The Proportioning of Shielded Circuits for Minimum High-Frequency Attenuation

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For given conditions of design there exists an optimum proportioning or configuration which makes the high-frequency attenuation of a given type of individually shielded circuit a minimum. Determination is made of such optimum proportioning for a wide variety of types of individually shielded circuits including several novel types designed to make the high-frequency attenuation low in comparison with the cross-sectional area occupied by the circuit, and the attenuation of different types is compared. The following topics and specific circuit structures are considered:

Coaxial Circuits—Basic Coaxial Circuit; Effect of Dielectric; Effect of Frequency on Optimum Ratio; Thin Walls; Stranded Conductors; Optimum Proportioning as a Function of Conductor Resistance.

Balanced Shielded Circuits—Shielded Pair (Cylindrical Conductors and Shield)—Condition for Minimum Attenuation, Condition for Maximum Characteristic Impedance, Effect of Dielectric, Effect of Frequency; Pair in Space; Shielded Stranded Pair; Pair with Shield Return; Double Coaxial Circuit; Shielded Pair (Round Conductors and Oval Shield); Shielded Pair (Quasi-Elliptical Conductors); Shielded Quad.

Introduction

Since the very beginning of mathematics, problems of maximizing and minimizing have possessed a marked fascination. The Greeks were successful in solving a few geometric problems of this character. Later, algebra was found to be another method of attack. Finally, the powerful methods of the calculus became available for the determination of maxima and minima in manifold variety. The reasons for the continued interest in such problems are not hard to find. It is but natural to seek the ideal, and here, at least, is one phase of mankind's search for perfection in which a goodly measure of success may be achieved. In addition, a knowledge of the optimum dimensioning of things, or of the optimum relations between things, frequently holds much practical significance.

It is mainly with problems of maxima and minima that this paper is concerned. These problems have to do with transmission circuits which are surrounded by individual shields. Recent literature [1, 2] has pointed out that circuits of this type have properties which render them especially suitable for the transmission of broad bands of frequencies. Such circuits are also finding application as "lead-ins" to connect radio antennas with transmitting or receiving apparatus.[3, 4]

[1] For numbered references, see end of paper.

It is well at this juncture to understand the function of shielding in a high-frequency transmission circuit. Such shielding serves one or both of these purposes: (*a*) keeping interference due to external sources from entering the circuit, and (*b*) preventing the circuit from causing interference in external circuits. The shielding may either supplement or completely replace the use of electrical balance to reduce interference. The design of shield, that is, its construction, material, thickness, etc., is determined by the degree of shielding required and by considerations of mechanical performance and cost. The degree of shielding needed depends in turn upon such factors as the type and length of circuit, the nature and frequency of the signals to be transmitted, and the magnitudes of external interference. These interesting aspects of shield design, some of which have been dealt with elsewhere,[1, 2, 5] will not be discussed here.

Attention will rather be directed to an intriguing property of any individually shielded circuit, namely, that, for given conditions of design, there always exists an optimum proportioning or configuration which makes the transmission efficiency of the circuit a maximum, or, in other words, makes the attenuation a minimum. One such condition of design which may be imposed is that the cross-sectional area enclosed within the shield is to be a constant. In what follows, determination will be made of such optimum proportioning for a wide variety of types of individually shielded circuits. Since the attenuation is generally of outstanding importance in a high-frequency transmission line, the results should be not only of theoretical interest but also of practical value. Moreover, the different methods which are used in solving these problems should find further application, both in the many other known problems which must perforce be omitted for lack of space, and in those problems which may be conceived in the future.

The principal types of individually shielded circuits to be discussed are:

(1) Coaxial or concentric circuits, in which an outer conductor, which serves also as a shield, completely surrounds a centrally disposed inner conductor.
(2) Shielded pairs, consisting of a pair of conductors which form the transmission circuit, these being surrounded by an individual conducting shield.

The coaxial circuit is unbalanced, and relies solely upon shielding for protection against interference from or into its exterior. In contrast to this is the balanced type of circuit, in which the go and return conductors are designed to be substantially alike and are located substantially symmetrically with respect to earth and surrounding conductors.

In the past, telephone transmission circuits have been largely of the balanced type. It has been found possible to operate such balanced circuits up to fairly high frequencies,[2] without incurring excessive interference. However, as the frequency is raised it becomes increasingly difficult to maintain a sufficiently high degree of balance, and shielding may then be desirable. The shielding may eliminate balance entirely, as in the coaxial circuit, or may be combined with balance in what may be termed a shielded balanced circuit, of which the shielded pair is an outstanding example.

For the simplest forms of circuits, the optimum relations may be precisely derived with the aid of the propagation formulas. In more difficult cases it is necessary to use approximate methods of one kind or another. These methods, however, can generally be made to yield sufficiently accurate results for practical purposes.

Coaxial Circuits

Coaxial circuits, which furnish the least difficult problems in optimum proportioning, make a natural starting point for this subject.

Basic Coaxial Circuit

The first type of circuit to be considered is the basic circuit consisting of two tubular conductors arranged coaxially, whose cross-section is shown diagrammatically in Fig. 1.

Before trying to find out how to proportion such a circuit, it must be noted that in the design of any shielded circuit there enter a number of variables, including the overall size of the structure, the type and thickness of shield, the type of conductor or conductors, the type of insulation, and the frequencies to be transmitted. Some of these factors exert an important influence on the optimum proportioning, so that it is necessary, in order to arrive at a unique solution in a given case, to keep certain factors fixed. Thereafter, however, the effect produced upon the result by varying these factors may be examined.

First, therefore, let the following assumptions be made:

1. That the tubular conductors of Fig. 1 are composed of solid material.
2. That the dielectric is gaseous, with zero dielectric loss. This is a condition which may be approached in practice.
3. That the inner diameter of the outer conductor is fixed. This is a convenient assumption, having for its basis the fact that it is ordinarily desirable, for economic or other reasons, to limit the

size of the outer conductor, and the further fact that the thickness of the outer conductor will ordinarily be determined by mechanical considerations or by shielding requirements.

4. That the frequency is high enough to permit the use of certain approximate formulas as noted below. Practically, this means that at the frequency considered the currents are largely crowded toward the inner surface of the outer conductor and the outer surface of the inner conductor.

The problem is to discover the proportioning which will make the high-frequency attenuation of the circuit a minimum under such conditions. It is well known that the attenuation of a transmission

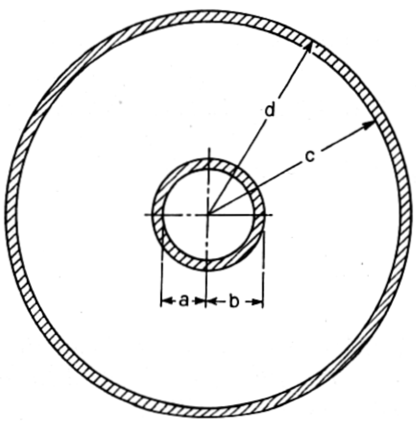

Fig. 1—Coaxial conductor circuit.

circuit at high frequencies may be represented by the following approximate formula:[6]

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} \text{ nepers per cm.,} \qquad (1)$$

where R, L, G and C designate, respectively, the linear resistance, inductance, conductance and capacitance of the circuit. Except as otherwise indicated, values in this and subsequent formulas are expressed in c.g.s. electromagnetic units.

When the dielectric loss is negligible, the second term of formula (1) evidently disappears.

Let a and b represent, respectively, the inner and outer radii of the inner conductor, c and d the inner and outer radii of the outer conductor, f the frequency, λ_1 and μ_1, respectively, the conductivity and permeability of the material of the inner conductor, and λ_2 and μ_2 the corresponding values for the outer conductor. The ratio λ_1/λ_2 will be designated by n.

