Copper Oxide Modulators in Carrier Telephone Systems *

By R. S. CARUTHERS

Copper oxide modulators are widely used in telephone systems for translating either single speech channels or groups of speech channels to carrier frequency locations on the lines. A number of simple circuit arrangements have been developed that enable suppression of certain undesired frequencies to a degree that is impractical in tube modulators. These modulators transmit equally well in either direction and the modulating elements are more non-linear than in tube modulators. As a result numerous effects are found that ordinarily are not important in the tube arrangements. Analytical studies have been considerably simplified by the use of a small signal, and a large carrier controlling the impedance variation of the copper oxide. It is found in this case that the superposition and reciprocity theorems hold for all the circuits that it has been possible to analyze even though the modulator is made up of non-linear elements. Open and short-circuit impedance measurements can be made use of as in four-terminal linear networks, and a generalized reflection theory developed. Performance data are given for an idealized modulator under a variety of operating conditions.

Introduction

At least as early as 1927, copper oxide rectifiers were being tried as modulators for the speech channels of carrier telephone systems in this country. At this time only a rather large type of rectifier was available, better adapted for power use rather than in modulating the few milliwatts of a speech signal. Largely because of instabilities these early units were found to be unsatisfactory for modulator use. Further developments in copper oxide rectifiers made in various laboratories extended the variety and improved the quality of the product available, so that by about 1931 they began to be promising as serious competitors for vacuum tubes in modulators. Since 1931 continued improvements in copper oxide rectifiers have rapidly increased their field of application until now they are employed in practically all modulators of the latest types of carrier telephone systems.

In the new systems a copper oxide modulator is used instead of the previous push-pull arrangement of two vacuum tubes. In cable,[1]

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[1] The general features of carrier telephone terminals have been described in a paper "Cable Carrier Telephone Terminals" by R. W. Chesnut, L. M. Ilgenfritz and A. Kenner, *Electrical Engineering*, January 1938.

open wire and coaxial carrier systems, from twenty-six to twenty-eight of these modulators are needed in each direction for translating each twelve-channel group of speech bands from voice to carrier and back again. These copper oxide modulators have no power costs, tube replacements or possibilities of power failures. In Fig. 1 the four 3/16 inch diameter copper oxide discs generally used in a carrier telephone modulator are shown individually, assembled with connections, and potted in a can.

The carrier terminals have tended to become increasingly complex as it became their function to place more and more channels on a single pair of wires. The extreme simplicity and reliability of copper oxide modulators have been of great value in helping to overcome this tendency. Copper oxide modulators have been used from zero frequency to nearly four million cycles. Certain modulators for coaxial carrier systems have been designed to modulate simultaneously as many as sixty speech channels spaced over a 240,000-cycle band of frequencies.

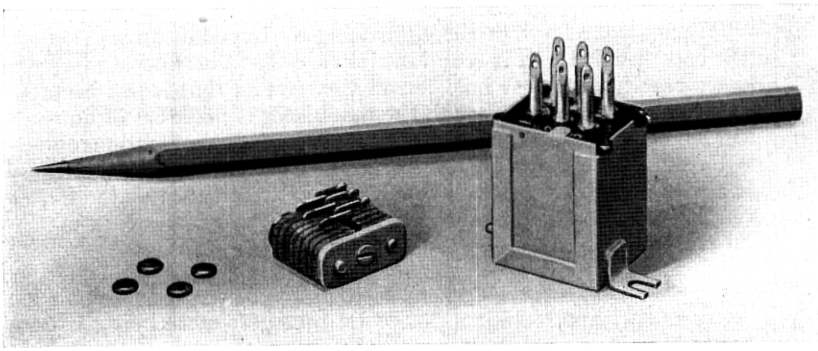


Fig. 1—Four disc copper oxide modulator.

Copper oxide modulators probably differ most from tube modulators because the simplicity of the rectifier elements allows a much greater variety of circuit arrangements to be used. Although the underlying principles of operation are not new, it has become necessary to investigate numerous transmission effects that could be neglected in tube modulators. This has resulted not only from the newer circuit arrangements with their smaller losses, but also from higher transmission standards for the overall system along with the greatly increased numbers of modulators in long circuits. Copper oxide modulators, unlike tube modulators, transmit signals equally well in either direction. While this is a simplification in allowing a modulator also to be used as a demodulator, the modulator becomes complicated by the effects of reflections back and forth into the signal bands of numerous frequency bands of modulation products.

Circuit Arrangement

The circuit arrangements used in copper oxide modulators generally are concerned either with carrier suppression, with carrier transmission along with the signal, or with balancing action to suppress certain unwanted bands of signal frequencies. In most carrier telephone systems economy of frequency space and amplifier load capacity demands the use of single-sideband, carrier-suppressed transmission.

In Figs. 2(a), 2(b), and 2(c) three types of copper oxide modulators are shown, each arranged to suppress the carrier in both the signal input and the signal output circuits. In Figs. 2(d) and 2(e) the carrier is balanced out in only one signal branch. In the usual arrangements a signal band selective filter must be used in each signal branch to restrict transmission to that of the wanted frequency band. Largely in this way interferences are guarded against, not only into other channels or systems to which the modulator output circuit is connected on the line or at the distant end, but also back into the complex array of facilities to which the input circuit may be connected.

In any of the circuits shown, modulation results from either the reduction or reversal of the current flow between the input and output signal circuits at periodic intervals as the carrier varies the copper oxide resistance back and forth between high and low values. In Fig. 2(a) where the connections of the input and output signal circuits are periodically short-circuited by the carrier-actuated copper oxide, transmission of the modulated signal into the input circuit or the unmodulated signal into the output circuit is prevented by filters, each of high impedance at the frequency of the other signal. In Fig. 2(b) the connections between the signal and modulated signal circuits are open-circuited periodically by the carrier. In this case each filter should have a low impedance at the other signal frequency. In Figs. 2(c), 2(d) and 2(e) the copper oxide rectifiers are made to become alternately low and high resistance in pairs as the polarity of the carrier is either in the same direction as the arrows or in the opposite direction. As a result, current flow from the input signal circuit into the output is periodically reversed by provision of a periodically reversing low impedance path. In effect each signal is balanced from the other's circuit.

