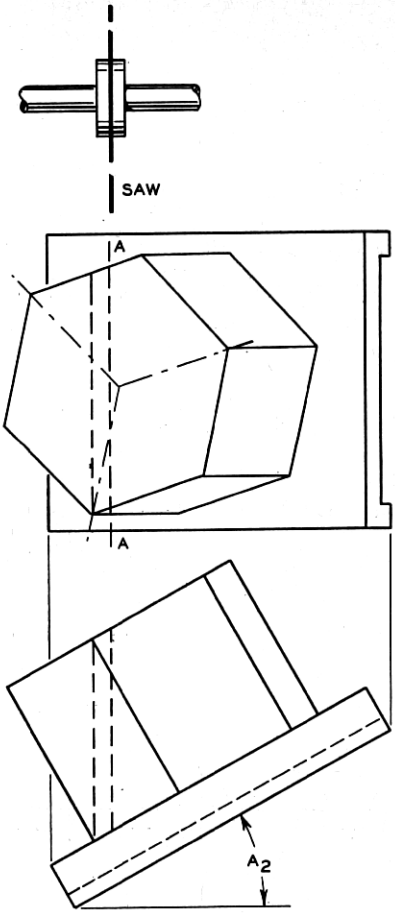

Fig. 2.16—The rotation A_2

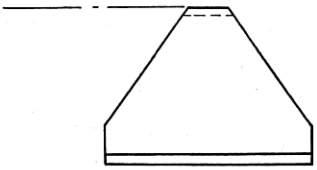

Fig. 2.17—The slab after the rotations A_1 and A_2

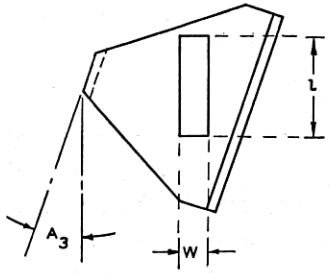

Fig. 2.18—Making the A_3 rotation

The section is then rotated clockwise on its base, through angle A_1 as in Fig. 2.15 and cemented in this position.

The carriage plate is then transferred to a diamond saw angle bracket of tilt A_2, as in Fig. 2.16, and the crystal is sawed into slices slightly thicker than the required final thickness t.

The operator turns these slices down flat on the table of a dicing saw as in Fig. 2.17 by rotating the slices 90° clockwise about the axis AA, then turns the slice through angle A_3 as in Fig. 18 and makes a cut. The plate is finished as shown in Fig. 2.18.

Since the angle bracket is not reversible, negative A_2 angles are cut by adding $\pm 180°$ to A_1 and reversing the sign of A_3.

THE A ANGLES FOR SOME STANDARD PLATES

Cut	A_1	A_2	A_3
x	90°	0	0
y	0	0	0
z	0 or 90°	90°	0 or 90°
−18	90°	0	+18°
+5	90°	0	−5
AT	0	35¼°	0
BT	180°	+49°	0
CT	0	38°	0
DT	180°	52°	0
GT	0	51° 7′	±45°
MT	96° 40′	39° 32′	−10° 24′
NT	99° 25′	49° 20′	−12° 20′

2.5 THE RELATION BETWEEN THE I.R.E. ANGLES $\Phi\theta\psi$ AND THE Z SECTION ANGLES A_1, A_2, A_3

It can be shown that:

$$A_1 = 90 + \phi$$

$$A_2 = 90 - \theta$$

$$A_3 = -90 + \psi$$

2.6 POLARIZED LIGHT AS APPLIED TO CRYSTALS

Light consists of electromagnetic "vibrations." The vibrations are perpendicular to the direction of propagation but ordinarily helter-skelter in all directions perpendicular to the propagation. The color of the light is determined by the vibration frequency, blue vibrating more rapidly than red. In a vacuum, light travels at 186,000 miles per second (3×10^{10} cms per second) all colors at the same velocity. On entering a transparent medium the velocity is reduced, ordinarily blue being slowed more than red. The frequencies are unaltered on entering the medium.

Light traveling through a uniaxial crystal in the direction of Fig. 2.19 breaks up into two components that travel at different velocities. For one

of these components the vibration is all in the plane of *poz*, of the other the vibration is all perpendicular to the plane of *poz*.

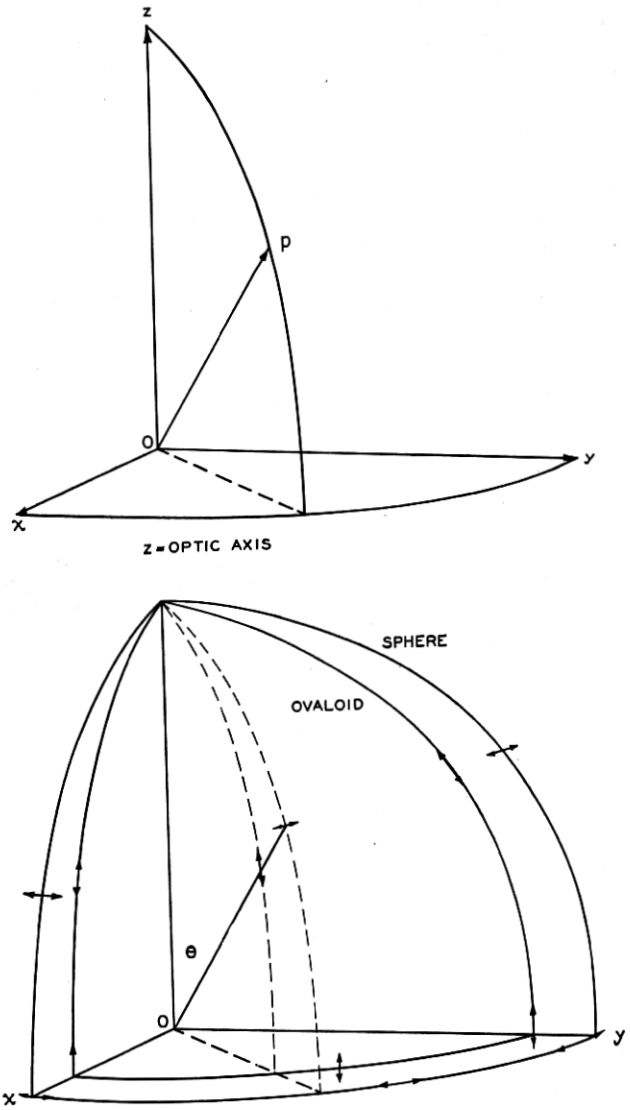

Fig. 2.19—The velocities of light in a uniaxial crystal

A plot of the propagation velocities for all directions is a surface of two sheets, one octant of which is shown in Fig. 2.19. One sheet is a sphere;

the other sheet, which is an ovaloid of revolution, touches the sphere at the two points where the double sheet is pierced by the optic axis. Of the two rays traveling along one line, the one with a velocity corresponding to the sphere is called the ordinary ray; the one with a velocity corresponding to the ovaloid is called the extraordinary ray. For quartz the ovaloid is prolate and lies inside the sphere. For tourmaline the ovaloid is oblate and lies outside the sphere. The small arrows show the direction of vibration. Each of the components is said to be polarized since for each all the vibration is in one direction.

Since both sheets are surfaces of revolution with the optic axis as the axis of revolution, we can never tell the x axis from the y axis by optical means.* Only the z axis is a unique direction and can be determined optically. If this figure is taken to represent the case for blue light there will be a slightly larger but similar figure for red light since, in the crystal, red light travels faster than blue light.

Some kinds of crystals have velocity plots for which the double sheet surfaces touch at four points. Hence they have two optic axes and are called biaxial. All hexagonal, rhombohedral and tetragonal crystals are uniaxial, all others except the isometric ones are biaxial. Rochelle Salt is biaxial.

