The Transmission Characteristics of Toll Telephone Cables at Carrier Frequencies

By C. M. HEBBERT

IN THE design of a new telephone transmission system a knowledge of the characteristics of the medium over which the waves are to pass is, of course, a prerequisite. What painstaking experimentation is necessary to accumulate such knowledge, however, what voluminous data are involved, what minutiae of detail, and what extremes of accuracy, are things far less obvious.

Recent papers have described a new 12-channel carrier telephone system for operation over cable pairs.[1] For this system a knowledge of the maximum cable losses is needed in order to determine the necessary repeater gains. Accurate data on the insertion loss slope versus frequency are required so that compensating equalizers can be designed to give uniform transmission over the frequency band. In order to design a regulating system to compensate for the variations in attenuation which result from changes in cable temperature, precise knowledge of these variations as a function of frequency is essential. It is necessary to know the impedance of the cable pairs in order that the amplifier impedance may be matched to it, thereby avoiding reflections which would aggravate cross-talk effects. For various purposes, e.g., testing the cables, designing the coils to balance out crosstalk, etc., it is also necessary to know the fundamental parameters (resistance, inductance, capacitance and conductance) or so-called primary constants of the pairs. The velocity of transmission also plays a part in determining the characteristics of the channels. In addition to all these transmission characteristics, it is, of course, essential to know the cross-talk couplings between different pairs. This subject has been treated elsewhere[2], however, and is not considered herein.

In order that the cable carrier systems may be applied in the plant without requiring extensive transmission measurements on each individual carrier pair in each repeater section, it is important that the differences in the transmission characteristics between different pairs be known. The problem therefore becomes one of statistical analysis. In most cases the

[1] "A Carrier Telephone System for Toll Cables," C. W. Green and E. I. Green, *B.S.T.J.*, Vol. 17, January 1938, page 80. "Experience in Applying Carrier Telephone Systems to Toll Cables," W. B. Bedell, G. B. Ransom and W. A. Stevens, *B.S.T.J.*, Vol. 18, October 1939, page 547.

[2] "Crosstalk and Noise Features of Cable Carrier Telephone System," M. A. Weaver, R. S. Tucker and P. S. Darnell, *B.S.T.J.*, Vol. 17, January 1938, page 137.

effects involved are cumulative with distance and the accuracy involved in the determination of the various characteristics is therefore set by the maximum distance over which the system is designed to operate. For a distance of 4000 miles the total loss at the top frequency of 60 kc. will be approximately 16,000 db, the attenuation difference between the top frequency of 60 kc. and the bottom frequency of 12 kc. nearly 6000 db if the cable is at about the average temperature, 55°F. The range of variation in loss with temperature, assuming aerial cable over the whole distance, will be about ±8 per cent of the total at 60 kc. It is desired to correct these frequency differences in loss and variations with temperature so accurately that individual channels will be constant to within ±2 db.

Prior to the beginning of experimentation with cable carrier systems limited use had been made, in connection with carrier systems operated over open-wire lines at frequencies up to 30 kc., of conductors in relatively short entrance and intermediate cables. The available data, however, were quite inadequate for the cable carrier problem. Accordingly, an extensive series of tests was undertaken. Reels of standard toll cable were placed in temperature controlled rooms where the extreme temperature variations of the mid-west could be substantially duplicated (the actual laboratory temperatures ranged from just below 0° F to 120° F) and measurements were made to determine the changes in the parameters of the cable accompanying these wide temperature variations at frequencies from 1 kc. to 100 kc. and higher in some cases. Certain of the tests even studied the effect of varying the humidity content of the cables. Further measurements were then made on suitable lengths of pairs in actual commercial cables in which carrier systems were to be installed. These results corroborated and extended the data from the laboratory measurements; the subsequent operation of equalizers, regulators, etc., based upon these data, showed no essential discrepancies.

The present paper, after referring to the types of toll cables employed for the new carrier systems, outlines the methods employed in determining their characteristics both in the laboratory and in the field, summarizes these characteristics for typical 19-gauge cable at frequencies up to 100 kc. and finally extends them to frequencies as high as 700 kc. for 16 and 19-gauge cables.

Types of Carrier Toll Cables

The type K carrier system has been designed so that it may be applied to existing cables, thus in many cases avoiding the installation of expensive new cables. Most of the standard toll cable in the Bell System contains chiefly 19-gauge paper insulated conductors in "multiple twin quads," i.e., two conductors are twisted together to form a pair and two pairs twisted together to make a quad. The nominal capacitance of a pair is .062 mf. per mile. There are various twist lengths of both pairs and quads in a given cable as well as cables ranging in size from 12-quad cable to the oversize 19-gauge cable containing 225 quads. Some type K is operating over "paired" cable, i.e., cable in which only the wires of each pair are twisted together.

Operation in two directions is accomplished by using either a separate cable for each direction or a single cable with two groups of conductors separated by a layer shield. This avoids serious near-end crosstalk effects which would result from the large level difference existing between opposite directions at a repeater point.

Methods of Measurement

As mentioned above, 250-foot reels of standard toll cable were placed in a special room which could be accurately maintained at any desired temperature from about zero to 120 degrees, Fahrenheit, and measurements made for various frequencies and temperatures. For the most part, these consisted of open-circuit admittance and short-circuit impedance measurements on part of the pairs in the cable at temperatures of about zero, 30, 50, 90 and 120 degrees F., over the frequency range from 4 to 100 kc. From these measurements computations could then be made to determine the resistance, inductance, capacitance and conductance as well as the attenuation, phase constant and characteristic impedance of this type of cable at the different temperatures and frequencies. Detailed data on frequency and temperature variations of these quantities are given below. Most of these data were obtained from measurements made on 16- and 19-gauge pairs in a typical reel-length of standard toll cable. The temperature is difficult to maintain at a constant level and d-c. resistances of certain pairs were measured at frequent intervals during the process in order to get accurate temperature readings by comparing with resistance-temperature curves of these pairs. Thermocouples were also attached to the cable at various points along its length and sheath temperatures determined from them. After stabilizing the room temperature as closely as possible, the variations in cable temperature took place slowly enough to be allowed for in the computations.

After the selection of the Toledo-South Bend route for a trial installation, further measurements were made on certain of the pairs in this cable. The test sections, extending out of Lagrange, Indiana, were made about 10 miles long in order to obtain the averaging effect of length. For this distance it was not possible to use open and short-circuit measurements as was done in the laboratory, and a substitution method[3] was devised (Fig. 1).

[3] This was devised by H. B. Noyes and will be described by him in a paper in the *Bell Laboratories Record.*

This consists essentially of first measuring the input and output a-c. currents at the two ends of the test pair by means of thermocouple-milliammeter arrangements and then immediately sending d-c. over another pair (called reference pair in Fig. 1) built out to a convenient fixed d-c. resistance, the same for all measurements, and adjusting resistance networks at both ends of the line until the meter readings are the same as for the a-c. Suitable charts then enable readings of attenuation (insertion loss) corresponding to the d-c. (and therefore also to the a-c.) readings to be made very rapidly.

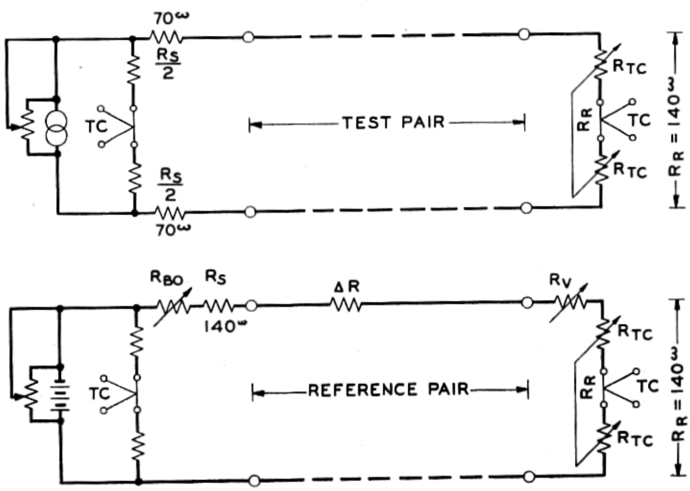

Fig. 1—Simplified attenuation measuring circuit

Toll Cable Characteristics Below 100 Kc.

Primary Line Parameters

The four primary line parameters, R, L, G and C—series resistance, series inductance, shunt leakage and shunt capacitance—are of the same sort for all kinds of transmission lines, but the relative importance of the various elements changes considerably with frequency and the type of structure considered. The old name primary "constants" is obviously a misnomer, and it is simpler to speak of them as line "parameters," since this does not necessarily imply anything regarding their constancy or inconstancy under various conditions.

The "true" or distributed values of these parameters are usually obtained from measurements of the open circuit admittance, $G' + j\omega C'$, and the short-circuit impedance, $R' + j\omega L'$ of the actual pair in a short length of cable, by means of the following formulas:

$$R \text{ (resistance)} \quad = R'(1 - \tfrac{2}{3}\,\omega^2 L'C') + \cdots$$
$$L \text{ (inductance)} = L'(1 - \tfrac{1}{3}\,\omega^2 L'C' \cdots) + \tfrac{1}{3}\,R'^2C' + \cdots$$
$$G \text{ (conductance)} = G'(1 - \tfrac{2}{3}\,\omega^2 L'C' \cdots) - \tfrac{1}{3}\,R'\omega^2 C'^2 + \tfrac{2}{5}\,R'\omega^4 L'C'^3 \cdots$$
$$C \text{ (capacitance)} = C'(1 - \tfrac{1}{3}\,\omega^2 L'C' + \tfrac{2}{3}\,R'G' \cdots) \qquad (1)$$

These formulas give accuracies within one per cent for reel-lengths of 500 feet or less and frequencies up to 100 kilocycles for 19-gauge cables having a capacity of .062 mf per mile. All the curves of R, L, G, C herewith are based on true values obtained from such computations.

Resistance

The quantity R, series resistance in ohms per mile, has a large variation with frequency produced by the well-known phenomenon called skin effect and another large increment, resulting from the closeness of the wires in cables, known as the proximity effect.[4–7]. The magnitude of the proximity effect varies with the diameter of the conductors as well as with their separation. The curves in Fig. 2 show the increment in resistance resulting from skin effect and the total increase including proximity effect as computed for a pair of wires separated by various multiples of their diameters. The abscissa, B, in Fig. 2 is a sort of universal parameter used in data on skin effect so that a single curve will suffice for various gauges. If f is frequency in cycles per second and R_0 is the d-c. resistance for 1000 feet of the wire (not a 1000-foot loop), the parameter B is given by the equation

$$B = \sqrt{f/R_0} \doteq \sqrt{f/8} \qquad (2)$$

for 19-gauge wire so that $B = 80$ corresponds to 51,200 cycles. According to the curves at $B = 80$ (51.2 kc), the skin effect increases the a-c. resistance to about 12 per cent more than the d-c. resistance.

For a separation of two diameters between centers of the wires of a pair ($k = .25$) the proximity effect adds another 6 per cent to the resistance ratio making the total a-c. resistance about 1.18 times the d-c. resistance at 51.2 kc. If the wires are closer together ($k = .4$) the a-c. resistance is computed to be about 1.30 times the d-c., which is about the ratio actually measured. The effects caused by the presence of the adjacent pair in a

[4] J. R. Carson, "Wave Propagation over Parallel Wires—The Proximity Effect," *Phil. Mag.*, Vol. 41, April 1921, pp. 607–633.
[5] A. E. Kennelly, F. A. Laws and P. H. Pierce, "Experimental Researches on Skin Effect in Conductors," *A.I.E.E. Trans.*, Vol. 34, Part 2, 1915, pp. 1953–2021.
[6] A. E. Kennelly and H. A. Affel, "Skin Effect Resistance Measurements of Conductors at Radio Frequencies," *I.R.E. Proc.*, Vol. 4, No. 6, Dec. 1916, pp. 523–574.
[7] Günter Wuckel, "Physics of Telephone Cables at High Frequencies," *EFD* 47, (Nov. 1937) pp. 209–224.

quad, the surrounding wires and the lead sheath are not included in these computations.

These values assume a temperature of 20° Centigrade (68° Fahrenheit) but if the temperature varies, so also does the resistance. Figure 3 shows the a-c. temperature coefficient of resistance and its variation with temperature for 19-gauge pairs in ohms per ohm per degree, Fahrenheit, i.e.,

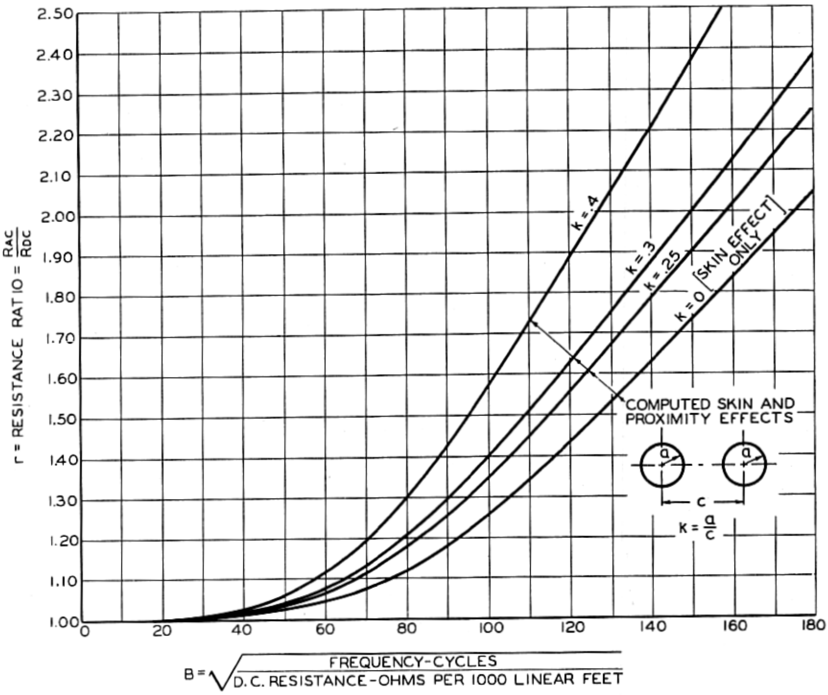

Fig. 2—Skin effect and proximity effect on a.-c. resistance of toll cable pairs

$$A = \frac{1}{R_1}\frac{dR}{dt} \tag{3}$$

where A is the a-c. temperature coefficient of resistance of copper at t_1 degrees, Fahrenheit. The a-c. resistance R at temperature t is given by the formula

$$R = R_1 [1 + A(t - t_1)] \tag{4}$$

where R_1 is the a-c. resistance at temperature t_1 degrees, Fahrenheit. The coefficient A decreases with increasing frequency, but not indefinitely; it approaches 1/2 the d-c. coefficient as its asymptotic limit with frequency.

