Contemporary Advances in Physics, XXXII Particles of the Cosmic Rays

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Even after fifteen years of intensive research following on two decades of more desultory study, the cosmic rays are still a store of new and remarkable data. The question of their ultimate origin, though by no means extinct, has been set aside by many physicists in favor of a fuller inquiry into their qualities. The distinctive mark of the cosmic-ray particles is the immensity of their energies; for, great by all previous standards as are the energy-values which physicists now can impart in their laboratories, those manifest in the cosmic rays are greater by factors not of thousands merely. but often of millions. To this remote and exalted energy-range belong the penetrating particles capable of cleaving through a metre of lead, and the wonderful and beautiful phenomenon of cosmic-ray showers. It is not to be wondered at that with energies so high, particles so familiar as electrons and photons should be invested with unfamiliar powers. So evidently they are; but some of the charged corpuscles of the cosmic rays have properties such that their strangeness cannot be ascribed to high energy alone, but apparently must be based upon some fundamental difference (perhaps a difference of mass) from all the particles thus far identified.

WHEN a new member is admitted to a small and jealously-restricted club supposedly already filled for all time, the event has a dramatic aspect. When a concept is formed in a nebulous way and rapidly gains precision with the passage of the years, the story is of philosophic interest. When physicists extend their knowledge into ranges of energy heretofore unsuspected, and find them inhabited by particles classifiable as electrons but in possession of powers ordinarily unknown, and also by particles which must be put in a class by themselves—when such things are available for telling, the tale has scientific value. When evidence comes in the form of pictures so striking as those which can here be shown, the science of lifeless matter has an aesthetic splendor such as rarely embellishes it. All of these features appear in the recent advances of the study of cosmic rays.

The small and exclusive club consists of the subatomic particles, long supposed to comprise only the negative electron and the proton and other positive atom-nuclei. Into it the positive electron had been forced in 1932, and the neutron in 1933; a vacant chair was

other terminal equipment. In the present state of the art, it is difficult to make practical use of transfer admittances in predicting the performance of a telegraph circuit.

The significant point is that the satisfactory transmission of the selected characters is an indication of the ability to transmit the desired telegraph signals satisfactorily. Also, the measurement of the distortion on the selected characters is particularly useful when it is desired to equalize individual circuits of varying length and makeup to secure a minimum of distortion.

The testing procedures suggested by the considerations of the foregoing have been incorporated into the testing instrumentalities discussed in the main paper. These methods have been used for several years in the adjustment and maintenance of telegraph circuits and found to be of considerable utility.

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being reserved for the negative proton, which as yet has not turned up to claim it; few if any expected the actual applicant. The concept now hardening into the definite form of this applicant is that of the "mesotron." This is a particle presumed to be equal in charge to the electron, but in mass a couple of hundreds of times as great. In so naming it I follow (C. D.) Anderson's recent proposal, though other titles such as "barytron" and "heavy electron" are already more or less firmly rooted in the literature. The quality which marks it out, when it appears with enormous energy among the cosmic rays, is an extreme and almost incredible power of penetration. This means that the so-called mesotrons are able to traverse decimetres, nay even metres of lead (or of dense matter generally). Like electrons, mesotrons may be of either sign of charge. As for the cosmic-ray particles still classified as electrons, they are marked out by their power of producing one of the most magnificent phenomena of Nature, the "shower of cosmic rays," or "shower" for short. Shower-production by the supposed electrons, penetration by the supposed mesotrons. ionization along the course of either corpuscle through air: these are the three phenomena which will furnish most of the illustrations. much of the text of this article. The story of their incorporation into the structure of physical theory will furnish the remainder.

(But negative electrons and protons, not to speak of other atomnuclei, have been identified through having their charge-to-mass ratios measured with the aid of electric and magnetic deflecting fields in elementary classical ways. Why then do I not cut this introduction short by giving the results of such a measurement upon the mesotron? The reason is, that no such measurement has yet been made. Probably one will be made ere long. Should it give something near to the result expected, the delay will not have been regrettable; for the end of the delay will mark the beginning of the time, when the story to be related in these pages will be regarded as being "of historical interest" only—which is to say, that it will then be liable to be forgotten.)

So that the reader may see at once the three phenomena which are to bulk so largely in this story, I draw his attention at once to some of the pictures which decorate this article.¹ Nearly all of them were made (of course) with the aid of the cloud-chamber or expansion-chamber of C. T. R. Wilson, that device so precious in physics and precious in so many ways.

¹They decorate it with particular clarity, thanks to the kindness of Messrs. Anderson, Auger, Brode, Corson, Fowler, Fussell, Neddermeyer, Stevenson and Street in supplying me with prints of their splendid photographs.

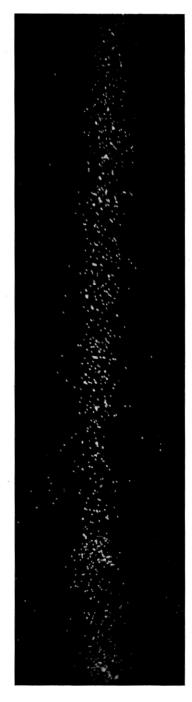


Fig. 1—Track of a cosmic-ray particle (probably an electron) in the expansion-chamber, time having been allowed for the ions to drift apart before the expansion. (Corson and Brode, University of California)

At the beginning I place, as Fig. 1, a picture of the track of a cosmicray particle believed to be an electron. Anyone who has ever studied the pictures of cloud-chamber tracks will at once be impressed by seeing how distinctly the droplets stand apart. This separation was achieved by letting half a second elapse from the instant when the electron shot through, to the instant when by expansion the gas of the chamber grew suddenly cool and the water-vapor suspended in the gas condensed itself as dewdrops on the ions. These ions, formed by the passage of the electron, had been diffusing through the gas during the half-second intervening, and the diffusion-process had served in the main to carry them apart (though there must also have been cases of ions approaching and possibly even combining with each other). The counting of these droplets is germane to the question as to whether the traversing particle was or was not an electron. This question, however, we leave till later, and turn to photographs in which the droplets of the tracks lie close together and are uncountable, because the expansion took place before there had been time for much diffusion. Tracks so formed have the advantage of sharpness over what they lose in detail.