2.7 Polarizers and Analyzers

In the Nicol prism means were found to eliminate the ordinary ray; the other is transmitted as polarized light. That is, ordinary light of any or all colors upon passing through a Nicol prism emerges as plane polarized light with no change in color.

Transparent colored media appear colored because they absorb some colors of light more than other colors. In colored crystals the two rays themselves often differ in their color absorption so that the crystal as viewed by means of the ordinary ray seems to be of a different color than as viewed by the extraordinary ray. Quinine iodo-disulfate, or Herapathite, absorbs most visible colors of one ray almost completely; transmits about 60% of the visible colors in the other ray. Hence light emerging from this crystal is almost completely polarized. In the commercial product called "polaroid", myriads of such crystals, microscopic in size, are contained in a celluloid-like sheet and oriented by stretching the sheet. This material now replaces Nicol prisms for all but the most exacting uses.

If we put two identical polaroid sheets together with their transmission vibration directions parallel as in Fig. 2.20 we can see through them but if their transmission vibration directions are at right angles we cannot see

* Methods depending on etch pits are excluded. They are optical only in the sense that observing natural faces is optical.

through them because the second sheet can transmit none of the vibrations transmitted by the first sheet. As we rotate the second sheet back from complete extinction to "best transmission" the transmitted light increases sinusoidally. In any such arrangement as Fig. 2.20 the first sheet is called the polarizer, the second is called the analyzer. The name analyzer is chosen because light that can be extinguished by means of a suitably rotated analyzer must be plane polarized, and it must be vibrating at right angles to the transmission vibration direction of the analyzer when set for extinction. The transmission vibration direction of a polaroid plate will hereafter be called its vibration axis.

Let us go back to Fig. 2.19 and cut out from around the point p, the small tangential crystal plate shown magnified in Fig. 2.21. Here p is the direction of propagation as before, and z is parallel to the optic axis. Also s which is

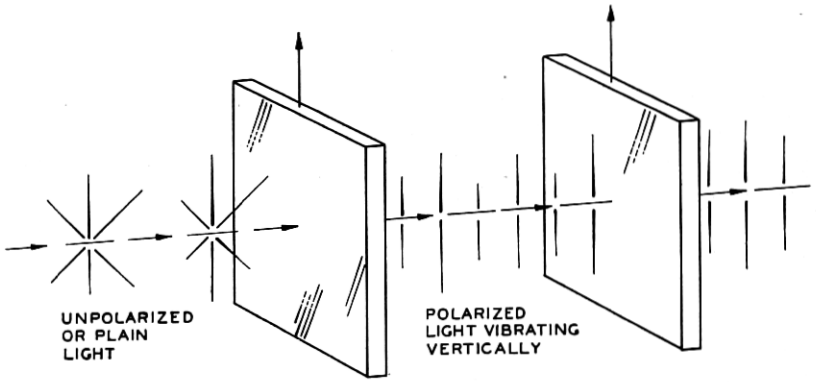

Fig. 2.20—Light polarization

in the plane of p and z, is the direction of slow vibration while f which is perpendicular to this plane is the direction of fast vibration. The vibration frequency is really the same for both. "Slow vibration" means "vibration direction for slow transmission." All directions of propagation that have this vibration axis have the same velocity.

In Fig. 2.22 we have placed this plate between "crossed polaroids"—that is polaroids set for extinction. The slow direction makes an angle α with the polarizer vibration axis. When this vertical-polarized ray of intensity I enters the crystal it breaks up into components, one of intensity:

$$I \cos \alpha \text{ vibrates along s} \tag{2.1}$$

and one of intensity

$$I \sin \alpha \text{ vibrates along } f \tag{2.2}$$

as illustrated in Fig. 2.23.

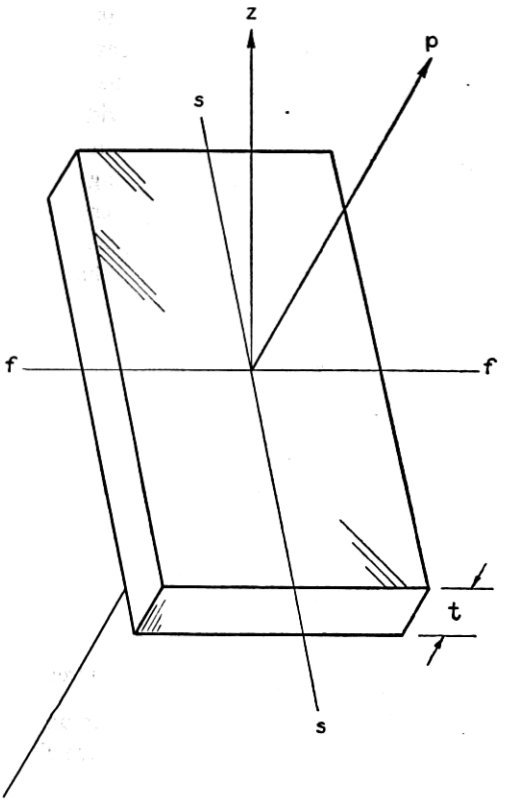

Fig. 2.21—Light breaks into components in the crystal

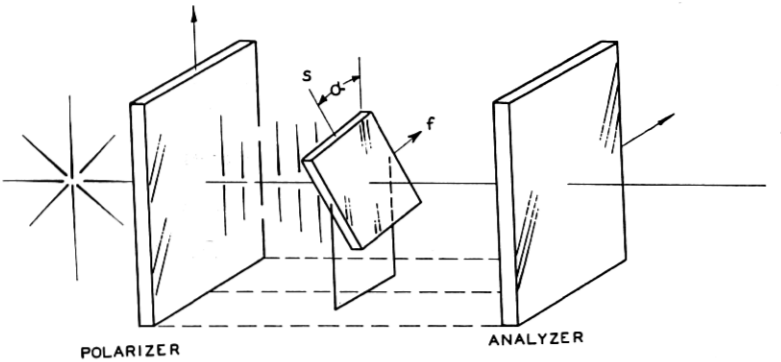

Fig. 2.22—Transmission when a crystal is placed askew between crossed polarizers

If $\alpha = 0$ the fast component reduces to zero and the slow component goes through the crystal unchanged hence emerging plane polarized. It can

then be extinguished by the analyzer. If $\alpha = 90°$ the slow component reduces to zero and the fast one goes through unchanged and again can be extinguished by the analyzer. This effect can be used to check crystal orientations. Such an instrument fitted with a divided circle used to measure α is called a stauroscope. The stauroscope often uses a special analyzer capable of better determination of extinction setting.

If α is not 0 or 90° two components traverse the crystal and recombine at the boundary. These two components are at right angles to each other; they are of unequal intensities, and they differ in phase because they traveled at different speeds.