The normal variation of air temperatures in the middle western part of the country is from about 1° Fahrenheit, (−17° Centigrade) to plus 109° Fahrenheit (43° Centigrade). Extremely hot summers like that in 1936, which was preceded by a severe winter, show even higher temperatures and there are occasional periods in mid-western winters when the temperature hovers continuously around −30° F., for a week or two. These temperatures are almost the temperatures assumed by open wires, but wires inside a lead sheath like those in an aerial cable are subjected to much higher than air temperatures in hot weather when exposed to direct sunlight in the absence of wind. Some observations made at Lagrange, Indiana, in 1936

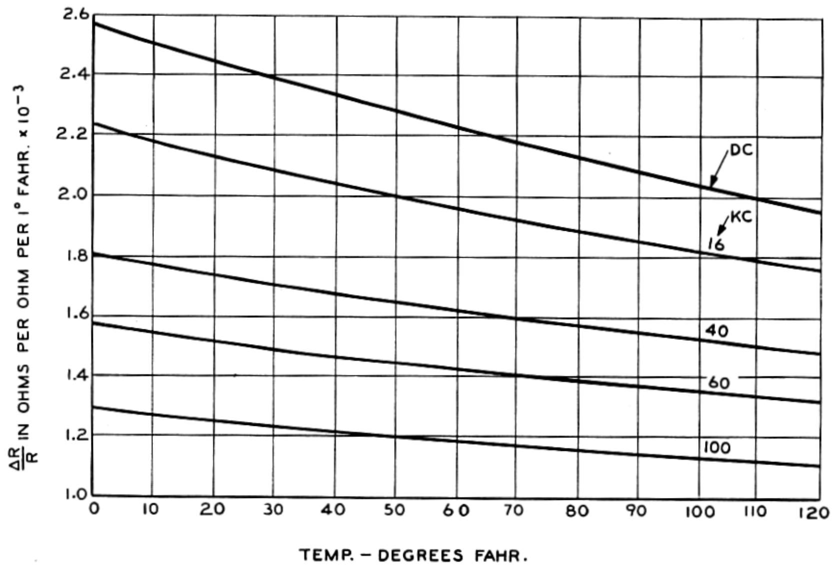

Fig. 3—Resistance-temperature coefficient—$\frac{\Delta R}{R}$ in ohms per ohm per 1° fahr.-19 gauge cable

on d-c. resistance of cable pairs showed that temperatures in the ten miles of cable averaged about 122° Fahrenheit (50° Centigrade) when the air temperature read on thermometers was about 104° Fahrenheit (40° Centigrade). Similar data taken at Chester, New Jersey, showed temperatures in the cables 20° to 25° Fahrenheit higher than the air temperature on hot bright days.

The actual observed cable temperature range in that season (1936) as indicated by the d-c. resistance measurements was thus from −4° Fahrenheit (−20° Centigrade) to 122° Fahrenheit (50° Centigrade). In terms of a-c. resistance changes, this amounts to a resistance change of about 20 ohms per mile at 50 kc., the resistance at the lower temperature being about 96 ohms per mile and at the higher temperature about 116 ohms per mile. This amounts to about ± 10 per cent variation from the mean.

In addition to the wide annual variations, there are daily variations of as much as 50° Fahrenheit at times, that is, almost half as much as the normal annual variation. The practical importance of these large resistance changes lies in their large contribution to changes in attenuation as will be brought out more fully in connection with variations of attenuation with temperature.

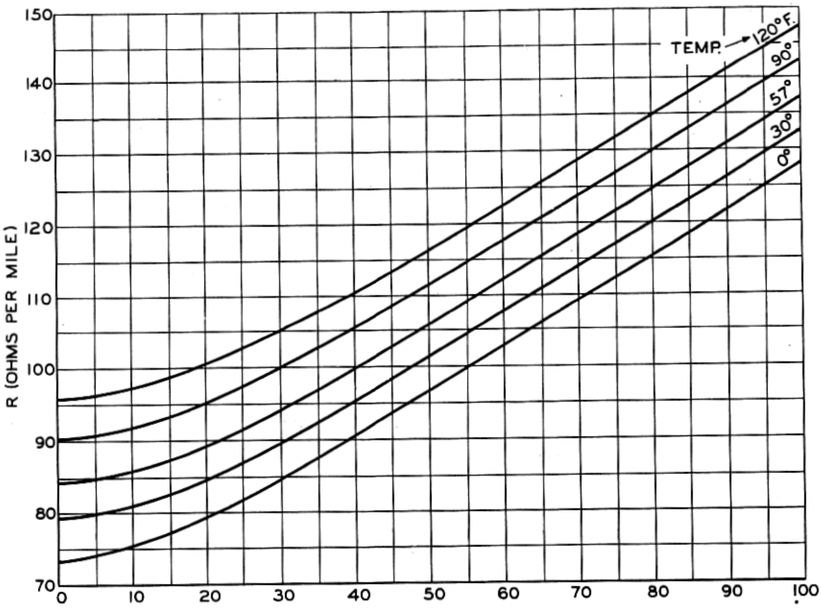

Fig. 4—Resistance per mile vs. frequency—19 gauge pairs

Underground and buried cable are, of course, not subjected to such wide annual variations and daily variations are almost entirely eliminated by the attenuation of heat changes by the soil. Cable in ducts usually lies well below the freezing line and this depth at the same time protects it from the summer's heat. The normal range for cable in ducts is from about freezing to about 70 degrees, F. Cable buried only a foot or so underground would have a considerably larger annual temperature range but a great deal of such cable is buried two to three feet deep.

Curves in Fig. 4 show the actual a-c. resistance variation with frequency and in Fig. 5 are shown temperature variations of resistance at typical frequencies for 19-gauge toll pairs in a reel-length of standard toll cable.

In addition to the variations with frequency and temperature there are the initial differences between pairs on account of manufacturing processes,

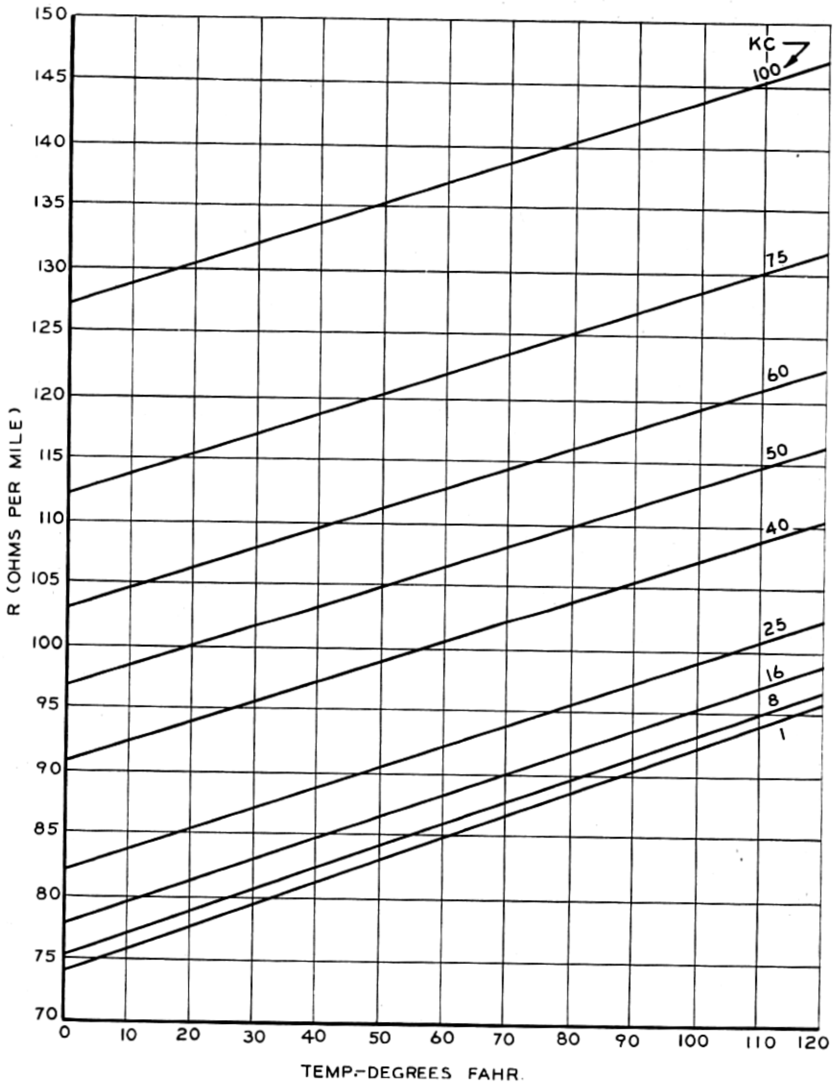

Fig. 5—Resistance per mile vs. temperature—19 gauge pairs

etc. One such source of variation in resistance of pairs is the difference in wire diameters caused by wear of the dies used in drawing wire.[8] The

[8] John R. Shea and Samuel McMullan, "Developments in the Manufacture of Copper Wire," *B.S.T.J.*, VI (April 1927), pp. 187–216.

permissible variation in diameter for ordinary toll cable is ± 1 per cent which means a d-c. resistance variation of about ± 2 per cent.

Still another cause of resistance variation is the presence of small quantities of impurities in the copper which show up as a reduction of as much as 2 per cent in the conductivity. This causes trouble in calibrating temperature-resistance curves in the laboratory setup.

Finally, in a single reel length the outside pairs are longer than the inside pairs. The total pair-to-pair variation in resistance from the average of the reel caused by all these factors amounts to about ± 3 per cent with a standard deviation of about 1.5 per cent.

Inductance

The inductance of a circuit formed by two parallel wires closely spaced relative to their length is

$$L = 0.64374\left[2.3026 \log_{10} \frac{2D}{d} + \mu\delta\right] \times 10^{-3} \text{ henrys per loop mile} \quad (5)$$

where d, the wire diameter, and D, the separation of the wires, are measured in the same unit; μ is the permeability, and δ is a frequency factor.

As is well known, the tendency of alternating currents to concentrate on the surface of a wire reduces the magnetic flux within the wire and decreases the internal inductance of the wire. This internal inductance is given by the term $\mu\delta$ in Equation (5). In like manner, the "proximity effect" produces a concentration of current density in the adjacent portions of the two wires of a pair.

Another term might well be added to formula (5) to represent this proximity effect. The procedure outlined by J. R. Carson on pages 625 and 626 of the *Philosophical Magazine* paper[4] of 1921 has been carried out with the results given in an Appendix to this paper. Formula (11a) of the Appendix gives the ratio, K, of the a-c. inductance of the pair (less the "geometric inductance") to the a-c. inductance of a wire with concentric return, which is given by a well-known formula (7a in the Appendix). It will be seen that the factor introduced by proximity effect decreases with frequency but is asymptotic to a definite value, depending upon the separation of the two wires, as the frequency increases indefinitely. Similar curves are given in an extensive study of the mutual inductance of four parallel wires of a quad by R. S. Hoyt and Sallie Pero Mead[9]. Their theoretical studies agree closely with experimental values given by R. N. Hunter and

[9] Ray S. Hoyt and Sallie Pero Mead, "Mutual Impedances of Parallel Wires," *B.S.T.J.*, XIV (1935), pp. 509–533.

R. P. Booth[10] who made measurements on 18-gauge and 20-gauge pairs in various arrangements and on a 55-foot length of 19-gauge quadded cable.

Overall inductance variations of 19-gauge pairs with frequency and temperature are shown by the curves of Figs. 6 and 7.

The magnitude of inductance variations from pair to pair in reel lengths of cable is about ±3 per cent from the mean with a standard deviation of about 1.5 per cent.