Although an indefinite number of other circuit configurations can be used, no novel transmission feature would be found which was not

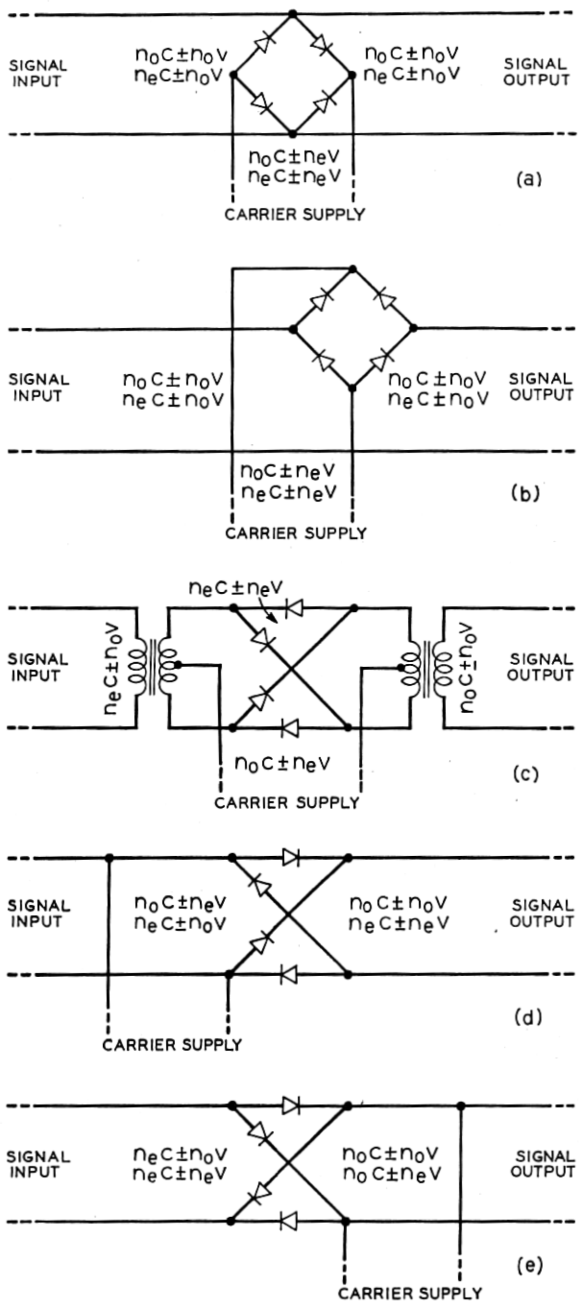


Fig. 2—Types of copper oxide modulator circuits.

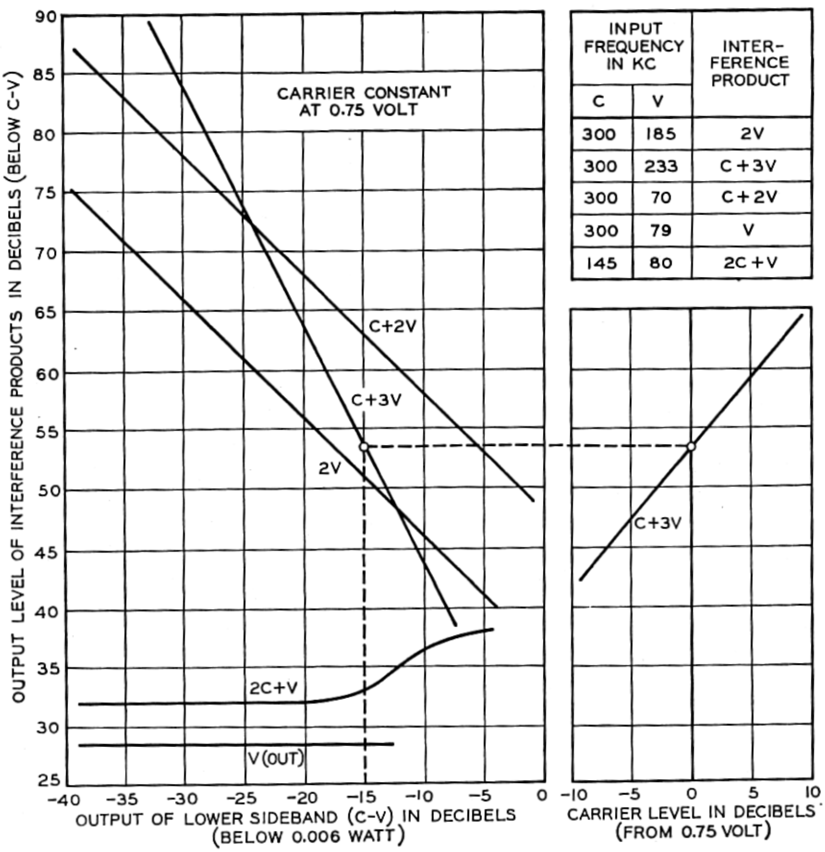
present in the five circuits already shown. Third order modulators in which the copper oxide is arranged to give equal interruptions to the signal in both positive and negative half cycles of the carrier are exceptions not considered here. In addition, circuits like Hartley's, in which phase discriminations have been obtained in the sideband outputs from two modulators by altering both the carrier and signal input phase of one, can be viewed as composed of two modulators of any of the types illustrated.

In these copper oxide modulators all modulation product frequencies can be grouped into four classes:

$$n_0c \pm n_0v,$$
$$n_0c \pm n_ev,$$
$$n_ec \pm n_0v,$$
$$n_ec \pm n_ev,$$

in which c and v are the carrier and input signal frequencies and n_0 is any odd number 1, 3, 5 etc., while n_e is any even number 0, 2, 4, 6, etc. If c and v contain more than one frequency each, n_0 and n_e are respectively the odd and even combinations of all multiples of the c and v frequencies. All frequencies of one of these four types appear together in a specific branch of the modulator circuit; and they will not appear in another branch unless from a dissymmetry among the copper oxide units or unless inherent in the circuit configuration. The branches in which the modulation products appear are shown in the circuits illustrated. It is apparent that only in the case of the double-balanced circuit of Fig. 2c, are all of these types of products completely separated in different parts of the circuit. In the other circuits shown the classes of products appear together in combinations of two types. In any balanced circuit that can be drawn the above relationships will be found to hold.