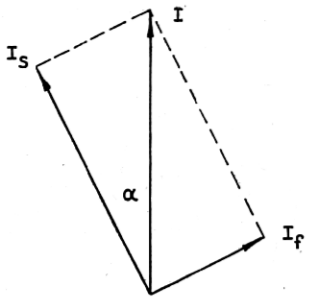

Fig. 2.23—The intensity of the two components from Fig. 22

Now v_s and v_f have the same frequency F so that in unit time each makes F wave-lengths. This requires that the slow ray have F wave-lengths in a distance v_s and hence that each wave have a length:

$$\lambda_s = \frac{v_s}{F} \tag{2.3}$$

Similarly

$$\lambda_f = \frac{v_f}{F} \tag{2.3'}$$

In a distance t there are $\frac{t}{\lambda_f}$ fast waves and $\frac{t}{\lambda_s}$ slow ones. Let us say that there are N more fast waves than slow ones in the distance t. Consequently $N = \frac{t}{\lambda_f} - \frac{t}{\lambda_s}$ which, from (2.3) and (2.3') may be written:

$$N = \frac{tF}{v_f} - \frac{tF}{v_s} \tag{2.4}$$

All the data on light are given in terms of wave-lengths in a vacuum, not in terms of frequency, so we will assume that in a vacuum the wave-length

of this light is λ, and as in a vacuum its velocity is $V(= 3 \times 10^{10}$ cms per second) an equation similar to (2.3) would tell us that $\lambda = \frac{V}{F}$ and hence that:

$$F = \frac{V}{\lambda} \tag{2.5}$$

With (2.5) we can rewrite (2.4) as

$$N = \frac{t}{\lambda}\left(\frac{V}{v_f} - \frac{V}{v_s}\right) \tag{2.6}$$

The ratio of the velocity in a vacuum to the velocity in a medium is called the refractive index of the medium commonly given the symbol n. For most transparent materials n is between 1.3 and 1.8.

We write these refractive indices as

$$\frac{V}{v_f} = n_f \quad \text{and} \quad \frac{V}{v_s} = n_s \quad \text{respectively.}$$

Now (2.4) becomes:

$$N = \frac{t}{\lambda}(n_f - n_s) \tag{2.7}$$

After passing through the crystal plate of thickness t, Fig. 2.21, the two light components recombine. They are polarized at right angles each to each; they are of unequal intensities, and they differ in phase by N wavelengths as given by equation (2.7).

If the crystal were vanishingly thin the two components that recombine would be effectively in step or in phase. In Fig. 2.24 we have plotted vibration amplitude against time for these two components. They are separated for clarity. In the upper curve the slow vibration is shown as vertical, in the center curve the fast one is shown as horizontal. In the lower curve corresponding points have been added vectorially. From actual construction we see that in the resultant curve the vibration is always parallel to the line AA'. Hence it is plane polarized and can be extinguished by means of an analyzer.

Let us now consider the case of a thicker plate for which the slow wave gets a quarter-wave-length behind the faster one. This case is plotted in Fig. 2.25 in the same way that the previous case was plotted in Fig. 2.24. It turns out to be a space curve like a slightly flattened corkscrew. Viewed along the axis it looks like an ellipse, as shown to the right of the space curve. If the slow ray had lost but a little with respect to the fast ray, we would have gotten a very flat ellipse. If the two components had had the

same amplitude with the quarter wave phase difference the end view in Fig. 2.25 would have been a true circle. Now since the vibration, in these cases, is not all in one plane, the light cannot be extinguished by an analyzer; it is not plane polarized light. In the one case it is called "elliptically polarized" light and in the other, "circularly polarized" light. If the slow ray loses an integral number of wave-lengths, it makes no difference; only frac-

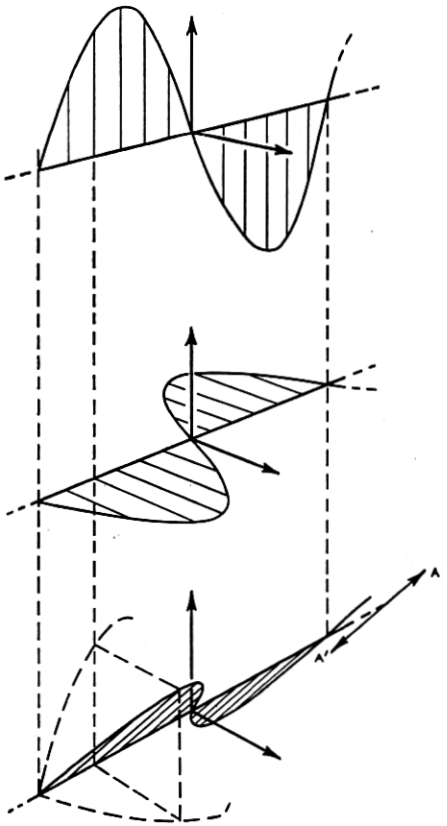

Fig. 2.24—The recombination of the light components after passing through a thin crystal

tions of wave-lengths count, except that if several wave-lengths are present a thickness that is right to give an integral number of wave-lengths for one color may give an integer plus a fraction for some other color. If the thickness is fairly small, this may cause spectral colors from white light. For thick plates the wave-lengths so overlap that the field appears colorless but dark or white according to the value of the angle α; i.e., if α is zero or

90° the field is dark; if $\alpha \neq 0$ or 90° the field is bright. In Fig. 2.26 we illustrate how, for one color of light the polarization starts out as plane at the

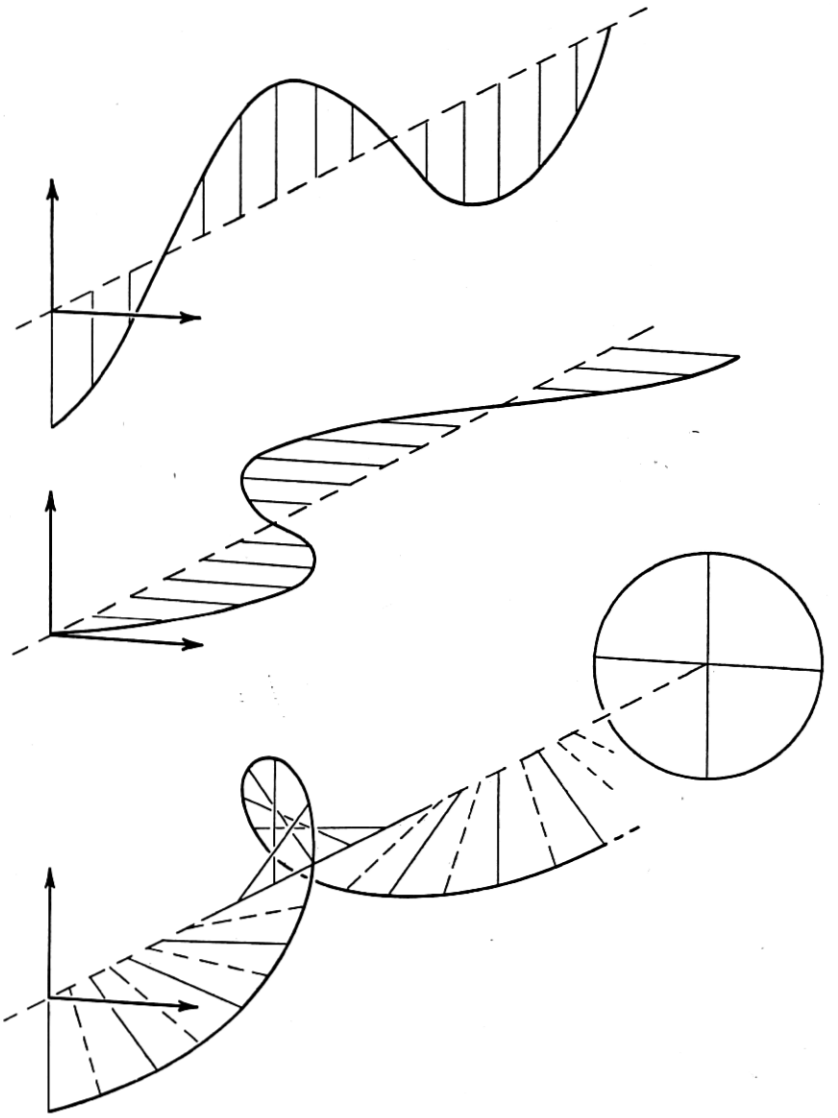

Fig. 2.25—Recombination more generally

crystal boundary, passes through elliptical to circular polarization, then flattens out the other way through elliptical to plane polarization at a distance

in the crystal corresponding to the slow wave being one wave-length behind the fast one.