The high-frequency resistance of the inner conductor may then be approximately expressed by the formula:[5, 7]

$$R_1 = \frac{1}{b}\sqrt{\frac{f\mu_1}{\lambda_1}}\ \text{abohms per cm.} \tag{2}$$

Similarly the high-frequency resistance of the outer conductor is approximately:

$$R_0 = \frac{1}{c}\sqrt{\frac{f\mu_2}{\lambda_2}}\ \text{abohms per cm.} \tag{3}$$

The high-frequency inductance of the circuit is approximately [7]

$$L = 2\log_e \frac{c}{b}\ \text{abhenries per cm.} \tag{4}$$

The capacitance of the circuit is [8]

$$C = \frac{\epsilon}{2\log_e \dfrac{c}{b}}\ \text{abfarads per cm.,} \tag{5}$$

where ϵ is the dielectric constant of the dielectric material between conductors, equal to $1/9 \times 10^{-20}$ for gaseous dielectric, corresponding to unity in the practical system of units.

The high-frequency attenuation of the coaxial circuit with negligible dielectric loss, obtained by combining the above formulas, is

$$\alpha = \frac{1}{2c}\sqrt{\frac{f}{\lambda_1}}\left(\frac{c}{b} + \sqrt{n}\right)\frac{\sqrt{\epsilon}}{2\log_e \dfrac{c}{b}}\ \text{nepers per cm.} \tag{6}$$

The value of permeability assumed in the above equation, and hereafter, is unity, but the methods may be used also for other values.

If the inner diameter of outer conductor be assumed fixed, this expression may be minimized with respect to the ratio c/b, which is the ratio of the radii (or diameters). For convenience this ratio may be designated as ρ. It is found that the high-frequency attenuation is a minimum when the value of ρ is that given by

$$\log_e \rho = \frac{\rho + \sqrt{n}}{\rho}. \tag{7}$$

Figure 2 shows the values of the ratio ρ which satisfy this relation plotted as a function of the conductivity ratio n.

It is noteworthy that the optimum ratio of radii or diameters is independent of (*a*) the diameter and thickness of outer conductor, (*b*) the inner diameter of the inner conductor, and (*c*) the frequency, provided the frequency is high enough for the approximate formulas to hold. It follows from (*a*) that, assuming a fixed thickness of outer conductor, moderately small in comparison with its diameter, relation (7) makes it possible to find the minimum size of outer conductor with which a given value of high-frequency attenuation may be realized. It follows from (*b*) that the inner conductor may be either hollow or solid, provided that the approximate resistance formulas are valid.

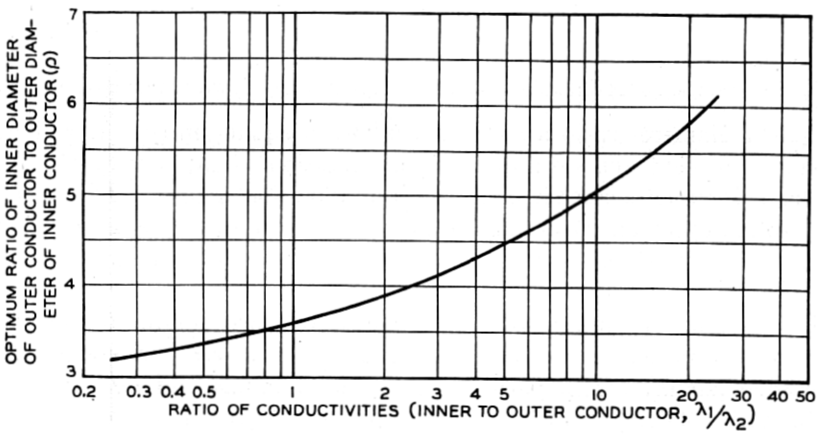

Fig. 2—Variation of optimum diameter ratio of coaxial circuit with conductivity ratio.

A case of special interest arises when the two conductors have the same conductivity, that is, when n equals unity. For this condition the solution of (7) is *

$$\rho = \frac{c}{b} = 3.59. \tag{8}$$

A practical example of the case of different conductivities is a coaxial structure in which the inner conductor is of copper and the outer conductor of lead. For a lead outer conductor containing about 1 per cent of antimony, the ratio of conductivities of inner and outer

* The existence of an optimum relation of this kind was first noted by C. S. Franklin, who gave the value as 3.7. (See Reference 3.) Subsequently the precise value was derived independently of Franklin. (See Reference 10.)

conductors is approximately 13, and the optimum diameter ratio for such a structure, as found from Fig. 2, is about 5.25.

The behavior of the attenuation in the vicinity of the optimum diameter ratio is illustrated in Fig. 3, which shows attenuation plotted

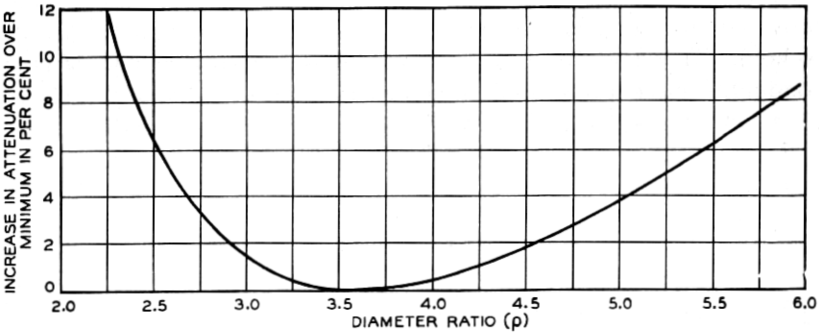

Fig. 3—Variation of optimum diameter ratio of coaxial circuit with conductivity ratio.

against diameter ratio for the case where n equals unity. It will be seen that near the optimum the attenuation changes very slowly. This is fortunate, since it means that unavoidable departures from the optimum diameter ratio may be permitted without appreciable effect on the attenuation. Other small departures from ideal design are also allowable. Thus, for example, it has been assumed in deriving the condition for minimum attenuation that the two conductors of the circuit are perfectly coaxial or concentric. However, for moderately small departures from perfect concentricity occasioned by practical difficulties of construction, the conditions for minimum attenuation are substantially the same as for a circuit with no eccentricity. The situation is similar for other types of shielded circuits to be considered later, in these cases also only a reasonably close approximation to the ideal being necessary.

Effect of Dielectric

Suppose now that the capacitance and leakage conductance introduced by the insulation are substantial.[9] First, it will be assumed that the space between the two conductors is filled with a substantially uniform non-gaseous dielectric material having a dielectric constant ϵ and a power factor p. Such would be the case, for example, if the two coaxial conductors were separated by a continuous rubber insulation. The leakage conductance of the circuit now becomes

$$G = p\omega C = \frac{p\omega\epsilon}{2 \log_e \frac{c}{b}} \text{ abmhos per cm.,} \tag{9}$$

where, as usual, ω equals $2\pi f$.

By substituting in formula (1), the high-frequency attenuation is found to be

$$\alpha = \frac{1}{2c}\sqrt{\frac{f\epsilon}{\lambda_1}}\,(\rho + \sqrt{n})\,\frac{1}{2\log_e \rho} + \frac{p\omega\sqrt{\epsilon}}{2} \text{ nepers per cm.} \tag{10}$$

Since ω, p, and ϵ are not functions of the ratio c/b, the second term of this expression is constant for purposes of differentiation with respect to that ratio, and the condition for minimum attenuation is identical with that previously found, as given in formula (7).

A high-frequency transmission property of smaller interest than the attenuation is the characteristic impedance. This is given by the familiar formula [6]

$$Z_0 = \sqrt{\frac{L}{C}} \text{ abohms.} \tag{11}$$

For the coaxial circuit with dielectric constant ϵ the high-frequency characteristic impedance is

$$Z_0 = \frac{2\log_e \rho}{\sqrt{\epsilon}} \text{ abohms.} \tag{12}$$

There now comes the case where the space between the conductors consists of a combination of gaseous and non-gaseous dielectrics. Perhaps the simplest example occurs when the conductors are separated by insulating discs or washers extending continuously between the two conductors with flat sides perpendicular thereto. Such a construction is illustrated in Fig. 4. Let the thickness of each insulating

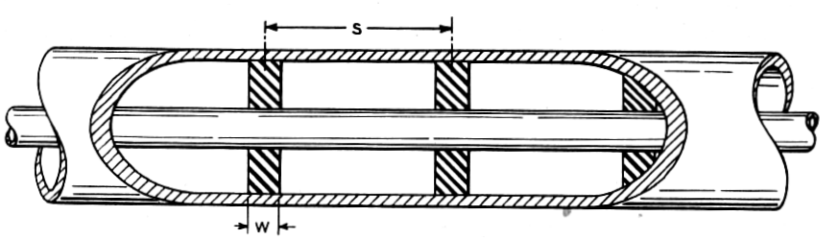

Fig. 4—Coaxial structure with disc insulation.

disc be designated w, the spacing between centers of adjacent discs, s, the dielectric constant of the air dielectric, ϵ_1, and that of the disc material ϵ_2.