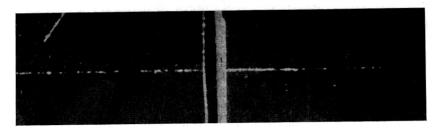


Fig. 2—Track of a particle, presumably a mesotron, traversing a metal plate without sensible deflection. (Auger; Universite dé Paris)

Figure 2 presents the track of a particle which traversed a plate of lead as it shot across the chamber. In passing through the lead, it underwent no sensible deflection; no other particle sprang from the lead; and there is nothing in the aspect of the track which differs on the two sides of the metal. It would be more impressive yet to present a similar picture for a particle traversing ten or fifty centimetres of lead, but here the practical limitations on the size of a Wilson chamber defeat the physicist, or at any rate no one has overcome them yet. Ehrenfest has lately circumvented them by the laborious scheme of setting up two Wilson chambers, one above the other, with as much as 9 cm. of lead or gold between them. However, the passage of single

charged particles through thicknesses as great or even much greater is amply attested by the scheme of apparatus sketched in Fig. 3, even without the cloud-chamber there indicated by "Ch."

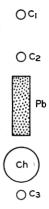


Fig. 3—Scheme of apparatus for observing very penetrative particles with counters and cloud-chamber.

In this sketch of Fig. 3, the objects C_1 and C_2 and C_3 are Geiger-Müller counters: that is to say, gas-filled discharge-tubes of a very special design, the two electrodes of each being an axial wire and a coaxial cylinder, and the electrode-size, voltage, and gas-content being very carefully adjusted. These long large cylinders, usually called simply "counters" without the prefixed names, are familiar sights in almost every laboratory where cosmic rays are studied. If a charged flying corpuscle penetrates such a tube, a momentary discharge takes place in the gas. If such discharges spring up simultaneously in all the three tubes of such a system as Fig. 3 exhibits, the event is recorded by a mechanism. ("Simultaneously" is of course a word which requires detailed exegesis; it meant at first that in all tubes discharges began within 0.01 second of each other, but this interval has been pushed down to .0001 second and lower.)

These events, the "threefold coincidences," do actually occur. Of course, since in each of the tubes a discharge occurs now and then by itself, some of the coincidences must be the result of chance; but the probable number of these meaningless ones can easily be estimated from the frequency and the duration of the individual discharges, and in the best experiments they are a small minority. For the great majority, the simplest of explanations is to attribute each of them to a single vertically-flying particle cutting through all of the counters in succession. Yet there are other thinkable causes, and confirmation

of this simplest idea is needed. It was supplied when the cloudchamber, "Ch" in the figure, was inserted. The chamber was compelled by mechanism to expand, always when and only when a threefold coincidence happened; and at the great majority of its expansions it showed a vertical track. Figure 3 exhibits the arrangement of Street, Woodward and Stevenson at Harvard, who found the track of the traversing particle at 202 expansions out of 219. Auger and Ehrenfest at Paris had already set up four counters and a cloudchamber and a block of lead in a vertical line, and found the track of the single traversing particle at fifty-five expansions out of sixty-nine. Another test is made by displacing one of the counters out of line with the others, whereupon it is found that the coincidences fall off in number sharply. And now to come to the point which most concerns us: there were 45 cm of lead between the counters in the experiment of Fig. 3, and 50 cm in the experiment by Auger and Ehrenfest, and no fewer than 101 cm in an early experiment of Rossi's with counters though without the chamber! Such is the power of penetration of some of the charged corpuscles of the cosmic rays.

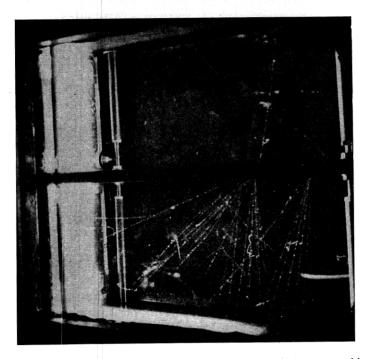


Fig. 4—Three showers, two evoked by charged particles and one presumably by a photon. (Street and Stevenson, Harvard University)

The reader has now been introduced to charged particles which bore through quantities of lead, apparently without doing or suffering anything. Next he is to be introduced to particles which begin to do something startling, when they have scarcely more than entered into a thin metal plate. This is vividly shown to him in Fig. 4, in which—after he can detach his eyes from the pretty sight beneath the transverse leaden plate—he will see that two of the "showers" beneath spring from the places where the metal was entered by two charged particles coming from above. These are accordingly called "shower-producing particles."

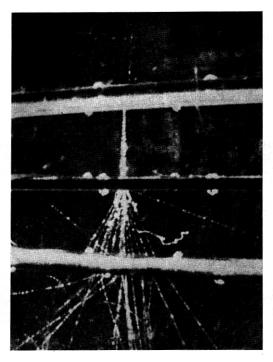


Fig. 5—Shower begun by a charged particle impinging on a 6.3-mm lead plate, and multiplied as it passes through a second such plate; in the third plate, 0.7 mm thick, only deflections occur. (Fussell, Harvard University)

Figures 5 and 6 and 7 show examples of showers even more gorgeous—regular cloudbursts, to continue with the metaphor (and indeed the term "burst" is often used as a synonym for "very large shower"). Of these, the special value of Fig. 6 is that the tracks that start in the gas itself bear witness to corpuscles of light—photons—included in the shower; for these are the tracks of electrons ejected by photons

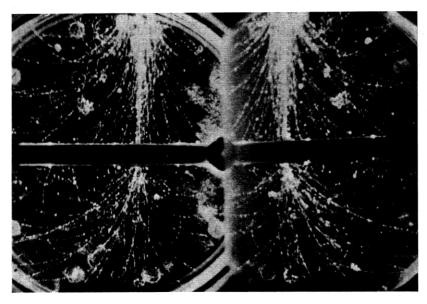


Fig. 6—Shower comprising photons attested by the (curled) tracks of slow electrons released in the gas. (Anderson and Neddermeyer, California Institute of Technology)

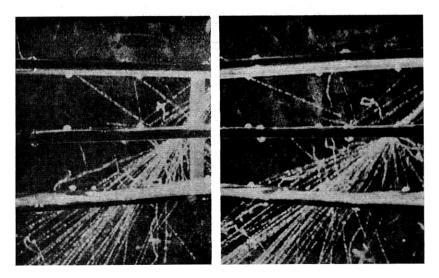


Fig. 7—Another example of a shower undergoing multiplication as it passes through metal plates. (Fussell)

from atoms of the gas. (The agent which bends them into curlicues is, of course, a magnetic field applied to the whole of the Wilson chamber.) Showers, then, comprise photons as well as charged particles. The