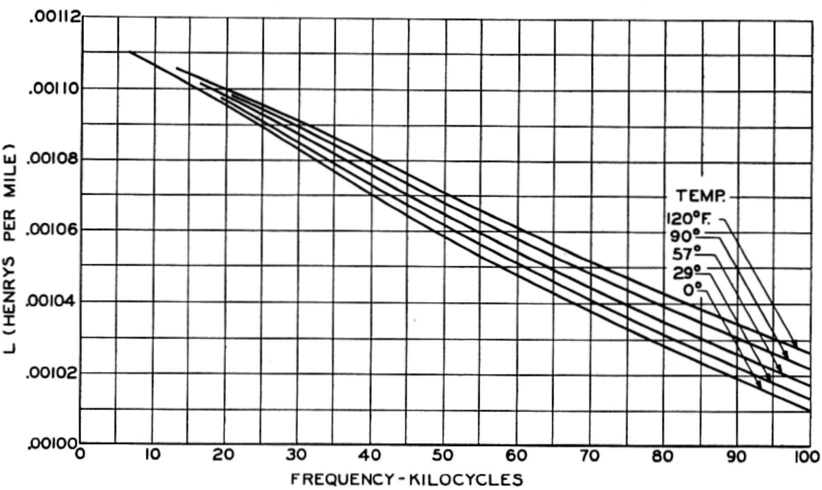

Fig. 6—Inductance per mile vs. frequency—19 gauge pairs

Capacitance

The usual formula for capacitance of two parallel wires in space, separated by a distance negligible compared with their length, is

$$C = \frac{0.019415}{\log_{10} \dfrac{2D}{d}} \times 10^{-6} \text{ farads per loop mile} \tag{6}$$

Conditions in a cable are vastly different from those assumed in this formula which assumes that the two wires are at a great distance from other wires and from the ground. In the cable, pairs are twisted and, in addition, other wires are very near and the sheath is effectively at ground potential, resulting in a considerable modification of the capacitance. Moreover, the formula assumes that the wires are in air, which has a dielectric constant almost equal to unity. (1.00059 at 0° Centigrade) The dielectric constant of

[10] R. N. Hunter and R. P. Booth, "Cable Crosstalk—Effect of Non-Uniform Current Distribution in the Wires," *B.S.T.J.*, XIV (1935) pp. 179–194.

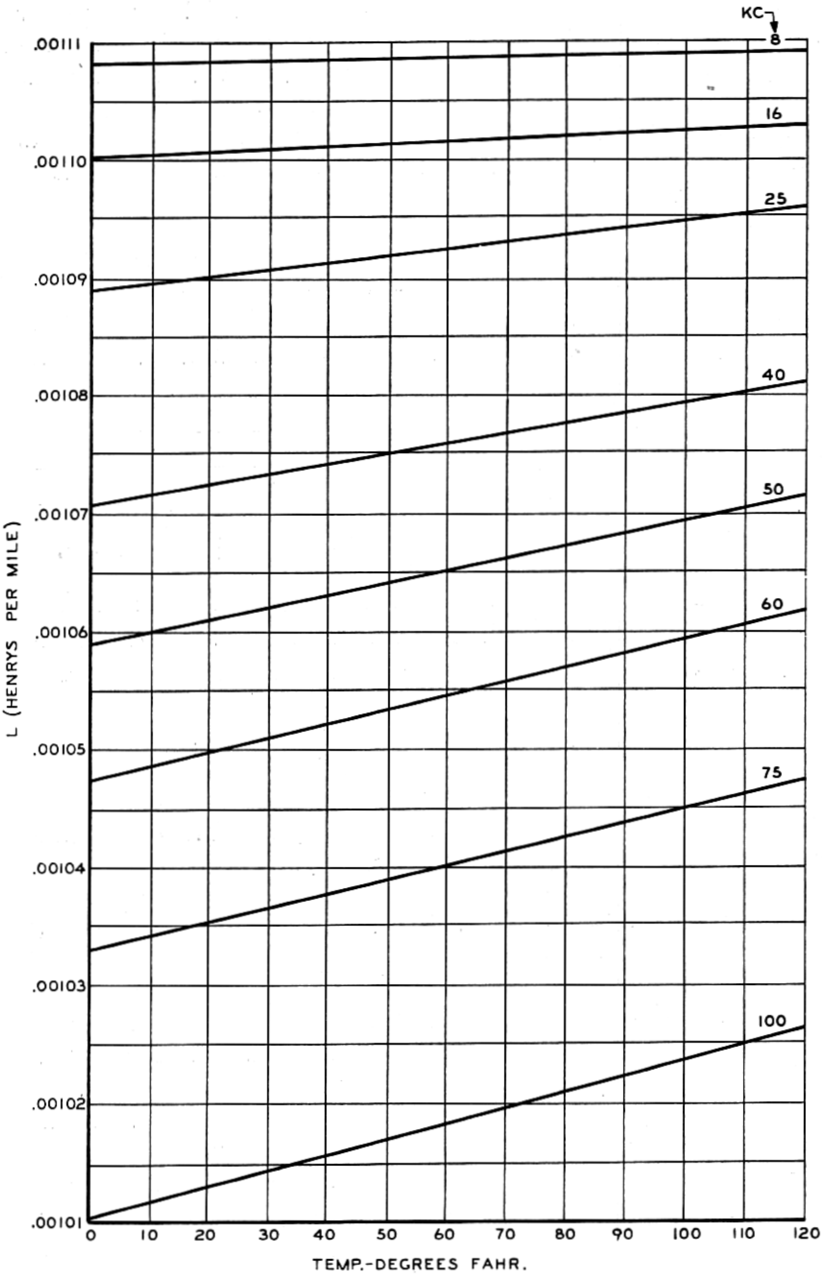

Fig. 7—Inductance per mile vs. temperature—19 gauge pairs

the paper in cables varies from 1.7 to 1.9 depending upon the amount of air and impurities contained in the paper. Of the space around the wires inside the sheath about 40 per cent is filled with paper and the remaining 60 per cent with air. Frequency and temperature of the cable affect the true dielectric constant in a complicated way. Slight amounts of moisture remaining in the cable even after drying affect the dielectric constant and the capacitance as well as the leakage conductance and introduce further changes in the frequency-temperature characteristics.

Results of an extensive study of the dielectric constant were given by E. J. Murphy and S. O. Morgan in a series of recent papers[11]. They point out (first paper, p. 494; second paper, p. 641) that a dielectric may be thought of as an assemblage of *bound charges*, that is charged particles which are so bound together that they are not able to drift from one electrode to the other under the action of an applied electric field of uniform intensity. But the applied field disturbs the equilibrium of the forces acting on the bound charges and they take up new equilibrium positions, thereby increasing their potential energy when the applied field is removed. Then when the applied field is removed, some of this energy is dissipated as heat in the dielectric. If the applied field is alternating, the bound charges swing back and forth with certain amplitudes and the sum of the product of the amplitude by the charge extended over all the bound charges in a unit volume determines the *dielectric constant* of the material. The energy dissipated as heat by the motions of the bound charges is the *dielectric loss*, which is proportional to the a-c. conductivity after the d-c. conductivity has been subtracted from it.

Considering the fact that positive and negative charges will be displaced in opposite directions and such a motion constitutes an electric current, there is thus what is called a *polarization current* or *charging current* flowing while the polarization (or displacement of charges) is being formed. If the current alternates too rapidly for the polarization to form completely before the field reverses its direction, the magnitude of the dielectric polarization and the dielectric constant will be reduced. The result of this lag, therefore, is that the dielectric constant (and likewise the capacity) decreases with increasing frequency. This is the phenomenon known as anomalous dispersion from its relation to the anomalous dispersion of light, i.e., at visible frequencies.

A further important concept in dielectric theory is that the molecules of all dielectrics except those in which the positive and negative charges are symmetrically located, possess a permanent electric moment characteristic

[11] E. J. Murphy and S. O. Morgan, "The Dielectric Properties of Insulating Materials," *B.S.T.J.* XVI (1937) pp. 493–512; XVII (1938) pp. 640–669; XVIII (1939) pp. 502–537. These are referred to as "First Paper," etc.

of those molecules. These polarized molecules are called *dipoles* and when an electric field is applied the dipole axes tend to line up in the direction of the applied field. It is probable that for a combination dielectric such as the paper and air in cables with possible traces of moisture, in spite of oven-drying, the dipoles constitute only part of the charges. The frequency also is too low, in most of the data, to emphasize the effects due to dipoles. The paper-air combination introduces another slowing up of the polarization process on account of *interfacial polarization*. Maxwell showed that if the dielectric in a condenser consisted of two layers of materials having different constants, the capacity depends upon the charging time because of time required in charging the interface between the two dielectrics. For a-c. this means a decreasing capacity with increasing frequency and, since there

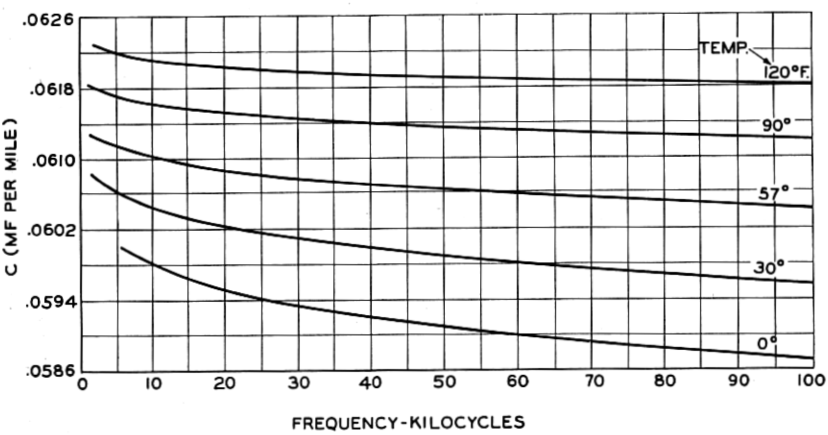

Fig. 8—Capacitance per mile vs. frequency—19 gauge pairs

are effectively an immense number of interfaces between paper and air in the cable, this effect must be of some importance.

Increasing the temperature increases the thermal energy of the molecules and their consequent thermal motion which helps maintain the random orientation of the molecules. Thus, the thermal motion opposes the action of the electric field in maintaining the alignment of the dipoles so that as the temperature rises, the polarization is reduced. But in the cable there are unequal expansions of the copper and the lead sheath which may act to increase the internal pressure as the temperature rises, increasing dielectric densities as well as bringing the wires closer together.

The final result of all these effects on the capacitance of the cable pairs is shown by the curves of Fig. 8, which give the 19-gauge capacitance-frequency relations for several temperatures. Figure 9 shows the variation of capacitance with temperature for several frequencies. The largest change

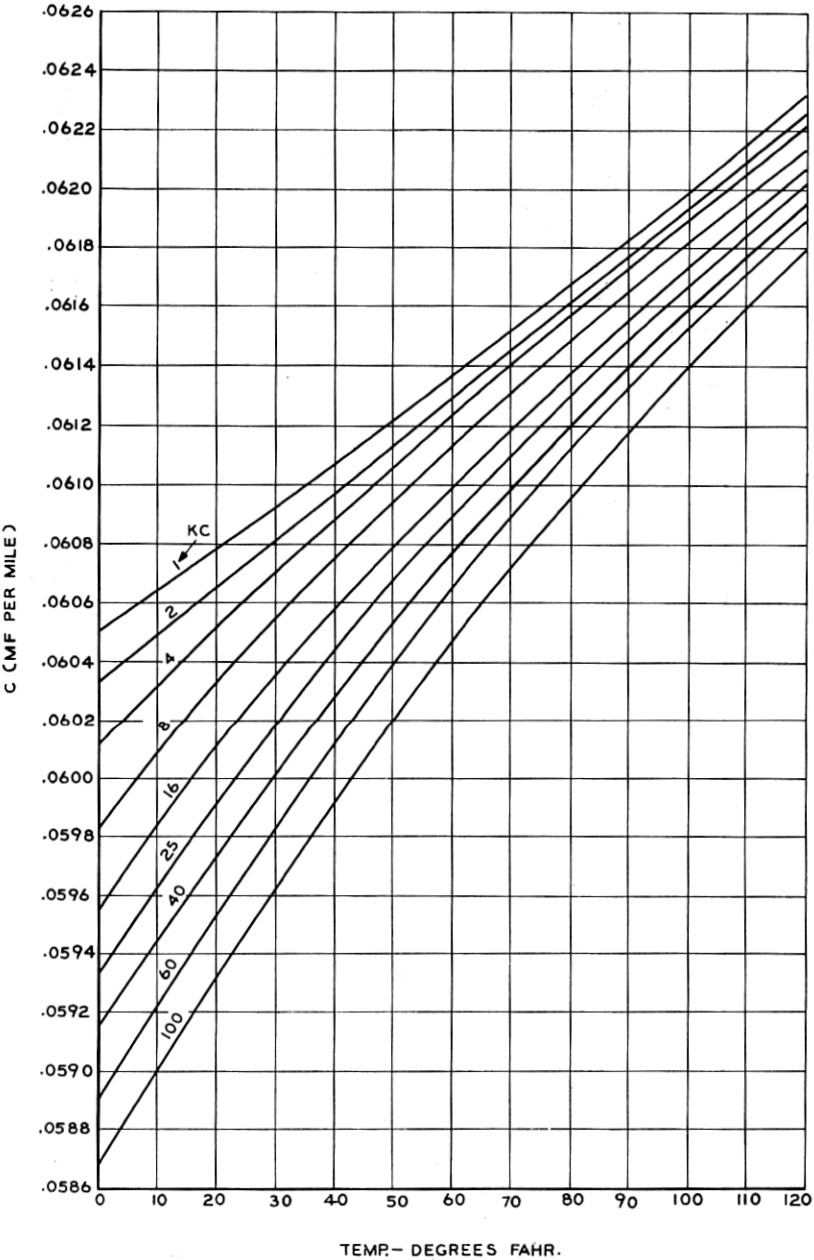

Fig. 9—Capacitance per mile vs. temperature—19 gauge pairs

shown is at 100 kc. and amounts to about 6 per cent increase for 120 degrees increase in temperature.

Leakage Conductance

The variation in the dielectric constant of the insulating layers of paper is further reflected in the leakage conductance, G. This is probably the most inconstant of the parameters and is a function of separation of the wires and their diameters, as well as frequency and temperature and the nature of the dielectric. Humidity, if present, is a highly important contributor to high leakance, but in properly dried cables the humidity is not very great. It will be seen in the later discussion of attenuation and the factors affecting it that conductance is a much less important factor, relatively, than it is for open-wire lines.