In Fig. 2.27 we show two AT plates resting on a reference surface. In the first crystal the propagation is perpendicular to x, in the second crystal it is

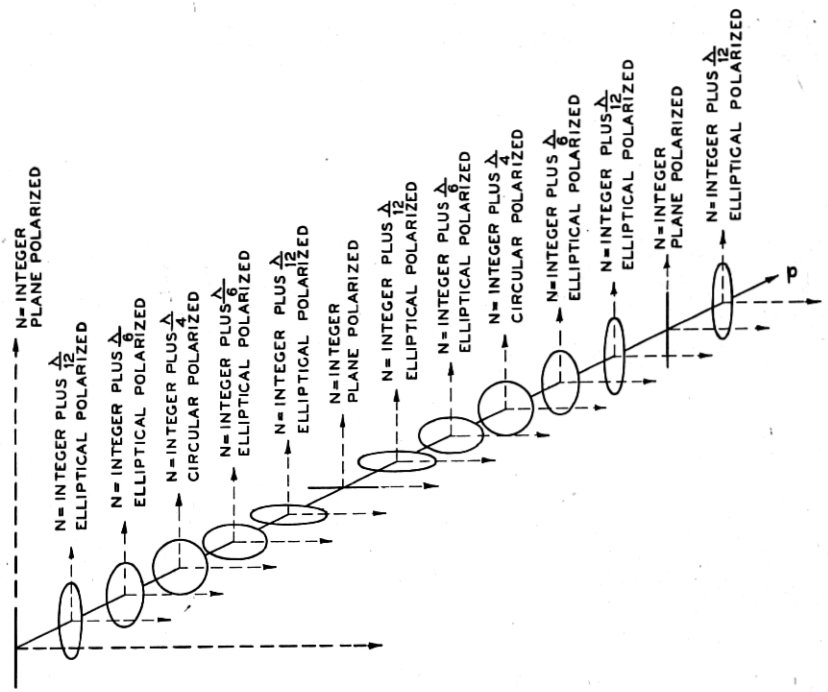

Fig. 2.26—How the kind of polarization changes with crystal thickness

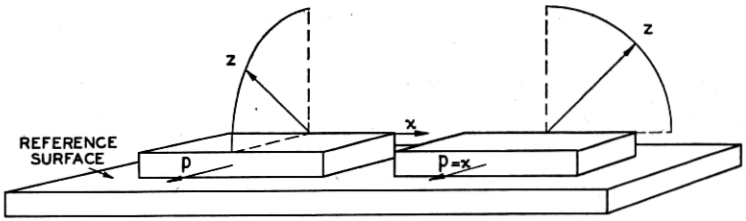

Fig. 2.27—The normascope principle

along x. If the reference surface is the reference table of a simple stauroscope the edge of the first crystal will appear dark because $\alpha = 0°$ or $90°$; the edge of the second crystal will appear bright because α is not $0°$ or $90°$. Actually this α for an AT plate can be $\pm 35° \pm$ any multiple of $90°$ because we don't know whether z stands out to the right or to the left. Hence the

reading might be, for instance, $-35°$, $+35°$, $+55°$, $+125°$, etc. This is the principle of the *normascope* used to identify the x direction for crystal adjustment.

Let us study these relative phase shifts at different angles near the optic axis. Now quartz has an optical complication beyond that just described—it rotates the plane of polarization of plane polarized light traveling along the optic axis. This complicates our present attempt to build up a background sufficient for an understanding of the conoscope. But the conoscope finds the optic axis for other crystals that do not rotate the plane of polarization, tourmaline for example; so we will ignore this rotation, to be-

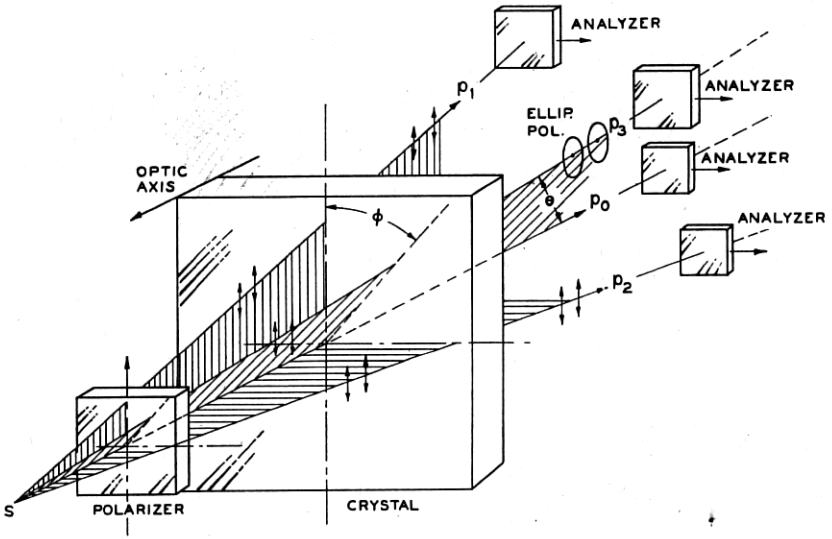

Fig. 2.28—How the polarization changes with propagation direction in a crystal plate

gin with, in order to arrive quickly at some useful conclusions. We will later explain how optical rotation modifies these conclusions.

Consider then the crystal z section shown in Fig. 2.28. A source s sends monochromatic light through the polarizer which passes only vertical vibrations. We will assume that the light passes in and out of the crystal without a deviation of path. Since the vibration is in the plane of z and p_1 (its direction of propagation) the ray does not break up inside the crystal but is propagated as plane polarized light, unchanged. An analyzer set for vertical extinction could then extinguish this ray.

This is true for propagation from s anywhere in this vertical plane. Also since the vibration is perpendicular to the plane of z and p_2 the ray p_2 does not break up inside the crystal but passes through and out un-

changed. All rays from s in the horizontal plane emerge plane polarized and can be extinguished by an analyzer set for vertical extinction. The ray p_0 is in both these planes so it can be similarly extinguished.

With the ray p_3 the situation is different. Here the vibration is not in the $z\ p_3$ plane so the ray breaks up inside the crystal into two components which travel with different velocities and recombine in or out of phase to give the various degrees of elliptical polarization (including plane and circular). Hence, an eye looking back along p_3, through an analyzer set for vertical extinction, will see light or dark depending on the phase shift N. Now this phase shift for a given thickness of plate is zero along p_0 but increases as θ increases (without changing ϕ; see Fig. 2.28), passing through one integral

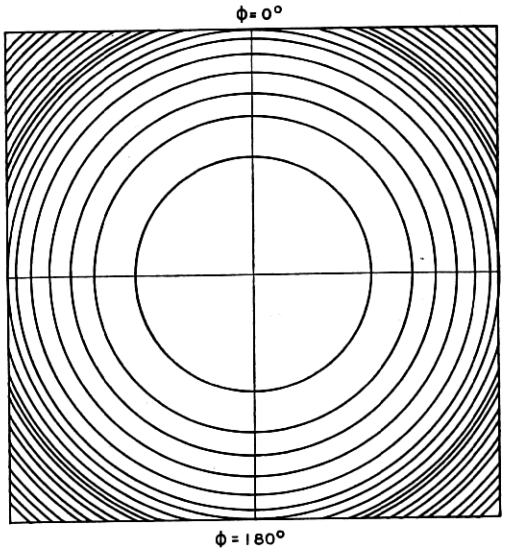

Fig. 2.29—A plot of phase as a function of ϕ and θ

value after another. Therefore, as we allow θ to increase, the eye should see alternate dark and bright regions. Moreover, since the crystal is optically symmetric about z, if ϕ is changed without changing θ, the apparent brightness will not change (except that if $\phi = 0, 90°, 180°$ or $270°$ the field is dark as we previously explained). Consequently, if we could see along all directions at once we would see a pattern of concentric dark rings on a dark cross as shown in Fig. 2.29.