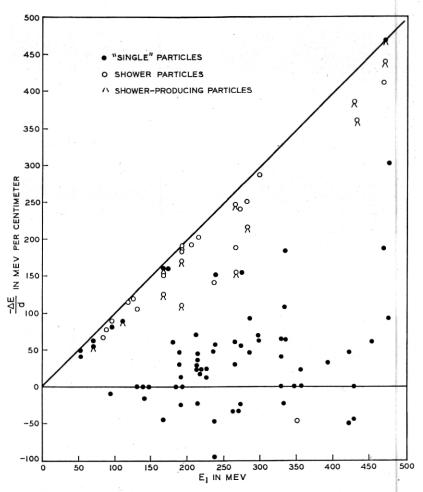


Fig. 8—Energy-losses per unit length of path (in Mev/cm) suffered by 94 cosmic-ray particles in traveling through platinum. (Anderson and Neddermeyer)

special value of Figs. 5 and 7 is, that they show the progressive aggrandizement of showers as these pass onward through dense matter. This is called "the multiplication of showers." Shower particles are themselves capable of being shower-producing particles. One could not tell from these figures whether the multiplication is due to the charged particles or the photons, to either singly or to both. Here again the

reader may consult Fig. 4, in order to notice that one of the three showers there depicted sprang from a place in the plate to which no charged particle came. This suggests that a photon may cause a shower, and that the multiplication of a shower already begun is due to the action of its charged particles and of its photons both.

Two classes of charged particles begin to take shape: the penetrating ones on the one hand, the shower particles and the shower-producing particles classified together on the other. To bring out another aspect of the distinction, I now turn to the data underlying Fig. 8.

These data are derived from cloud-chamber photographs such as Fig. 9 exemplifies. If the track of a charged particle is sensibly curved in such a magnetic field as it is possible to apply to a Wilson chamber, it may be possible to infer the momentum and the energy of the particle. I digress to give the formulae, so as to make it clear just what can be deduced from what amount of knowledge. The elementary procedure consists in pointing out that the charged body describes a circle in the plane perpendicular to the magnetic field, and that consequently the force exerted on it by the field is to be equated to the product of its mass by its centrifugal acceleration. Putting ne for the charge (in electrostatic units) of the corpuscle, m for its mass, v for its speed and p for the magnitude of its momentum in the plane normal to the field, p for the radius of the circle and p for the field-strength, and writing down the two members of the equation, one finds:

$$Hnev/c = mv^2/\rho, \tag{1}$$

$$p = (ne/c)H\rho. (2)$$

These equations remain valid when (as usually is the case with cosmicray electrons) the speed is so great that relativistic mechanics must be used instead of ordinary. At such high speeds equation (2) retains its aspect. Equation (1) may also be left unaltered, but one must be sure to remember that m is a certain function of v:

$$m = m_0 \sqrt{1 - v^2/c^2}, (3)$$

 m_0 being known as the "rest-mass" of the body.

² Curvatures of tracks being so very important in this field of research, it is necessary to examine with the greatest of care into all of the causes (apart from magnetic field) which may produce or affect them. Notable among these are currents in the gas, which are especially obnoxious if there is a metal plate in the chamber. Indeed it seems strange that the currents should not be more hampering than they are, considering the expansions which occur. Sometimes people observe that in the absence of magnetic field, there is a slight curvature of the tracks; then in the presence of magnetic field, they deduct this amount from the curvatures observed. The papers of Anderson and Blackett abound in information on these delicate questions.

Equation (2) does not involve the mass at all. In the usual loose phrasing, $H\rho$ gives the momentum of the particle provided that its charge is known. The like cannot be said for the energy, which is given by $H\rho$ only if both the charge and the rest-mass are known. For particles of the cosmic rays it is best to disregard the ordinary expression for kinetic energy $(\frac{1}{2}mv^2)$ and adopt for good the relativistic expression mc^2 , to wit, $m_0c^2/\sqrt{1-v^2/c^2}$. Of this the portion m_0c^2 is not kinetic energy: it is the "rest-energy" associated with the "rest-mass" m_0 , inseparable from the particle so long as this exists; it amounts to about half-a-million electron-volts or 0.5 Mev for the electron, to about 1000 Mev for the proton. The remainder may be called kinetic energy. For nearly all of the electrons and most of the other cosmic-ray particles, this remainder is by far the greater part. The dependence of the kinetic energy upon $H\rho$ is exhibited, for electrons and for protons, by Fig. 13 (page 213). One sees that for different

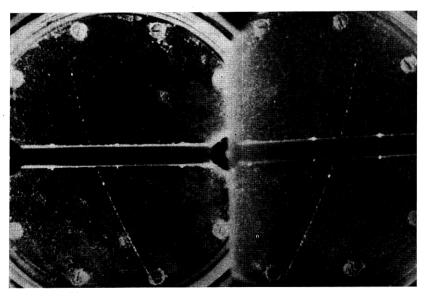


Fig. 9—Track exhibiting measurable and unequal curvatures on the two sides of a metal plate, thus indicating changes of energy and momentum suffered in the traversal. (Anderson)

masses a given $H\rho$ -value leads to different energy-values, but also that the error due to an incorrect estimate of rest-mass becomes proportionately smaller as the $H\rho$ -value increases. Yet the possibility of the error is always there, if the mass of the particle is not certainly known; and it affects many published "energy-values"

based on the presumption—often admitted in the context to be more than doubtful—that the particles to which they refer are electrons. The danger might be mitigated by describing these as "quasi-energy-values" expressed in "quasi-Mev."—For actual electrons with momenta as great as those figuring in the cosmic rays, the energy-value in electron-volts is practically equal to 300 times the $H\rho$ -value expressed in gauss-centimetres.

Many a cosmic-ray particle suffers no deflection that can be detected in its entire course across a Wilson chamber (diameter, 15 cm. or even more) in a magnetic field as strong as can be applied over so great a volume (field-strength, 20,000 gauss or thereabouts). One might well be tempted to think such a particle chargeless, for if this were the case, the field would have no grasp at all upon it; but if it were chargeless it could not ionize the molecules of the gas and therefore could not form the chain of ions on which the droplets are founded. In some of the finest of the experiments (those in Pasadena and those in Paris) a detectable curvature of the track would be shown if this were made by an electron of energy so enormous as $2 \cdot 10^{10}$ electronvolts (20,000 Mev!). The uncurved tracks accordingly speak of electrons of energies greater than 20,000 Mev, if these particles are electrons; and the inference is not much less drastic, if they are more massive than an electron.

