The Production of Ultra-High-Frequency Oscillations by Means of Diodes

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The general problem of obtaining oscillations by the use of diodes with critical electron transit time is outlined. Some of the properties of a 10 cm. oscillator tested experimentally are included. Extraneous losses were reduced when the oscillator was enclosed within a wave guide.

THE theory of the production of negative impedance by means of an electron discharge between two parallel planes has been known for some years.[1] The negative resistance appears whenever the electron transit time is approximately 1¼, 2¼, 3¼, etc. cycles of a given high-frequency current. Using this property, Müller was able to construct tubes giving 100 cm. oscillations.[2] The operating efficiencies were quite low, and in the frequency range covered by these tubes it seems fairly conclusive that other methods of producing oscillations are more effective than the critical transit time diode. However, there is promise in the application of diode operation to much higher frequencies than those of Müller.

In a diode where the electron discharge occurs between two parallel planes where one performs the function of electron emitting cathode and the other constitutes an anode biased at a positive potential, the effective impedance presented to an external source is inherently low in magnitude. This is because of the capacitance between the two planes which causes the decrease in impedance at high frequencies. For the production of oscillations, the capacitance must be combined with a resonant structure having the proper inductance to resonate at the desired frequency and having a resistance which effectively is less in magnitude than that of the electron stream. Because of the low losses thus required of the coupling or tuning circuit the properties of concentric lines and of tuned cavities offer a favorable method of attack. These structures also have the property that the impedance presented to the diode proper may be made low to match its capacitive reactance at the high frequencies desired.

The two most important sources of circuit resistance are ordinary ohmic loss modified in the usual way by skin effect in the conducting

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[1] For numbered references see end of paper.

material forming the resonant system and secondly the losses caused by radiation of energy. These latter are extremely important where the negative resistance is only a few ohms as in the present instance and necessitate the use of nearly closed structures. This again directs attention to the properties of cavities and concentric lines tuned by internal capacitive resonance, the low capacitance being formed by the electrodes between which the electron discharge flows. It was on the basis of these principles that the actual diode models were constructed.

The general aspect of these tubes is shown in Fig. 1 which presents a section through the axis of revolution. The cylinders of radii r_1 and r_2 respectively constitute the outer and inner conductors of a concentric line. At one end of the inner conductor a flange partly closes the system thus confining most of the energy within the cavity. At the other end of the inner conductor the flat surface of the inner conductor constitutes an emitting cathode while the opposing surface of the outer conductor constitutes the positively biased anode which also completely closes the end of the cylinder. The system is tuned by the capacitance between cathode and anode and the effective inductance of the coaxial line of length h. The emitter was coated in the experiments with an oxide of the uncombined type and was heated by a filament located within the inner cylinder. The spacers for separating the inner cylinder from the main body of the outer conductor were composed of fused quartz in order to obtain low losses and good mechanical rigidity. A water jacket was supplied to assist in cooling the anode.

In reference to Fig. 1, the tuning relation between the cathode-anode capacitance and the inductance of the resonant circuit connected to it requires the following relation to be satisfied,

$$\frac{1}{\lambda}\tan\frac{2\pi h}{\lambda} = \frac{x}{\pi r_2^2 \log_e \dfrac{r_1}{r_2}}. \tag{1}$$

Here λ stands for the free space wave-length. The other quantities in the formula are illustrated in Fig. 1 and all dimensions are in centimeters. The radii r_1 and r_2 refer respectively to the inner surface of the outer cylinder and the outer surface of the inner cylinder. Improved formulas for the resonant frequencies of cavities of this type have recently been published by Hansen.[3]

The formula (1) is based on the approximation that the presence of electrons between cathode and anode does not affect the dielectric constant of the resulting capacitance. This approximation is a good one when the electron transit time is greater than a cycle, as is the case here.