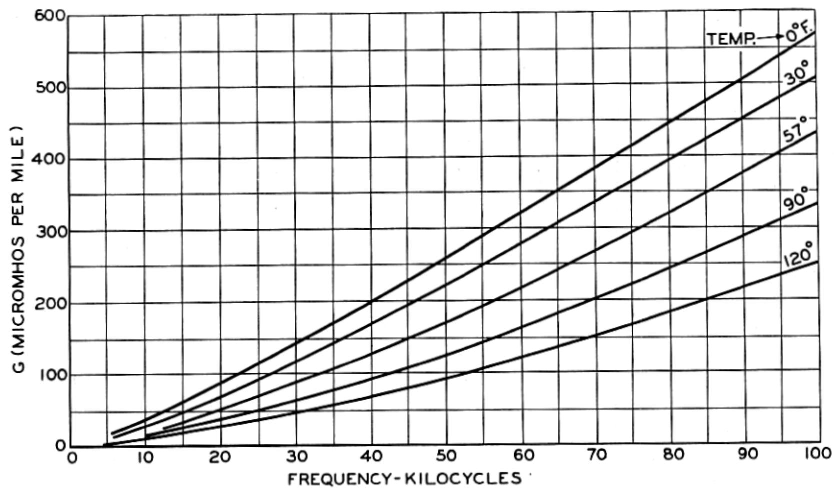

Fig. 10—Conductance per mile vs. frequency—19 gauge pairs

The curves of Fig. 10 show the variation of conductance of 19-gauge pairs with frequency at five temperatures from zero to 120° F. When plotted on log-log paper these curves are nearly linear, showing that conductance varies with frequency approximately according to a formula of the type

$$G = aF^k \tag{7}$$

where a is about $.0001 \times 10^{-6}$ and k is about 1.33 for the 57° data. F. B. Livingston in a paper[12] on conductance in cables stated that for the data there given k averaged about 1.3.

The range of conductance from pair to pair in a reel is about ± 11 per cent from the average and the standard deviation about 5.5 per cent.

[12] F. B. Livingston, "Conductance in Telephone Cables," *Bell Laboratories Record*, Vol. XVI (Dec. 1937) p. 141.

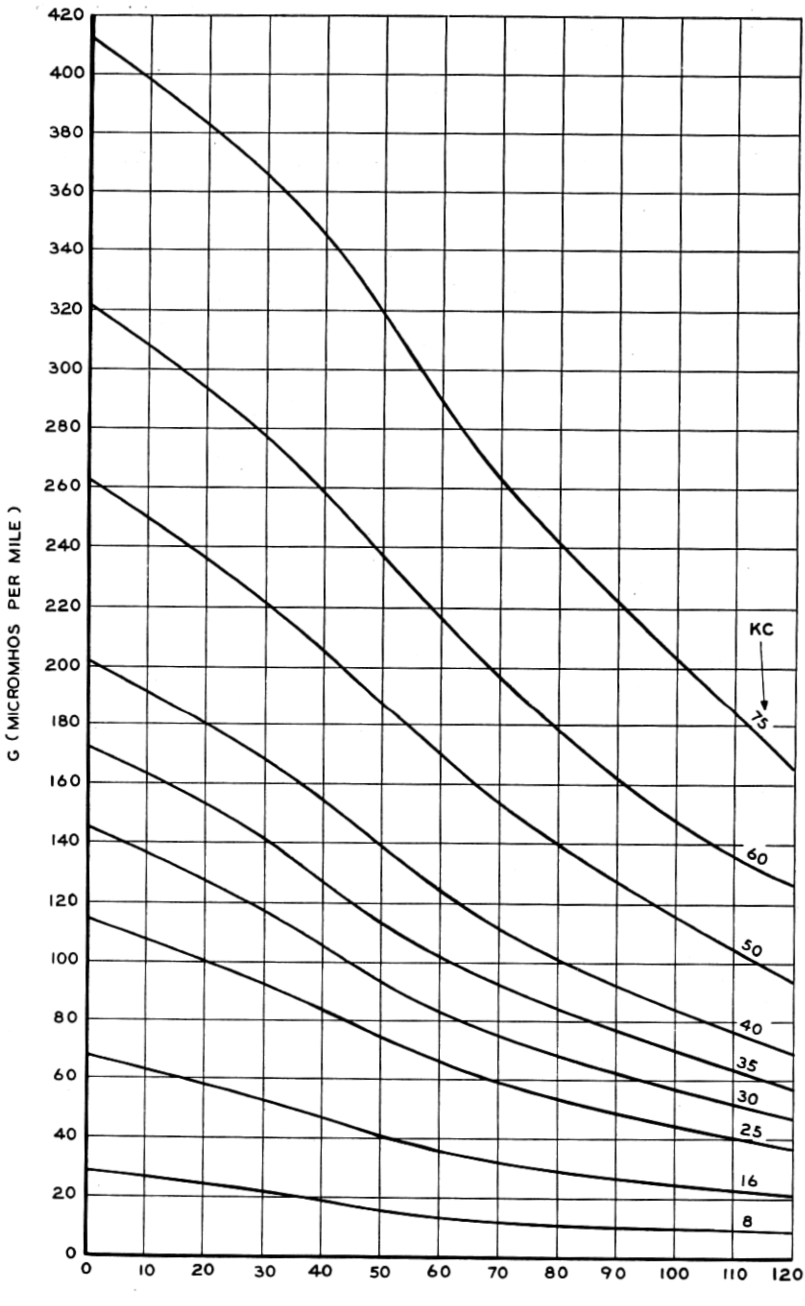

Fig. 11—Conductance per mile vs. temperature—19 gauge pairs

As might be expected, temperature has a great deal to do with the value of G. Variations with temperature shown by the curves in Fig. 11 may be expressed for small temperature ranges by the equation

$$G = G_1[1 + k(t - t_1)] \tag{8}$$

where G_1 is the value of G at the temperature t_1 and k is the temperature coefficient of leakage conductance. Curves of k based on measurements on a 61-pair, 16-gauge cable are given in Fig. 12 in the neighborhood of 70 degrees Fahrenheit. It will be noticed that k is negative below 1200 kc. but at high frequencies the coefficient increases rapidly from its minimum value reached at about 500 kc.

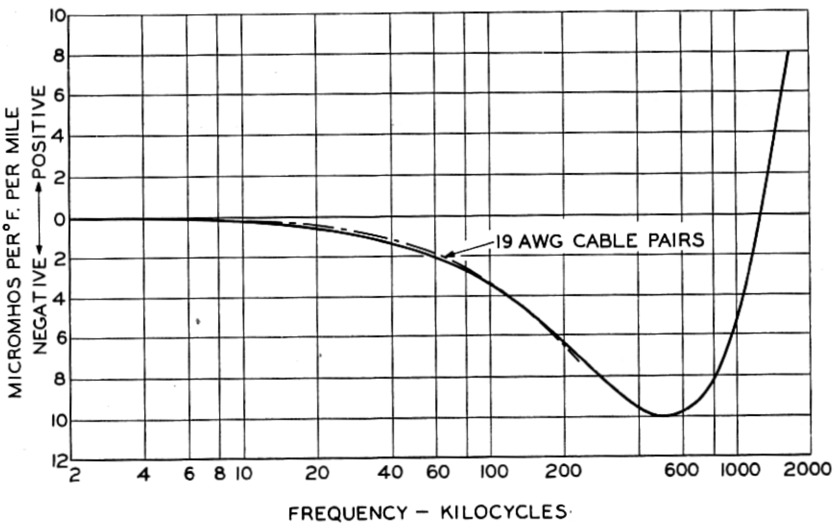

Fig. 12—Conductance-temperature coefficient; micromhos per mi. per 1°F. 16 AWG 61-pair paper insulated cable at 70°F.

It was mentioned above that moisture in the cable has a pronounced effect on the conductance. To drive out excess moisture during manufacture the reels of paper covered cable (or cores) are placed in vacuum driers[13] and then stored in a room maintained at about 110° F. and at a relative humidity of 1/2 of 1 per cent or less until the cables are covered with their lead-antimony sheaths. The lead presses are adjacent to the ovens and the cable is fed through the wall directly into the press so that it emerges at the opposite side covered with the sheath. This procedure minimizes the amount of moisture entering the paper of the cables after they have been dried. The practical measure of the moisture content and the effectiveness of the drying

[13] C. D. Hart, "Recent Developments in the Process of Manufacturing Lead-Covered Telephone Cable," *B.S.T.J.*, VII (1928) pp. 321–342.

is the value of the quantity $G/2C$ = conductance divided by twice the capacitance, both measured at the room temperature in the factory. The quantity $G/2C$ is used because it is the coefficient in the leakage component of attenuation as explained in connection with the formula (12) below for high-frequency attenuation. The average value of $G/2C$ for 1000 cycles at 70° F. is about 8.3. This quantity increases with frequency and at the same time decreases with temperature in the same way G changes, since the capacitance changes are relatively so much smaller than the conductance changes.

Layer to Layer Variations of Primary Parameters

The values of the primary parameters vary from inside layers to outside layers of cables, in addition to variations mentioned under specific parameters above. There are three basic reasons for this variation with location in the cable. The first is that the length of an outside pair is usually considerably greater than the length of an inside pair. Unusual twisting of the inside layers might make up for this difference but in the ordinary construction this is not done. This increase in length amounts to as much as 1 or 2 per cent and is reflected at once in the d-c. resistance as well as in the a-c. parameters.

The second reason is that, particularly in the outside layer, the sheath being made of lead-antimony, has electrical properties considerably different from the properties of copper wires. The large size of the sheath relative to the wires is an important factor. High-frequency currents in the wires near the sheath produce fields cutting the sheath which affects the fields in a different fashion from the way adjacent copper wires affect the field of a pair of conductors near the center.

The third reason, closely allied to the second, is that the conductors in the core of the cable are surrounded by a practically symmetrical mass of copper conductors and paper plus the sheath, while conductors in any other layer are surrounded by an unsymmetrical arrangement of conductors and paper.

A fourth factor is the variation in the amount of space allowed pairs in the core by the pressure of the outside layers.

The magnitude of these effects is indicated by the curves of Fig. 13, showing layer-to-layer variations in per cent for Resistance, Inductance, Conductance and Capacitance. Such large variations would be of considerable importance were it not for the fact that in the process of splicing pairs are made to pass, in effect, from inside to outside of the cable and vice versa. A long study of these variations will be found in the paper by Wuckel[7]. The effects of splicing together sections slightly different in their character-

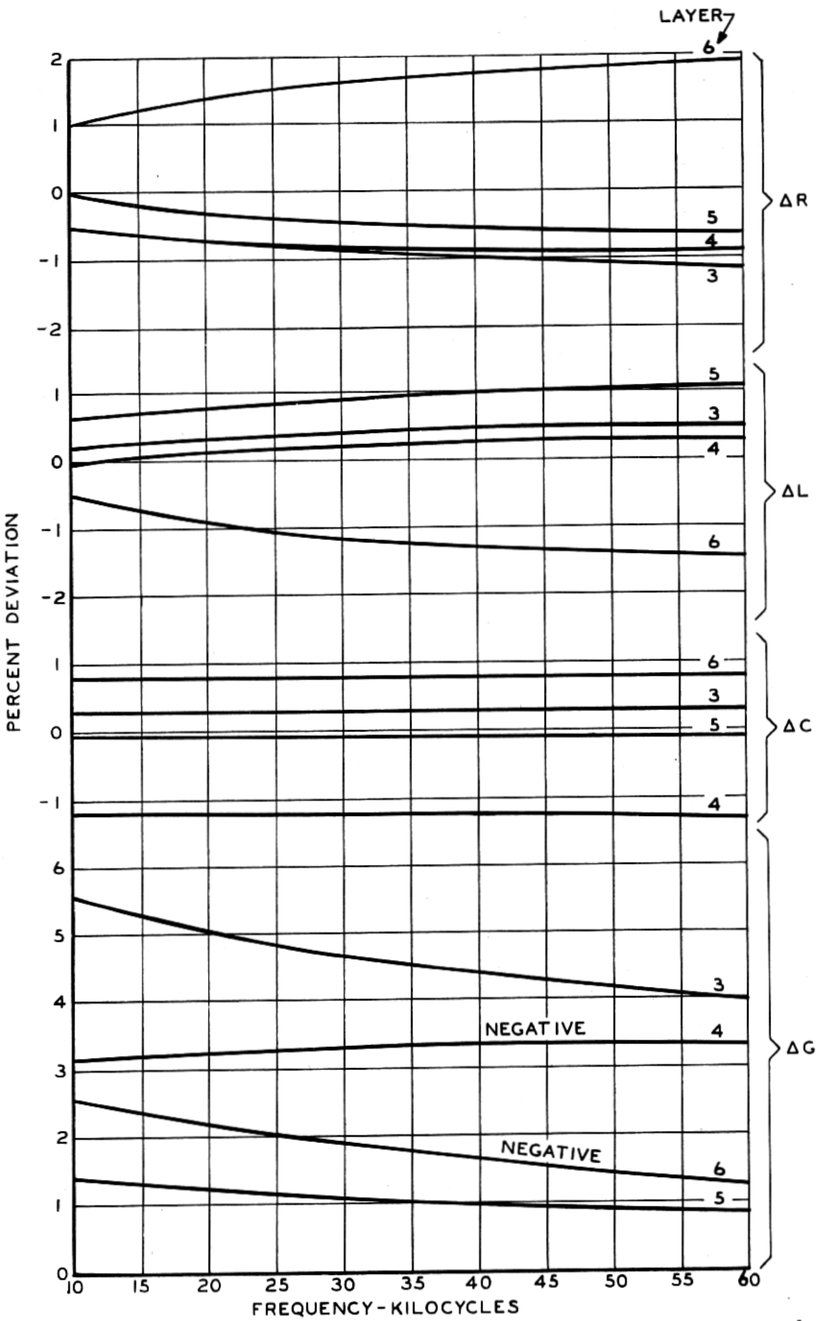

Fig. 13—Percentage deviations of layer average values of *R*, *L*, *G*, *C* from grand average—19 gauge pairs

istics were given for impedance, attenuation and delay distortion by Pierre Mertz and K. W. Pfleger[14].

