A Representation of the Sunspot Cycle *

By C. N. ANDERSON

ALTHOUGH sunspots had been observed occasionally back to ancient times, their study may be said to date from their rediscovery by Galileo in the spring of 1610 with the then newly invented telescope. Since then much has been written about their nature, their periodicities and possible influence on human affairs.

The purpose of the study reported on in this paper was to analyze the components of the sunspot data and thereby to reconstruct a curve which would not only represent the variation in sunspot numbers from 1749 to date but would also be consistent with times of maxima and minima from 1610 to 1749. A number of attempts along this line have been made in the past,[3, 4, 5, 6, 9, 11] all of which have neglected the data previous to 1749 and all have used a slightly different method of analysis. It is believed that the agreement in the present study is somewhat better than in those of the past; nevertheless, no claim is made for any great accuracy in predicting future sunspot activity. Harmonic analysis based on a fraction of a period is always a source of danger and, furthermore, we have no assurance that all the components of the sunspot curve are periodic functions.[8] In fact, some papers have appeared in which each cycle was treated as a more or less independent outburst.[2, 10, 12, 13] Nevertheless, because of the long base line over which agreement is obtained in this present study, it is hoped that the results may not be too much in error for at least a few cycles.

The data used are the series of relative sunspot numbers begun by Rudolph Wolf,[1] Professor of Astronomy at Zurich and continued by his successors Wolfer and Brunner. Wolf began his systematic observations of sunspot numbers in 1849. He endeavored to make some allowance for the area of the spots and to avoid having a small spot of short duration count as much as a large group. With this in mind, he applied the following formula to his observation:

$$\text{Relative sunspot number} = k(10g + f),$$

where g and f are the group and total spot numbers respectively,

---

* Presented before the Astronomy Section of A.A.A.S. at Richmond, Va., December 28, 1938.

and k is a constant depending on the type of telescope and other factors affecting the observation. The figure 10 is an arbitrary one arrived at by Wolf from investigation of a number of individual cases and which seemed to give him the proper relationship.

A careful study was then made by Wolf of all existing records of prior data. Hofrat Schwabe of Dessau supplied data for the period 1826 to 1855 to which a correction was applied determined from a study of the overlapping data and from the percentage of spotless days. Johann Casper Staudacher of Nürnberg had made a total of 1131 observations (from one to ten every month) by means of a helioscope during the period 1749 to 1799. He often gave detailed descriptions and included many sketches. Imagine a man making observations for fifty years without any attempt at analysis and then have the data resurrected fifty years after his death to form an important contribution to the record. Flaugergues (1794–1830), Tevel (1816–1836), Adams (1819–1823) and Arago (1822–1830) supplied most of the data for the intervening period between the observations of Staudacher and Schwabe. Wolf lists about a hundred references (225 up to 1866) to sunspots prior to 1850. In most cases, the observations were incidental to other solar observations such as culminations, solar diameter, eclipses, transits, or on the nature of sunspots rather than their number. Each record was carefully studied, and from them all Wolf obtained a representation of sunspot numbers for as far back as 1749 and established the times of maxima and minima with an accuracy of ± 2 years or better back as far as 1610 A.D. Although the data prior to 1849 include a certain element of unreliability and all the data represent *relative* numbers which have been obtained from the observations by applying a weighting factor, it is, however, not only the best record but the only one for such a long period of time. In the aggregate it is probably a good indication of the variation of solar activity.

The method employed in the present analysis is briefly as follows:

(*a*) The yearly averages of sunspot numbers from 1749 to the end of 1937 were first plotted in the conventional way as shown in Fig. 1; it was noted that in certain sections of the curve, notably after 1840, the maximum amplitudes of alternate eleven-year periods were higher than the intervening ones.