The capacitance of the coaxial circuit now becomes

$$C = \frac{\epsilon_1 + \dfrac{(\epsilon_2 - \epsilon_1)w}{s}}{2 \log_e \rho} \text{ abfarads per cm.,} \tag{13}$$

while the leakage conductance is

$$G = \frac{w}{s} \cdot \frac{p\omega\epsilon_2}{2 \log_e \rho} \text{ abmhos per cm.} \tag{14}$$

On substituting these values in formula (1) the following expression results:

$$\alpha = \frac{1}{2c}\sqrt{\frac{f}{\lambda_1}} \frac{\sqrt{\epsilon_1 s + (\epsilon_2 - \epsilon_1)w}}{\sqrt{s}} \frac{\rho + \sqrt{n}}{2 \log_e \rho} + \frac{p\omega\epsilon_2}{2} \frac{w}{\sqrt{s}} \frac{1}{\sqrt{\epsilon_1 s + (\epsilon_2 - \epsilon_1)w}} \text{ nepers per cm.} \tag{15}$$

Once more the second term is independent of c and b, and the condition for minimum attenuation is, as before, that given by equation (7).

The high-frequency characteristic impedance in this case, however, is

$$Z_0 = \frac{2 \log_e \rho}{\sqrt{\epsilon_1 + \dfrac{(\epsilon_2 - \epsilon_1)w}{s}}} \text{ abohms.} \tag{16}$$

The quantity in the denominator of the above expression is evidently the weighted average dielectric constant of the insulating medium.

In the case just considered, the gaseous and non-gaseous dielectrics were separated from each other by planes perpendicular to the axis of the conductors. Consequently, each line of dielectric flux passed through only one kind of material. It can be shown that, as long as this latter condition holds, the condition for minimum high-frequency attenuation as given by equation (7) is valid, or, in other words, the optimum diameter ratio is that shown in Fig. 2. Cases arise, however, in which a line of dielectric flux, in going from one conductor to the other, may pass through more than one kind of dielectric material. It is extremely difficult to obtain a mathematical solution for the diameter ratio which results in minimum attenuation for such cases, since this involves a three-dimensional field problem. Consideration of the problem, however, indicates that the optimum diameter ratio will not differ appreciably from that given by Fig. 2, especially if the dielectric is mostly gaseous, which, of course, is highly desirable.

Effect of Frequency on Optimum Ratio

It has been seen that, at the higher frequencies where the approximate transmission formulas may be employed, the optimum diameter ratio is substantially independent of frequency. In so far as the practical application of individually shielded circuits is concerned, it is in these higher frequencies that interest primarily centers. Even when it is desired to transmit a wide band extending from high frequencies down to comparatively low ones, it is advantageous to proportion the circuit so as to minimize the attenuation at the highest transmitted frequency, since the attenuation at all lower frequencies will be less than the value thus obtained.

It may, however, be worth while to consider briefly the question of optimum proportioning when low frequencies only are involved. The appropriate transmission formulas to be used instead of the approximate high-frequency expressions are known,[5] and the optimum diameter ratio in any specific case may be derived from these. It will be evident that, since skin effect is present to a lesser degree at the low frequencies, the diameter and thickness of the outer conductor and the thickness of the inner conductor will, as the frequency is decreased, have an increasing influence on the optimum proportioning.

Without attempting to derive precise values for the different conditions, it may be noted that the optimum diameter ratio for low frequencies is invariably less than that for high frequencies, the high-frequency value being approached asymptotically as a limit. The reason for this will be readily apparent. Let the inner diameter and thickness of the outer conductor be assumed fixed. At high frequencies the resistance of the inner conductor varies inversely with the first power of its diameter. At lower frequencies, however, this resistance varies inversely with some power of the diameter greater than unity, and finally, at zero frequency, assuming a solid wire, with the square of the diameter. Hence it is, that, in varying the size of the inner conductor in order to obtain a balance between the change of resistance and change of capacity, it is advantageous to make the inner conductor somewhat larger, or, in other words, to make the diameter ratio smaller, at low frequencies than at high frequencies.

Thin Walls

What is the result if the walls of the two coaxial conductors are made very thin? Under this condition the conductor resistance, and hence the attenuation, will remain substantially constant over a wide range of frequencies. This constancy is realized, however, at the expense of an increase in the attenuation as compared with that for thicker conductor walls.

Using the notation of Fig. 1, the resistances of the inner and outer conductors, both with conductivity λ, at frequencies where the walls are sufficiently thin to avoid skin effect, are

$$R_i = \frac{1}{\pi\lambda(b^2 - a^2)} \text{ abohms per cm.} \tag{17}$$

$$R_0 = \frac{1}{\pi\lambda(d^2 - c^2)} \text{ abohms per cm.} \tag{18}$$

Let the inner conductor have a fixed thickness $b - a$, the outer conductor a thickness $d - c$, and let the ratio $(b - a)/(d - c)$ be represented by t. For small values of wall thickness

$$b^2 - a^2 \doteqdot 2b(b - a) \tag{19}$$

and

$$d^2 - c^2 \doteqdot 2c(d - c) = 2c\,\frac{(b - a)}{t}. \tag{20}$$

Substituting these relations and the values of L and C from (4) and (5) in equation (1), it is found that the attenuation for the circuit with thin walls is

$$\alpha = \frac{\sqrt{\epsilon}}{4\pi\lambda c}\frac{(\rho + t)}{(b - a)}\frac{1}{2\log_e \rho} \text{ nepers per cm.} \tag{21}$$

Differentiation shows that minimum attenuation in the case of thin walls is obtained when

$$\log_e \rho = \frac{\rho + t}{\rho}. \tag{22}$$

The values of diameter ratio which satisfy this relation may be found from the curve of Fig. 2, if the values of abscissæ on that curve are interpreted as values of t^2.

If the conductor walls are thin, as above, and if in addition the conductivities of the two conductors are not the same, that of the inner conductor being n times that of the outer one, the condition for minimum attenuation becomes

$$\log_e \rho = \frac{\rho + nt}{\rho}. \tag{23}$$

Figure 2 may be used to find the values of diameter ratio which satisfy this relation also, the abscissæ scale markings in this case being taken as values of n^2t^2.

Stranded Conductors

With conductors having solid walls, or composed of non-insulated strips or filaments, the currents at high frequencies are largely crowded toward the inner surface of the outer conductor and the outer surface of the inner conductor, due to skin effect. Since the losses in the conductors themselves ordinarily comprise the major portion of the attenuation in a coaxial circuit, interest attaches to the possibility of counteracting the increase in conductor resistance due to skin effect by using a conductor composed of a number of individually insulated strands so twisted or interwoven as to distribute the current more nearly uniformly over the cross-section.[11] Chief attention naturally focuses upon the inner conductor, which is by far the greater contributor to the resistance, and this discussion will be largely limited to the case where only the inner coaxial conductor is stranded.*

Types of stranded conductors suitable for use as the inner conductor of a coaxial circuit include both those in which the conductor cross-section is completely filled with insulated strands and those in which the insulated strands form an annular cross-section, surrounding a core of non-conducting or conducting material. Of various possible methods of stranding, one simple and effective process is similar to that used in the construction of rope. A few strands are twisted together to form a group, several such groups are twisted into a larger group, and so on until the desired conductor cross-section is obtained.

The high-frequency resistance of a stranded conductor may be determined either by measurement or computation. For a completely stranded inner conductor of any diameter, size, number of strands, and thickness of insulation, the high-frequency resistance is given by S. Butterworth [12] and in unpublished material by J. R. Carson. The resistance values obtained in measurements of stranded conductors approximate very closely the theoretical results.

