Oscillations in an Electromechanical System

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Experimental results are given on an oscillating electromechanical system in which, under a single frequency impressed electromotive force, mechanical vibrations are sustained at a frequency near the resonant frequency of the mechanical system and electrical oscillations at the difference between the frequency of the mechanical vibration and that of the impressed force.

The system is the one studied analytically by R. V. L. Hartley in an accompanying paper. Its performance conforms to the principal operating features predicted in his analysis.

IN AN accompanying paper [1] an analytic investigation is made of a system involving a non-linearity in the coupling between an electrical and a mechanical system. The electro-mechanical system under discussion is, in its simplest form, a condenser, with one plate sharply resonant mechanically, a generator, and an impedance, all connected in series. If the charge on the condenser is q, there will be a force on the mechanical system proportional to q^2. While the mechanical system and the electrical system involved are individually linear, there is a non-linearity in this electrostatic coupling, and hence the possibility exists of mechanical and electrical vibrations at other frequencies than the impressed frequency. On this basis the possibility of the generation of a mechanical vibration, *not at a harmonic of the impressed electromotive force*, and electrical currents at the difference between the frequency of the mechanical vibration and that of the impressed electromotive force was predicted by the analysis.

That the phenomenon discussed can occur was first verified by Mr. Eugene Peterson. A condenser microphone was given a mechanical resonance at 600 cycles per second, by cementing a small metal ball to the center of the diaphragm. An alternating electromotive force at 2200 cycles per second was impressed and the system given a series resonance at the difference frequency, 1600 cycles per second, by means of an inductance. When the impressed voltage was increased beyond a critical value mechanical vibrations suddenly built up and current of the difference frequency, larger in amplitude than the current of the impressed frequency, appeared in the electrical system. The same result was obtained using a prong of a tuning fork as the vibrating plate.

[1] "Oscillations in Systems with Non-Linear Reactance" by R. V. L. Hartley, in this issue of the *Bell Sys. Tech. Jour.*

These tests, while they exhibited the most important characteristic predicted, did not give any other check on the validity of the mathematical results because the systems differed so greatly from that assumed by Mr. Hartley. To obtain simple results in the theoretical discussion it was found necessary to make some severely restricting assumptions. The mechanical system was assumed rigid except to motion at frequencies near the resonant frequency. Similarly the electrical system was assumed to have infinite impedance to all frequency components except that impressed and the difference frequency between the impressed and the mechanical. Thus the only currents and velocities present were of the frequencies (in radians per second), ω_m (mechanical), $\omega_d = \omega_g - \omega_m$ (difference), ω_g (impressed).

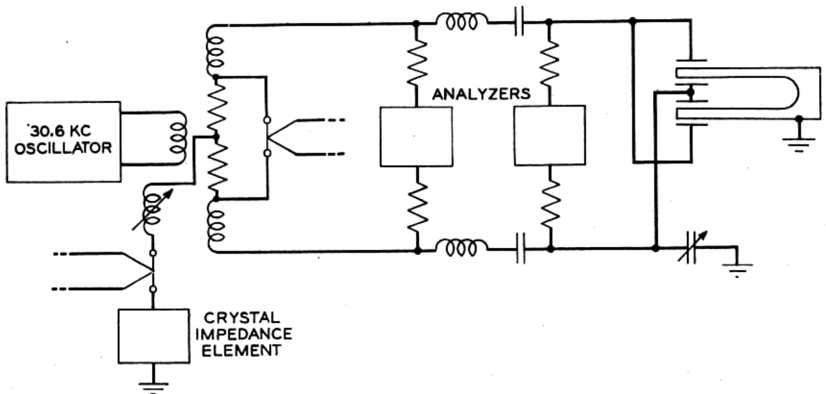

Fig. 1—Circuit diagram.

Under ordinary conditions a non-linear system, such as this one, involving two frequency components (ω_g and ω_m) would have, as modulation products, currents and velocities of all the possible combination frequencies ($r\omega_d + s\omega_m$, r, $s = 0, \pm 1, \pm 2, \cdots$). There would be dissipation of energy at each of these frequencies. These components (other than the three of interest) are the ones which must be suppressed if the system is to be a good approximation to the assumed one.

In order to satisfy the above conditions the circuit of Fig. 1 was constructed.[2] The use of the parallel system instead of a simple series circuit had several advantages. The forces on the tuning fork were so balanced that any constant displacement was avoided. Since the tuning fork had a very sharp resonance [3] the mechanical system was

[2] The tuning fork with condenser plates was designed by Mr. W. A. Marrison.
[3] The damping effect of the air was avoided by operating the fork in a vacuum.

a close approximation to the assumed one. On the electrical side the parallel system had the advantage that the current of the impressed frequency (ω_g) flowed around the outside while the sum ($\omega_g + \omega_m$) and difference ($\omega_d = \omega_g - \omega_m$) frequency components flowed through the mid-branch. The sum frequency component was the most difficult one to suppress. This could be done in the parallel system by means of a piezo-crystal impedance element,[4] tuned to ω_d without at the same time putting in a high impedance to ω_g. This piezo-crystal element had so sharp a resonance that, while its impedance at resonance (30 kilocycles per second) was only 125 ohms, it was about 60,000 ohms only 1000 cycles per second away from resonance. Thus the very high impedance to the sum frequency was obtained. There were, however, some modulation products which flowed around the outside with the impressed current. Only the impedance of a simple tuned circuit was available around this circuit so any product near in frequency to the impressed frequency would not face a very high impedance. The nearest one was $\omega_g - 2\omega_m$. This was not as completely suppressed as the other unwanted products but changing the impedance so that this component was considerably different in magnitude had no appreciable effect on the other electrical components. While this circuit gives a good approximation to the hypothetical one, the result is a highly critical system demanding very fine adjustment and a highly stable generator, since a very slight frequency change has an appreciable effect on the impedances presented by the very sharply tuned circuits.