(*b*) The data were redrawn with alternate eleven-year periods above and below the time axis; this not only smoothed out the envelope of the maxima but also simplified the analysis by eliminating a computed mean value base line which has been employed in previous analyses; the maximum-amplitude component becomes approximately

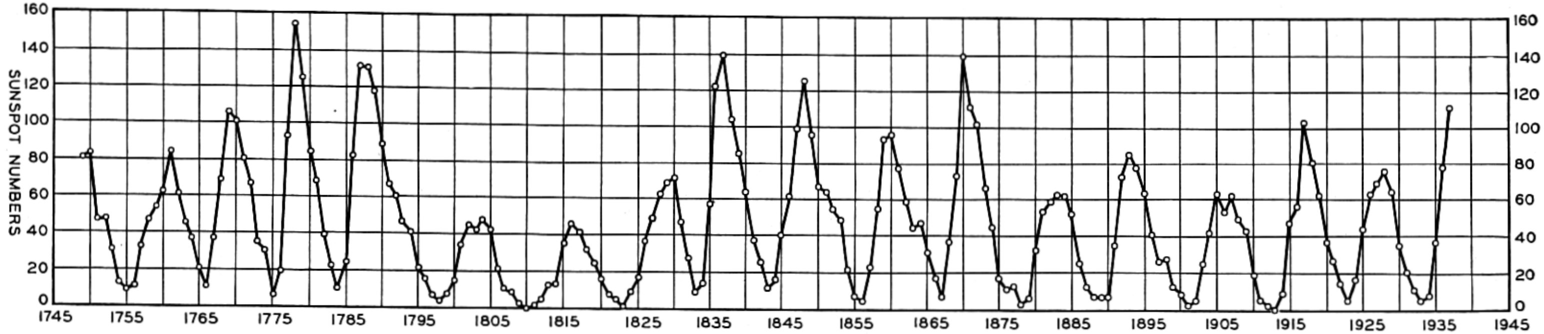

Fig. 1—Yearly averages of monthly sunspot numbers. This series of relative sunspot numbers was begun by Wolf of the Zurich Observatory and continued by his successors, Wolfer and Brunner.

twenty-two years instead of eleven years and the physical justification is the similarity in the polarity of the leading spots in alternate eleven-year periods.

(*c*) Quarterly values were obtained from the curve in order to obtain a smaller unit and to approximate better such periods as 22.25 years, 17.33 years and other periods not an integral number of years; the values were assigned + or − signs in accordance with whether they were above or below the time axis.

(*d*) A periodogram was computed and the component of maximum amplitude appeared to be 22.75 years; this component was eliminated, a new periodogram computed and so on; after many trials it was finally decided that no solution could be found which would reduce the residue satisfactorily with 22.75 years as the main component.

(*e*) Inspection of the analysis indicated an improvement would be obtained by a decrease in the period and accordingly a change was made to 22.5 years and the computations repeated; a solution was finally obtained which fit the 1749–1937 curve of sunspots fairly well but, when extrapolated back to 1600 A.D., did not fit particularly well the observed times of maxima and minima for the period 1610 to 1749; a still further reduction in the chief component was necessary.

(*f*) The computations were repeated with 22.25 years as the chief component; after the computation of the series was completed, it was discovered that the components were either harmonics or nearly integral harmonics of 312 years; many of these components had been in use since the original periodogram.

(*g*) A search was made for the 312-year period, since it should be possible to check in the cases of two minima and two maxima, with the following results:

Minima	1610.8 ± 0.4	1923.1	Diff. 312.3 ± 0.4
Maxima	1615.5 ± 1.5	1928.6	313.1 ± 1.5
Minima	1619 ± 2	1933.6	314.6 ± 2
Maxima	1626 ± 0.5	1937.7	311.7 ± 0.5

giving a weighted average of 312.5 years or an average of 312.0 years using the two most reliable values.

(*h*) Computations were again repeated using harmonic components of 312 years. This gave a very good representation of the data from 1749 to date and also agreed with the times of maxima and minima for the period 1610 to 1749. The resultant curve is shown in Figs. 2 and 3.