Closely allied to these effects is the possibility of temperature differences across the section of cable in actual installed cables. Splicing usually takes care of this, too, but there are traces of such a lag in cases where the pairs remain in the outer part of the cable for a long distance and then pass to the inner group for the remaining part of the line. No such cross-sectional variation entered into the laboratory measurements as the temperatures were sufficiently well maintained close to given desired values.

Attenuation

The propagation constant γ is given by the familiar formula

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$= j\omega\sqrt{LC}\sqrt{\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)} \qquad (9)$$

The real part, α, is the attenuation in nepers and the imaginary part, β, is the phase in radians. Expressing the attenuation in terms of reals, gives

$$2\alpha^2 = \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} - (\omega^2 LC - RG)$$
$$= \omega^2 LC\sqrt{(1 + R^2/\omega^2 L^2)(1 + G^2/\omega^2 C^2)} - (\omega^2 LC - RG) \qquad (10)$$

In cables, $G/\omega C$ is small as compared to unity, in which case (10) may be reduced to the approximate form

$$2\alpha^2 = \omega^2 LC\sqrt{1 + R^2/\omega^2 L^2} - (\omega^2 LC - RG) \qquad (11)$$

The formula for β^2 is the same as for α^2 except for the sign of the last two terms in (10) and (11), that is, the sign in front of the parenthesis is + instead of −.

By expanding the square root term in (10) and using only first order terms in the expansion, another approximate form, frequently found useful in checking high-frequency values, is obtained, viz.,

$$a \doteqdot \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} = \left(\frac{R}{2L} + \frac{G}{2C}\right)\sqrt{LC} \qquad (12)$$

(Terms neglected in this approximation all include powers of ω^2 in their denominators and so become negligible at high frequencies.) The first term is commonly called the "resistance component of attenuation" and represents series losses. The second term represents shunt losses and is called the "leakage component of attenuation". The quantity $\sqrt{L/C}$, as is well known, represents the nominal characteristic impedance of the circuit.

[14] Pierre Mertz and K. W. Pfleger, "Irregularities in Broad-Band Wire Transmission Circuits," *B.S.T.J.*, XVI (Oct. 1937) pp. 541–559.

The shapes of attenuation-frequency curves at high, low and intermediate temperatures are shown by the curves of Fig. 14. These curves do not appear to be strikingly different in shape but more detailed study of the variations with temperature show the rate of change (decibels per degree per mile) to vary with frequency according to the curve of Fig. 15. The frequency of maximum rate of variation depends upon the gauge, as does the

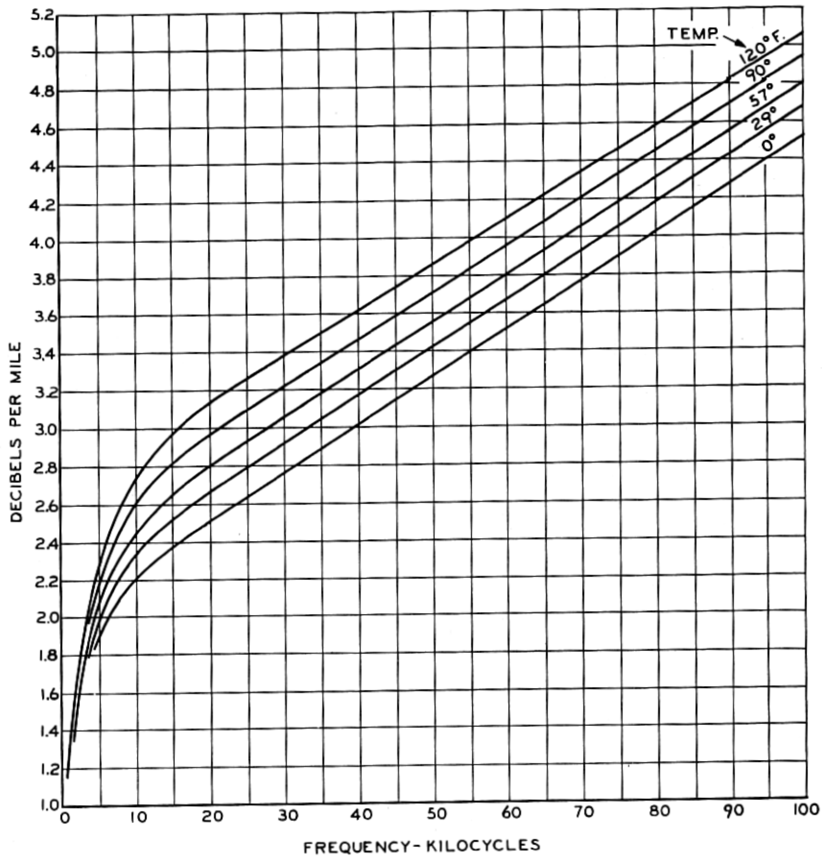

Fig. 14—Attenuation; decibels per mile—19 gauge pairs

actual rate of change. If the curves are plotted as attenuation coefficients (db per db per degree, Fahrenheit) with the abscissa

$$B = \sqrt{\frac{\text{frequency}}{R_{dc} \text{ per } 1000 \text{ ft}}},$$

the same as for skin and proximity effects in Fig. 2, the peaks are brought together as indicated in Fig. 16.

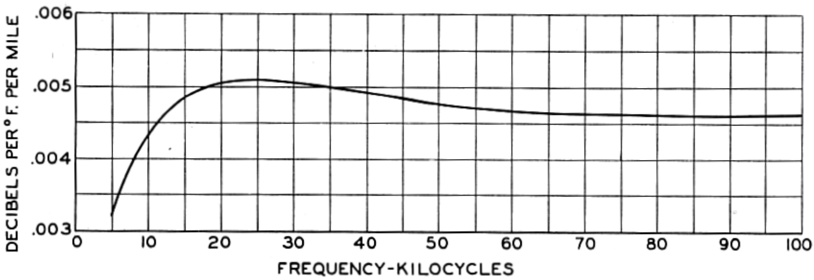

Fig. 15—Temperature variation of attenuation; decibels per degree Fahrenheit per mile vs. frequency—19 gauge pairs

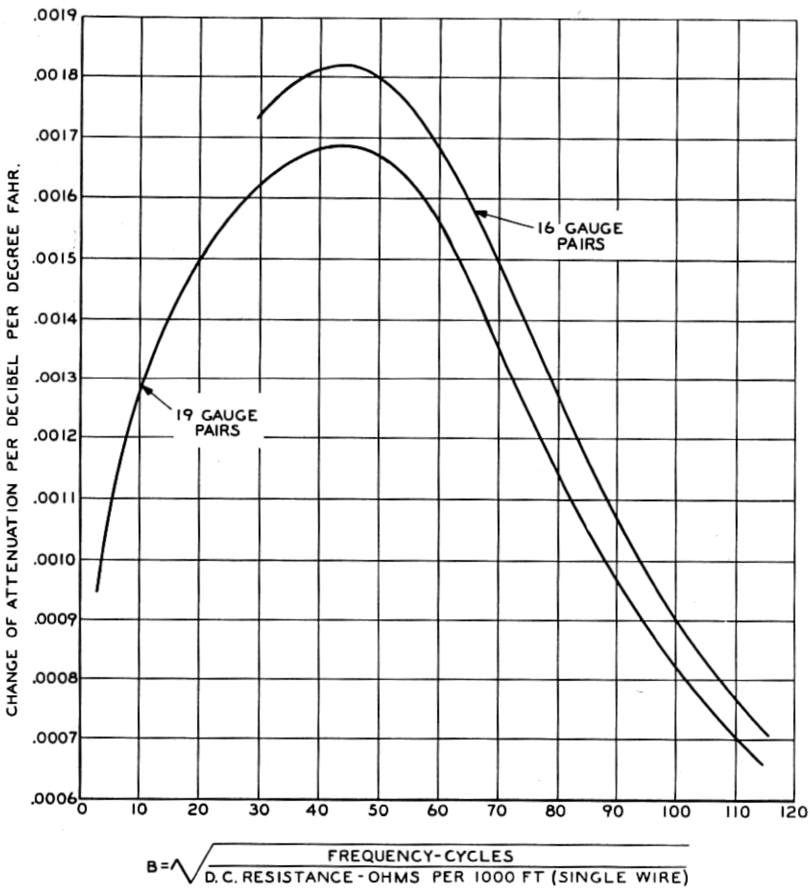

Fig. 16—Temperature coefficient of attenuation, α. $A = A_{50}[1 + \alpha(T - 50)]$

The change in attenuation with temperature may be formulated in various ways as a function of its component variables, R, L, G, C, the most obvious way being to take the partial derivatives of one of the equations (9), (10), or

(11) with respect to R, L, G and C, in order to get a differential expression $d\alpha$ in terms of dR, dL, dC and dG. This may then be interpreted as a change with temperature or a manufacturing variation, or a variation of attenuation from pair to pair in the cable. This procedure applied to (10) results in the formula

$$\begin{aligned}4\alpha\cdot d\alpha = &\left[G + R\sqrt{(G^2+\omega^2C^2)/(R^2+\omega^2L^2)}\right]dR\\ &+\left[R + G\sqrt{(R^2+\omega^2L^2)/(G^2+\omega^2C^2)}\right]dG\\ &-\left[\omega^2C - \omega^2L\sqrt{(G^2+\omega^2C^2)/(R^2+\omega^2L^2)}\right]dL\\ &-\left[\omega^2L - \omega^2C\sqrt{(R^2+\omega^2L^2)/(G^2+\omega^2C^2)}\right]dC \qquad (13)\end{aligned}$$

Curves of Fig. 17 show the components of the temperature variation produced by changes in R, L, G and C for standard 19-gauge cable.

A better formula from the point of view of equalizer design results from applying Taylor's series expansion to equation (9) and taking the real part of the resulting expressions. In this method, the variables are taken to be LC, R/L and G/C which effectively reduces the number of variables by one. There is a further advantage which appears in equation (14), below, namely, that the coefficient of the per cent variation in LC is just 1/2 the attenuation constant α and this means, therefore, only a slight addition to the basic equalizer which matches the curve for α vs frequency. There are thus added only two new types of temperature equalizers, one for R/L and one for G/C correction. Since equation (10) is already in the real form, it is more straightforward to expand it by Taylor's series and use the required number of terms. The formula thus obtained is naturally the same as that obtained from (9) and is as follows:

$$\begin{aligned}\Delta\alpha = &\frac{\alpha}{2}\frac{\Delta(LC)}{LC} + \frac{1}{2}\frac{R}{\omega L}\sqrt{\frac{\alpha^2+\omega^2LC}{1+R^2/\omega^2L^2}}\frac{\Delta(R/L)}{R/L}\\ &+\frac{1}{2}\frac{G}{\omega C}\sqrt{\frac{\alpha^2+\omega^2LC}{1+G^2/\omega^2C^2}}\frac{\Delta(G/C)}{G/C}\\ &-\frac{1}{8}\frac{R^2}{\omega^2L^2}\frac{(R/\omega L)\sqrt{\alpha^2+\omega^2LC}+\sqrt{\alpha^2-RG}}{\sqrt{(1+R^2/\omega^2L^2)^3}}\left[\frac{\Delta(R/L)}{R/L}\right]^2+\cdots \qquad (14)\end{aligned}$$

Application of this formula gives slightly different values for the temperature-attenuation coefficient at different parts of the temperature range for most frequencies. This means that the change of attenuation with temperature is not quite linear. The nonlinearity is so small below 100 kc. that it has not been measured with any certainty on lengths of cable varying from 500 feet up to 10 miles, but on long cable carrier circuits corrections for it may become necessary.

The formula has another use, however, in determining the effects of small manufacturing variations on the probable attenuation of cables made up of the resulting product, as well as computing fairly accurately the attenuation of all the pairs in a layer by means of the average values of the constants and the departures of the values of the constants of the individual pairs from the average values. Actual attenuation variations of pairs in a reel

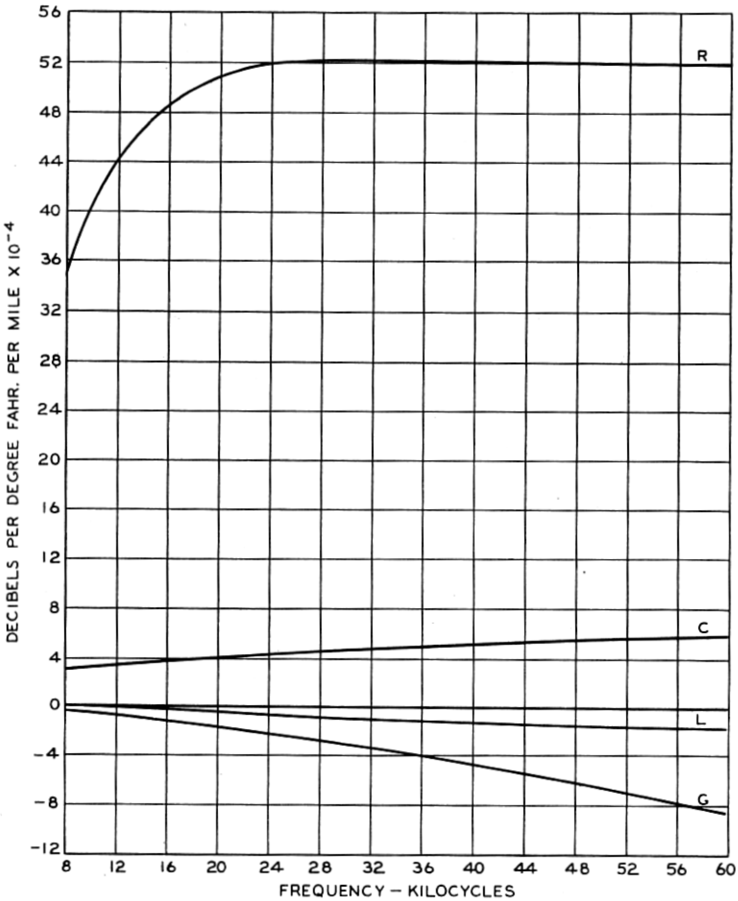

Fig. 17—Analysis of variation of attenuation with temperature; variations due to the components R, L, G, C—19 gauge pairs

are about ±5 per cent and the standard deviation of the variations is about 2.5 per cent.