In evaluating the results obtained with stranding, it is convenient to compare the resistance of a stranded conductor with that of a non-stranded conductor of the same overall size. For the case of a stranded inner conductor, the ratio of the resistance of the stranded conductor at any given frequency to the resistance at the same frequency of a solid conductor having the same outer diameter and composed of the same material used in the strands may be designated as m.

The values of the resistance ratio m which may be realized in practice depend upon the frequency and the design of stranded conductor. Some idea of these values for two specific conductors may be obtained

* "Stranded" is used to mean "composed of insulated strands."

from the curves of Fig. 5. It will be seen that there is ordinarily a frequency at which the resistance ratio is a minimum. Above this frequency the improvement due to stranding rapidly vanishes, the performance thereafter being worse than that of the corresponding non-stranded conductor. The minimum value of resistance ratio attained in the range of some hundreds of kilocycles may be in the order of 0.6, a very substantial improvement. In order to secure any marked advantage in the frequency range above 700 or 800 kilocycles, the number and fineness of the individual strands would be such as practically to preclude their use.

Another result obtained with stranding is an increase in the internal inductance of the conductors, which likewise serves to reduce the high-

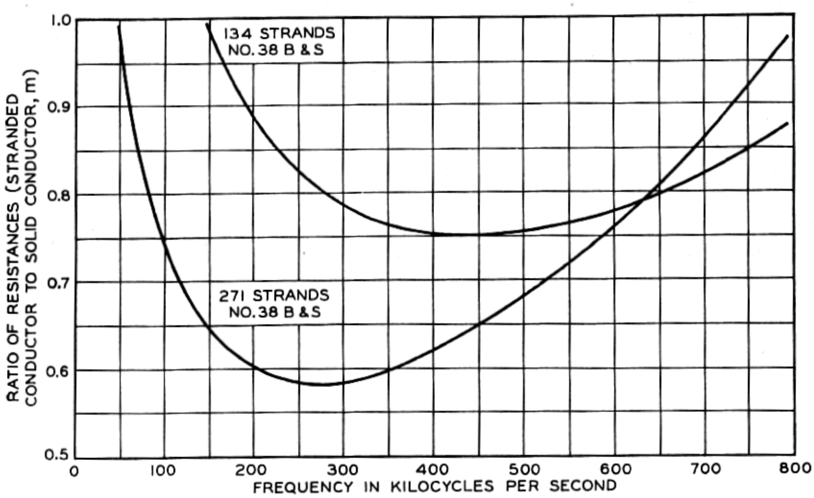

Fig. 5—Resistance ratios of stranded conductors.

frequency attenuation. For a round conductor which is completely stranded, the internal inductance at all frequencies where the current is uniformly distributed over the conductor cross-section approximates .5 abhenry per centimeter, which is the internal inductance of a solid round wire at zero frequency. In general, this value of internal inductance will hold up to frequencies somewhat above that for which the resistance ratio m is a minimum. The internal inductance of a stranded conductor of annular cross-section, for all frequencies where the current is uniformly distributed over the cross-section, is

$$L_i = \frac{b^2 - 3a^2}{2(b^2 - a^2)} + \frac{2a^4}{(b^2 - a^2)^2} \log_e \frac{b}{a} \text{ abhenries per cm.} \qquad (24)$$

This is the same as the internal inductance at zero frequency of a solid tube of the same dimensions.

Since either the inner or outer conductor of a coaxial circuit, or both, may be stranded, and since, in addition, the dielectric loss may either be negligible or may be appreciable, there are six different cases of optimum proportioning which might be considered.[13] Only one case, however, that of a coaxial circuit with only the inner conductor stranded and with negligible dielectric loss, will be taken up here. The high-frequency attenuation of such a coaxial circuit is

$$\alpha = \frac{m}{2c}\sqrt{\frac{f}{\lambda_1}}\left(\rho + \frac{\sqrt{n}}{m}\right)\sqrt{\frac{\epsilon}{(4 \log_e \rho)^2 + 2L_i \log_e \rho}} \text{ nepers per cm.} \qquad (25)$$

While the value of m varies with frequency and with the design of the stranded conductor, this value is, for a particular frequency and a particular design, definitely determinable. As has been noted, it is generally desirable to proportion a transmission circuit so as to minimize the attenuation at the highest frequency to be transmitted. Furthermore, the value of m will not vary rapidly with changes in conductor diameter provided the number of strands be changed as the conductor size is varied. It therefore becomes possible to treat m as a constant in deriving the relation for optimum proportioning.

Using ρ to designate c/b, the condition for minimum high-frequency attenuation is found to be

$$\frac{2\dfrac{m}{\sqrt{n}}\rho \log_e \rho}{\dfrac{m}{\sqrt{n}}\rho + 1} = \frac{4 \log_e \rho + L_1}{2 \log_e \rho + L_1}. \qquad (26)$$

Figure 6 shows graphs of equation (26) for two values of L_i, namely, $L_i = 0.5$ abhenry per centimeter, which corresponds to the case where the cross-section of the inner conductor is completely stranded, and $L_i = 0$. When the stranded inner conductor is of annular cross-section the optimum value of the diameter ratio lies somewhere between the two curves shown. The useful range of m probably lies between about 0.5 and unity and that of n between about 1 and 15.

As to the practical use of stranding, it is apparent from the resistance ratio curves of Fig. 5 that in order to take advantage of stranding it would be necessary to limit the transmission band to a maximum frequency well below that possible with non-stranded conductors. Further drawbacks to the use of stranded conductors are their greater cost as compared with non-stranded ones, and greater mechanical difficulties in using them. For these reasons stranded conductors do not seem likely to find early application in broad band transmission circuits.

Optimum Proportioning as a Function of Conductor Resistance

The optimum diameter ratio of a coaxial circuit may also be expressed broadly as a function of the two conductor resistances. Assume a coaxial circuit in which the high-frequency resistance of the inner conductor varies, at least over a limited range, inversely as its

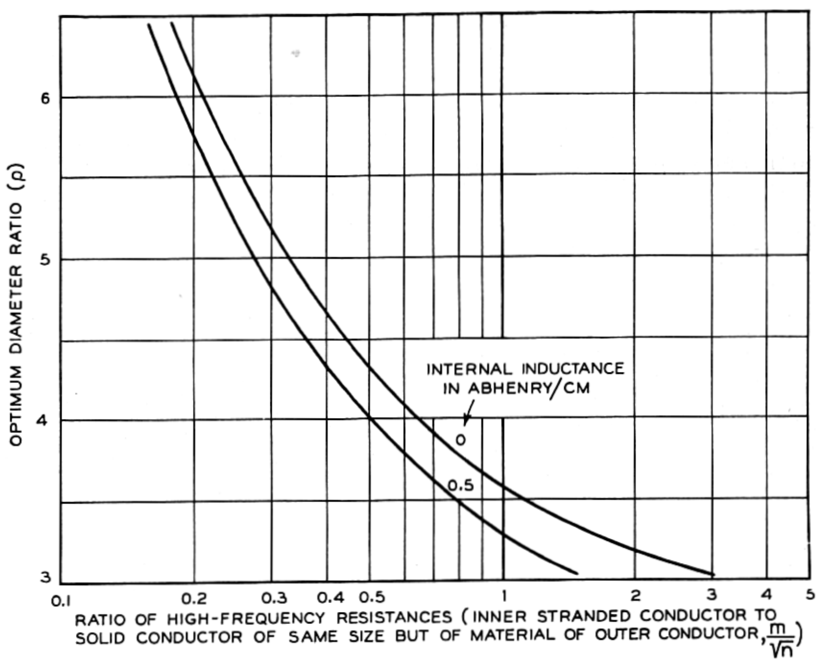

Fig. 6—Optimum diameter ratio of coaxial circuit with stranded inner conductor.

outer radius and that of the outer conductor as its inner radius, thus

$$R_i = \frac{k_1}{b} \qquad \text{and} \qquad R_0 = \frac{k_2}{c}. \tag{27), (28}$$

These relations are approximately true for all the types of circuits which have been discussed. Let

$$r = \frac{k_2}{k_1} = \frac{R_0}{R_i} \cdot \frac{c}{b} = \frac{R_0}{R_i} \rho. \tag{29}$$

If the internal inductance of the conductors is assumed to be zero, which is the most usual case, the high-frequency attenuation of the circuit may then be written

$$\alpha = \frac{k_1}{2c}(\rho + r)\frac{1}{2 \log_e \rho}. \tag{30}$$

Upon minimizing with respect to c/b the condition for minimum high-frequency attenuation is found to be

$$\log_e \rho = \frac{\rho + r}{\rho} = 1 + \frac{R_0}{R_i}. \tag{31}$$

These relations have been found useful in certain instances.