The components of the reconstructed curve are shown in Fig. 4. It is of interest to note that the 22.25-year component which is largely

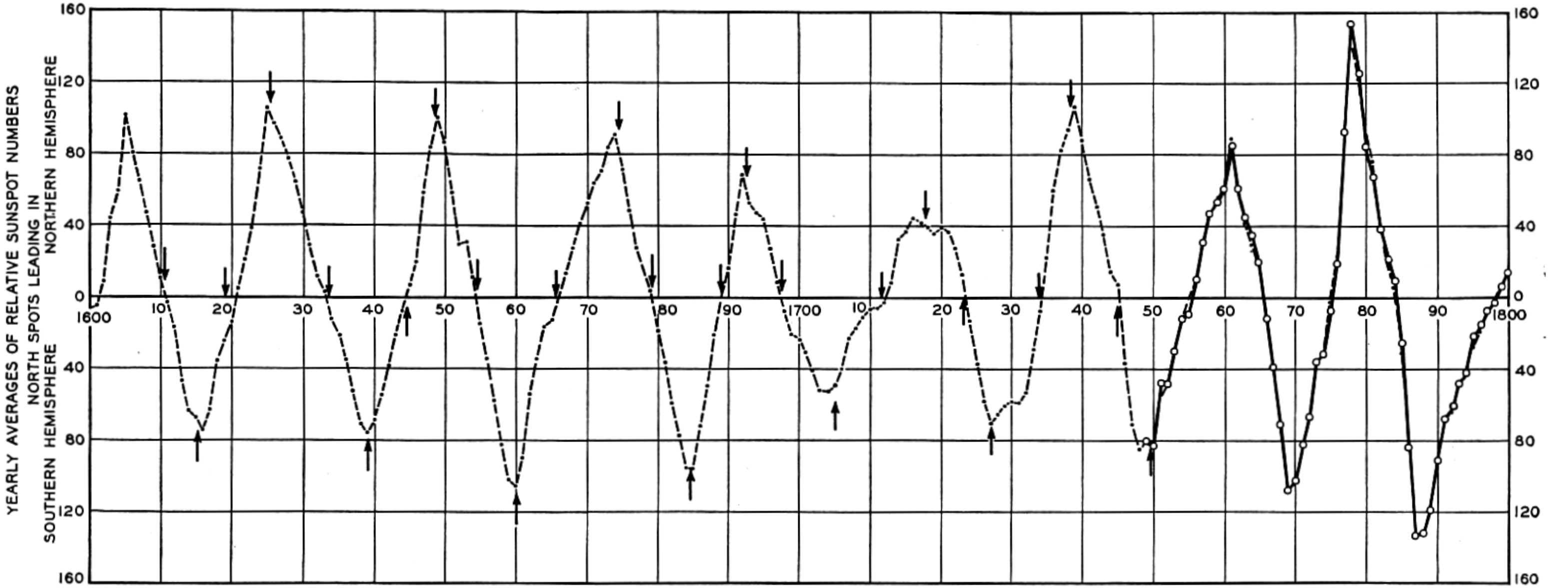

Fig. 2—Measured and computed sunspot numbers, 1600–1800 A.D. Solid line indicates measured values; light dashed line indicates values obtained by adding up the components computed from the measured values; components are harmonics of a 312-year period; arrows indicate the years when maxima and minima were observed.