But we *can* see along all these directions at once if we employ a properly placed lens for a lens can converge all these rays to one point where an eye can be placed for viewing.

Thus an eye at e, Fig. 2.30, will see, in the direction $e\ p_0'$, the ray that

started along s p_0. It will see along e p_1' the ray that started along s p_1'. Every point on the lens will have associated with it a different direction in the crystal. Therefore the eye will see a pattern like that of Fig. 2.29. This is the principle of the conoscope. In the conoscope (the name means "conical viewing") the source s is replaced by the image of a source, the image being cast by a lens; see Fig. 2.31. Thus by the use of two similar lenses we get twice as much working space as one lens would give.

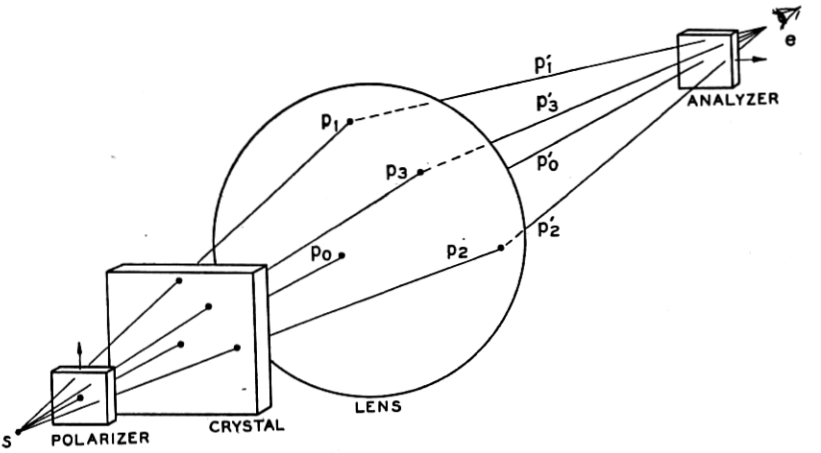

Fig. 2.30—The principle of the conoscope

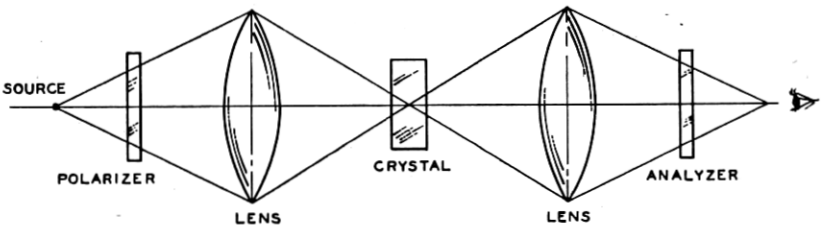

Fig. 2.31—A practical conoscope

Figure 2.32 shows a cross-section of the Western Electric conoscope. The graduated dial shaft goes out through the bottom of the tank to give more working room—older instruments had the shaft overhead and it was in the way. The light source is a mercury arc lamp with filters to isolate the 5461A line. The lenses have a converging power corresponding to f:0.6. The focus is not changed by changes in the refraction of the oil—in fact, the focus is the same with no liquid in the tank as when filled with liquid. This is of some interest for those who might wish to use the instrument for Rochelle salt and accordingly use a fluid of refractive index about 1.495

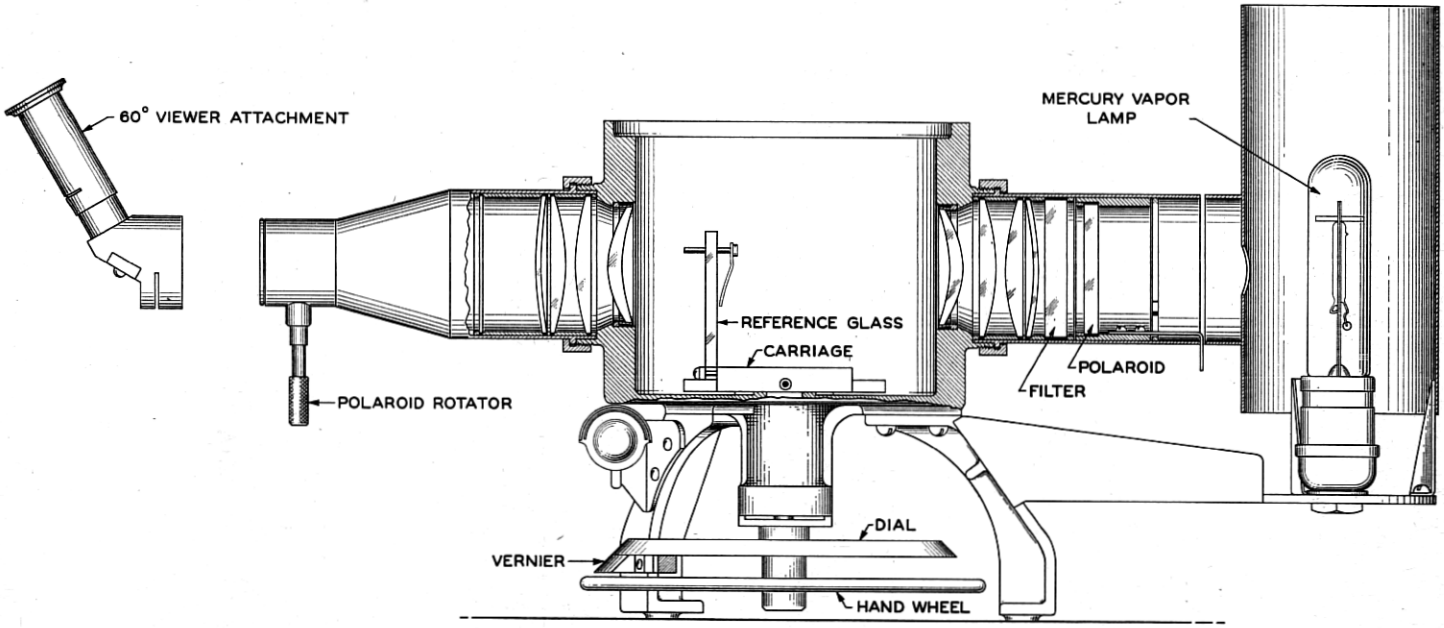

Fig. 2.32—The Western Electric conoscope

instead of the 1.546 of quartz. The dial is graduated into degrees and a vernier allows readings to tenths of degrees.