Balanced Shielded Circuits

Though arrangements of three or more coaxial conductors are possible,[14] practical interest is almost wholly limited to coaxial circuits employing but two conductors. With balanced shielded circuits, however, the number of conductors, counting the shield as one, is necessarily three and may be more. With a coaxial circuit, moreover, the cylindrical shape is the natural and usual one for the conductors. With balanced shielded circuits, on the other hand, there enter a number of possibilities. Not only are cylindrical shapes of conductors and shield to be considered, but a variety of other shapes as well. More complex, therefore, than the foregoing problems in optimum proportioning are those for balanced shielded circuits, now to be discussed.

Shielded Pair—Cylindrical Conductors and Shield

The simplest form of balanced shielded circuit is a shielded pair comprising two cylindrical conductors surrounded by a cylindrical shield. Such a circuit is shown diagrammatically in cross-section in Fig. 7. For the present, attention will be directed to the circuit obtained when the two enclosed conductors are connected one as a return for the other.

Condition for Minimum Attenuation [15]

As before, it is desired to minimize the high-frequency attenuation. Let it be assumed first, as in the coaxial circuit, that the area within the shield is fixed, the conductors are of solid material and the dielectric is gaseous. Let b represent the radius of each conductor in Fig. 7, c the inner radius of the shield, h the distance from the center of either conductor to the center of the shield, λ_1 the conductivity of each conductor, λ_2 that of the shield, and n the ratio of λ_1/λ_2. Expressions for the high-frequency attenuation of this circuit have been given in unpublished formulas developed by S. A. Schelkunoff and by Mrs. S. P. Mead. The approximate formula given below is due to the latter.

$$\alpha = \frac{\rho\left[1 + \frac{1+2\nu^2}{4\nu^4}(1-4\sigma^2)\right] + 4\sqrt{n}\,\sigma^2\left[1+\sigma^4 - \frac{1+4\nu^2}{8\nu^4}\right]}{\log_e\left[2\nu\frac{1-\sigma^2}{1+\sigma^2}\right] - \frac{1+4\nu^2}{16v^4}(1-4\sigma^2)} \times \frac{1}{4c}\frac{\sqrt{f\epsilon}}{\sqrt{\lambda_1}} \text{ nepers per cm.,} \qquad (32)$$

where $\sigma = h/c$ and $\nu = h/b$.

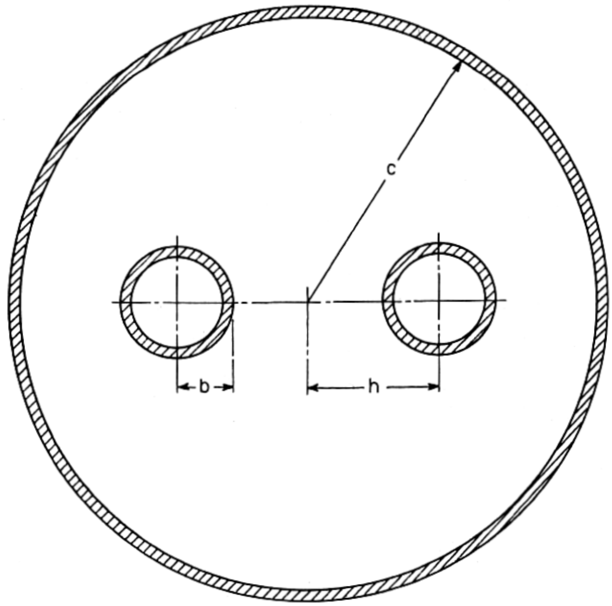

Fig. 7—Shielded pair.

The values of the diameter ratio (ρ) and what may be termed the spacing ratio (σ), which make this expression a minimum for different values of the conductivity ratio n, can be determined in different ways. One possible method is to find the values of h and b which satisfy the equations $\partial\alpha/\partial h = 0$ and $\partial\alpha/\partial b = 0$. The partial derivatives are, however, very complicated. Accordingly a preferable alternative is to substitute various pairs of values of ρ and σ in (32) and determine, graphically or otherwise, the particular pair which makes it a minimum. In this way it is found that when the conductors and shield are of the same material, so that n equals unity, the optimum values are approximately

$$\rho = \frac{c}{b} = 5.4; \qquad \sigma = \frac{h}{c} = .46. \qquad (33), (34)$$

The optimum diameter and spacing ratios for different values of the conductivity ratio n are shown in Figs. 8 and 9. For copper conduc-

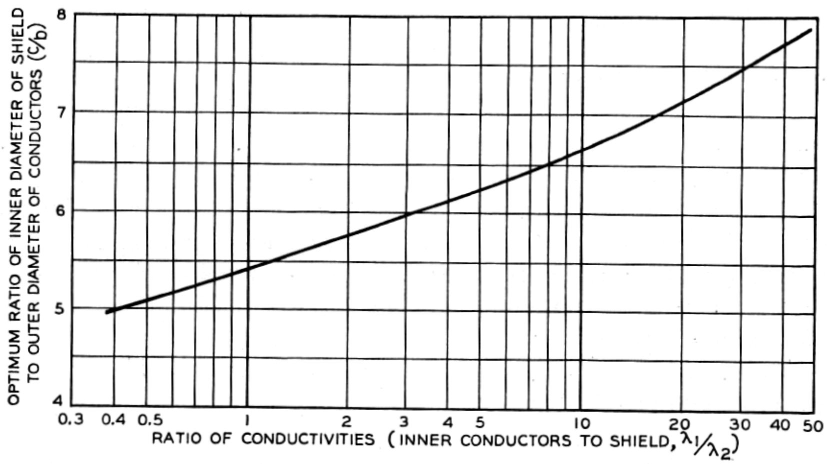

Fig. 8—Variation of optimum diameter ratio of shielded pair with conductivity ratio.

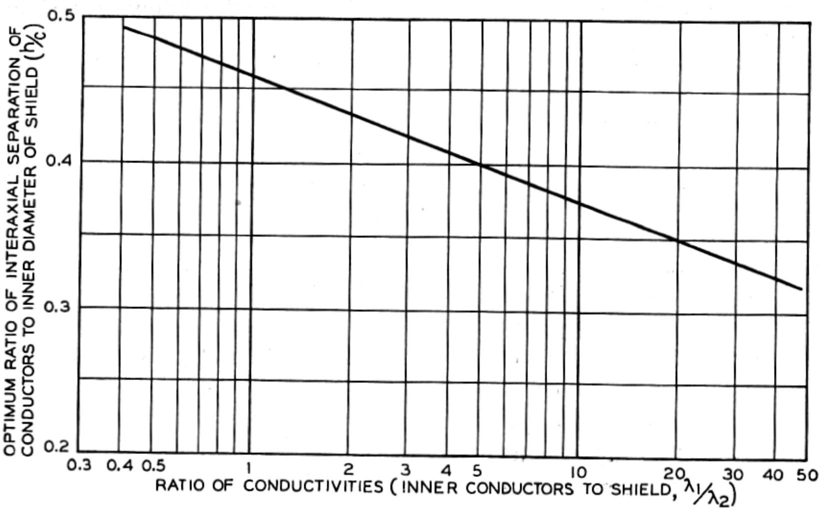

Fig. 9—Variation of optimum spacing ratio of shielded pair with conductivity ratio.

tors and a lead shield, the values are approximately 6.9 and .36, respectively.

As with the coaxial circuit, these optimum relations are independent of the diameter and thickness of the shield. Hence they make it possible to find the minimum size of shield necessary for a given value of high-frequency attenuation. The optimum relations are also independent of the frequency, provided the frequency is high enough for the approximate formulas to hold. The inner conductors may be either hollow or solid.