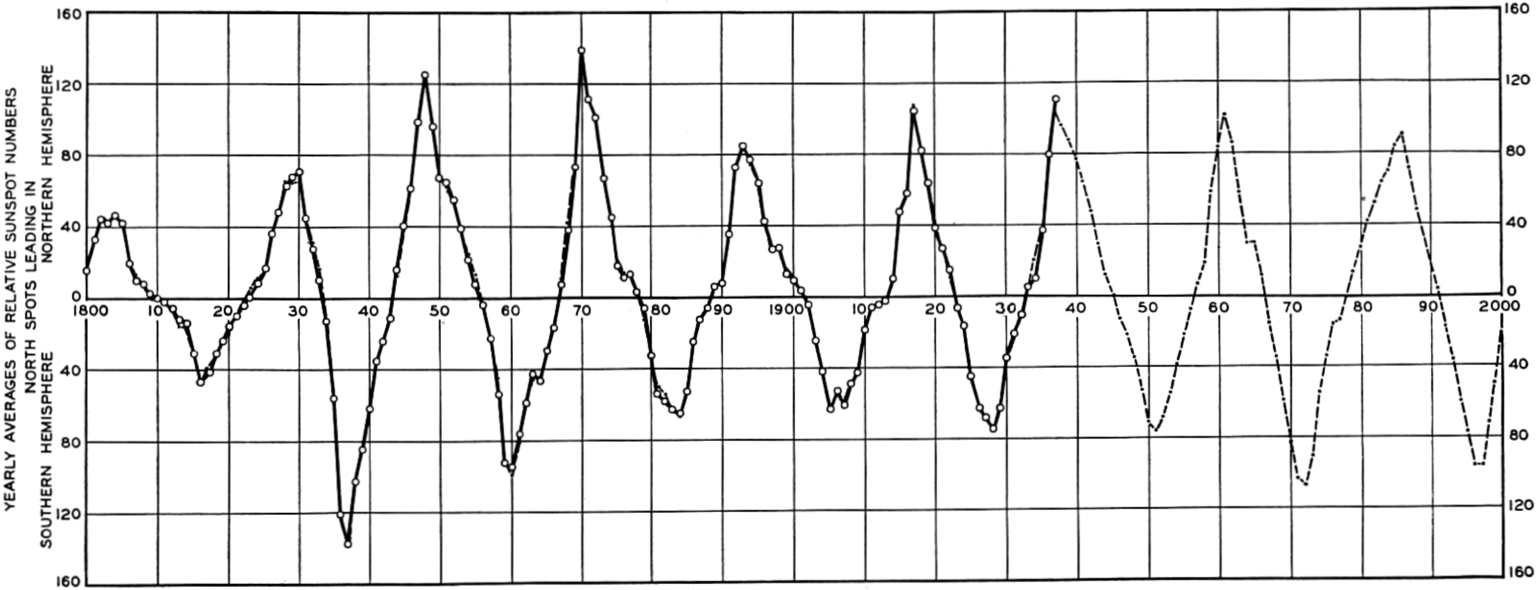

Fig. 3—Measured and computed sunspot numbers, 1800–2000 A.D. Solid line indicates measured values; light dashed line indicates values obtained by adding up the components computed from the measured values; components are harmonics of a 312-year period.