We, however, are more interested, for the present, in the tracks which are sensibly curved; and most of all, in the tracks which are intersected by a metal plate and which show a curvature on one side of the plate and a larger curvature on the other (Figure 9). From the two ρ -values one can deduce the momentum-loss Δp and the energy-loss ΔE suffered by the particle in passing through the plate. (Yet I emphasize again that Δp is computable only if the charge is correctly guessed, and ΔE only if the rest-mass is correctly guessed in addition to the charge.) With this ambition Anderson inserted such plates for the first time into a Wilson chamber, in 1931. The idea had a wonderful and unforeseen result, some years ago recounted in these pages. Notice that above I spoke of the momentum-loss and the energy-loss suffered by a particle in going through a plate. In so doing I was making the assumption that it is a loss and not a gain which happens. If this highly plausible assumption is correct, then the sense in which the particle is traveling its path is knowable; it is from the side of the plate on which the curvature is less, to the side on which the curvature is greater. If the sense of the motion is knowable, so also the sign of the charge of the particle is knowable. being positive or negative according as the track is bent with its concavity toward the left or toward the right of an observer looking into the chamber from the north-seeking pole of his magnet. Without the plate, neither sense nor sign would be knowable except in the rarest of cases.³ Anderson in August 1932 found on one of his photographs the track of a particle which by this criterion was positive, and which by the density of droplets along its track (we take up this topic later) he identified as an electron. He thus became the discoverer of the positive electron.

Concentrating on the measuring of ΔE after the excitement of the positive electron had subsided, Anderson presently found that its values are very fluctuating. Thus in 1934 he published the details of nine traversals, made by particles assumed to be electrons, through thicknesses of lead from 7 to 15 mm. (Even with a single metal plate the effective thickness varies, since corpuscles traverse the plate with varying degrees of obliqueness.) These were by no means identical in initial energy, this ranging from 38 to 240 Mey; nevertheless one might have expected the energy-loss per unit length of path in lead to be about the same for all, and yet the nine values thereof were scattered all the way from 18 to 120 Mev/cm! Such fluctuations suggest that the energy is lost in great amounts at a few events, and not in driblets at many. They did not deter Anderson and Neddermeyer from making such measurements on hundreds of later particles, classifying the particles into groups according to their energy-values, and averaging the energy-losses within each group. What then was found has a bearing upon the problem; but we pass over it for the time being, and consider in Fig. 8 the record of ninety-four particles which, during a later experiment, passed through a plate of platinum one centimetre thick.1

Plotted horizontally are the energy-values of the particles while above the plate, vertically the energy-changes divided by the lengths of path in the platinum. The axis of abscissæ is the locus of energy-losses imperceptibly small; the line slanting at 45° is the locus of energy-losses which are total, the particles shown on this line having been stopped by the plate. The fact that some of the representative points lie below the horizontal axis means only that for every particle the observers subtracted its energy below the plate from its energy above, irrespective of its direction of motion. Suppose that these

⁸ One might be misled by the adjective "cosmic" into believing that all cosmicray particles come from above, their sense of motion making an angle of less than 90° with the downward-pointing vertical. Many, however, including Anderson's first positive electron, have been found by this criterion to be moving upward (i.e. at more than 90° to the downward-pointing vertical). The showers of Figs. 6 and 7 show that this is not a forced interpretation. ¹ I am indebted to Dr. Anderson for a plate exhibiting data thus far unpublished.

subjacent points correspond to upward-going corpuscles, and transfer them across the horizontal axis. Then, the sprinkling of points extends all the way from axis to slanting line; and this is the sign of fluctuations such as Anderson from the start had observed. Notice however that the representative points are of four aspects: solid dots and hollow circles, with or without downward-pointing barbs. dots refer to tracks which were seen in the chamber singly; the circles, to particles which "entered the chamber accompanied by other particles." The lonely particles are prevailingly able to pass through matter without suffering energy-losses nearly so great as those which the others incur! Thus by itself and without any theory, Fig. 8 establishes a distinction between the singly-appearing corpuscles on the one hand, and those which appear in company on the other. Moreover the barbs are often attached to the hollow circles, bearing out the inference from Figs. 5 and 7 that shower particles are likely to be shower-producing particles; but rarely are they attached to solid dots, never to those which lie far off from the slanting line.

(This seems the best place for mention of the similar work now being done in England by Blackett and (J. G.) Wilson, in France by Ehrenfest. The Englishmen have set plates of gold, lead, copper and aluminium, of various thicknesses from 3.3 mm to 2 cm, into the middle of an expansion-chamber in Anderson's fashion; Ehrenfest, using a pair of cloud-chambers one over the other, was able to put between them a block of gold no less than 9 cm thick! Their way of reducing their data for plotting is not the same as that employed at Pasadena, and their diagrams therefore look very different 1 from Fig. 8. Their energy-range runs much further upward, as far as 5000 Mev, and the great majority of the particles which they plot lie beyond the limit of Fig. 8. Many of Ehrenfest's particles got through the great thickness of gold without losing anywhere nearly the whole of their energy, and are therefore to be classed as much more penetrating than electrons should be. So did nearly all of the particles of energy greater than 250 Mev observed in England, but there were a few of these which lost most of their energy in 0.33 cm of lead, and of these few about half seemed to belong to showers.

¹ For the benefit of those who may consult the original papers, I give the difference. Let E_1 and E_2 stand for the (quasi) energy-values of a particle before and after passing through a thickness d of metal; ΔE for (E_1-E_2) ; x for $\frac{1}{2}(E_1+E_2)$. What is plotted by Anderson and Neddermeyer (Figure 8) is $\Delta E/d$ as ordinate and E_1 as abscissa. Blackett (in all his papers but the earliest), Wilson and Ehrenfest begin by subtracting from ΔE a quantity sd which is supposed to be the amount of energy spent by the particle in detaching electrons from atoms while traversing the metal (Blackett assigns the value 15 Mev/cm to s in lead, Ehrenfest takes 28 for gold); they then plot ($\Delta E - sd$)/xd as ordinate and x as abscissa. Their ordinate (denoted by them as R) is then more nearly ready for comparison with theory.