Modulation products will be of a type to which the circuit offers some degree of balance, of a type that can be made to vary in importance relative to the signal by adjustment of either the carrier or signal voltage, or of a type to which neither balance nor level adjustment is of any benefit. Satisfactory operation of such modulators requires large carrier voltages relative to those of the signal, so that products like $c \pm v$, $2c \pm v$, $3c \pm v$, etc. tend to be of large magnitude while products like $c \pm 2v$, $c \pm 3v$, etc. tend to be small. Furthermore, the former types can be made to predominate even more over the latter types either by increasing the carrier amplitude or by decreasing the signal levels. A 6 db reduction in signal results in 12 db reduction of $c \pm 2v$ and 18 db reduction in $c \pm 3v$. In any circuit application interferences of this type lend themselves to reduction in so far as carrier power is available for high signal level operation, or in so far as noise does not limit for low signal levels. Laboratory measurements of some of these modulation products made during the development of a group modulator for a twelve-channel open-wire carrier system are shown in Fig. 3 for a double-balanced modulator



INPUT FREQUENCY IN KC		INTERFERENCE PRODUCT
C	V	
300	185	2V
300	233	C+3V
300	70	C+2V
300	79	V
145	80	2C+V

Fig. 3—Intensity of modulation products in a representative double-balanced copper oxide modulator.

like that of Fig. 2(c). Single 3/16 inch diameter discs as shown in Fig. 1 were used in each bridge arm. About 20 to 30 db reduction in interference by balance alone is quite readily obtained in the normal run of manufactured copper oxide rectifier elements, for those products to which the circuit arrangement offers a balance. Any further improvement must be obtained either by closer control of manufacture or disc selection, by artificial balancing with some means such as condenser-resistance potentiometers, or by statistical averaging through use of numbers of discs in each bridge arm.

In single-channel modulators interferences caused by the signal into its own signal band will occur only in the presence of the signal. In such cases they need be only 20 to 30 db below the signal, except in special cases, as for example, modulators for broad-band program channels. In multi-channel systems interferences may be produced in the silent channels by the active channels. This kind of interference or crosstalk is ordinarily made to be 70 db or more below the wanted signals for commercial telephone service. In such cases overlapping bands of frequencies not improved by level adjustment are avoided by judicious choice of the carrier frequencies.

Circuit Impedance and Loss

In all modulators the carrier serves merely as a means for obtaining a simple periodic variation of the impedance presented to the signals. It is not only immaterial to the signals how this variation is obtained, but the signals also are totally unaware of whether electrical, mechanical or other means are used, just so long as the signals themselves are unable to affect the time variation of this impedance. In a copper oxide modulator, only by making the carrier amplitude large compared to the signal amplitudes across the rectifier elements, can the impedance of the rectifiers be made to vary at carrier rather than signal rates. Too large a signal amplitude not only results in the production of undesired frequencies, but also the impedance and loss characteristics of the modulator vary with the signal amplitude. With small signals the carrier energy is used up in maintaining the copper oxide at prescribed impedance values at each instant of time, and none of the modulation products involving the signal receive more than a negligible amount of energy from the carrier. As a result the output signal energy will always be less than that of the input signal, partly because of i^2r losses within the copper oxide, and partly from the diversion of the input signal energy into the energies of the many modulation product frequencies.

The signal impedance of a copper oxide modulator is a combination of a characteristic impedance of the rectifier elements and the impedance of the connected circuits at all the modulation product frequencies. The characteristic impedance of the rectifier can be viewed crudely as an average of the impedance encountered by a small signal over a cycle of the carrier, treating each instantaneous value of the carrier voltage as a d-c. bias. If the impedance for small superimposed a-c. voltages is measured on a single copper oxide disc at various d-c. bias voltages, this impedance generally changes with both bias and frequency. Measurements to 200 kilocycles made on a 3/16 inch diameter disc are shown in Fig. 4. At all negative bias

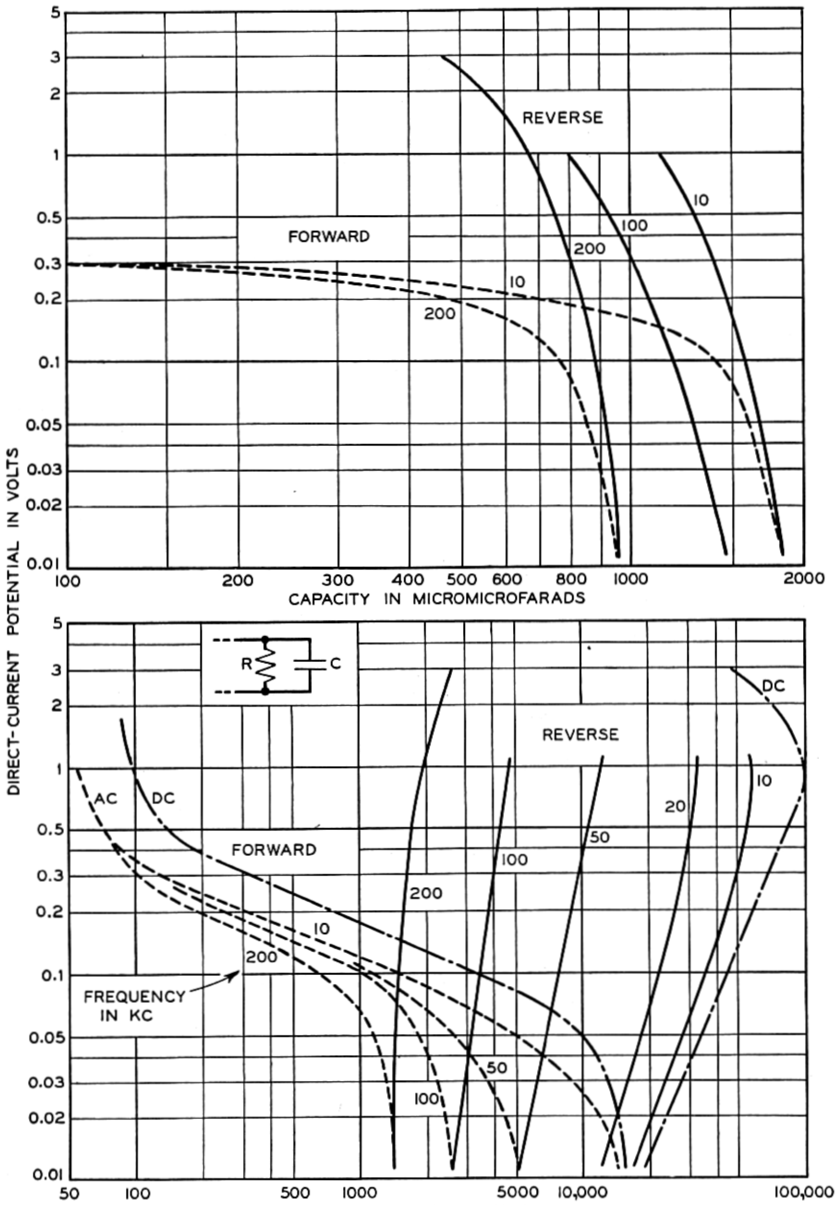


Fig. 4—Impedance of a copper oxide disc at various forward and reverse d-c. voltages for small superposed a-c. voltage.

voltages the impedance is a resistance in parallel with a condenser. This resistance decreases rapidly with increasing frequency while the shunt capacity decreases only a moderate amount. At moderate positive voltages (about 1/2 volt) the impedance becomes resistive and does not change appreciably with frequency. Experimentally it is found that the signal impedances in resistance terminated modulators can be made largely free of reactance at high frequencies by using either large carrier amplitudes, inductive tuning of the copper oxide capacities, or lower impedance connected signal circuits to accentuate the importance of the low-resistance part of the copper oxide characteristic. Very much lower circuit impedances must be used at the higher frequencies. Where 600 to 1000 ohms is a satisfactory impedance at speech frequencies, less than 50 ohms may be the best impedance to use, at three megacycles.