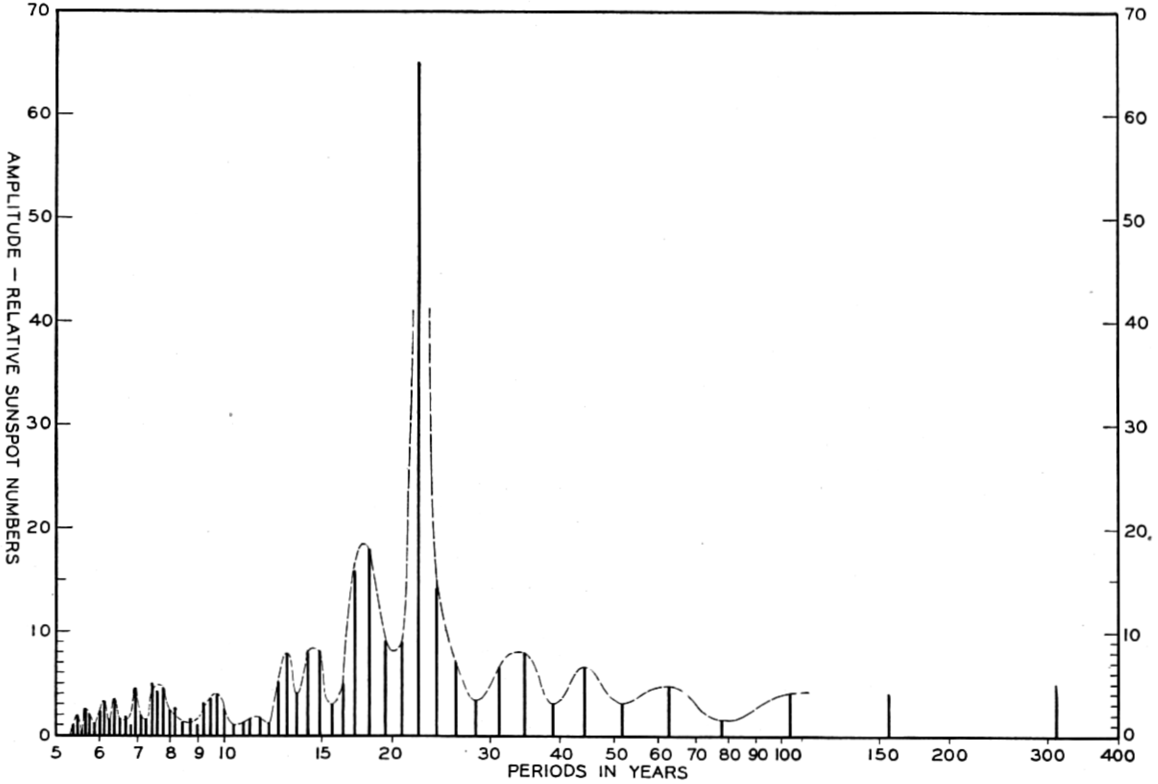

Fig. 4—Analysis of components of the sunspot cycle. Vertical lines indicate amplitudes of harmonic components of an apparent 312-year cycle.

responsible for the eleven-year periodicity of sunspots has an amplitude of only about 2/5 of the greatest amplitudes of the resultant maxima. Next in importance are the two adjacent periods at 17.3 and 18.4 years, respectively.

In conclusion, the study has resulted in a representation of yearly sunspot averages which agrees as well as could be desired with the data and which is also consistent with the reported times of maxima and minima back as far as 1610 A.D. The chief component has been treated as a 22-year (22.25 years) component instead of eleven years. In the course of computation the components appeared to be harmonics, or nearly so, of a 312-year period. A substantiation of this 312-year cycle was found in a check of the overlapping data (maxima and minima from 1923 to date).

It had been hoped that the resultant distribution of amplitudes versus frequency of the components might be capable of simple interpretation and that a rather simple explanation of the phenomena of sunspot periodicity might result, such as one or two forces acting upon a nonlinear system with a given fundamental period. Further studies may still indicate this to be the case.

The author wishes to express his appreciation of the assistance of Miss Helen Grant with the rather tedious computations.

REFERENCES

1. "Mitteilungen über die Sonnenflecken," Rudolph Wolf, *Naturforschende Gesellschaft in Zurich Vierteljahrsschrift*, 1856–1865.
2. "Über Eine Neue Theorie zur Erklärung der Periodicität der Solaren Erscheinungen," J. Halm, *Astronomisches Nachrichten*, Vol. 156, 44, 1901.
3. "On the Periodicities of Sunspots," A. Schuster, *Phil. Trans. Royal Soc.*, London, Vol. 85, 1911.
4. "Determination of Periodicities by Harmonic Analyzer with Application of the Sunspot Cycle," A. A. Michelson, *Astrophysical Journal*, Vol. 38, 1913.
5. "On the Harmonic Analysis of Sunspot Relative Numbers," Hisashi Kimura, *Monthly Notices of R.A.S.*, Vol. 73, 1913.
6. "On the Harmonic Analysis of Wolf's Sunspot Numbers," H. H. Turner, *Monthly Notices R.A.S.*, Vol. 73, 549, 1913.
7. "On the Expression of Sunspot Periodicity as a Fourier Sequence," H. H. Turner, *Monthly Notices R.A.S.*, Vol. 73, 714, 1913.
8. "On a Method of Investigating Periodicities in Disturbed Series with Special Reference to Wolfer's Sunspot Numbers," G. Udny Yule, *Phil. Trans. Royal Soc.*, London, Ser. A226, 1926–27.
9. "A New Analysis of the Sunspot Numbers," Dinsmore Alter, *Monthly Weather Review*, October 1928.
10. "Untersuchungen über die Häufigkeitskurve der Sonnenflecke," H. Ludendorff, *Zeitschrift für Astrophysik*, Vol. 2, May 1931.
11. "Über Perioden der Sonnenflecken," S. Oppenheim, *Astronomisches Nachrichten*, Vol. 232, 369, 1932.
12. "Forecasting Sunspots and Radio Transmission Conditions," A. L. Durkee, *Bell Laboratories Record*, Vol. XVI, December 1937.
13. "The Mathematical Characteristics of Sunspot Variations," John Q. Stewart and H. A. A. Panofsky, *Astrophysical Journal*, Vol. 88, November 1938.