Practically, since $G/\omega C$ is small, formula (11) may be used in the Taylor series expansion with the variables R/L, LC and RG. This gives the formula

$$4\alpha\cdot\Delta\alpha = \frac{RC}{\sqrt{1 + R^2/\omega^2 L^2}}\cdot\Delta(R/L) + \Delta(RG) + \omega^2(\sqrt{1 + R^2/\omega^2 L^2} - 1)\Delta(LC) \quad (15)$$

Components of $\Delta\alpha/\Delta T$ computed by formula (15) are given in Fig. 18 It is evident that changes in R/L are responsible for most of the change in attenuation since the small changes in attenuation introduced by ΔLC and ΔRG tend to annul each other over most of the frequency range shown. This is to be expected from the approximate high-frequency formula (12) in which $G/2C$ is much smaller than $R/2L$.

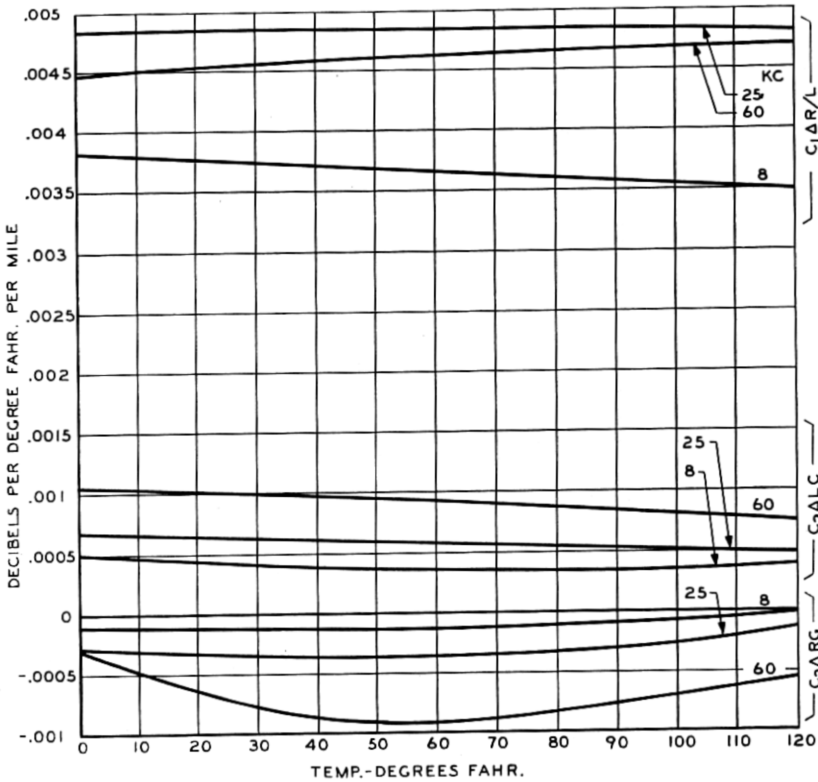

Fig. 18—Components of $\frac{\partial\alpha}{\partial T}$ = DB per °F. per mile = $C_1\Delta R/L + C_2\Delta LC + C_3\Delta RG$ vs. temperature—19 gauge pairs

Phase Change and Velocity

As pointed out above, merely changing the signs of the last parenthetical expression in equations (10) and (11) gives corresponding formulas for phase angle in radians. Fortunately, the phase change is nearly, though not quite, linear with frequency (Fig. 19). The velocity, $V = \omega/\beta$, for 19-gauge pairs is about 105,000 miles per second at 10 kc. and increases slowly to about 125,000 mps. at 100 kc. At high frequencies the internal inductance is small and, if the inductance L is expressed in abhenries and capacitance C in abstat-microfarads, then $V^2 = 1/LC = 1/k$, where k is the dielectric constant[15].

Impedance

The characteristic impedance is

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \tag{16}$$

which has a large reactive component at low frequencies as shown by the curves of Fig. 20, based on the same reel-length measurements as the curves

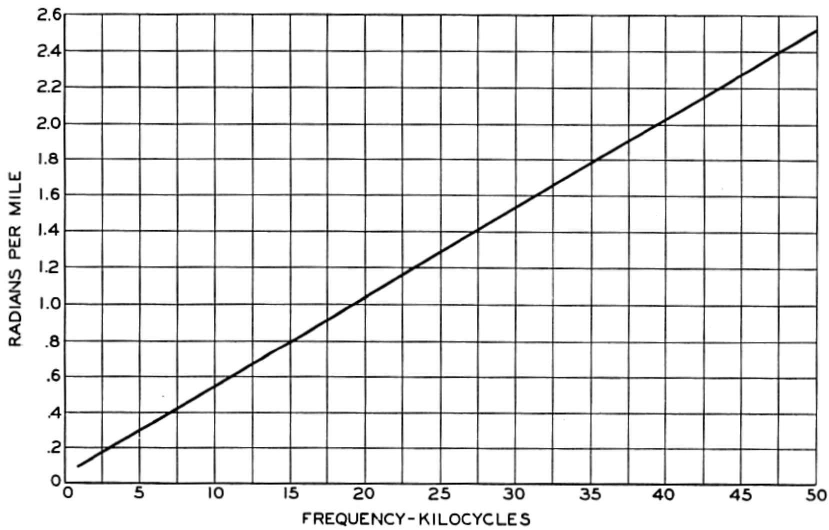

Fig. 19—Phase; radians per mile—16 gauge pairs 36°F.

for R, L, G and C in Figs. 3 to 11. In actual long cables the curves are irregular with frequency as a consequence of small irregularities along the line[13]. (See, for example, Fig. 26.) There are also small variations with temperature; for the resistance component about ± 1.5 per cent from the average for the temperature range zero to 120° F. at 10 kc., and about ± 1 per cent at 100 kc. The reactive component varies ± 10 per cent at 10 kc. over the same temperature range*.

[15] G. H. Livens, "The Theory of Electricity," p. 456 and p. 539.

* K. Simizu and I. Miyamoto, "Effect of Temperature on the Non-Loaded Carrier Cable, *Nippon Elec. Comm. Eng.*, May 1939, p. 596–599. Give similar data on the variation of parameters and attenuation for spiral-four cable at frequencies 0-30 kc. and temperatures 0–50°C. They do not specify the length measured but state that the wire diameter was 1.5 mm. From their d-c. resistance data the length appears to have been about 160 feet.

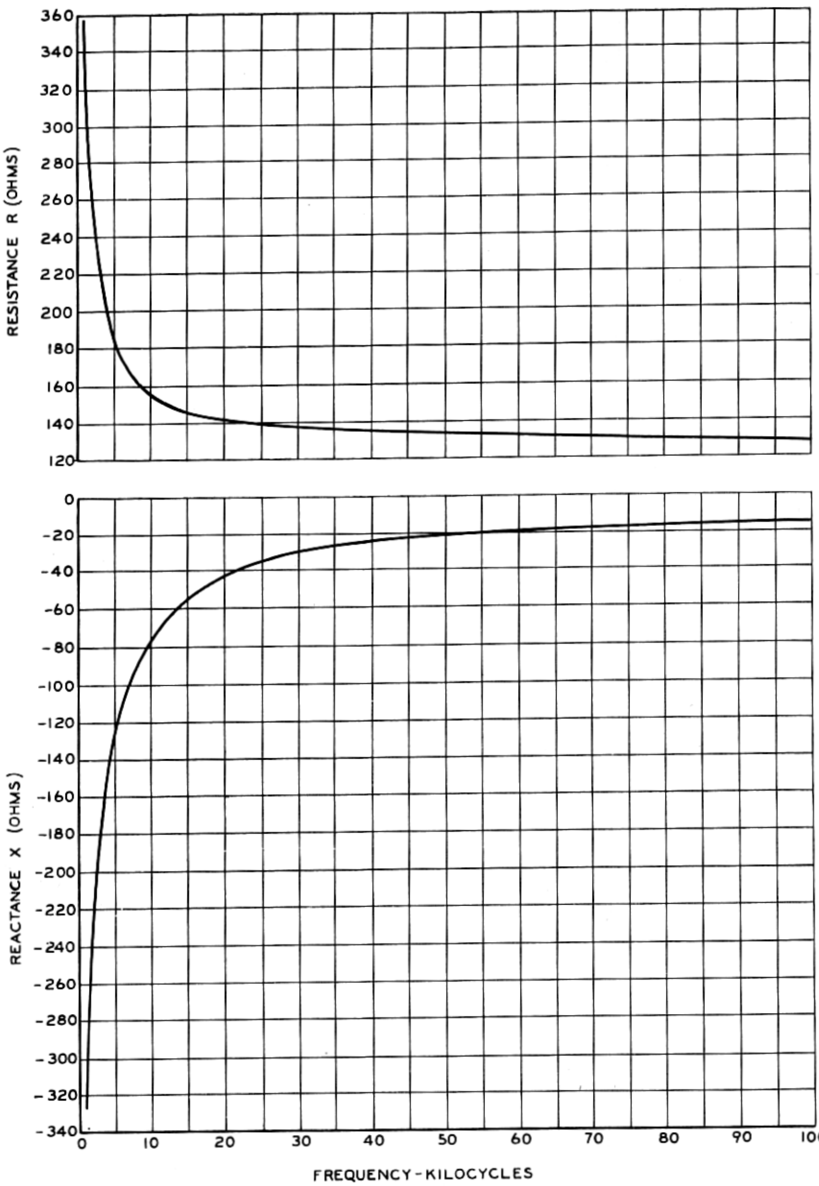

Fig. 20—Characteristic impedance, $Z = R - jx$; temperature 90°F.—19 gauge pairs

Characteristics of Toll Cable Above 100 Kc.

The preceding discussion has dealt largely with the characteristics of toll cables up to 100 kc. However, some measurements have been made extending to much higher frequencies. In the laboratories measurements were made on 16 and 19-gauge pairs in reel-lengths at frequencies up to about 3000 kilocycles. Field data at frequencies from 100 kc. to 2000 kc. were obtained on 16-gauge and 19-gauge pairs in cables about 3 miles long at

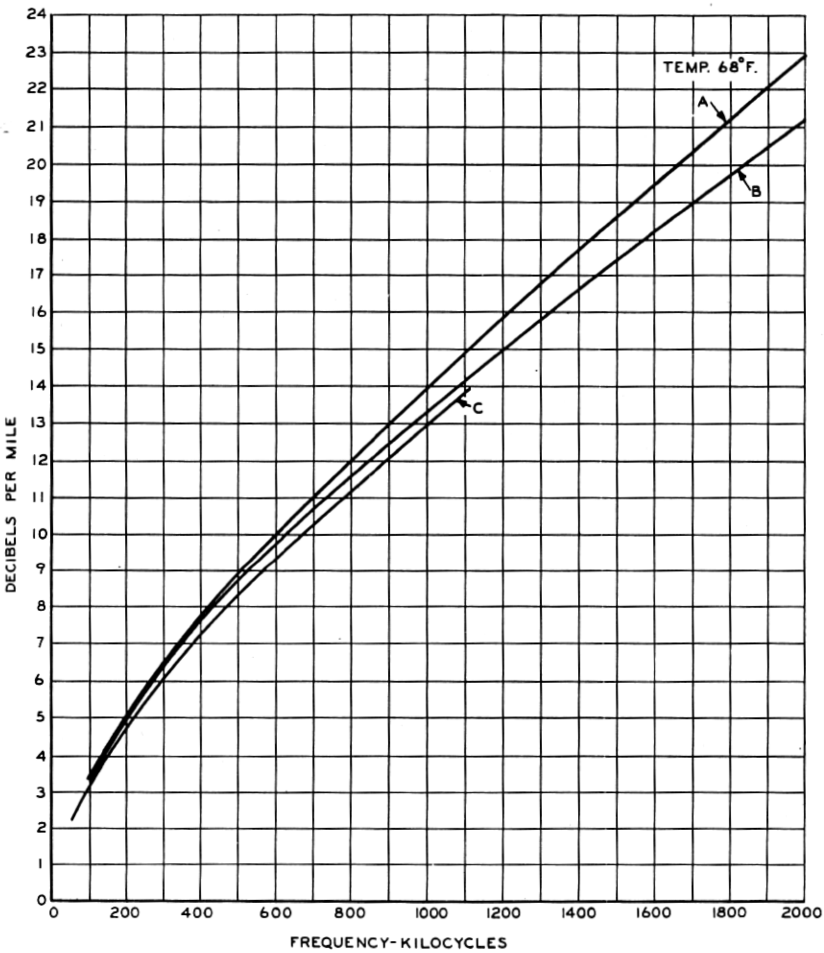

Fig. 21—Attenuation of 16 gauge cable pairs

A, on reel, 247 feet; B, fourteen reels, 1.3 miles; C, aerial cable, 3.6 miles, Ticonderoga, N. Y.