Impedance measurements on a double-balanced modulator designed to translate a twelve-channel group of frequencies for cable carrier systems from a band at 60 to 108 kilocycles to a 12- to 60-kilocycle band are shown in Fig. 5 for several resistance terminations. Absence of any impedance irregularities with frequency is apparent. Also, the tendency is shown for the modulator impedance to become less reactive with lower resistance terminations.

Inasmuch as copper oxide discs are available in sizes from 1/16 inch to more than an inch in diameter, a wide range of circuit impedances are possible varying from only a few ohms to thousands of ohms. Large area discs roughly are equivalent to small area discs in parallel. Thus by using a disc of n times the area of a small one or n of the small ones in parallel, the best circuit impedance becomes $1/n$th at the same carrier voltage. Either discs in series or ones of smaller diameter enable the impedance to be raised in a corresponding manner. The lower impedance and greater energy dissipations of larger discs, or paralleled smaller ones at the same carrier voltage, obviously allow greater input signal energies before the signal impedance and loss begin to vary with the signal, and overload distortion appears. Similarly series stacks of discs, or series-parallel combinations, offer wide choice in both the signal levels that can be satisfactorily modulated and in the impedance levels. Usually r.m.s. carrier voltages across individual discs in the conducting direction will best be made somewhere between 3/10 and 3/4 volts.

The impedances of the connected circuits at the modulation product frequencies react back on the signal impedances in a way similar to the way that the two terminating impedances of a four-terminal linear network react on each other. In the case of the copper oxide modulator a reaction from some modulation product back into the signal impedance is less and less as the product becomes of higher order, or as the circuit loss to it becomes greater. Where the impedances of the connected circuits have bothersome interactions with the copper oxide impedance either at the signal frequencies or the lower loss modulation products, resistance pad separation is usually the simplest solution if the increased loss can be tolerated.

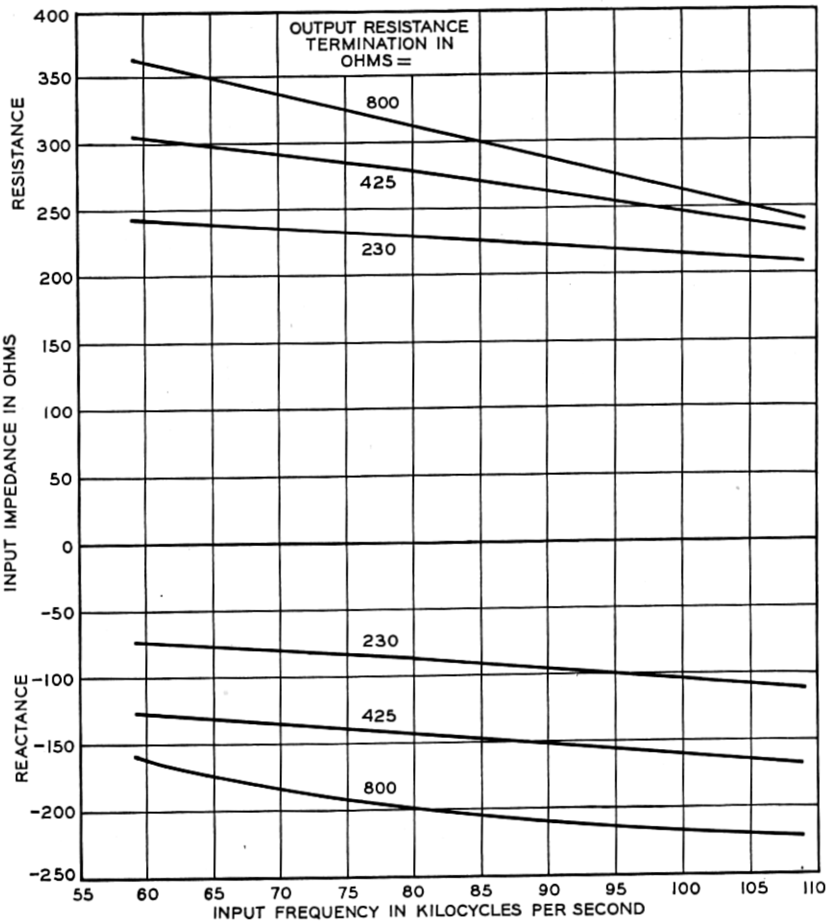


Fig. 5—Impedance of a representative double-balanced modulator.

Energy losses in copper oxide modulators between signal input and single sideband signal output have been found to be no greater than 8 or 9 db even at frequencies of 3 or 4 megacycles. At lower frequencies 5 or 6 db losses are normal, but losses as low as 2 db have been obtained under less practical operating conditions. Experimental loss measurements are shown in Fig. 6 for a double-balanced modulator using single 3/16 inch diameter discs in each bridge arm. This modulator was designed to simultaneously modulate sixty speech channels occupying a 240,000-cycle band width. The modulator loss, like the impedance, depends on the impedance terminations of the modulator at all the modulation product frequencies as well as on the internal losses of the modulator. Short circuit, open circuit, or reactive terminations at the unwanted frequencies, permit energy losses only through reflections at the signal circuit junctions to the modulator or within the modulator. With proper terminations and loss-free copper oxide, 100 per cent efficiency frequency translations are theoretically possible. In a practical case, a larger carrier amplitude results in a smaller percentage of the time in which the rectifier

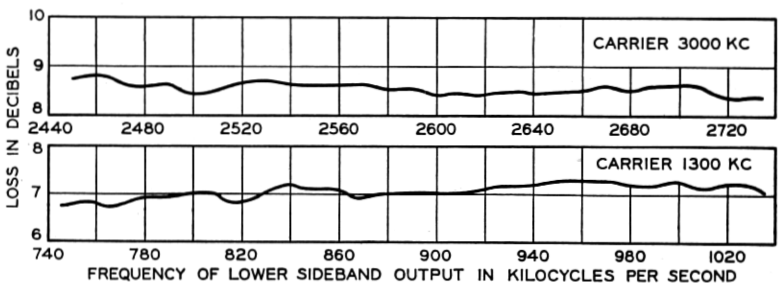


Fig. 6—Loss in a double-balanced group modulator for coaxial systems.