At energy-values below 200 Mev Blackett finds almost no penetrating particles, a singular contrast with the Pasadena observations; he suspects that the penetrating particles become ordinary electrons when they are slowed down into this energy-range. I mention also the measurements made on some twenty penetrating corpuscles by Leprince-Ringuet and Crussard, leading to the exceptional conclusion that positives suffer smaller energy-losses than negatives.)

But granting that there are two sorts of particle with a right to different names: has either a right to the name "electron"? To settle this question, and for several other reasons, it is time to call upon theory.

It is now some thirty years since there entered into physics a German word, Bremsstrahlung, which can be translated literally into English as "braking radiation," and would no doubt be so translated if "braking" did not sound like another English word of entirely different meaning. This is chiefly observed emerging from X-ray tubes, being emitted from their metallic targets when these are struck by the stream of bombarding electrons. It consists of photons or corpuscles of light, each containing at least a part of the kinetic energy of one of the incident electrons. The distribution-in-energy of the photons makes it clear that the electrons frequently lose large fractions of their initial energy en bloc, throwing it off in individual parcels which are these photons (indeed it sometimes happens that the entire kinetic energy of an incident electron is shed in the form of a single corpuscle of light). This radiation forms the so-called "continuous X-ray spectrum" or "X-ray continuum" emerging from targets of X-ray tubes. With the spectrum-lines which are sometimes seen superposed on this continuum we have nothing here to do.

By the classical theory of thirty years ago this continuous spectrum is attributed to the slowing-down of the electrons as they penetrate into the metal, whence the name *Bremsstrahlung*. By the quantal theory of today it is still ascribed to the slowing-down, which must now be conceived as taking place in instantaneous jerks, occurring probably in the close vicinity of atom nuclei. At each of the jerks, the electron-speed is suddenly reduced and the kinetic energy goes forth in the form of light. The later theory in its quantitative form gives a competent account of the continuous X-ray spectrum as it springs from the tubes of the laboratory, with their bombarding electron-streams energized by voltages of a few tens or hundreds of thousands. For a long time nobody seemingly troubled to extend it to voltages of the order of thousands of millions; a futile extension indeed this would have been, so far as X-ray tubes are concerned.

When finally the extension was made by people interested in the cosmic rays, it turned out that according to the quantal theory the liability of electrons to these "radiative energy-losses" goes up so greatly with increasing speed, that electrons of even the cosmic-ray energies should not be able to bore their way through as much as five centimetres of lead!

After the meaning of this inference sank in, there ensued a period lasting for months (in 1935 and 1936) in which several eminent theorists were willing to concede that Nature must have set a limit to the scope of quantal theory. It was beginning to be believed that somewhere between the energy-range attainable in the laboratory and the energy-range manifest in the cosmic rays, there is a critical energyvalue beyond which the electron escapes from the sway of the quantal laws, and is exempted from losing its energy by the process of Bremsstrahlung. This belief was an artifice for permitting the penetrative particles of the cosmic rays to be called by the name of electron. It might have remained a credible artifice, if the penetrative particles had been the only ones-if, that is to say, there had never been any evidence for the existence of particles among the cosmic rays having the properties required of electrons by the quantal theory. Such a situation may have seemed to exist at the time when the belief was dominant. It exists no longer, as the description of Fig. 8 has just suggested; but before considering further the data, I must introduce something more of what the theory has to say.

Since 1934 it has been known that a photon of energy greater than about one million electron-volts is capable, when in the vicinity of an atom-nucleus, of converting itself into a pair of electrons of opposite sign. About one million electron-volts-1.02 Mev, to be somewhat more precise-becomes "rest-energy" of the twin electrons, being incorporated with their rest-masses; the remainder ($h\nu - 1.02$, if by hv we denote the photon-energy in Mev) becomes kinetic energy of The process may be produced at command and the electrons. exhibited to the eye, by projecting the photons known as gamma-rays against metal targets contained in expansion-chambers. The gammarays originally used for this purpose proceeded from natural radioactive substances; mostly they were those emitted by a certain substance (thorium C'') with a photon-energy of 2.62 Mev. Nowadays gamma-rays of energy several times as great can be produced by effecting certain transmutations, in the course of which (or afterward) they emerge from the new-born nuclei. Figure 10 shows an admirable example of an electron-pair formed out of such a photon. Moreover, the converse process is well-known: positive electrons falling against

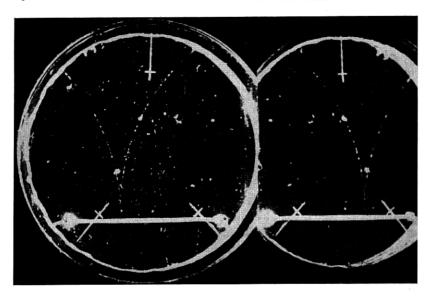


Fig. 10—An electron-pair born from a photon. (W. A. Fowler, California Institute of Technology)

a plate of dense matter bring about the emission of photons of energy 0.51 Mev, and these are just what are to be expected if the positive electrons (after being slowed down) unite with some of the innumerable negative electrons already in the plate and produce, at every such union, a pair of equal photons.⁴ Much too abundant to be here described is the evidence for the ability of electron-pairs to pass into light and light to pass into electron-pairs, making it permissible to imagine a continual alternation of energy between these two so sharply contrasted forms.

Formation of the photons of *Bremsstrahlung* by electrons of enormous energy, and formation of electron-pairs out of such photons: these reciprocal processes engaged the attention of several theorists (Bethe, Heitler, Sauter, Weiszaecker, Oppenheimer) in the years 1933 and 1934. The problem was, to evaluate by quantal theory the chance that electron or photon would spend its energy in producing photon or electron-pair, while traversing given thickness of given element.

⁴ Evidently this is not quite the converse of the process previously described, which if reversed would consist in the merger of a positive and a negative electron with the formation of a single photon bearing away all of their energy. Some evidence exists for the occurrence of this process. There is no sign of the fourth conceivable process (the meeting and merger of two photons to form two electrons) which must obviously be very rare in practice owing to the feeble concentration of photons in actual beams of gamma-rays. Nevertheless this last is the process first predicted by the theorist Dirac.