Ticonderoga, New York. A third set of data was obtained from measurements on 7000 feet of a special type (61-pair) of 16-gauge cable on reels in the laboratory under controlled temperature conditions. Figure 21 shows the attenuation values to 2000 kc. obtained in the three sets of 16-gauge

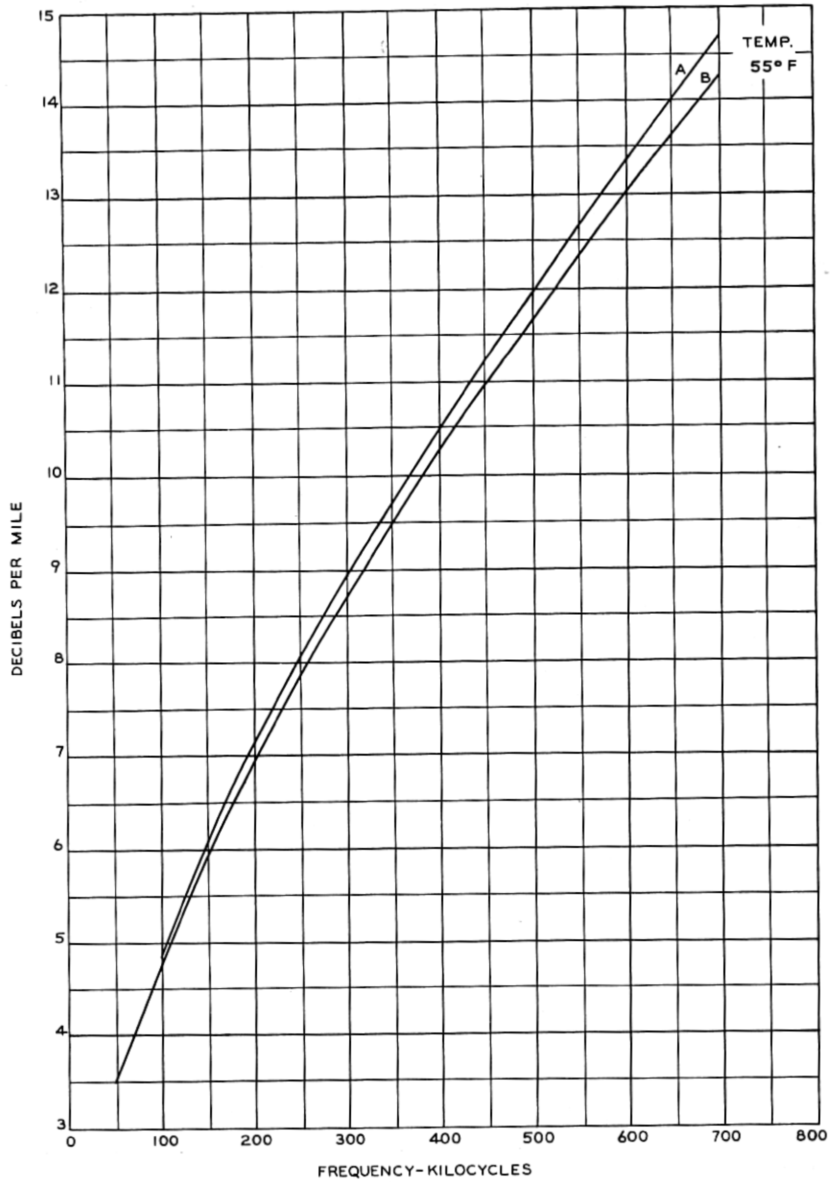

Fig. 22—Attenuation of 19 gauge cable pairs

A, on reel, 247 feet Bell Tel. Labs., Inc.; B, aerial cable, 3.6 miles, Ticonderoga, N. Y. Curves A and B show average of 10 pairs.

data at 68 degrees Fahrenheit. Figure 22 shows results on 19-gauge pairs at 55 degrees from field and laboratory data up to 700 kc.

The curves of the change in attenuation per degree F. per mile (db/1°F./mi) as shown by Figs. 23 and 24 are highly dependent upon the temperature, showing that at these high frequencies the attenuation is decidedly nonlinear with temperature in the toll cables.

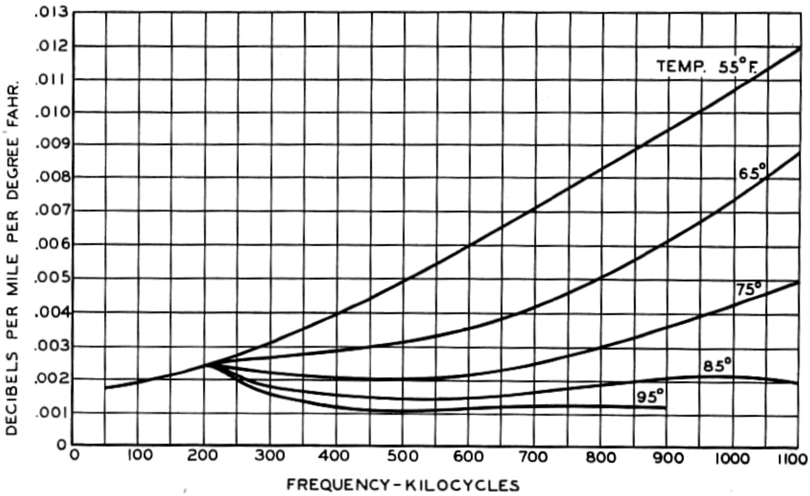

Fig. 23—Variation in attenuation at different temperatures for 1°F. change in temperature; aerial cable—16 gauge pairs

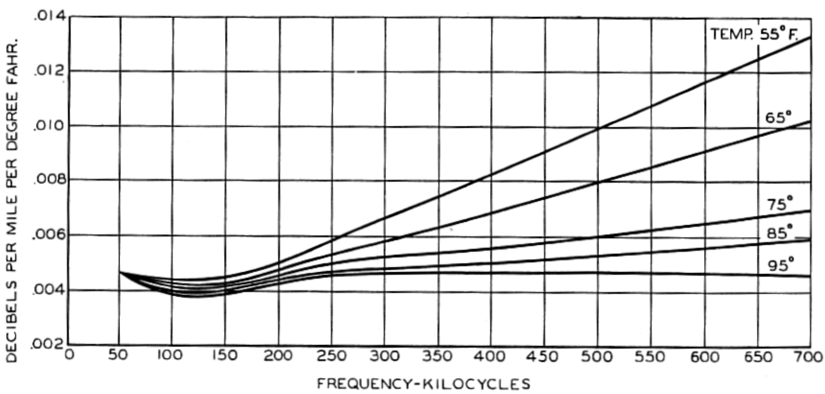

Fig. 24—Variation in attenuation at different temperatures for 1°F. change in temperature; aerial cable—19 gauge pairs

Toll Entrance Cable

The insertion losses measured between 125-ohm resistances on various lengths of 13, 16 and 19-gauge toll entrance cables at Denver, Colorado, are shown in Fig. 25. The data have been reduced to a per-mile basis by

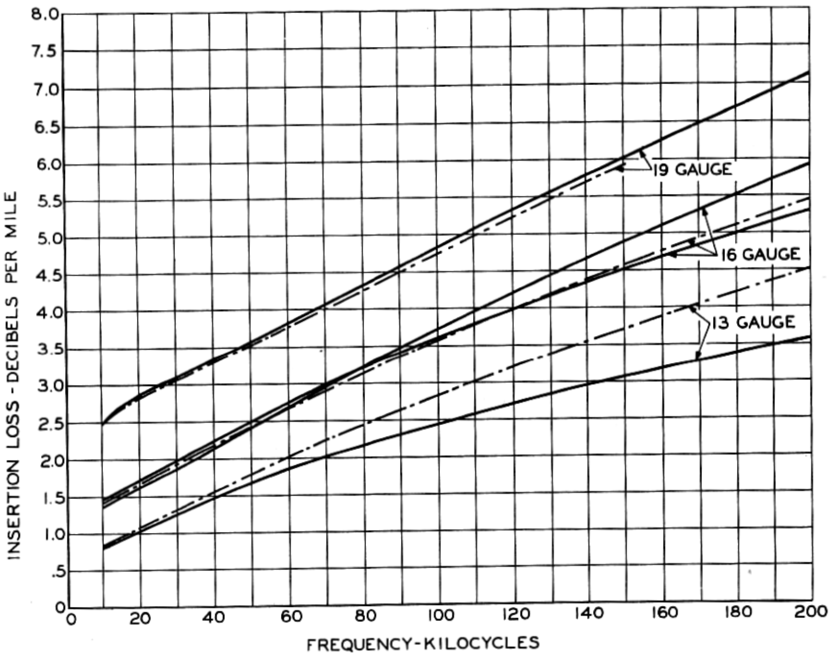

Fig. 25—Carrier frequency loss* of toll entrance cable; non-loaded, quadded—temperature 60°F., approx.

*Insertion loss between 125-ohm resistances

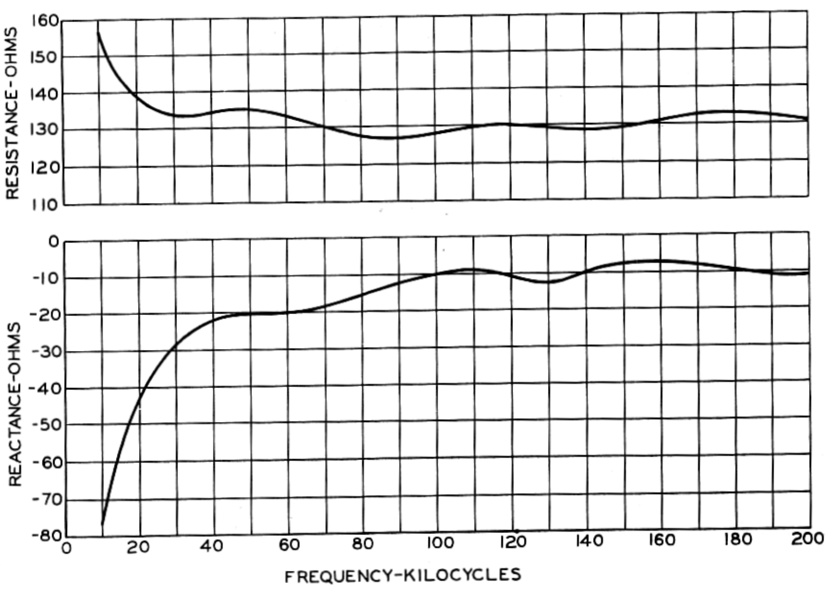

Fig. 26—Carrier frequency impedance of toll entrance cable, Denver, Colo.—19 gauge, quadded, non-loaded—terminated in 125 ohms

direct division of the measured attenuations by the lengths. This is, of course, not strictly accurate, but the errors are very small at these frequencies. This is nonloaded cable and frequencies measured were from 10 kc. to 200 kc. The values check closely the values shown in the previous figures for frequencies below 100 kc. The 13-gauge figures are the first of such data given herein but the first laboratory measurements on reel-lengths (begun in 1921) included reels of 13-gauge cable, and curves of 13-gauge attenuation and impedance were given in a paper[16] by E. H. Colpitts and O. B. Blackwell.

Corresponding data on impedance show the values given on Fig. 26. The wavy characteristic of these curves, as mentioned in the section on Impedance above, is caused by small irregularities in the pairs, particularly differences between pairs in successive reel-lengths giving rise to reflection currents at certain frequencies[14]. In new construction smoother impedance characteristics can be obtained when it is important to do so, by close control of the product during manufacture, followed by suitable splicing methods.

ACKNOWLEDGMENT

It is practically impossible to name all my associates in the Bell Telephone Laboratories whose work has been drawn upon in assembling these data, but I am especially indebted to Mr. Pierre Mertz and Mr. E. I. Green for their helpful suggestions and continued encouragement.

APPENDIX

Wave Propagation over Two Parallel Wires: The Proximity Effect—Inductance*

In his paper[4] on the Proximity Effect, J. R. Carson carried out the detailed computations for the ratio C of the a-c. resistance of two parallel wires to the a-c. resistance of a wire when the return conductor is concentric. He gave a formal expression for the impedance (equation 64 of his paper), viz.,

$$R + iX = 2Z + ipL \tag{1a}$$

This simple equation is complicated by the fact that Z and L are given by two complex expressions involving Bessel functions and the set of harmonic coefficients of the Fourier-Bessel expansion for the axial electric force in

[16] E. H. Colpitts and O. B. Blackwell, "Carrier Current Telephony and Telegraphy," *Jour. A. I. E. E.* XL, Feb. 1921, pp. 205–300.

* This work, done under the direction of Mr. J. R. Carson, was completed in April, 1922. For the general theory of wave propagation on parallel conductors see a paper by Chester Snow, "Alternating Current Distribution in Cylindrical Conductors," Proc. Int. Math. Congress, Toronto (1924) Vol. II, pp. 157–218.

one of the wires and the separation of the wires. The Bessel functions are of order zero to infinity and the argument, b, is given by

$$b = ia\sqrt{4\pi\lambda\mu ip} \tag{2a}$$

where

a = radius of the wire in cm.
λ = conductivity of wire in c.g.s. units
μ = permeability of wire in c.g.s. units
p = 2π times the frequency in cycles per second
i = $\sqrt{-1}$

The separation comes in by way of the quantity k, the ratio a/c of the radius to the interaxial separation of the wires, and a function s which can be expressed as a continued fraction in k^2, viz.,

$$s = \cfrac{1}{1 - \cfrac{k^2}{1 - \cfrac{k^2}{1 - \cdots}}} \tag{3a}$$

which results in

$$s = \frac{1}{1 - k^2 s} \tag{4a}$$

from which

$$s = \frac{1 - \sqrt{1 - 4k^2}}{2k^2} \tag{5a}$$

as given by Carson's equation (38).