Condition for Maximum Characteristic Impedance[15]

Occasionally it is of interest to know the condition that must be satisfied to obtain maximum high-frequency characteristic impedance for a solid pair with circular shield. At high frequencies the value of $1/\sqrt{LC}$ approaches a constant value equal to the velocity of light divided by the square root of the ratio of the dielectric constant of the circuit to that of air. Hence the condition for maximum characteristic impedance is also, from equation (11), that for maximum inductance and minimum capacitance.

Accordingly, the high-frequency characteristic impedance of the shielded solid pair circuit is given by the formula:

$$Z_0 = \frac{4}{\sqrt{\epsilon}}\left(\log_e \left[2\nu \frac{1 - \sigma^2}{1 + \sigma^2} \right] - \frac{1 + 4\nu^2}{16\nu^4}(1 - 4\sigma^2) \right) \text{abohms.} \quad (35)$$

Let it be assumed first that the wires are very small compared with the shield. Then equation (35) may be written

$$Z_0 = \frac{4}{\sqrt{\epsilon}} \log_e \left[\sigma \frac{1 - \sigma^2}{1 + \sigma^2} \right] + \frac{4}{\sqrt{\epsilon}} \log_e 2\rho \text{ abohms.} \quad (36)$$

For a given ratio of inner diameter of shield to outer diameter of conductor, the second term of this expression is constant. By minimizing the first term with respect to σ, it is found that, so long as the ratio of inner diameter of shield to conductor diameter is large, maximum characteristic impedance is obtained when σ has a value of .486.

If the conductors are large compared with the shield, equation (36) no longer holds. However, since the capacitance and high-frequency characteristic impedance are inversely proportional to one another, the position of the conductors with respect to the shield must be such as to minimize the capacitance. It is clear that as the conductor diameter approaches the inner radius of the shield, σ approaches 0.5 for minimum capacitance. Hence, for any ratio of inner diameter of shield to diameter of conductor, the ratio of the interaxial separation of the conductors to the inner diameter of the shield which gives maximum characteristic impedance lies between the limits 0.486 and 0.500. For practical purposes a value of about 0.49 may generally be used.

Effect of Dielectric

The effect of dielectric for a shielded pair is similar to that for a coaxial circuit. When the insulation is so disposed between conductors and shield that a line of dielectric flux passes through only one kind of dielectric material, the second term of the attenuation formula is independent of the proportioning of conductors and shield, so that the optimum proportions as given in Figs. 8 and 9 are unchanged. These values will also serve for most practical cases where a line of dielectric flux may pass through more than one kind of material.

Effect of Frequency

At frequencies where the approximate formulas no longer hold, the conditions for minimum attenuation as given by Figs. 8 and 9 undergo some change, especially the former. As the frequency is decreased the attenuation is minimized by increasing the size of conductor for a given size of shield. In other words, the optimum diameter ratio grows less. The optimum spacing ratio increases from 0.46 toward the value which gives minimum capacitance, i.e., approximately 0.49.

Pair in Space

It is interesting to digress for a moment to consider briefly the case shown in Fig. 10 of a pair of round conductors in space. This may

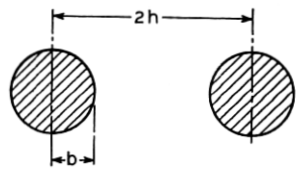

Fig. 10—Pair in space.

be regarded as a pair surrounded by a shield of infinite diameter. If the conductors are of solid material, the attenuation of the circuit at high frequencies is

$$\alpha = \frac{P}{4b}\sqrt{\frac{f\epsilon}{\lambda}}\frac{1}{\cosh^{-1}\nu} \text{ nepers per cm.,} \tag{37}$$

where P is the proximity effect factor, given in a paper by J. R. Carson.[16] At high frequencies this factor reduces to the asymptotic value

$$P = \frac{\nu}{\sqrt{\nu^2 - 1}}. \tag{38}$$

For a given high frequency and given wire separation, assuming the dielectric constant and conductivity to be fixed, equation (37) becomes

$$\alpha = \frac{K_3 \nu^2}{\sqrt{\nu^2 - 1} \cosh^{-1} \nu}, \tag{39}$$

where K_3 is a constant.

For a given wire separation this expression is minimized when

$$\nu = \frac{h}{b} = 2.27. \tag{49}$$

For open-wire pairs, which may be considered as approaching pairs in space, it is ordinarily cheaper to obtain any desired attenuation at a given frequency by using a wide separation and relatively small conductors rather than a narrow separation and conductors of such size as to satisfy (40). This relation is of considerable utility, however, in that it is a reasonably close approximation to the optimum for many kinds of shielded pairs. The corresponding ratio for the shielded solid pair, as given by (33) and (34), is approximately 2.5.

Shielded Stranded Pair

The preceding discussion of shielded pairs has been limited to types of enclosed conductors such that high-frequency currents are crowded toward the conductor surfaces. There will now be found the optimum proportioning when the enclosed conductors are stranded.[17]

The capacitance and inductance between two shielded stranded wires when surrounded by a cylindrical shield are approximately

$$C = \frac{\epsilon}{4 \log_e \left[2\nu \frac{1 - \sigma^2}{1 + \sigma^2} \right]} \text{ abfarads per cm.,} \tag{41}$$

$$L = 4 \log_e \left[2\nu \frac{1 - \sigma^2}{1 + \sigma^2} \right] + 2L_i \text{ abhenries per cm.,} \tag{42}$$

where L_i is the internal inductance of each conductor.

If it be assumed that the current distribution is uniform over the cross-section of the enclosed conductors, the resistance of each is the same as if its return were coaxial. Hence the high-frequency resistance of each conductor is

$$R_i = \frac{2m}{b}\sqrt{\frac{f}{\lambda_1}} \text{ abohms per cm.} \tag{43}$$

The high-frequency resistance of the shield can be shown to be

$$R_0 = \frac{8ch^2}{c^4 - h^4}\sqrt{\frac{f}{\lambda_2}} \text{ abohms per cm.} \tag{44}$$

The high-frequency attenuation of the shielded stranded pair, found by substituting equations (41) to (44) in (1), is, with zero dielectric loss,

$$\alpha = \frac{\frac{m}{2c}\sqrt{\frac{f}{\lambda_1}}\left[\rho + \frac{4\sqrt{n}\sigma^2}{m(1-\sigma^4)}\right]}{\sqrt{\left[\log_e 2\nu\frac{1-\sigma^2}{1+\sigma^2}\right]\left[4\log_e 2\nu\frac{1-\sigma^2}{1+\sigma^2} + 2L_i\right]}} \text{ nepers per cm.} \tag{45}$$

The optimum proportions of the shielded stranded pair at high frequencies depend, therefore, on the two quantities $m/\sqrt{n}$ and L_i. For any given shield radius c, the values of h and b which give minimum attenuation may be found by setting

$$\frac{\partial \alpha}{\partial h} = 0; \qquad \text{and} \qquad \frac{\partial \alpha}{\partial b} = 0. \qquad (46), (47)$$

By imposing the first condition it is found that

$$\frac{\partial M}{\partial h}\frac{(c^4 - h^4)^2}{8ch(c^4h^4)} = M\frac{2\log_e M(4\log_e M + 2L_i)}{8\log_e M + 2L_i}\,\frac{1}{\frac{m}{\sqrt{n}b} + \frac{4ch^2}{c^4 - h^4}}. \tag{48}$$

Imposing the second condition we find that

$$-\frac{\sqrt{n}b^2}{m}\frac{M}{b} = M\frac{2\log_e M(4\log_e M + 2L_i)}{8\log_e M + 2L_i}\,\frac{1}{\frac{m}{\sqrt{n}b} + \frac{4ch^2}{c^4 - h^4}}, \tag{49}$$

where $M = 2\nu(1 - \sigma^2)/(1 + \sigma^2)$.