Approximations had to be made in the calculation, as nearly always in quantal problems; but they are supposed not to affect the rightness of the main result. To quote Oppenheimer's description of this result: "a beam of high-energy electrons should have a good part of its energy converted into photons in a centimetre of lead; in an equal distance these photons will be largely reconverted into pairs."

Such was the result from which, in 1935, it was inferred that quantal theory must be wrong because it was predicting something which could not be found in Nature; and from which, in 1936 and thereafter, it was concluded that quantal theory not only was correct but had made a splendid triumph, in explaining the phenomena of showers! It is not altogether clear why the later conclusion was not drawn at the start; perhaps the reason is, that as lately as the summer of 1936 fine photographs of showers were still rather rare, while such pictures as Figs. 5 and 7 with their examples of self-augmenting showers had not as yet been made. On the other hand it would be premature to say and misleading to imply that the process which the theory describes is in exact and quantitative accord with the observations There are at any rate good grounds for hoping that as the mathematics of the theory is more fully worked out and the art of the experiments refined, the agreement will grow better and better. The most that seems safe to say is, that now we have a general scheme for the interpretation of showers of a certain type, and a very hopeful prospect that this general scheme will be converted into a detailed and quantitative explanation as the mathematics of the theory on the one hand, the aptness and precision of the observations on the other hand are gradually improved.

By inserting the words "of a certain type" in the foregoing sentence, I leave open the possibility that showers may be classified into more than one type, and all of these but one be ascribed to other processes. This is no mere possibility but already almost a certainty. Certain showers which include "heavy tracks" due to protons or still more massive particles are ascribed to nuclear explosions provoked by cosmic rays. If a shower fails to undergo the "multiplication" illustrated in Figs. 5 and 7, it is taken as belonging to this other type. Exception made for such cases, it is strongly plausible to say that shower particles and shower-producing particles are electrons; that accordingly high-energy electrons exist among the cosmic rays, behaving as the quantal theory says that they should; and that consequently the other particles, setting themselves apart from electrons by their penetrative power and their failure to make showers, are of another sort.

Ability to penetrate matter, inability 5 to make showers: these are the complementary aspects of the property which distinguishes this other type of particle, the mesotron. If one wishes to contrive a particle having this property and differing otherwise as little as possible from the electron, how must it be done? The electron has the qualities of charge and mass; also those of spin and magnetic moment, but these are considered (perhaps wrongly) to be little or not at all concerned with shower-production. If we imagine the mass to be increased while the charge remains the same, the liability to Bremsstrahlung will diminish; for Bremsstrahlung occurs when sudden sharp deflections or decelerations occur, and these are less sharp and sudden the more massive the particle is. Now Bremsstrahlung is the prelude to the entire manifold process of the forming of a shower, and hence a mere increase in the mass of the hypothetical particle leads in the desired direction. The theory indicates that a particle with the electronic charge and a few dozen times the electronic mass will be penetrating enough. We do not need, however, to be contented with such vague intimations, for there is yet another phenomenon in respect of which the mesotron differs from the electron, and from this the mass can be deduced more sharply.

So far, we have been considering the passages of particles through solids. There, the paths are concealed, the adventures of the particles can only be inferred—from the difference between energy before and energy after traversal, or from the photons and the secondary electrons which are driven out of the solid. Now we are to consider the passages of charged particles through the gas of the Wilson chamber, which, unlike the scriptural way of the eagle through the air, are preserved for our inspection by the droplets. Figure 1 has shown to us a track in which the number of droplets in unit length of path can rather readily be counted. What does this number signify? And is it truly an indication of the mass of the traveling particle, as I hinted on an early page?

The latter question might perhaps be sufficiently answered without reference to the former; but for completeness, and for the sake of its own interest, the former ought to be treated more fully than it was in that brief earlier mention. In the voyage recorded in Fig. 1, nothing so drastic happened to the traversing particle as would have been the losing of a large part of its energy in the form of a photon of *Bremsstrahlung*. It lost its energy in driblets, spent in detaching electrons from molecules and giving them a small extra bonus of kinetic energy

⁵ It is better to say "relative inability" since occasional showers are attributed to mesotrons, which perhaps operate by making a violent impact on an electron and so giving it the energy needful for starting the process.

with which to go wandering around in the gas. They had not speed enough to wander far, even in the half-a-second afforded them before the condensation. Probably they had already adhered to molecules before the condensing water immobilized them. One speaks of the droplets as being condensed partly on negative, partly on positive ions; the last-named are the molecules from which the electrons were reft. (If, during the half-a-second, an electric field of suitable strength is applied, the ions of the two signs drift in opposite ways, and when the water-vapor comes down there are seen two parallel trails of droplets with an empty space between.)

The simplest idea is that the traversing particle tears off one electron from each of many molecules through or near which it passes, and that half of the droplets are formed on these electrons and the other half upon the molecules bereft. This is too simple to be true. It is likely that sometimes the particle removes two electrons or more from a single molecule, so that there well may be more negative ions than positive. Much more serious is the certain fact that often when an electron is thus released by the direct action of the traversing particle, it shoots away with speed and energy enough to enable it to release one or several more from neighboring molecules. Now and then one comes on a cloud-chamber photograph in which there appears a track with branches (Fig. 11); each of these is the trail of an electron which



Fig. 11—Tracks of a charged particle bristling with short branching tracks, made by electrons ejected from atoms with energy sufficient to ionize. (Auger)

has received a truly abnormal and extraordinary amount of energy. Much commoner, in fact universal, is the "beaded" appearance of such trails as appear in most of the pictures of this article: it is presumed that each of the beads is an unresolved cluster of droplets formed on a cluster of ions, all but one pair of them made in the indicated way. Occasionally one sees a picture in which the interval allowed for diffusion has been so happily chosen that the droplets in the clusters are far enough apart for counting, and yet consecutive clusters do not overlap. In making Fig. 1 the interval allowed was a little too long, and yet perhaps it is possible to think that the ions are denser in some parts of the trail than in others, as though they had been formed in clusters which have broadened almost but not quite to the point of losing their identity.

It is therefore necessary to distinguish, in mind if not in fact, between the "primary ionization" consisting of the electrons and the molecules torn apart from each other by the direct immediate action of the traversing particle, and the "entire ionization" (sometimes called "probable ionization") consisting of these together with all the ions formed by the directly-ejected electrons. Under ideal conditions it is presumed that the measure of the former would be the total number of droplet-clusters, the measure of the latter would be the total number of droplets, in unit length of path. Not many physicists have tried to evaluate both of these numbers. Of those who have, the data have been scanty, but the consensus of opinion is that the latter is about or not quite twice as great as the former. It is, however likely that the value of the ratio of the two is not important when one wants only to distinguish between electron and mesotron, as we shall presently see.