The next design formula required is the resistance of the electron

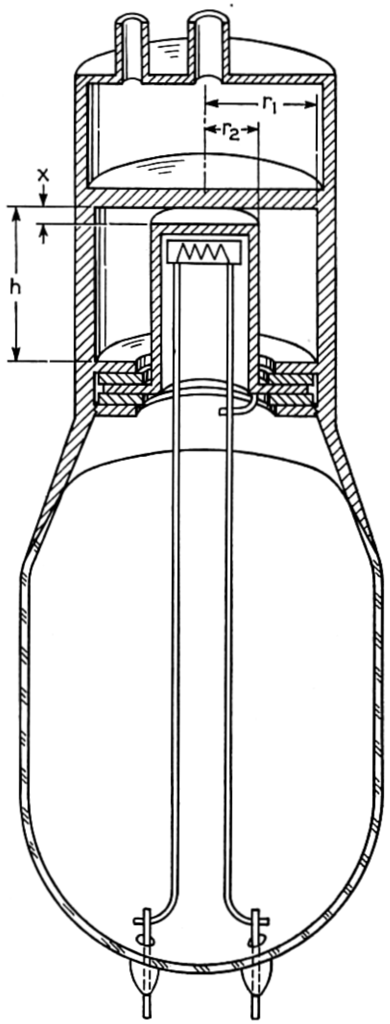

Fig. 1—Ten-centimeter diode used in tests.

Diode	r_1	r_2	x	h
No. 24	1.270	0.635	0.203	1.870
No. 37	1.220	0.635	0.105	1.870

Centimeters

stream. Reckoned per square centimeter of area this may be written,[4*]

$$r_p = \frac{1.78\ \lambda^4\ I_0}{10^4}[2(1 - \cos\theta) - \theta\sin\theta] \text{ ohms for cm.}^2, \qquad (2)$$

where I_0 is the direct current density in amperes per square centimeter flowing to the anode and θ is the electron transit angle, given by

$$\theta = \frac{Ax}{\lambda\sqrt{V_0}} \text{ radians.} \qquad (3)$$

Here V_0 is the constant potential difference in volts between the cathode and anode and A is a numerical factor which depends upon the amount of space charge within the electron discharge, being equal to 6300 for negligible space charge and to 9500 for complete space charge with intermediate values for intermediate space charge. As an alternative the resistance (2) may be written

$$r_p = \frac{12\ r_0}{\theta^4}[2(1 - \cos\theta) - \theta\sin\theta] \text{ ohms for cm.}^2, \qquad (4)$$

where r_0 is the low-frequency series resistance of the device. With space charge, r_0 is the slope of the static characteristic derived from Child's equation

$$I_0 = \frac{2.33}{10^6}\frac{V_0^{3/2}}{x^2} \text{ amperes/cm.}^2. \qquad (5)$$

More generally r_0 is given by the expression

$$r_0 = \frac{1.48}{10^5} I_0 \frac{x^4}{V_0^2} A^4 \text{ ohms for cm.}^2, \qquad (6)$$

where A is the same as was defined under (3).

Figure 2 shows a graph of the electron stream resistance as a function of transit angle and is repeated from previous papers.[1] However, it may not have been emphasized in the literature that the graph as well as equations (2) and (4) apply not only with complete space charge but with intermediate values when interpreted correctly, namely in terms of the d-c. current density I_0 rather than in terms of the applied potentials.

Whenever the transit angle is equal to $2\pi n + \frac{\pi}{2}$ where n is 1, 2, 3,

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[*] Equation 41 in this reference applies where the initial velocities are very small. With complete space charge $q = J$ and $a_a = 0$ whereas without space charge $q = 0$. Either condition gives the same series resistance in terms of I_0.

etc. then the electron stream exhibits a negative resistance. From this it may be inferred that oscillations are possible not only for values of n equal to unity but also for larger values, thus yielding the possibility of higher order oscillations when the circuit coupled to the electron stream is properly proportioned. If we start with a given cathode temperature with space charge and attempt to obtain the longer transit times by a decrease in applied potential, the smaller currents obtained will decrease the negative resistance. Hence, with space charge it is advisable to employ the smallest value of n possible in actual circuit design.

For computation work the general formula (2) may be greatly simplified because we need to know only the maximum values which the negative resistance attains. These occur in the neighborhood of transit angles given by

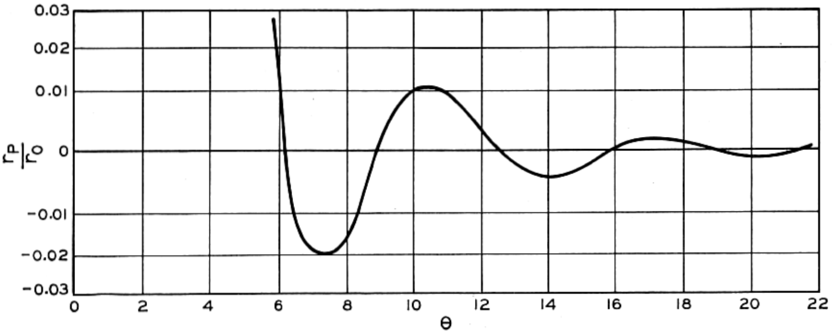

Fig. 2—Relation between transit angle θ and diode resistance.