If a crystal plate is held against the glass reference surface one may read the angle between the optic axis and the surface normal. One should occasionally check the instrument (against slippage of the dial) by reversing the crystal and recentering the pattern. If the readings are not identical, the dial should be adjusted till they are. Even if the readings are not identical the mean value should be correct. If one is using the method of ring

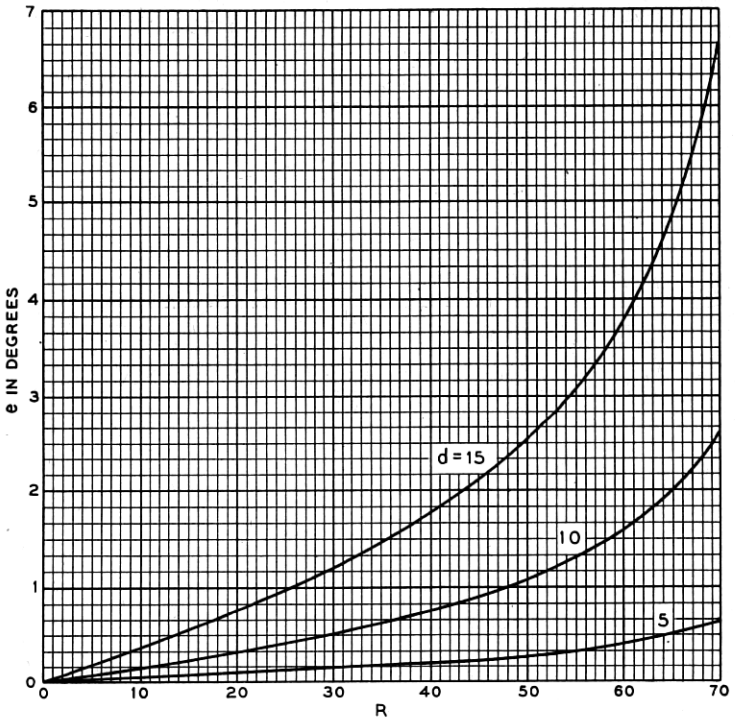

Fig. 2.33—The ring eccentricity correction chart

centering, the correction for eccentricity from Fig. 2.33 should be applied to this mean value.

The carriage may be slid back and forth and for very small crystals the carriage should be placed so that the crystal is near the center of the tank so that very little of the light cone by-passes the crystal. By the use of a block a thin crystal can be examined by viewing through its edge or length.

The carriage can be removed and "raw" crystals examined. The optic axis is plainly visible and quite accurate orientations can be made if there is not too much opaque material on the crystal. Excessive optical twinning

makes a confused pattern but good orientations can be made anyway. A "raw" crystal can be mounted adjustably in a jig that is lowered into the conoscope, the optic axis lined up, the jig transferred to a saw, and sections sawed directly.

Let us turn now to the quantitative analysis of the ring pattern seen in the eye piece when examining a uniaxial crystal. We wish to know the size of the smallest ring in the field, or rather the corresponding angle in the crystal. This first dark ring (analyzer and polarizer crossed) is the result of the slow wave falling *one* wave length behind the fast one. If the plate thickness (Fig. 2.34) is t' the path length in the crystal is

$$t = \frac{t'}{\cos\theta} \tag{2.8}$$

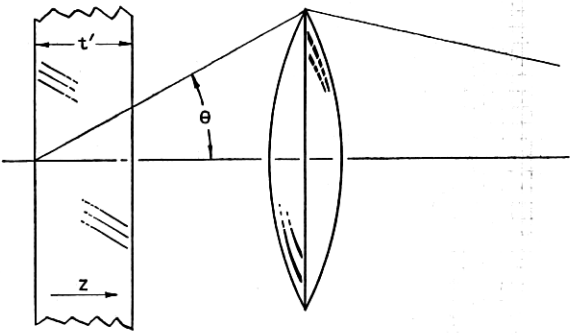

Fig. 2.34—The angle of the smallest ring

This is to be substituted in Eq. 2.7, namely:

$$N = \frac{t}{\lambda}(n_f - n_s) \tag{2.7}$$

Now it can be shown that, quite accurately, at the angle θ from the optic axis:

$$n_s - n_f = .00917 \sin^2\theta \tag{2.9}$$

where .00917 is the difference in the refractive indices for the ordinary ray and the extraordinary ray for green mercury light traveling at right angles to the optic axis. (These are generally given the symbols n_o and n_e or n_ω and n_ϵ respectively.)

$$N_1 = \frac{t'}{\lambda \cos\theta} \times .00917 \sin^2\theta = 1$$

and since $\lambda = .000546$ mm. for green mercury light this may be written

$$t' \sin\theta \tan\theta = 0.0595 \text{ mm.} \tag{2.10}$$

whence we solve for the values in this table

convergence	$\theta = 5°$	10°	20°	30°
thickness	$t' = 7.8$	1.94	0.48	0.21

This shows that if we wish to examine thin plates in a conoscope the lenses must be strongly convergent. The conoscope used in the Western Electric has a convergence corresponding to about the 20° entry of the table so it

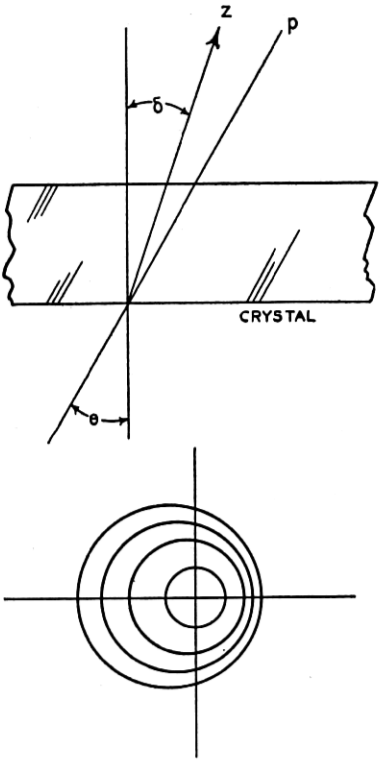

Fig. 2.35—Ring eccentricity

can be used on crystals down to a half millimeter thick—that is for *orientation* studies. In determining *handedness* we remember that this is a question of the rotatory power of quartz causing the rings to expand or contract on rotating the analyzer. Also we said that this rotatory power effectively disappears at 15° from the axis. If no ring is found within 15° of the axis there is no ring capable of expanding or contracting and we cannot test the handedness of such a thin crystal no matter how strong a lens we employ. We can then fall back on the succession of colors shown when we rotate the

analyzer using white light. Red, yellow, blue are observed for clockwise rotation with IRE right-hand quartz.

If the z section is not a true one, Eq. 2.9 will be replaced by one allowing for this error δ:

$$n_s - n_f = .00917 \sin^2 (\theta - \delta)$$

This will make the rings non-concentric and although the "cross" intersection is still the true optic axis the ring centers are not.

Hence if we are tempted to find the optic axis by centering a large sharp ring in preference to a small fuzzy one we find this eccentricity error must be allowed for.

2.8 Rotation of the Plane of Polarization

If we have a polarizer and an analyzer set for extinction (Fig. 2.36), then place a thin z section of quartz between them, the field brightens up but can be extinguished again by rotating the analyzer, Fig. 2.37. For the mineralogist's right-hand quartz the analyzer must be rotated 21.7° (yellow light assumed) clockwise to re-establish extinction, counter-clockwise 21.7° for left-hand quartz. The rotation is more for blue light, less for red. If the section is not a perfect z section the rotation is less than this, effectively disappearing at about 15° from the optic axis.

A thick slab can be examined in this way and, due to the color difference in rotation, "rainbows" will be seen in the quartz when held at just the right orientation. These rainbows will follow the contours of the specimen unless both right and left quartz are present in one piece. When this is the case the one kind generally occurs as spike- or blade-like intrusions in the other. It will then cause the rainbows to have sharp, jagged outlines bearing no relation to the specimen contour.

Also, since red, yellow, blue, are here in the order of increasing rotation, if we rotate the analyzer clockwise for right-hand quartz (I.R.E. RHQ) we will pass through best transmission for red, best for yellow and best for blue in that order so that the field will assume these colors in this order.

With uncut stones this examination is best made under an immersion fluid. The *inspectoscope* is made for this work. We spoke of the rotation of the plane of polarization and its complicating of the issue for the conoscope. Due to this the field at the center is not dark when the analyzer and polarizer are crossed. Also if we rotate the analyzer clockwise the rings of the pattern either expand or contract according to whether the crystal is right-hand quartz or left-hand quartz (IRE definition).