elements have impedances that are comparable to the connected circuits and that are neither blocking nor conducting. Signal energies are lost in this time interval, so a higher efficiency modulator results. The time spent on the intermediate resistance parts of the rectifier characteristic can be further reduced by introducing harmonics into the carrier wave, so that a square type of wave results. The resistance of the rectifier is abruptly switched back and forth between blocking and conducting values in this manner. When the connected circuit impedances at the unwanted frequencies are very high or low, best efficiencies result when transmission between the signal circuits is blocked most of the time. Thus in a circuit like that of Fig. 2(a) when the filters are high impedance at the unwanted products, highest efficiency results when the copper oxide is a low resistance short circuit for the major portion of the carrier cycle. In Fig. 2(b) an open circuit is desirable most of the time.

Linear Modulator Theory

The analytical studies that have been of most benefit in the development of copper oxide modulators have made use of a variable resistance characteristic controlled by the carrier. This assumption has made it possible to investigate modulator performance [2] for a wide variety of characteristics under a great many operating conditions. Copper oxide modulator performance in particular cases as well as the effects of the circuit elements on this performance can readily be inferred from the data at hand about these idealized modulators.

In limited space it is not possible to discuss the varieties of resistance modulators that have been analyzed. However, certain viewpoints will be discussed that have been very useful not only for obtaining solutions for some of the hypothetical cases, but also in supplementing laboratory experiments on actual modulators.

All of these analytical studies have assumed a signal sufficiently small compared to the carrier, that it can be varied in magnitude without noticeable changes in the signal impedance or in the linearity between input and output signal amplitudes. This is in agreement with design procedure, as the circuit impedances and losses are determined on such a linear basis.

Superposition Principle

All of the modulator circuits with which we have been dealing, though composed of non-linear elements, have been resolved into the equivalent of linear systems by virtue of using a large carrier and small signal amplitude. We may simultaneously apply any number of signal frequencies, but all have negligible effect on the periodic changing of the non-linear element resistance by the carrier. These frequencies may be modulation product voltages, some applied at the output terminals and some at the input terminals, but in all cases, even though frequencies may coincide, it can be shown that the principle of superposition will hold without interaction between the applied forces and the responses. This permits a great simplification in the mathematical approach to modulator analysis, because the modulation product or signal voltages can be applied one at a time and the current responses summed. The voltage-current ratios at each frequency can then be replaced by equivalent impedances.

Any non-linear resistance like copper oxide will have a current-voltage characteristic that can be expressed as accurately as de-

[2] A physical picture of modulator performance in terms of linear networks is developed in a paper published in the January 1939 *Bell System Technical Journal*, "Equivalent Modulator Circuits" by E. Peterson and L. W. Hussey.

sired by

$$i = a_1 e + a_2 e^2 + \cdots + a_n e^n. \quad (1)$$

If a carrier and signal voltage are applied to this non-linear element, each term beyond the first will independently produce currents of new frequencies composed of the intermodulation products of these two voltages. If in turn the external impedances at these new frequencies are not zero, new voltages will appear across the non-linear element to produce still more new frequencies. In this case the simplest consideration is to minimize the number of voltages by presenting zero impedance to the modulation products. If the carrier voltage is $C \cos ct$ and the input signal voltage $S \cos st$, the current flow in the nth term is

$$i = a_n(C \cos ct + S \cos st)^n. \quad (2)$$

In the binomial expansion of this expression it is obvious that linear response to the signal and freedom from distortion result when the ratio of carrier to signal is made sufficiently large so that only the first two terms are important.

$$i \approx a_n[(C \cos ct)^n + n(C \cos ct)^{n-1} \cdot S \cos st]. \quad (3)$$

The first term in equation (3) is the current flow at d-c. and harmonics of the carrier; it has no effect on the input signal and output signal except in so far as impedance termination presented across the non-linear element at the carrier harmonic frequencies may alter the carrier voltage harmonic content. The signal input current and the signal output current, as well as the unwanted modulation products of the signal, result from the second term. The even values of n produce the even order sidebands, second order being the output signal, while the odd values of n produce the input signal current and the odd order sidebands. These currents can be evaluated from

$$\cos^n b = \frac{K^n_{\frac{n}{2}}}{2^n} + \sum_{m=1}^{m=n} \frac{K^n_{\frac{n-m}{2}}}{2^{n-1}} \cos mb,$$

in which $K^n_{\frac{n-m}{2}}$ is equal to the combination of n things taken $\frac{n-m}{2}$ at a time for $\frac{n-m}{2}$ integral and is equal to zero for $\frac{n-m}{2}$ non-integral.

The signal input current is

$$i_s = \frac{a_n n K^{n-1}_{\frac{n-1}{2}}}{2^{n-1}} C^{n-1} S \cos st, \tag{4}$$

while the second order output signal sideband is

$$i_{c\pm s} = \frac{a_n n K^{n-1}_{\frac{n-2}{2}}}{2^{n-1}} C^{n-1} S \cos (c \pm s)t. \tag{5}$$

Similarly, if the output signal voltage at second order sideband frequency $(c \pm s)$ had been applied along with the carrier in place of the input signal, and of an equal amplitude, then the following currents of the output and input signal frequencies would result:

$$i_{c\pm s} = \frac{a_n n K^{n-1}_{\frac{n-1}{2}}}{2^{n-1}} C^{n-1} S \cos (c \pm s)t, \tag{6}$$

$$i_s = \frac{a_n n K^{n-1}_{\frac{n-2}{2}}}{2^{n-1}} C^{n-1} S \cos st. \tag{7}$$

If both signal input and output frequency voltages are applied simultaneously, equation (3) then becomes

$$i \approx a_n[(C \cos ct)^n + n(C \cos ct)^{n-1} \cdot S \cos st + n(C \cos ct)^{n-1} \cdot S \cos (c \pm s)t]. \tag{8}$$

The current responses obviously are the sum of the separate responses from independent application of the two frequencies. Even if a complex array of terminating impedances are supplied so that voltages appear across the non-linear element at all the modulation product frequencies, each new voltage will individually produce its own current response, quite independently of the responses that are being produced by the other voltages. It can readily be seen then that superposition does not depend on any assumptions about what the terminating impedances may be.