$$\theta = 2\pi n + \frac{\pi}{2}, \qquad n = 1, 2, 3, \cdots \tag{7}$$

and under these conditions the effective negative resistance is

$$R_p = \frac{r_p}{\text{area}} = -\frac{1.4\,\lambda^4\,I_0}{10^3\,\pi r_2^2}\left(\frac{4n+1}{5}\right)\text{ ohms.} \tag{8}$$

The detailed steps in circuit design are these: First the allowable value of current density I_0 must be determined. This depends upon the ability of the cathode to emit electrons and as a practical limit something in the neighborhood of 300 mils per square centimeter cannot be exceeded. When this current has been decided upon then the value x of the separation between cathode and anode may be found from (3) and (7) with the lowest value of n which will give practical figures. The space charge condition (4) also gives the lowest allowable potential for which the required current can flow and hence the best efficiency in the simple diode of Fig. 1. From these values the negative resistance may be computed from (8).

As a next step the remainder of the circuit must be proportioned. The tuning relation (1) yields the values of height h for a given diameter. The next consideration is to insure that the sum of all positive resistances is less than the negative resistance of the diode. For a circuit with dimensions small compared with the wave-length, approximate formulas for the resistances associated with the losses in the circuit conductors can be readily derived from classical circuit analysis.

A most important resistance, not so readily computed, is caused by radiation of energy through the gap between the insulating flanges which separate cathode and anode. In most uses of the device, this radiated energy constitutes the useful load on the oscillator but care must be taken that the load is not so heavy as to stop the oscillations altogether. An important distinction must be made as to whether the tube is to radiate into free space or into some enclosure such as a hollow wave guide, for example. In the latter case the radiation may be regulated to a large extent by the geometry of the enclosure. For values of radiation resistance when energy is directed into free space an article by S. A. Schelkunoff [5] may be referred to.

For oscillation, as pointed out, the sum of all these positive resistances must be less than the negative resistance of the electron discharge and for high efficiency the radiation resistance should be much greater than the sum of all of the other positive resistances. This is usually found to be the case, and in fact the radiation resistance is likely to be so great as to stop oscillations unless the gap is made sufficiently small.

In designing a hollow wave guide mounting for diodes of the sort pictured in Fig. 1 it was recognized that since the high-frequency wave energy issues from the coaxial resonator as a wave guided along the heater leads, the natural and probably most effective thing to do was to dispose these leads so that the field associated with them would conform as nearly as possible to one of the wave types which can be supported in a hollow wave guide. Of these wave types, the so-called H_1 type[6] is readily generated by high-frequency current in a wire extending across a diameter of the guide, and the wave guide mounting shown in cross section on Fig. 3 is such as to give rise to this type of wave. For mechanical reasons a brass pipe of circular cross sections was chosen for the guide, and its diameter (3⅞ inches) was chosen large enough so that it would freely transmit an H_1 wave of the expected frequency. In the mounting, the high frequency circuit is completed from the anode to the wall of the guide through a stopping condenser.

Preliminary experiments with diode no. 24 had shown that when wave power issuing from the tube was allowed to radiate into free space, a space current of 500 milliamperes with anode voltage of about 300 volts was required to maintain oscillations. An interesting and instructive experiment is then to determine by how much this current is reduced when radiation is held to as low a value as possible. For this purpose an assemblage as in Fig. 4 was used. Here a wave guide mounting of form comparable to Fig. 3 is clamped into sections of wave guide closed at the two ends by closely fitting but longitudinally ad-