The problem of the primary ionization is one of the major tasks of theoretical physics. Classical and quantal theorists alike have spent great labor on the question: given a charged particle of specified charge and mass and speed traversing air (or any other gas), how many electrons will it set free from the molecules in unit length of path? At this point I will give only one of the results-or rather, something which is not a result at all, but a part of the assumptions. It is assumed that as the traversing charged particle flies along through or close to a molecule, it operates upon the electrons thereof by virtue of the ordinary electric forces between its charge and the charges of the electrons. It follows, then, that whatever expression finally may be derived for the primary ionization must depend only upon the charge and the speed of the traversing particle, and not upon its mass. (Mass and momentum of the particle must indeed be great enough to hold it on a sensibly straight course as it plows onward through the gas, despite its losses of energy as it detaches electrons; but this condition is always realized, with the corpuscles of the cosmic rays.)

I seem to have said that the primary ionization gives no power of distinguishing between an electron on the one hand, a particle of equal charge and different mass on the other. However, it *does* confer on us this power, for the reason that the curvature of a particle-track in a known magnetic field is a measure not of particle-speed but of

⁶ Unluckily called "secondary ionization" by some of the German theorists.

⁷ Best to observe the droplet clusters as individual entities, one would wish the expansion to occur before the ions have any time at all to diffuse. To attain this, Williams and Pickup caused the chamber to expand at moments taken at random, and trusted to luck for the appearance of cosmic-ray tracks formed at just the right instants. Luck served them with no fewer than four tracks betokening particles of a distinctive mass.

particle-momentum (equation 2). If by luck an experimenter should happen upon two tracks having the same curvature but made by particles having masses § standing to one another in the ratio (say) 100:1, the speeds would stand to one another in the ratio 1:100, and this might well entail a perceptible difference in the primary ionization. It would come to the same thing, if someone should take the data for a large number of tracks, and plot primary ionization as function of curvature: if there are really two kinds of particle differing in mass, there should be two sets of points lying along two curves, and from the ordinates of these curves at any abscissa the ratio of the masses would be derivable.

Perhaps the last sentence suggests that someone already has made this correlation, and has found that the points for all of the single or penetrating particles lie upon one curve, and all the points for showerparticles and shower-producing particles lie on another. not been done. The reason is, that many of the penetrating particles exhibit no perceptible curvature of track at all, and most of the others a very small curvature. The former are moving so fast that their momentum cannot even be estimated, except as being beyond a certain critical value. As for the latter, the speeds of even these are so great as to approach the speed of light; for a given momentum-value the speed varies only a little with the mass, and the primary ionization varies too little to serve as an index of mass. To make a profitable correlation, one must use only the particles of which the tracks are notably curved. Nearly all of these are shower-particles, which already are presumed to be electrons. To find a penetrating particle with a highly-curved track, one must find it when it is near to the end of its course and its energy wellnigh gone. Such is the principle which directed some of the recent successful searches for particles proclaiming themselves by their ionization to be more massive than

Before looking at the track of one of these particles, we ought to notice a couple of questions concerning ionization. One of them is: is the distinction between primary and entire ionization—or rather, our lack of perfect ability to make it in practice—likely to lead to trouble? Many observers are far from clear in reporting whether what they observe is more like the one or more like the other; but it seems probable that the second like the first is dependent only upon the speed and the charge of the traversing particle, not on the mass thereof; and this diminishes the dangers from confusing the two. The question is implicated with the second: to what extent do experi-

⁸ Allowance being made for the relativistic dependence of mass on speed.

ment and theory aid us in identifying the shower-particles with the electrons? As to experiment, there exist the records of a few studies made by the Wilson chamber upon particles acknowledged to be electrons, of energy-values ranging from about 2 Mev downward to some 25000 electron-volts. In respect of the trend with energy, they agree fairly well with the assertions of the quantal theory; but when one inquires whether the absolute value for the number of clusters of ions in unit length agrees with the absolute value of the quantal expression for the primary ionization at any particular energy, one is confronted with the fact that the quantal expression contains a multiplying factor which depends on intimate details of the structure of the molecule, and is not exactly known. The quantal theory, however, predicts a minimum in the curve of primary ionization vs. energy, at an energy of about 2 Mev. Such a minimum (Fig. 12) was

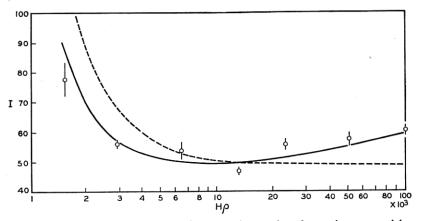


Fig. 12—Ionization-density (entire) along the tracks of cosmic-ray particles, plotted as function of $H\rho$. The continuous curve is that of a theoretical function containing a multiplying factor which has been adjusted to get the best fit to the data. (Corson and Brode)

actually found by Corson and Brode in their study of some fifty particles of the cosmic rays, and probably is to be ranked as evidence for the electronic nature of these particles quite as forcible, as would be an absolute agreement between the observed ionization and the predictions of a reliable theory.

Street and Stevenson, with a row of counters and an interposed cloud-chamber such as appeared in Fig. 3, adjusted their counters in such a way that the chamber expanded only when the counters above the chamber had simultaneous discharges and the counter below did *not*. A thousand photographs yielded to them the track

of one particle having a notable curvature and displaying an ionization six times as great as that attributable to an electron; they inferred a "mass 130," *i.e.* a rest-mass one hundred and thirty times as great as that of an electron. Neddermeyer and Anderson transposed the bottommost counter into the very centre of the cloud-chamber itself,

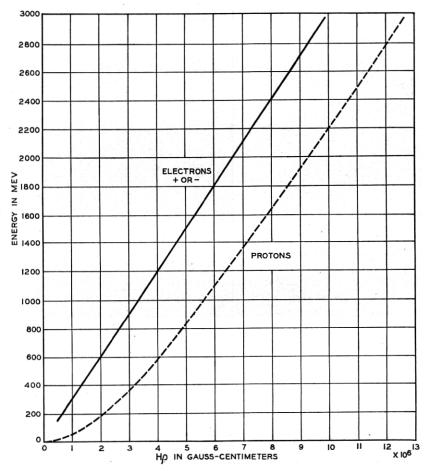


Fig. 13—Relation between energy and H_{ρ} -value for electrons (of either sign) and protons. (Anderson)

and there it appears in Fig. 14, neatly intersected by the course of a particle which above it made a track lightly curved and thinly studded with droplets, and beneath it made a track sharply curved and densely congested. Comparing ionization with curvature along the track above and the track below, they found 240 to be a satisfactory ratio

of the mass of the traversing particle to the electron-mass. Williams and Pickup, to whose technique I have already alluded (footnote 7 on page 210), observed four tracks of which three were compatible with a rest-mass of about 200, the remaining one requiring a mass-value between 430 and 800. A few more such tracks have appeared in the literature, but instead of describing them I turn for the climax to another and an exacter way in which Fig. 14 furnishes the desired value of mass.