Reciprocal Theorem

Equations (5) and (7) show that the sideband response to an input signal voltage is exactly equal in magnitude to the input signal response to the same amplitude sideband voltage. It can readily be seen that any two modulation products also bear such a reciprocal relationship between their voltages and currents, as a result of using the same amplitude and frequency of carrier harmonic multiplier of their respective voltages to modulate between the two frequency positions. Although reciprocity has been proved valid here only for short-circuit terminations at the modulation product frequencies, it can also be proved under numerous other conditions of circuit operation. It seems that, regardless of modulator complexity of impedance terminations or frequency loss effects, *the reciprocal theorem is a necessary attribute of such a linear and bilateral system in which there are no internal energy sources.* Two-way systems in which an amplifier for example, is included as an internal energy source in one or both directions will, of course, violate the reciprocal theorem if the gains in the two directions are different. This arrangement is, however, both bilateral and linear.

Complete Performance Criteria

The laws for transmission between a signal input frequency and a signal output frequency can be completely specified from open and short circuit impedance measurements at the signal input and output frequencies, regardless of the complexity of the modulator. (From such measurements optimum impedance terminations can even be determined for linear-bilateral systems with internal energy sources.)

The four-terminal network of Fig. 7 is assumed to represent a modu-

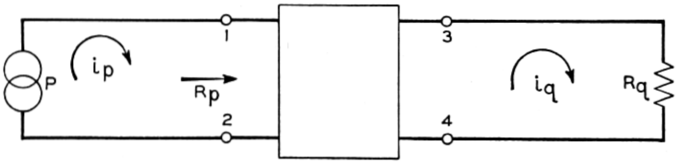


Fig. 7—Four terminal network equivalent of a linear modulator.

lator with a large carrier amplitude and having small signal voltage P of frequency p applied at the input terminals 1–2 at the left. Current of the output signal frequency q flows out of the terminals 3–4 into the impedance R_q. The generator P is assumed to have zero internal impedance at its own frequency. Impedance terminations at the 1–2 and 3–4 terminals at all other modulation product frequencies are perfectly general; whatever they are in a particular case, it is assumed that they are undisturbed as the terminations of the input and output at the signal frequencies are varied between open circuit and short circuit. The following symbols are used for the impedances looking into the modulator at the input terminals at input signal frequency p and at the output terminals at signal output frequency q.

Z_{p0} = impedance at p for open circuit at q;
Z_{q0} = " " q " " " " p;
Z_{ps} = " " p " short " " q;
Z_{qs} = " " q " " " " p;
R_q = impedance termination at frequency q;
R_p = impedance of modulator at frequency p with R_q at 3–4 terminals for frequency q;
K_a = transfer admittance between voltage at frequency p applied at the 1–2 terminals and short-circuit current at frequency q flowing from 3–4 terminals;
K_b = transfer admittance between voltage applied at 3–4 terminals at frequency q and short-circuit current at frequency p flowing from 1–2 terminals.

For R_q first short-circuited

$$i_{q_1} = PK_a, \tag{9}$$

$$i_{p_1} = \frac{P}{Z_{ps}}. \tag{10}$$

If the short circuit on R_q is removed, the following two additional currents will flow due to superposition of a new voltage $-i_qR_q$

$$i_{q_2} = \frac{-i_qR_q}{Z_{qs}} \tag{11}$$

and

$$i_{p_2} = -i_qR_qK_b, \tag{12}$$

$$i_q = i_{q_1} + i_{q_2} = \frac{PK_a}{1 + \dfrac{R_q}{Z_{qs}}}, \tag{13}$$

$$i_p = i_{p_1} + i_{p_2} = \frac{P}{Z_{ps}} - \frac{PK_aK_bR_q}{1 + \dfrac{R_q}{Z_{qs}}}. \tag{14}$$

The efficiency of the frequency translation, measured by the ratio of the power delivered to R_q to the power into terminals 1–2, is

$$\eta = \frac{i_q^2R_q}{i_pP} = \frac{\left(\dfrac{K_a}{1 + \dfrac{R_q}{Z_{qs}}}\right)^2 R_q}{\dfrac{1}{Z_{ps}} - \dfrac{K_aK_bR_q}{1 + \dfrac{R_q}{Z_{qs}}}}, \tag{15}$$

which is maximum for

$$R_q = \frac{Z_{qs}}{\sqrt{1 - K_a K_b Z_{ps} Z_{qs}}}. \tag{16}$$

The maximum efficiency is then

$$\eta_{\max.} = \frac{K_a^2 Z_{ps} Z_{qs}}{(1 + \sqrt{1 - K_a K_b Z_{ps} Z_{qs}})^2} \tag{17}$$

When equation (16) is substituted in (14) it is found that

$$R_p = \frac{P}{i_p} = \frac{Z_{ps}}{\sqrt{1 - K_a K_b Z_{ps} Z_{qs}}}. \tag{18}$$

In order to evaluate $K_a K_b$, open circuit impedance measurements must also be made. By superposition methods like those used in obtaining equations (11) and (12) it can readily be shown that

$$\frac{1}{Z_{p0}} = \frac{1}{Z_{ps}} - K_a K_b Z_{qs}, \tag{19}$$

$$\frac{1}{Z_{q0}} = \frac{1}{Z_{qs}} - K_a K_b Z_{ps}. \tag{20}$$

From these two equations, it follows that

$$\frac{Z_{ps}}{Z_{p0}} = \frac{Z_{qs}}{Z_{q0}}, \tag{21}$$

$$K_a K_b = \frac{\dfrac{1}{Z_{ps}} - \dfrac{1}{Z_{p0}}}{Z_{qs}}, \tag{22}$$

and $K_a K_b$ can be determined from any three of the open-short measurements by using (21) and (22). It follows that

$$K_a K_b Z_{ps} Z_{qs} = 1 - \frac{Z_{ps}}{Z_{p0}}. \tag{23}$$

Upon substitution in (16) and (18) it is found that

$$R_q = \sqrt{Z_{q0} Z_{qs}} \tag{24}$$

and

$$R_p = \sqrt{Z_{p0} Z_{ps}} \tag{25}$$

when 3–4 is terminated in R_q for maximum efficiency.

Open and short-circuit measurements enable us to compute the optimum efficiency from equation (17) only if the transfer admittance

K_a is known. If the reciprocal theorem holds, $K_a = K_b$ and K_a can be determined from (23). The optimum efficiency is then

$$\eta_{\max.} = \frac{1 - \sqrt{\frac{Z_{ps}}{Z_{p0}}}}{1 + \sqrt{\frac{Z_{ps}}{Z_{p0}}}}. \tag{26}$$

If the input signal generator has an internal impedance, most efficient energy delivery to the modulator will, of course, result if this impedance is made equal to R_p.