A different kind of pattern is visible in the conoscope when viewed perpendicular to z, a double set of hyperbolae as shown in Fig. 2.38. This pat-

tern has been used to check the orientation but much grief has ensued due to not recognizing one of its properties. This property is that, if z does not lie parallel to the crystal boundary the center of the pattern is not perpendicular to the optic axis and a rather involved correction must be used. This correction reduces the actual angle to about half the observed value.

This conoscope is an immersion instrument. The fluid is chosen to have an index of refraction to match the "ordinary" one for quartz. When this is done light is not bent in passing between fluid and quartz. When the fluid does *not* match there is a bending and all readings are subject to a

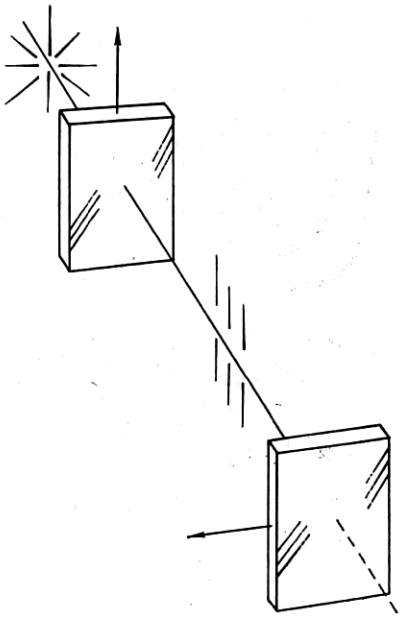

Fig. 2.36—Crossed polarizers

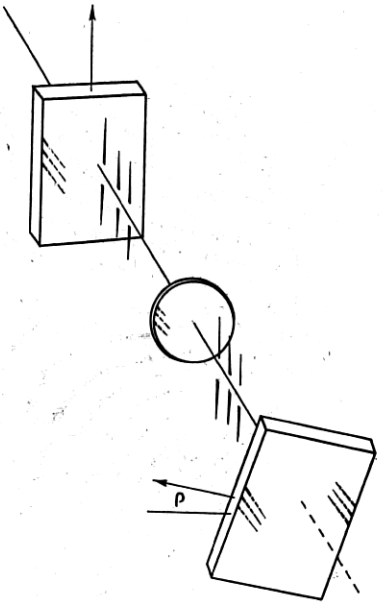

Fig. 2.37—Rotation of the plane of polarizer

correction. For example if we measure the angle of an AT plate in a fluid that is too low by .0048 (since n_0 for quartz in green mercury light is 1.5462 this fluid has $n = 1.5414$), we will get a reading that is too high by a quarter degree (the 35° angle will then appear low). A temperature rise of 12° C will lower the relative refractive indices by this amount.

Also the more perfectly the fluid matches, the more nearly will the rough quartz surface disappear and seem smooth and clear. This greatly enhances the sharpness of the rings.

The *refractoscope* (Fig. 2.39) was designed by G. W. Willard to tell when the

match is good. It uses the elimination of the bending as a test for refractive match. It also demonstrates the existence of two velocities in quartz, for two images are seen where a glass prism would cause but one. Also by viewing through an analyzer we see that the two images are caused by plane polarized light, the polarization planes being mutually perpendicular.

If the fluid has an index lower than that of the prism the rays will bend towards the base of the prism. For this reason, light that reaches the eye e from s must travel by the path $s\ q_0 p_0 e$ for the ordinary ray, $s\ q_\epsilon p_\epsilon e$ for the

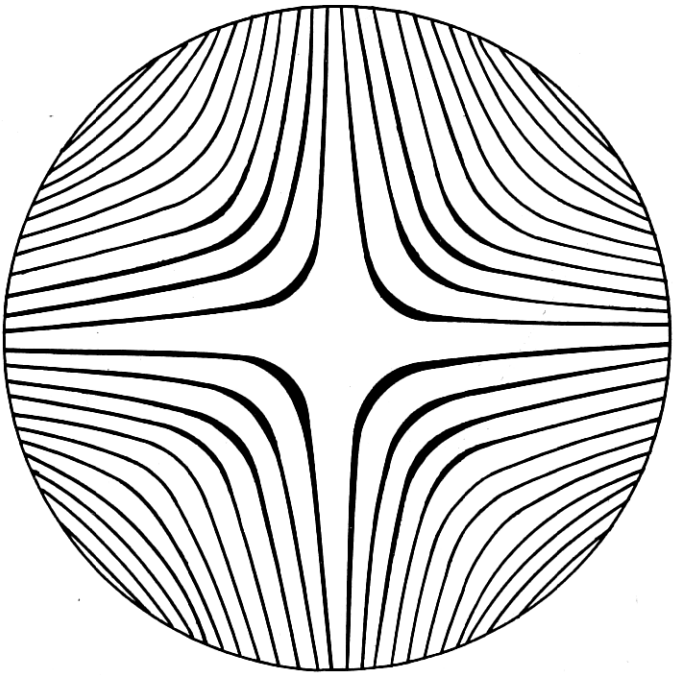

Fig. 2.38—Conoscope view normal to the optic axis

extraordinary ray. Hence the slit as seen through the prism will appear at s_0' for the ordinary ray, at s_ϵ' for the extraordinary while the slit as seen directly alongside the prism will appear at s. If the fluid index matches the quartz prism for the ordinary ray this ray will be unbent at p_0 and q_0 so that s_0' will appear as a continuation of s.

If the fluid index is too high the image s_0' will appear to the left of s with s_ϵ' still to the right of s_0'.

As the refractive indices of quartz for the ordinary and the extraordinary ray differ by .009 the apparent separation of s_0 and s_ϵ' represents .009 and

can be used to judge the difference between the liquid index and the quartz ordinary index.

The liquid can be adjusted by placing a cap face of a crystal on the reference glass and setting the dial to read 51.8° the liquid being then blended to center the pattern.

If the refractive index of the fluid is low by an amount L the observed reading R must be corrected by adding to it an amount ϵ.

Since

$$(N_q - L) \sin R = N \sin (R - \epsilon)$$

we can compute ϵ. The accompanying refraction correction nomograph was computed from the above equation. If we know that the fluid in the conoscope is high, by an amount H, and we wish to know the correction to be applied to a conoscope reading R we locate H on the diagonal line HL and locate R on the horizontal line R. We join these two points with a straight line and read the scale $\epsilon - \epsilon$ where this straight line crosses the curved line $\epsilon - \epsilon$. This value tells the size of the correction and whether to add it to R or subtract it from R. Conversely, if we wish to find how closely the index of the fluid must be held in order to have the correction less than say $\frac{1}{4}°$ at a reading of say $R = 50°$ we join the points $R = 50°$ and $\epsilon = \pm \frac{1}{4}°$ and find $H = \pm 0.005$. A ten-inch-long lucite strip with a straight line ruled on it is a convenient tool with which to read this nomograph.

We now inquire as to whether the refraction correction can be made to annul the ring eccentricity correction. In the appendix it is shown that this is done if $H = -.530 \tan^2 d$ where $2d$ is the distance between the vertical reticule lines.

Experimentally it is easy to achieve this balance by using a cap face parallel slice. With the cap face against the reference glass and the dial reading 51.8° the fluid is blended to make a single ring tangent to both reticule lines. When this is done for $d = 10°$ the fluid should have a refractive index of 1.5228 and the residual errors should be less than 2 minutes for R not over 60°.

2.9 Immersion Fluids

In order to match the refractive index of quartz we blend a substance which has an index that is too large with one that has an index that is too small. Such blended substances should be liquid at room temperature and hence should be perfectly mutually soluble. They should have low vapor pressure so that they do not evaporate quickly and should be harmless to the operator. Also they should be nearly colorless and clear. They should be fluent enough to be easily drained from the crystal and should have a

sufficiently high flash point that they would not present a fire hazard. The odor should not cause distress and finally the cost must be reasonable.