Upon equating the left hand members of (48) and (49), and substituting the values of the derivatives, the following expression results

$$\rho = \frac{8\sigma^2(1 + \sigma^4)}{\frac{m}{\sqrt{n}}(1 - \sigma^4)(1 - 4\sigma^2 - \sigma^4)}. \tag{50}$$

This expression is the locus of values of the ratio ρ which give minimum attenuation for different assumed values of the ratio σ. The unique values of $h/c = \sigma$ and $c/b = \rho$, which give minimum attenuation for a given value of $m/\sqrt{n}$ and L_i, may be obtained by taking pairs of σ and ρ which satisfy equation (50), substituting them in equation (45), and graphically determining the pair for which the attenuation is a minimum.

Figures 11 and 12 show a graph, obtained in this way, of the optimum

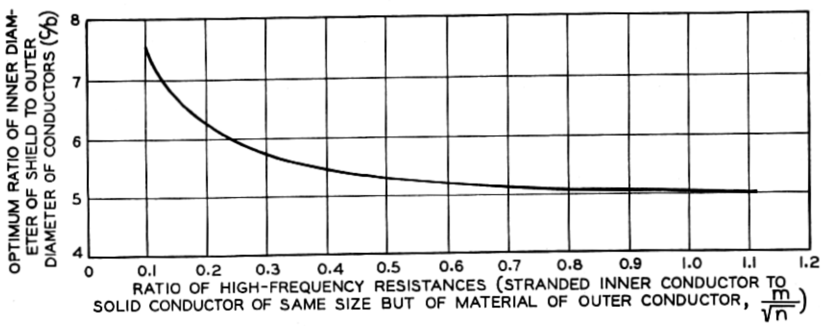

Fig. 11—Optimum diameter ratio of shielded stranded pair.

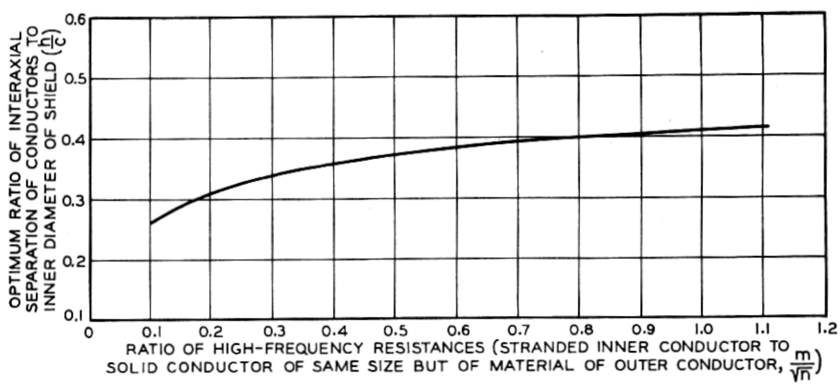

Fig. 12—Optimum spacing ratio of shielded stranded pair.

proportions for a shielded stranded pair, plotted as a function of $m/\sqrt{n}$ for a value of L_i equal to 0.5 abhenry per centimeter, which corresponds to the case where each conductor is completely stranded.

Pair with Shield Return

The discussion of the shielded pair thus far has been concerned solely with the circuit which employs one of the enclosed conductors as a return for the other. A second circuit may be obtained by transmitting over the two enclosed conductors in parallel with the shield as the return. This latter circuit alone is less efficient than a coaxial circuit formed by replacing the two inner conductors with a single one. If, however, the two circuits obtainable from the shielded pair structure can both be employed without excessive mutual interference, there will be a considerable increase in the usefulness of the system, measured in terms of the total frequency range that can be transmitted without exceeding a given attenuation. It is therefore of interest to determine the conditions making the total transmitted frequency range for the two circuits a maximum.[18]

The high-frequency attenuation of each circuit, assuming solid conductors, can be written

$$\alpha = K\sqrt{f}, \tag{51}$$

where K is a constant, different for each circuit, which depends on the size and material of the conductors, and the dielectric constant of the insulation. Leakage is assumed negligibly small.

Using subscripts 1 and 2, respectively, to designate the circuit comprising the two enclosed conductors one as a return for the other and the circuit comprising the two wires in parallel with shield return, it follows that

$$f_1 + f_2 = \frac{\alpha_1^2}{K_1^2} + \frac{\alpha_2^2}{K_2^2}\cdot \tag{52}$$

Letting $A = \alpha_2/\alpha_1$

$$f_1 + f_2 = \alpha_1^2\left(\frac{1}{K_1^2} + \frac{A^2}{K_2^2}\right)\cdot \tag{53}$$

Equation (53) gives the sum of the frequency ranges that can be transmitted in the above manner over any given shielded pair for any given attenuation at the highest frequencies of the bands. To obtain maximum total range, this equation must be maximized.

The attenuation of the circuit comprising one enclosed conductor as a return for the other is given by equation (32), from which the value of K_1 can be obtained immediately. An expression for K_2 has been given in an unpublished formula due to Mrs. S. P. Mead, as follows:

$$K_2 = \frac{U + V}{\log_e\left[\dfrac{\rho(1-\sigma^4)}{2\sigma}\right] - \dfrac{1+4\sigma^4}{1+4\nu^2}\left(1 + 4\sigma^4\left(\dfrac{5+4\nu^2}{1+4\nu^2}\right)\right)}\frac{1}{4c}\sqrt{\frac{\epsilon}{\lambda_1}} \tag{54}$$

in which

$$U = \rho\left[1 + \frac{8\nu^2(1+4\sigma^4)}{(1+4\nu^2)^2}\left(1 + 4\sigma^4\left(\frac{9+4\nu^2}{1+4\nu^2}\right)\right)\right],$$

$$V = 2\left[1 + 2\sigma^4 + \frac{8\sigma^4}{1+4\nu^2}\left(1 + 8\sigma^4\left(\frac{5+4\nu^2}{1+4\nu^2}\right)\right)\right]\cdot$$

For a given inner radius of shield and given dielectric constant and conductor material, the diameter and spacing ratios which make equation (53) a maximum can be obtained by the substitution method previously described. When the conductors and shield are of the same material and $A = 1$, computation shows that the total frequency range is a maximum when the radius ratio ρ equals 5.9 and the spacing ratio σ equals 0.33. This value of $A = 1$ represents an important practical case, since it will, as a rule, be desirable to employ the same repeater points for each circuit and permit the same attenuations between repeater points. It is also of interest, however, to determine the effect of other values of A.

When A is zero, the problem reduces to that of the simple shielded pair, which has been shown previously to be minimized by the proportions given in (33) and (34).

When A becomes large, or, in other words, when the phantom circuit alone is used, $1/K_2^2$ must be maximized. It is obviously necessary that the enclosed conductor be in contact and, accordingly, the spacing ratio must be the reciprocal of the diameter ratio. For this condition the following proportions result:

$$\rho = \frac{c}{b} = 6.0; \qquad \sigma = \frac{h}{c} = 0.17. \qquad (55), (56)$$

The above proportions are optimum only when the enclosed conductors and the shield are of the same conductivity. The relations for the case of unequal conductivities may be derived in a similar manner. For practical purposes the effect of dielectric loss on the optimum proportions is negligible.

Double Coaxial Circuit

Another form of balanced and shielded transmission circuits may be obtained by using two coaxial conductor units, the transmission path consisting of the two inner coaxial conductors in series, with the outer coaxial conductors serving only for shielding. Such a circuit is shown diagrammatically in cross-section in Fig. 13. Usually the outer conductors would be in practically continuous contact with each other. A circuit of this type will handle a frequency band extending to lower values than can be used with a single coaxial circuit, since it is balanced and the two coaxial units can be transposed by twisting or by periodic interchange of their positions. At high frequencies, where the shielding of the outer conductor of the coaxial circuit becomes effective, the outer conductors may be separated to any desired distance. It is essential, however, that they be connected together at the ends of the circuit.