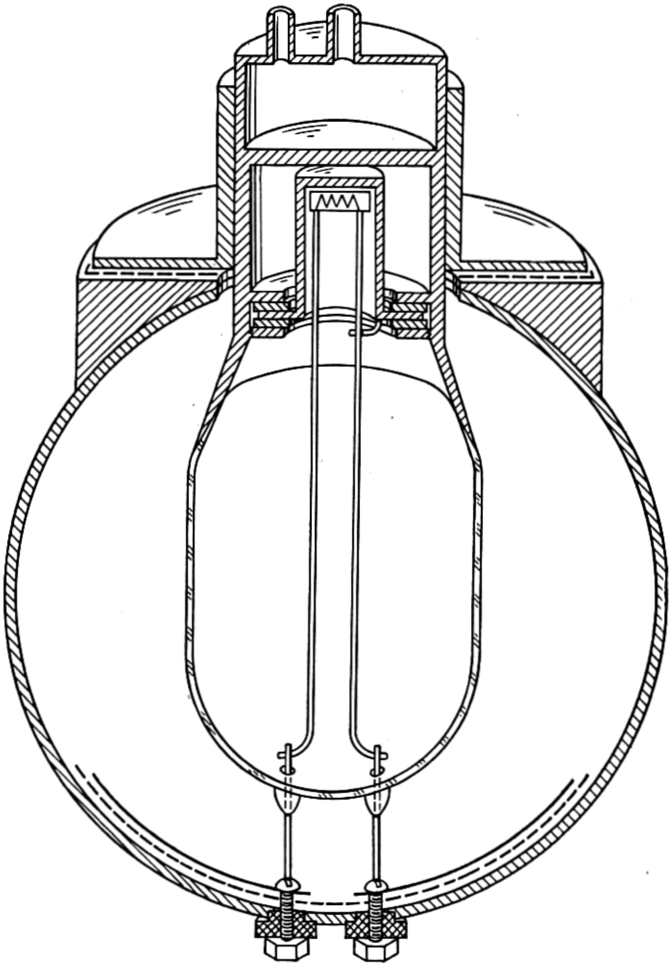

Fig. 3—Hollow wave guide mounting for diodes.

justable reflecting pistons. By thus completely enclosing the tube, escape of energy into free space is avoided, and the losses external to the tube are reduced to the ohmic losses (including dielectric losses) incident to the existence of the wave within the guide. The presence of wave power within the guide is indicated by a crystal detector-microammeter combination connected to an antenna extending a short distance within the guide, as shown in Fig. 4. Adjustment of the pistons closing the ends of the wave guide system allowed the attainment, for each value of anode current and voltage, of the most favorable impedance conditions for abstraction of energy from the diode.

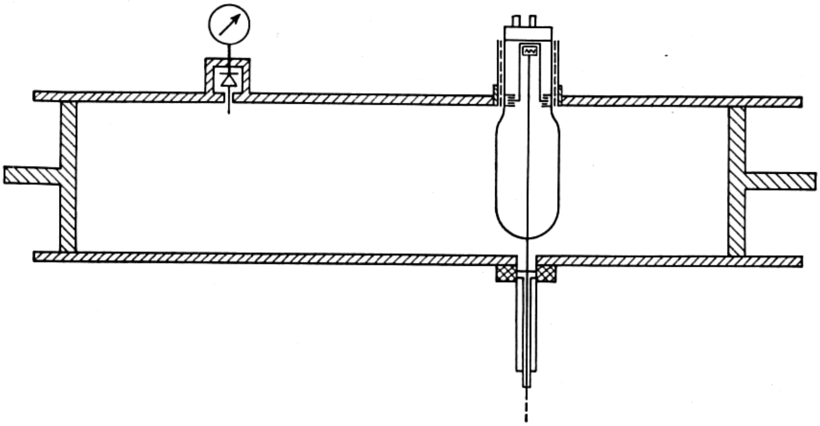

Fig. 4—Apparatus arrangement for "No Load" test of diode.

The results of this experiment are shown on Fig. 5, which gives the boundaries of the domain of oscillation of the two tubes whose dimensions are given on Fig. 1. The large gain in extent of the oscillation region of tube no. 24 is immediately apparent; the free space oscillation limit of $E_p = 300$ volts, $I_p = 500$ ma. has been lowered to $E_p = 210$ volts, $I_p = 110$ ma. For tube no. 37 oscillations occur at much lower voltages, as is to be expected from the smaller anode-cathode distance, and the minimum plate current required to maintain oscillations is also somewhat smaller.

In the arrangement of Fig. 4 no useful power is extracted from the diode. To examine the oscillation domain of the diodes when delivering useful power, the arrangement of Fig. 6 was used. Here in a section of wave guide closed at both ends by tightly fitting but longitudinally adjustable pistons there are placed the diode mounting shown on Fig. 3 and a power absorbing and measuring element. The assemblage constitutes in effect a wave guide transformer, for by suitably adjusting the positions of the pistons with respect to the source and the power absorber, and by a proper choice of the distance between the source and the power absorber, the impedance of the latter can be matched to that of the source, so as to ensure the maximum delivery of power.