Equivalent T, π and bridge networks can obviously be drawn from the open and short-circuit measurements as in four-terminal linear networks.

It appears that even in a plate or grid circuit modulator the formulae of equations (24), (25) and (26) can be applied to the plate or grid circuit, respectively, where the signals are small compared to the carrier, inasmuch as the modulating parts of these circuits are linear and bilateral with no internal energy sources.

Double-Balanced or Reversing-Switch Modulator [3]

A number of interesting conclusions can be reached about copper oxide modulators by assuming that the copper oxide acts like a switch having a low-resistance value when the positive half-cycle of the carrier voltage is across the disc and a high-resistance value during the negative half-cycle. The circuits of Fig. 2(c), 2(d) or 2(e) can then be represented by the equivalent circuit of Fig. 8.

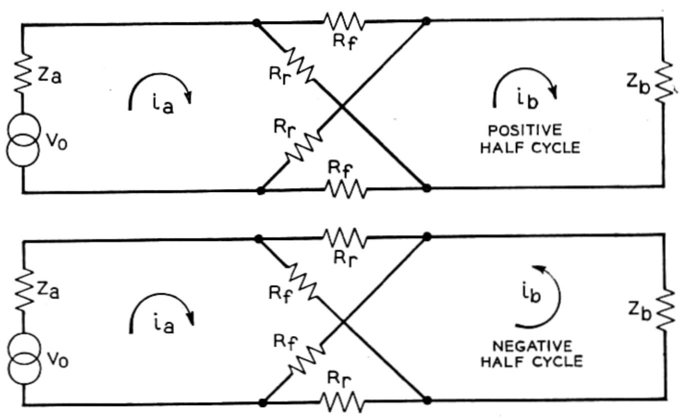


Fig. 8—Equivalent circuit of a double balanced modulator.

[3] Referred to in the German literature as the "ring modulator."

The input signal generator V_0 is applied out of an input circuit impedance Z_a, and the output signal is delivered to the output circuit impedance Z_b. Both Z_a and Z_b in general will be functions of frequency.

If $Z_a = Z_b = R$ at all frequencies it can easily be shown that the circuit will operate most efficiently for

$$R = \sqrt{R_r R_f}. \tag{27}$$

The signal input current will then be the only frequency in i_a:

$$i_a = \frac{V_0}{2R}. \tag{28}$$

From the properties of lattice networks it follows that

$$i_b = \frac{V_0}{2R} \cdot \frac{\sqrt{\frac{R_r}{R_f}} - 1}{\sqrt{\frac{R_r}{R_f}} + 1} \cdot f(t), \tag{29}$$

where

$$f(t) = \begin{cases} +1 \text{ for positive half-cycle of carrier} \\ -1 \text{ `` negative `` `` `` ``} \end{cases}.$$

Letting

$$f(t) = \frac{4}{\pi}\left(\cos ct - \frac{1}{3}\cos 3ct + \frac{1}{5}\cos 5ct \cdots\right)$$

and

$$k = \frac{\sqrt{\frac{R_r}{R_f}} - 1}{\sqrt{\frac{R_r}{R_f}} + 1}, \tag{30}$$

$$i_b = \frac{V_0 k}{2R} \cdot \frac{4}{\pi}\left[\cos ct - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5ct \cdots\right]. \tag{31}$$

The magnitude of the single-sideband output current is

$$i_{1+} = i_{1-} = \frac{2k}{\pi} \cdot \frac{V_0}{2R} \tag{32}$$

and the efficiency

$$\eta = \frac{4}{\pi^2} k^2. \tag{33}$$

In addition currents flow in the output circuit at both sidebands of all odd harmonics of the carrier frequency. These can be represented by i_{3+}, i_{3-}, i_{5+}, etc. No currents flow at the sideband frequencies of the even harmonics of the carrier, i_{2+}, i_{2-}, $\cdots$.

If V_0 should be replaced by an equal amplitude generator at any of the sideband frequencies, V_{1+}, V_{2-}, etc., then the input current would in any case be

$$I_{1+} = \frac{V_{1+}}{2R}, \qquad I_{2-} = \frac{V_{2-}}{2R}, \qquad \text{etc.}$$

The correspondence between the magnitudes of these entering currents and the magnitudes of the output currents at the modulation product frequencies are shown in Table I. Reciprocal relations between the

TABLE I

CORRESPONDENCE BETWEEN CURRENTS ENTERING AND LEAVING A REVERSING-SWITCH MODULATOR

Component on One Side of Modulator	Corresponding Components on Other Side of the Modulator								
	$\frac{2k}{\pi}$*		$-\frac{2k}{3\pi}$		$\frac{2k}{5\pi}$		$-\frac{2k}{7\pi}$		
I_0	I_{1-}	I_{1+}	I_{3-}	I_{3+}	I_{5-}	I_{5+}	I_{7-}	I_{7+}	$\cdots$
I_{1-}	I_0	I_{2-}	I_{2+}	I_{4-}	I_{4+}	I_{6-}	I_{6+}	I_{8-}	$\cdots$
I_{1+}	I_0	I_{2+}	I_{2-}	I_{4+}	I_{4-}	I_{6+}	I_{6-}	I_{8+}	$\cdots$
I_{2-}	I_{1-}	I_{3-}	I_{1+}	I_{5-}	I_{3+}	I_{7-}	I_{5+}	I_{9-}	$\cdots$
I_{2+}	I_{1+}	I_{3+}	I_{1-}	I_{5+}	I_{3-}	I_{7+}	I_{5-}	I_{9+}	$\cdots$
I_{3-}	I_{2-}	I_{4-}	I_0	I_{6-}	I_{2+}	I_{8-}	I_{4+}	I_{10-}	$\cdots$
I_{3+}	I_{2+}	I_{4+}	I_0	I_{6+}	I_{2-}	I_{8+}	I_{4-}	I_{10+}	$\cdots$
I_{4-}	I_{3-}	I_{5-}	I_{1-}	I_{7-}	I_{1+}	I_{9-}	I_{3+}	I_{11-}	$\cdots$
I_{4+}	I_{3+}	I_{5+}	I_{1+}	I_{7+}	I_{1-}	I_{9+}	I_{3-}	I_{11+}	$\cdots$
I_{5-}	I_{4-}	I_{6-}	I_{2-}	I_{8-}	I_0	I_{10-}	I_{2+}	I_{12-}	$\cdots$
I_{5+}	I_{4+}	I_{6+}	I_{2+}	I_{8+}	I_0	I_{10+}	I_{2-}	I_{12+}	$\cdots$
I_{6-}	I_{5-}	I_{7-}	I_{3-}	I_{9-}	I_{1-}	I_{11-}	I_{1+}	I_{13-}	$\cdots$
I_{6+}	I_{5+}	I_{7+}	I_{3+}	I_{9+}	I_{1+}	I_{11+}	I_{1-}	I_{13+}	$\cdots$
I_{7-}	I_{6-}	I_{8-}	I_{4-}	I_{10-}	I_{2-}	I_{12-}	I_0	I_{14-}	$\cdots$
I_{7+}	I_{6+}	I_{8+}	I_{4+}	I_{10+}	I_{2+}	I_{12+}	I_0	I_{14+}	$\cdots$
I_{8-}	I_{7-}	I_{9-}	I_{5-}	I_{11-}	I_{3-}	I_{13-}	I_{1-}	I_{15-}	$\cdots$