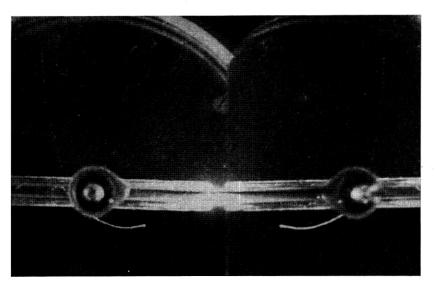


Fig. 14—Track of a mesotron slowed down by an obstacle in a Wilson chamber and finally brought to a stop in the gas of the chamber itself. (Neddermeyer and Anderson)

In Fig. 14, the track beneath the counter comes to a sudden end. One could take a sheet of coordinate-paper, and plot along the horizontal axis the curvature of the path as it emerges from the counter, and along the vertical axis the length of the path from that point of emergence onward to its end. This would give a single point of what is known as a "range-vs.-curvature relation" or a "range-vs.-momentum" relation. A second point can be found by measuring the thickness of the glass counter-wall twice traversed by the particle, converting it into an equivalent thickness of gas, adding this to the length of the path beneath the counter, and correlating the sum with the curvature of the path at the point where the particle enters the counter. Now, range-vs.-curvature relations are among the best-studied of the features of the charged particles already known—

electrons, protons, alpha-particles. These two points pertaining to the particle of Fig. 14 lie far from the curves appropriate to any of the three. An electron departing from the counter in a path of such a curvature as there is shown would have traveled 2000 times as far before reaching the end of its course! a proton, on the other hand, only one seventy-fifth as far! This at the moment is deemed the sharpest and most clear-cut evidence for the existence of a particle intermediate in mass between proton and electron, to which Anderson now assigns a mass of 220 (± 35) times the electron-mass.

It is fitting to end this article by mention of several other kinds of evidence which have bearing on the question of the mesotron; mainly they are relatively indirect, and would require much space to describe and assess. Inferences have been drawn from the number of electrons ejected with high energy from metal plates by penetrating particles traversing these: J. G. Wilson derives a mass-value greater than 100. A curious inference has been drawn from the deflections suffered by these particles in traversing metals: the magnitude of these should by theory be independent of the mass of the particle—since it does appear to be the same for penetrating particles as for electrons, it is deduced that the mesotron and the electron can differ only in mass. Inferences have been drawn from the trend of cosmic-ray intensity with elevation in the atmosphere, and from the trend of cosmic-ray intensity beneath metal screens as function of the material and thickness of these last (it was thus that Auger as early as 1934 was led to suspect the existence of two kinds of charged particle among the rays).

Inferences have also been drawn from nuclear theory. To enter adequately into this difficult field is impossible here: it must suffice to say that Yukawa conceived, as a constituent of nuclear structure, of a particle possessing the charge of an electron and a mass of about the magnitude which the mesotron appears to have, and possessing in addition the quantity of *instability*. The "Yukawa particle," that is to say, has the qualities demanded of the mesotron, and in addition is liable to emit an electron; what is left behind is then a neutral particle which could elude observation. The emission is expected to follow the law familiar in radioactivity, the durations of individual Yukawa particles being distributed according to the law of chance about a mean value. Is there evidence that the mesotron behaves in this way?

⁹ Values diverging from this by more than the estimated uncertainties have been published by other observers of other particles, and may betoken an underestimate of the uncertainty or the existence of particles of several masses. A "nomograph" for facilitating the evaluation of mass from curvature of path combined with ionization-density or range is given by Corson and Brode.

For this there is some evidence, of the following kinds. us compare (in imagination) the number (per unit time per unit area) of penetrating particles flying vertically downward and the number flying obliquely downward. The comparison can be readily made with such an apparatus as that sketched in Fig. 3, the cloud-chamber being superfluous and the lead absorber reduced to the least thickness sufficient to stop electrons; the axis is oriented first at 90° and then at various lesser angles θ to the horizontal plane. Even the whole of the atmosphere is insufficient to stop such mesotrons as the cloudchamber discloses; and yet the observations show a marked decline of the number thereof as θ decreases. But the particles which travel obliquely traverse a greater distance from the top of the atmosphere than those which come vertically down, and take a longer time in doing so; the decline of number with decrease of θ may therefore be ascribed to the perishing of the mesotrons en route to the apparatus as the route grows longer and longer. Second: Let us compare the effect of the obliquely-traversed atmosphere with that of a sheet of lead in cutting down the number of particles arriving at the apparatus. One must make a guess as to the thickness of lead which would be required to produce a falling-off of the number of particles equivalent to that observed in the atmosphere, if the falling off were due to actual stopping of mesotrons in air and lead respectively, and the impermanence of the mesotron did not enter in at all. It is commonly conjectured that the equivalent thicknesses of lead and air would stand to one another inversely as the densities of these materials. When, however, the effects of such "equivalent" thicknesses are compared, it is found that the falling-off beyond the lead is decidedly less than that beyond the air. Now the mesotrons take very much less time for traversing the sheet of lead than the wide expanses of the atmosphere; and the "anomaly," as it has been called, is tentatively explained by assuming that few of them perish in the lead, many in the long journey through the atmosphere.

Estimates of the mean life of the mesotron thus made yield values of the order of a millionth of a second. It is supposed by many that the mesotrons are born in the upper layers of the atmosphere. Such conjectures, however, lead beyond the scope of this article, which must be confined to these few recent fruits of the seemingly exhaustless cornucopia of the cosmic rays.

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