Dr. G. T. Kohman of the Bell Telephone Laboratories has prepared a list of such substances that can be mixed, any high one with any low one to

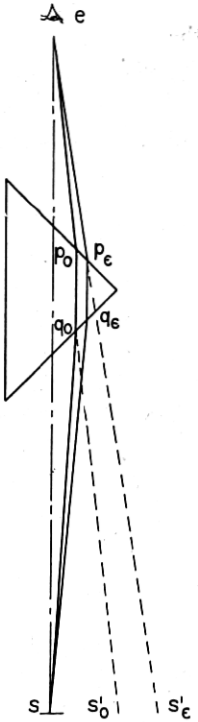

Fig. 2.39—The refractoscope

obtain a fluid satisfactory in all these respects. The following mixtures of substances are taken from his data.

Substance	Refractive Index	Mixture Parts by wt.	Density	Flash Point	Supplier
Dimethyl phthalate α monochlor naphthalene	1.51 1.63	73.9 26.1	1.193 1.194	255°F	Monsanto Chem. Co. Bakelite Co.
Dimethyl phthalate Dichlor naphthalene (solid at toom temp.)	1.51 1.63+	73.9 26.1	1.193 1.30	285°F	Monsanto Chem. Co. Hooker Chem. Co.
Decalin Dowtherm	1.467 1.586	35.3 64.7	0.895 1.1	170°F	Dupont Co. Dow Chem. Co.
Kerosene				170°F	

An immersion fluid for Rochelle salt can be made by mixing decalin with any of the other substances. For example *b* mixture of 34 parts of dimethyl phthalate and 66 parts of decalin should give the necessary index 1.495.

2.10 APPENDIX

THE RING ECCENTRICITY CORRECTION

Referring to Fig. 2.41, we see that, at an angle α, from the optic axis towards the plate thickness direction the phase relation is, by Equations (2.7) and (2.9):

$$N_1 = \frac{.00917\ t'\ \sin^2 \alpha_1}{\lambda \cos(\delta - \alpha_1)}$$

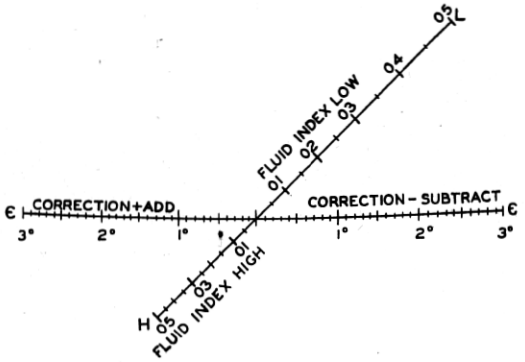

Fig. 2.40—Refraction correction nomograph

while at an angle α_2 away, it is:

$$N_2 = \frac{.00917\ t'\ \sin^2 \alpha_2}{\lambda \cos(\delta + \alpha_2)}$$

Whence, if these are equal, we have:

$$\frac{\sin^2 \alpha_1}{\cos(\delta - \alpha_1)} = \frac{\sin^2 \alpha_2}{\cos(\delta + \alpha_2)} \qquad (2.11)$$

These points, in the conoscope field, being of equal phase are parts of the same ring, and if matched to a pair of reticule lines, the optic axis is off their center line by an angle e, where

$$e = \frac{\alpha_1 - \alpha_2}{2}$$

If the separation of the reticule lines corresponds to an angle $2d$, we see that

$$d = \frac{\alpha_1 + \alpha_2}{2}$$

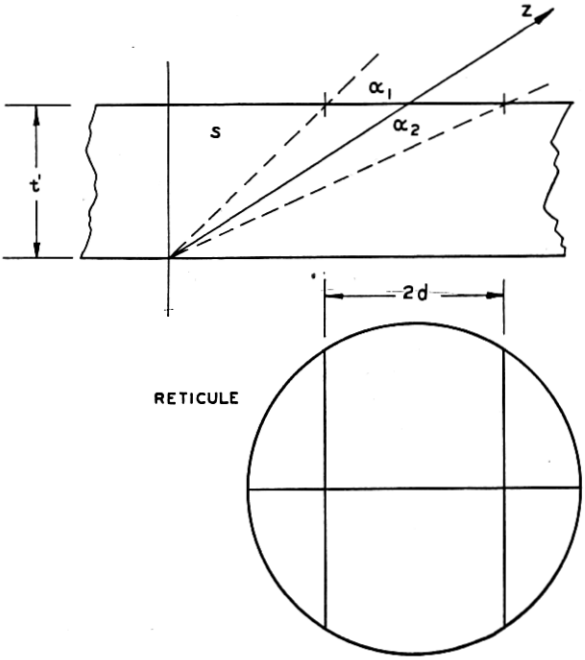

Fig. 2.41—Off center cross hairs

so that

$$\alpha_1 = d + e \quad \text{and} \quad \alpha_2 = d - e$$

The reading of the dial will be, at this match

$$R = \tfrac{1}{2}(\delta + \alpha_2 + \delta - \alpha_2) = \delta - \frac{\alpha_1 - \alpha_2}{2}$$

Hence we have

$$\frac{\sin(d + e)}{\sin(d - e)} = \sqrt{\frac{\cos(R - d)}{\cos(R + d)}} \tag{2.12}$$

For a given value of d, we can plot the values of e as a function of δ as given by (2.12). This plot is a chart of corrections to be added to the readings R to find the true angle δ.

Examination of Equation (2.12) shows us that the correction e is independent of the thickness t' and even of the birefringence; hence, the chart could serve for all uniaxial crystals. Equation (2.12) can be given an approximate solution:

$$e = 1820 \tan^2 d \tan R \text{ minutes}$$

$$e = 30.3 \tan^2 d \tan R \text{ degrees} \qquad (2.13)$$

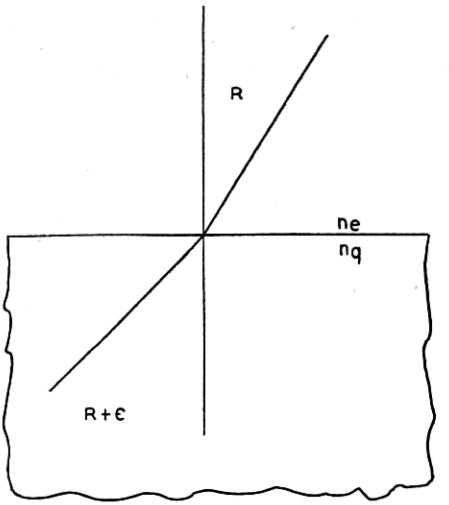

Fig. 2.42—Refraction at a surface

For R not more than 60 degrees and d not more than 15 degrees the error in e is not more than 5 minutes. Figure 2.33 is a chart of these corrections computed from the more exact equation (2.12).

Annulling the Ring Eccentricity Correction by Means of the Refraction Correction

The difference in quartz index and liquid index is

$$H = N_q - N_\ell$$

and by the law of refraction:

$$N_\ell \sin R = N_q \sin (R+\epsilon)$$

whence

$$-H = N_q \frac{\sin \epsilon}{\tan R} - (1 - \cos \epsilon)$$

and if ϵ is small

$$H = -\frac{N_q \sin \epsilon}{\tan R}$$

From Eq. 2.13, $e = 0.530 \tan^2 d \tan R$ radians, and putting this in the equation for H we find that the correction for ring eccentricity approximately annuls the correction for refraction if

$$H = - .530 \tan^2 d$$

For $d = 10°$ this gives $H = -.0255$, that is, a fluid index of 1.5207.