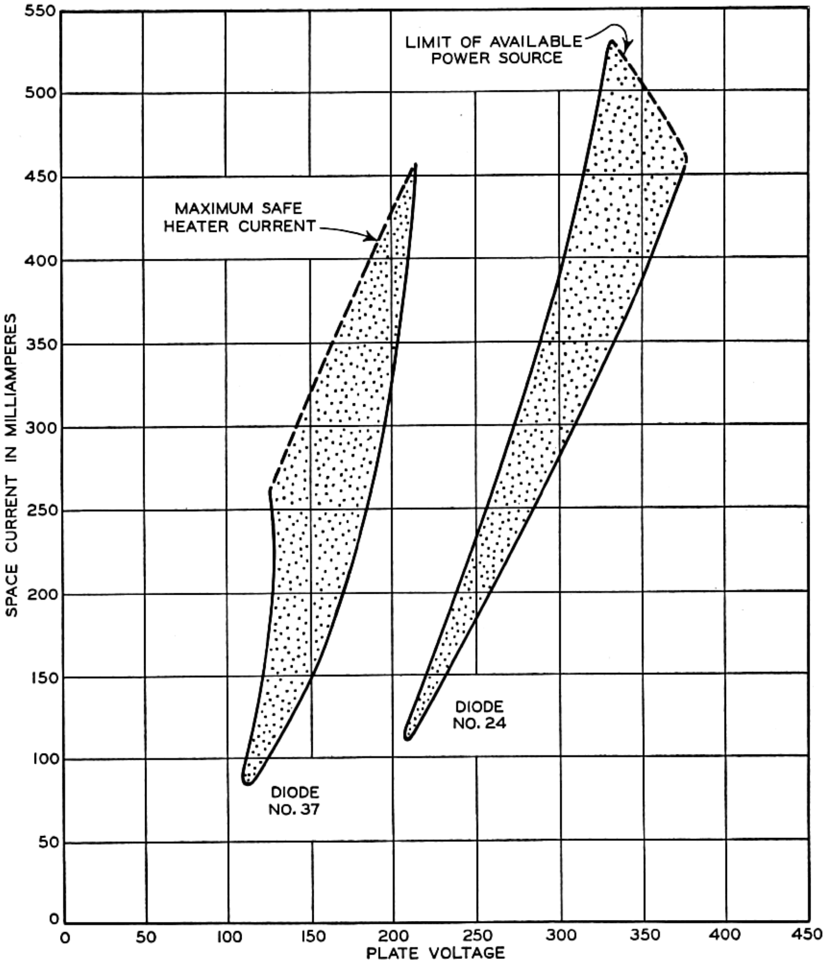

Fig. 5—"No Load" oscillation domain of diodes no. 24 and no. 37.

The power absorbing and measuring element shown on Fig. 6 represents a new and useful way of measuring power at the very high frequencies involved in this investigation. It makes use of the high negative temperature coefficient of resistance of boron. In the middle of a wire extending across a diameter of the wave guide, parallel to the lines of electric force in an H_1 wave, there is placed a small crystal of boron. Connection to the crystal is made by fine platinum wires, melted into two small globules on opposite sides of the crystal.* By virtue of its small size and the fine leads connected to it, small amounts of power dissipated in the resistance of the crystal will raise its temperature materially, with a consequent large change in its resistance. With a stopping condenser, an ohmmeter connected as shown in Fig. 6 serves to indicate the resistance of the crystal when absorbing high-frequency power, and calibration curves showing resistance as a function of power absorption can be obtained with direct current.

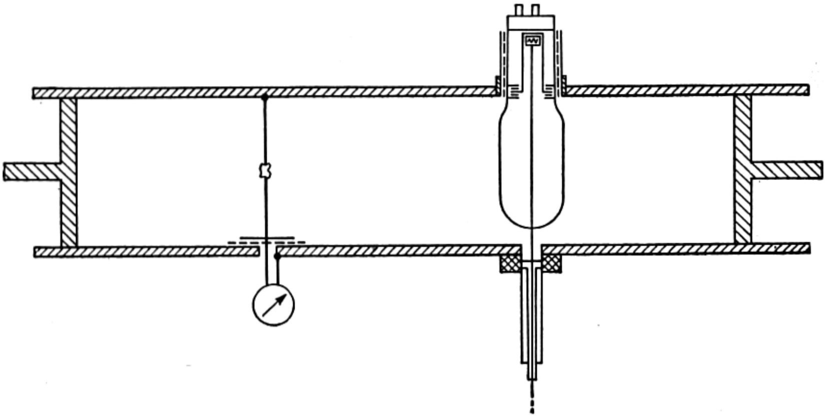

Fig. 6—Apparatus arrangement for "Loaded" test of diode no. 24.