The measurement of impressed current was made by the thermocouple across small resistors in the input transformer and checked by a current analyzer.[5] Corrections were made in the results when necessitated by the presence of the current component of frequency $\omega_g - 2\omega_m$. The corresponding voltage was obtained by measuring the current of that frequency through the large resistors R_1 by means of a current analyzer. The current of the difference frequency was measured by a thermocouple in the mid-branch. Measurement was also made of the voltage of frequency $\omega_g - 2\omega_m$ across the fork, by measuring the corresponding current through the resistances R_2. There was no simple means available for measuring the mechanical amplitude but it could be obtained from this measurement of $\omega_g - 2\omega_m$ voltage as will be explained later.

On Figs. 2 and 3 are shown curves computed from equations 28, 29 and 30 of the preceding paper for two cases, and the results of the

[4] Designed by Mr. W. P. Mason.

[5] See A. G. Landeen: "Analyzer for Complex Electric Waves," *Bell Sys. Tech. Jour.*, April, 1927.

experiment. The two cases computed are for the system operating with ω_d and ω_m exactly the frequencies of electrical and mechanical resonance and for operation with ω_d 21 cycles higher than resonance. In the latter case ω_m is determined by the requirement—noted by Mr. Hartley—that the phase angles of the impedances be equal. The actual change in ω_m is only a fraction of a cycle. The two cases are given because it was not possible, with the equipment available, to determine ω_d with sufficient accuracy to distinguish between them.

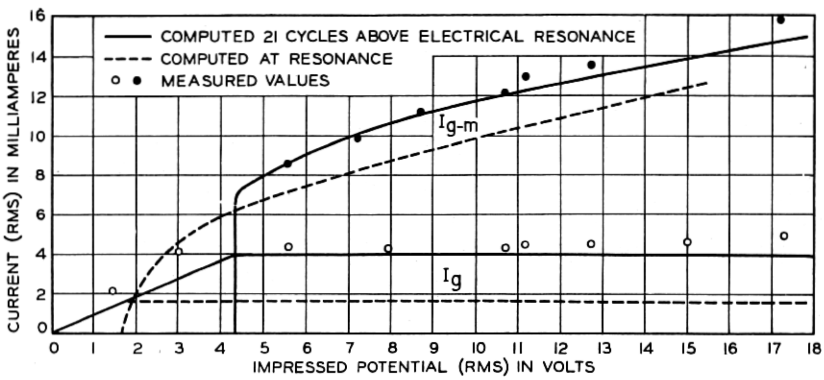

Fig. 2—Electrical current components.

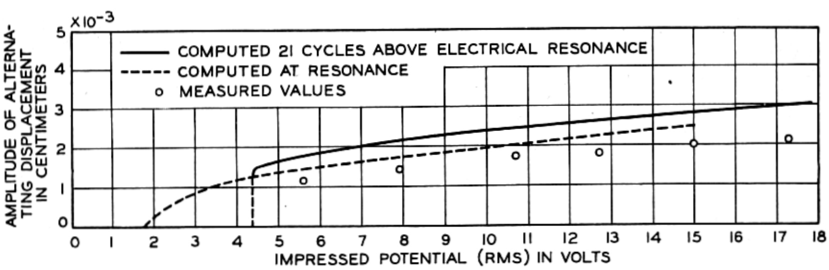

Fig. 3—Mechanical displacement.

While the frequency difference between them is only 20 cycles out of 30,000 it will be noted that, because of the critical character of the impedances, the results differ considerably in amplitudes of the components and in the threshold value.

The second case checks very closely as far as the electrical results are concerned and the mechanical results are of the same order of magnitude. The discrepancy can reasonably be laid to the inaccuracies in the indirect method of measuring the mechanical amplitude. More important is the verification of the outstanding properties predicted by the analysis. There is a threshold voltage above which the new frequency components suddenly appear and rapidly build up to large amplitudes as the voltage is increased. The current of the impressed frequency, ω_g, remains practically constant and independent of voltage above the threshold.

There remains to be described the method by which the mechanical amplitude was obtained. The current voltage relation for a condenser is

$$V = \frac{1}{C}\int i\,dt.$$

The current through the condenser involves only three frequency components in this case, so it can be written in the form

$$i = A_g \cos(\omega_g t + \varphi_g) + A_d \cos[(\omega_g - \omega_m)t + \varphi_d)]. \\ + A_e \cos[(\omega_g - 2\omega_m)t + \varphi_e)]$$

and the capacity of a condenser, in e.s.u., is

$$C = \epsilon\frac{A}{S} = \frac{\epsilon A}{(S_0 + S_m \cos \omega_m t)} \text{ farads}$$

where

$$\epsilon = 8.85 \times 10^{-14} = \text{permittivity},$$

A = plate area in cm.^{2},
S_0 = constant, or average spacing cm.,
S_m = amplitude of mechanical displacement.

From these equations the amplitude of the mechanical vibration in terms of the electrical amplitudes can be determined. Neglecting phase angles the relation is

$$S_m = \frac{V_e - \dfrac{S_0 A_e}{\epsilon A(\omega_g - 2\omega_m)}}{\dfrac{A_d}{2\epsilon A(\omega_g - \omega_m)}}$$

V_e = amplitude voltage component of frequency $\omega_g - 2\omega_m$.

The neglect of the phase angles will make some inaccuracy in the results. They would be exact, in the above formula, were A_e negligibly small. The term involving A_e is a correction term necessitated by the incomplete suppression of that frequency component.