The actual expression for $R + iX$ is as follows:

$$\begin{aligned} R + iX &= 2Z + ipL \\ &= -4ip \log ks + 2Z_0\left[1 + \sum_{n=1}^{\infty} (-ks)^n h_n J_n/J_0\right] \end{aligned} \tag{6a}$$

where

$$\begin{aligned} Z_0 &= R_0 + iX_0 \\ &= \frac{2p}{b}\frac{u_0 v_0' - u_0' v_0}{u_1^2 + v_1^2} + i\frac{2p}{b}\frac{u_0 u_0' + v_0 v_0'}{u_1^2 + v_1^2} \end{aligned} \tag{7a}$$

which is the impedance of a wire with concentric return expressed as usual[17] in terms of the ber and bei functions related to the Bessel functions by the formula

[17] Russell, "Alternating Currents," Edition 1904, Vol. I, p. 370.

$$u_n + iv_n = J_n (b\, i \sqrt{i}) \tag{8a}$$

and primes denote derivatives with respect to b.

Substituting Z_0 from (7a) in (6a) and carrying out the algebraic processes involved gives finally

$$R + iX = 2\, R_0\, C + i\,(-4\, p \log ks + 2\, K\, X_0) \tag{9a}$$

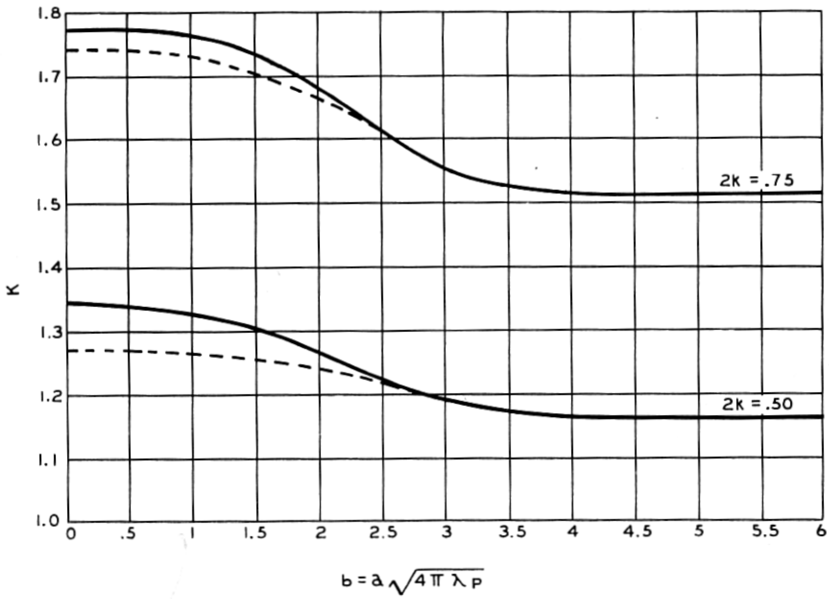

Fig. 27—Values of correction factor, K
For $2k$ = .75 and .50

where

$$C = 1 + \frac{4p}{bR_0} \sum (k^2 s^2)^n w_n + \frac{4p}{bR_0} \sum n(k^2 s)^{n+1} g w_n$$
$$- \frac{4}{b} \frac{X_0}{R_0} \frac{u_1^2 + v_1^2}{u_0^2 + v_0^2} \sum n(k^2 s)^{n+1} \frac{u_{n-1} v'_{n-1} - u'_{n-1} v_{n-1}}{u_{n-1}^2 + v_{n-1}^2} \tag{10a}$$

and

$$K = 1 + 2 \sum (k^2 s^2)^n \frac{u_{n-1} v'_{n-1} - u'_{n-1} v_{n-1}}{u_{n-1}^2 + v_{n-1}^2} \cdot \frac{u_0'^2 + v_0'^2}{u_0 u_0' + v_0 v_0'}$$
$$+ \frac{4p}{b^2 X_0} \sum n(k^2 s)^{n+1} bg \frac{u_{n-1} v'_{n-1} - u'_{n-1} v_{n-1}}{u_{n-1}^2 + v_{n-1}^2} + 2 w_n \frac{u_0 u_0' + v_0 v_0'}{u_0^2 + v_0^2}$$

$$= 1 + \frac{4p}{bX_0} \sum (ks)^{2n} \frac{u_{n-1}v'_{n-1} - u'_{n-1}v_{n-1}}{u^2_{n-1} + v^2_{n-1}}$$

$$+ \frac{4p}{bX_0} \sum n(k^2 s)^{n+1} g \frac{u_{n-1}v'_{n-1} - u'_{n-1}v_{n-1}}{u^2_{n-1} + v^2_{n-1}}$$

$$+ \frac{4}{b} \frac{u_1^2 + v_1^2}{u_0^2 + v_0^2} \sum n(k^2 s)^{n+1} w_n \qquad (11a)$$

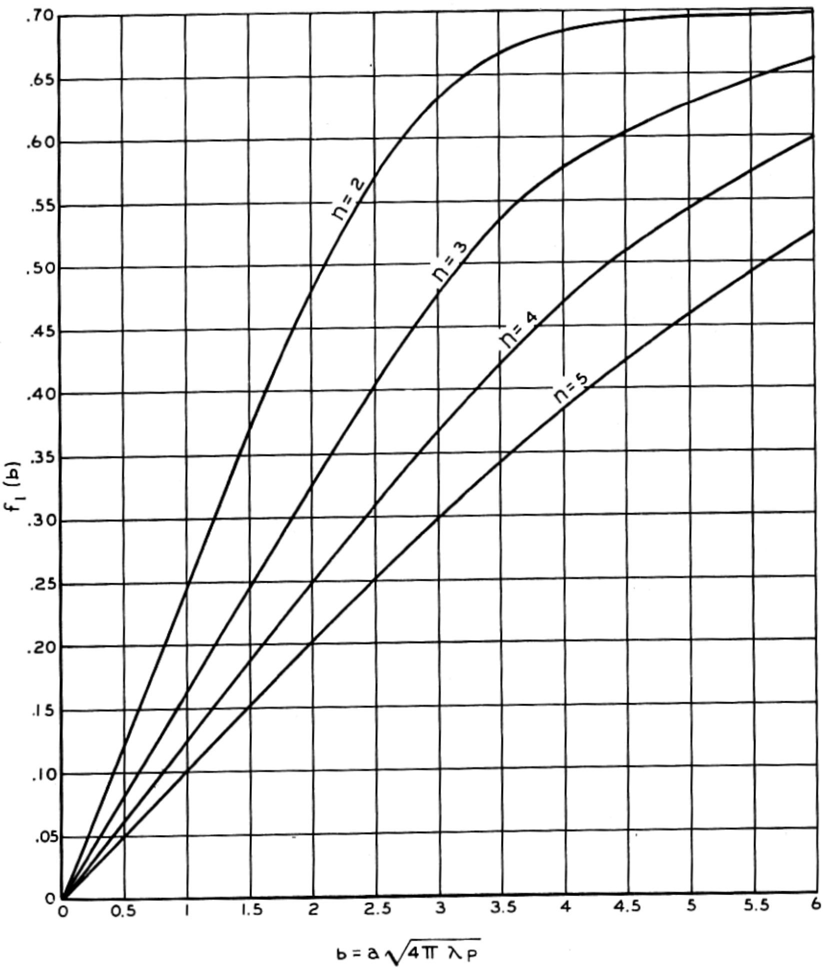

Fig. 28—Values of auxiliary functions

$$f_1(b) = \frac{u_{n-1}v'_{n-1} - u'_{n-1}v_{n-1}}{u^2_{n-1} + v^2_{n-1}}$$

In these formulas

$$g = \frac{2}{b}\,\frac{u_0' v_0 - u_0 v_0'}{u_0^2 + v_0^2} = -\frac{R_0}{p}\,\frac{u_1^2 + v_1^2}{u_0^2 + v_0^2} \tag{12a}$$

$$w_n = \frac{u_n v_n' - u_n' v_n}{u_{n-1}^2 + v_{n-1}^2} \tag{13a}$$

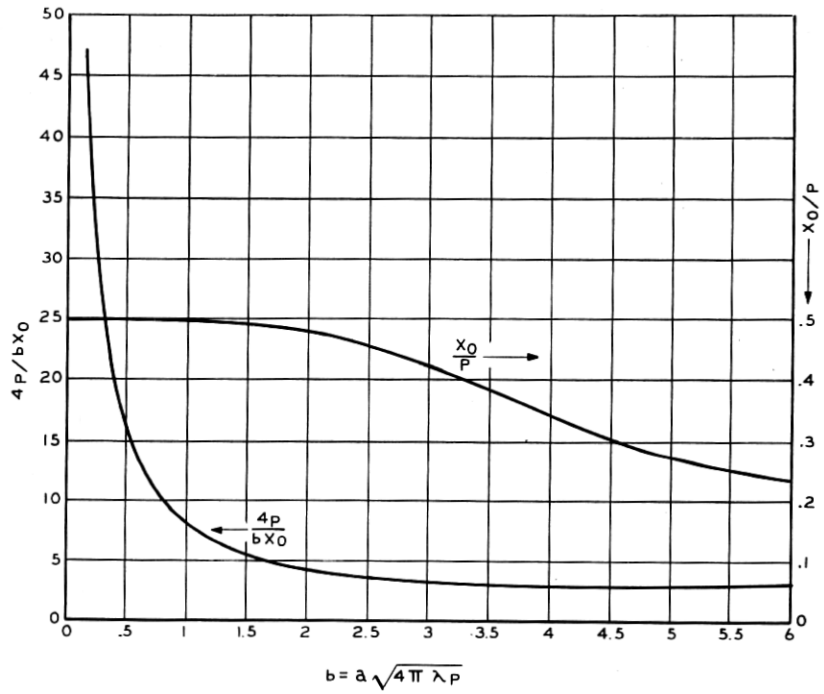

Fig. 29—Values of functions

$$\frac{4p}{bX_0} \quad \text{and} \quad \frac{X_0}{P}$$

The curves of figures 28–30 show the auxiliary functions vs b and figure 27 the correction factor K. The dotted curve for K is computed from Mie's formula[18] (14a) for small b. Two values of $2k$ are shown, .75 and .50, respectively.

G. Mie[18] gave formulas for small and large values of b, as follows:

For small b,

[18] G. Mie, *Annalen der Physik*, Vol. II, (1900) pp. 201–249.

$$L = 1 - 4 \log k - l_n - l'_n \tag{14a}$$

where $l_n = .417\ b^4/16 - .003\ b^8/256$

$$l'_n = b^4(1.33k^2 - .917k^4 - .652k^6 - .496k^8 \cdots)/16$$
$$- b^8(.633k^2 - 1.354k^4 + .539k^6 + .584k^8 \cdots)/256$$

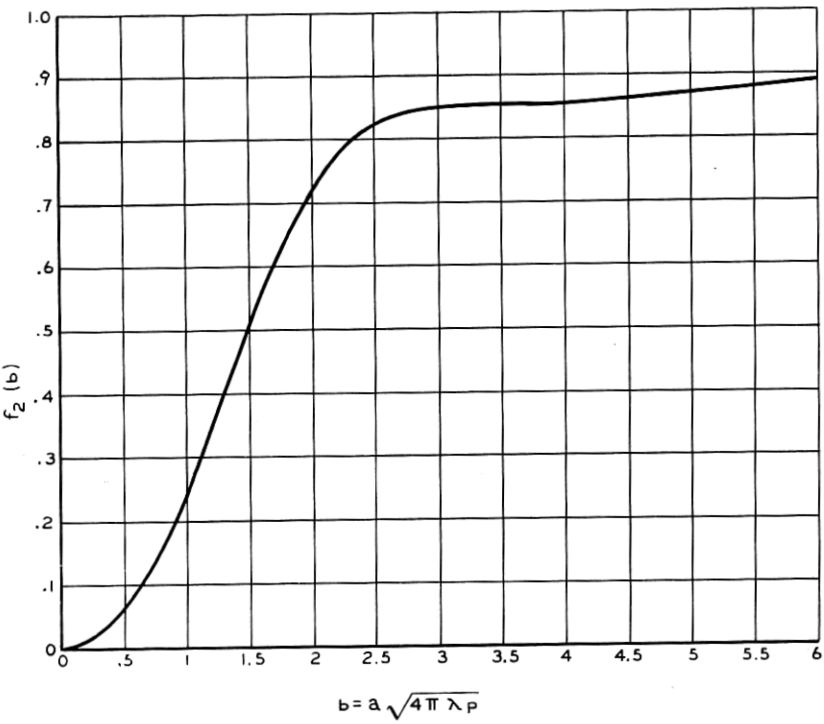

Fig. 30—Values of auxiliary functions

$$f_2(b) = \frac{u_1^2 + v_1^2}{u_0^2 + v_0^2} = \frac{ber'^2 + bei'^2}{ber^2 + bei^2}$$

For large b,

$$L = -4 \log ks + \frac{2\sqrt{2}}{b} C_m - \frac{3}{2b^2} \tag{15a}$$

where $C_m = s/(2 - s)$.