Power output and efficiency data obtained during these measurements are shown on Fig. 7. Power outputs of a few tenths of a watt at efficiencies ranging from one to two tenths of a per cent are obtainable.

In consideration of the wave-length of the oscillations generated by these two diodes, it will be recalled that they were designed nominally for a wave-length of about 10 centimeters. For diode No. 24 the wave-length was close to 10.6 cm. (2830 mc.) and for diode no. 37 it was somewhat higher, about 11.55 cm. (2600 mc.). This difference is of the order to be expected from the difference in the dimensions of the two tubes.

While the wave-length should be fixed largely by the dimensions of the coaxial resonant circuit built into the diode, it is to be expected that it will be affected to a small extent by the applied voltage and by

* These were developed by Mr. G. L. Pearson of the Bell Telephone Laboratories.

the position of the piston closing one or both of the ends of the wave guide. In the case of diode No. 37 the wave-length was found to vary over a range between 11.50 and 11.65 cm. with plate voltage and over a range between 11.52 and 11.56 cm. with piston position.

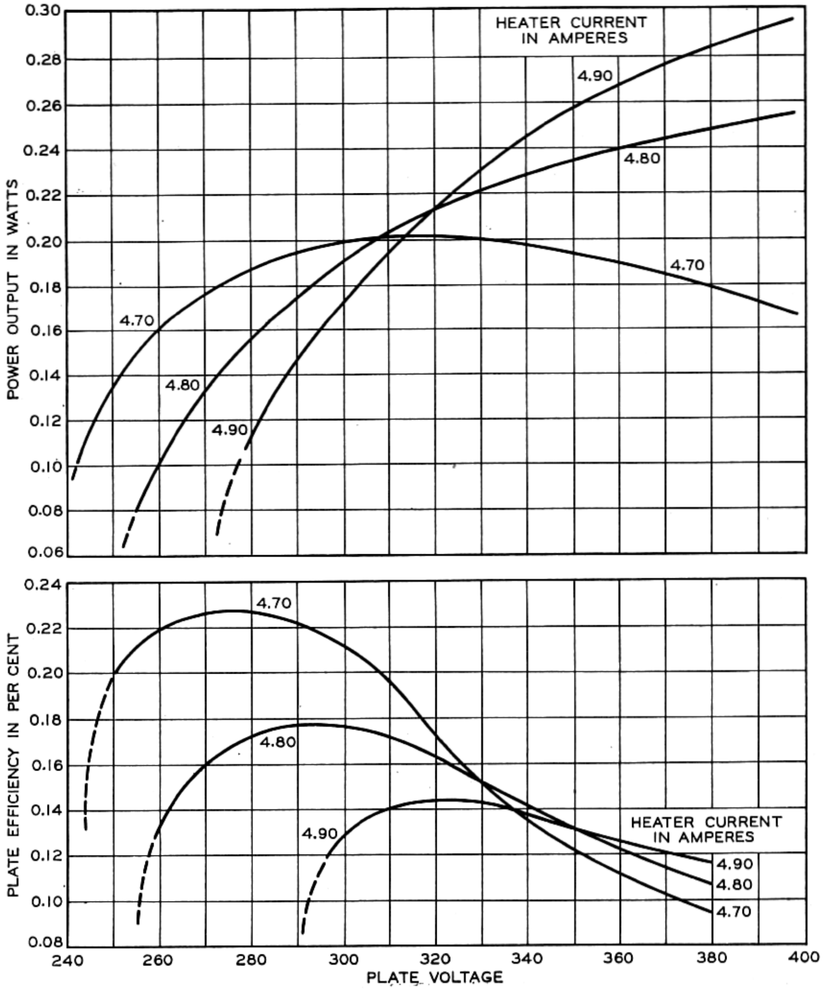

Fig. 7—Power output and plate efficiency for diode no. 24.

In conclusion, the writers wish to mention the work done by Mr. C. A. Bieling of the Bell Telephone Laboratories in working out suitable mechanical design features and in the actual assembly and processing of tubes which were built and tested.

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