NOTE: A current of the frequency indicated in the first column will be modulated to produce the components written on the same line, the magnitudes of which are the magnitude of the generating current multipled by the factors at the top of the columns.

$$*k = \frac{\sqrt{\frac{R_r}{R_f}} - 1}{\sqrt{\frac{R_r}{R_f}} + 1}.$$

driving voltage at one frequency and the output current at another frequency are obvious.

TABLE II

PERFORMANCE OF DOUBLE-BALANCED MODULATOR (IDEAL REVERSING SWITCH) FOR VARIOUS INPUT AND OUTPUT TERMINATIONS

Modulator Terminations				Modulator Impedance		Modulator Loss or Efficiency	
Input Circuit		Output Circuit		Input Signal	Output Signal	Voltage Ratio	db
Signal	Others	Signal	Others				
R	R	R	R	R	R	$\frac{2}{\pi}$	3.9
R	any value	R	R	R		$\frac{2}{\pi}$	3.9
R	R	R	0	$\frac{2R}{\pi^2 - 2}$	R	$\frac{2}{\pi}$	3.9
R	R	R	∞	$\frac{(\pi^2 - 2)R}{2}$	R	$\frac{2}{\pi}$	3.9
R	$\frac{(\pi^2 - 2)R}{2}$	$\frac{(\pi^2 - 2)R}{2}$	0	R	$\frac{\pi^2(\pi^2 - 2)R}{6\pi^2 - 16}$	$\frac{\pi}{\sqrt{2(\pi^2 - 2)}}$	2
R	$\frac{2R}{\pi^2 - 2}$	$\frac{2R}{\pi^2 - 2}$	∞	R	$\frac{(6\pi^2 - 16)R}{\pi^2(\pi^2 - 2)}$	$\frac{\pi}{\sqrt{2(\pi^2 - 2)}}$	2
R	0	R	0	0	0		∞
R	∞	R	∞	∞	∞		∞
R	0	R	∞	$\frac{\pi^2 R}{4}$	$\frac{4}{\pi^2} R$	$\frac{4\pi}{4 + \pi^2}$	.85
R	∞	R	0	$\frac{4}{\pi^2} R$	$\frac{\pi^2}{4} R$	$\frac{4\pi}{4 + \pi^2}$	.85
R	0	$\frac{4}{\pi^2} R$	∞	R	$\frac{4}{\pi^2} R$	1	0
R	∞	$\frac{\pi^2}{4} R$	0	R	$\frac{\pi^2}{4} R$	1	0
R_s	$R_{s'}$	R_r	$R_{r'}$	Z_i*	Z_0*	η*	

$$*Z_i = \frac{\pi^2 R_r'(R_r + R_s') + 4(R_r - R_r')R_s'}{\pi^2(R_r + R_s') - 4(R_r - R_r')},$$

$$Z_0 = \frac{\pi^2 R_s'(R_s + R_r') + 4(R_s - R_s')R_r'}{\pi^2(R_s + R_r') - 4(R_s - R_s')}.$$

$$\eta = \frac{4\pi(R_r' + R_s')\sqrt{R_s R_r}}{\pi^2(R_s + R_r')(R_r + R_s') - 4(R_r - R_r')(R_s - R_s')}.$$

Generalized Reflection Theory

Superposition permits us to apply simultaneously driving forces of the frequencies tabulated above in any relative phases and amplitudes that we care to choose on either side of the modulator. If simultaneously I_0 is applied on one side of the modulator and $(I_{1+})\frac{2k}{\pi}\cdot\frac{Z_{1+}-R}{Z_{1+}+R}$ is applied to the other set of modulator terminals, then the total current at the output terminals at the sideband frequency (1^+) will be

$$(I_{1+})\frac{2k}{\pi}\cdot\left[1-\frac{Z_{1+}-R}{Z_{1+}+R}\right]. \tag{34}$$

This is equivalent to saying that a resistance R at the sideband frequency $(1+)$ has been connected to the output terminals of the modulator and in this resistance is an internal zero impedance generator of voltage

$$2R(I_{1+})\frac{2k}{\pi}\cdot\frac{Z_{1+}-R}{Z_{1+}+R}. \tag{35}$$

This resistance R at sideband frequency $(1+)$ must be infinite at all other frequencies, if in parallel we assume another resistance of R at all frequencies except $(1+)$ at which it is infinite.

The equivalent impedance at frequency $(1+)$ at the modulator terminals connected to the $(1+)$ resistance with its internal generator, is the ratio of $(1+)$ voltage to $(1+)$ current.

$$Z=\frac{\frac{2k}{\pi}I_{1+}\cdot 2R-\frac{2k}{\pi}\cdot I_{1+}\cdot\left[1-\frac{Z_{1+}-R}{Z_{1+}+R}\right]R}{\frac{2k}{\pi}I_{1+}\cdot\left[1-\frac{Z_{1+}-R}{Z_{1+}+R}\right]}, \tag{36}$$

which reduces to

$$Z=Z_{1+}. \tag{37}$$

Z_{1+} may be real or complex as it involves only the amplitude and phase of the superimposed voltage of upper sideband frequency. It can readily be seen then that the solution for current flow at this frequency of equation (34) is identical with the case of linear networks in which the current is expressed as that flowing in a matched circuit modified by a reflection factor. Reflection from any modulation product frequency can be similarly treated.

A number of cases have been worked out of efficiencies and impedances in such modulators for transmission between an input signal and a single-sideband output signal. The modulating element has been assumed perfect ($k = 1$) and the terminations pure resistances. The results are shown in Table II.

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