In such a double coaxial circuit, used at high frequencies, equal and opposite currents will flow on the inner and outer conductors of each coaxial unit. The resistance and inductance of the balanced circuit will, therefore, be twice, and the capacitance and leakance one-half, the corresponding values for one coaxial unit. The attenuation of the balanced circuit is equal to the attenuation of one coaxial unit and may be expressed by the formulas previously given, where the various symbols are understood to refer to one unit of the circuit. Accordingly the optimum high-frequency proportions are the same as those previously derived for ordinary coaxial circuits of different types.[19]

As the frequency is reduced, the optimum proportions become different from those for coaxial units, since the circuit inductance ap-

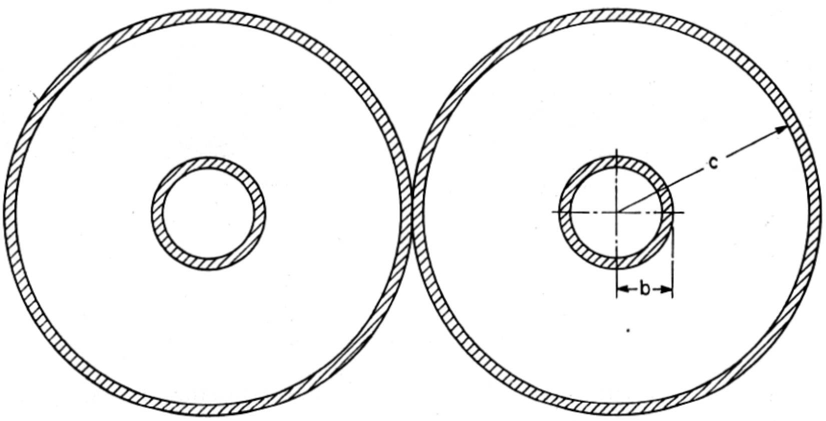

Fig. 13—Double coaxial circuit.

proximates more closely that for a simple pair of wires occupying the positions of the inner conductors, while the capacitance remains equal to one-half of that of one coaxial unit. As a result the optimum diameter ratio is larger than at high frequencies.

Shielded Pair—Round Conductors and Oval Shield

The shield around a pair does not have to be cylindrical. Upon consideration of a pair of round conductors with a cylindrical shield, as shown in Fig. 7, it is evident that the shield approaches quite close to the conductors at the sides, while it is well removed from them at the top and bottom of the figure. This means that for a given area enclosed by the shield the capacitance of the circuit is greater than would be the case if the shield were kept at a more nearly uniform distance from the conductors. Consequently, for a given area circumscribed by the shield, a reduction of attenuation can be secured by changing the shape of the shield.

The problem of determining the shape of shield which gives minimum high-frequency attenuation presents extreme difficulty, and a rigorous solution has not been obtained. However, it appears that a close approach to the ideal shape can be obtained by a shield having the cross-section shown in Fig. 14, which consists of two semi-circles

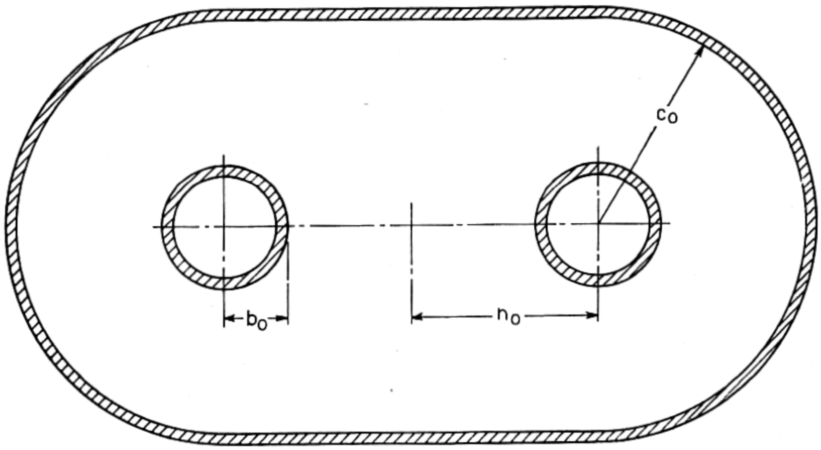

Fig. 14—Oval shielded pair.

joined by straight lines, the inner conductors being placed at the centers of the semi-circles. For convenience this shape of shield will be termed "oval."

The optimum proportioning [20] for a pair of conductors with such an oval shield may be closely approximated by comparison with the pair with circular shield and with the double coaxial circuit. In such comparison the cross-sectional areas of the different circuits will be assumed equal.

Consideration will first be given to the case where the enclosed conductors in Fig. 14 are of solid material. The conductivity of the conductors will be assumed the same as that of the shield, it being apparent that the same methods may be employed in the case of different conductivities. In arriving at the spacing ratio of the conductors for minimum attenuation, the condition for minimum capacitance will be used as a stepping stone. The spacing ratio of the conductors in Fig. 14 may be represented by $h_0/(c_0 + h_0)$. Comparison with Fig. 7 shows that the corresponding ratio for that figure is h/c, which, it has already been seen, should have a value of approximately .486 for minimum capacitance. It is evident that the value of the ratio $h_0/(c_0 + h_0)$ for minimum capacitance in Fig. 14 should be very close to .486, but in view of the concentricity of the conductors with the semi-circular parts of the shield it should be slightly less than this value. It has been found that to obtain minimum capacitance for an oval shielded circuit the spacing ratio should be approximately

$$\frac{h_0}{c_0 + h_0} \doteqdot .47. \tag{57}$$

It has been seen for Fig. 7 that to obtain minimum high frequency attenuation the spacing ratio is shifted from the value of .486, which gives minimum capacitance, to a value of about .46. For Fig. 14, however, the current density in the shield is more uniform, so that the proximity effect between conductors is less completely compensated by the shield currents. Hence the spacing ratio for minimum high-frequency attenuation for the oval shielded circuit should be approximately the same as that for minimum capacitance, as given in (57) above.

There remains to be determined the second condition for minimum high-frequency attenuation for an oval shielded circuit of given cross-sectional area, namely, the optimum value of the diameter ratio c_0/b_0. Comparison with Fig. 13 indicates that the optimum value of this ratio should be fairly close to the optimum value of 3.6 for the coaxial circuit. Comparison with Fig. 7, c_0 being equal to about $.69c$ for equal areas in the two cases, shows that the optimum value of the ratio c_0/b_0 should be slightly greater than 3.6. For practical purposes the optimum may be taken as

$$\frac{c_0}{b_0} \doteqdot 3.7. \tag{58}$$

With this ratio the size of the conductors with oval shield is, for the same cross-sectional area, approximately the same as that of the optimum size of conductors with circular shield.

The capacitance of the pair with oval shield is smaller than the capacitance of the pair with circular shield, because the inner conductors of the former are more widely separated and are farther from the shield. It is very slightly larger than the capacitance of the double coaxial circuit.

The part of the resistance of the oval shielded circuit which is due to the shield will be less than that for a circular shield because of the more uniform current density in the shield. However, as has been noted, the proximity effect between conductors is less completely neutralized by the shield currents than is the case for the circular shield. It appears that these two effects may approximately balance one another, and that the circuit resistance is approximately the same for both oval and circular shielded circuits.

It is found that a circuit of approximately optimum proportions comprising two solid round wires surrounded by an oval shield has about 12 per cent lower attenuation than a circuit with circular shield of equal cross-sectional area.

When the conductors enclosed within the oval shield are stranded there is no increase of conductor resistance due to proximity effect. On this account it is desirable to bring the conductors closer together in order to reduce the shield loss and the optimum spacing ratio will be less than for the case of solid conductors. With stranded conductors the attenuation reduction as compared with the circular shield is greater than in the case of solid wires; for example, if the resistance ratio (m) is .7, the attenuation with oval shield will be about 25 per cent less than that of the circular shield.

The circular form of shield is ordinarily the most convenient and practical one. A disadvantage of an oval shield as compared thereto is unequal stiffness or resistance to bending in different directions.

Shielded Pair—Quasi-Elliptical Conductors

It has been suggested at different times that the ordinary round form of conductor, while well adapted for manufacturing purposes, may not be the theoretically optimum shape for many types of high-frequency transmission circuits. Speculations in this respect have differed greatly, and a large variety of non-circular shapes of conductors have been proposed, including flat strips, strips with concave or convex faces opposite one another, angular forms, etc. However, except in the case of the coaxial circuit, for which the circular form is clearly the optimum, there has been, so far as the authors are aware, no exact analytical determination of the optimum conductor shape for a given type of circuit.

A complete treatment of possible problems of this kind would extend to great length. It is worth while, however, to consider a single problem, namely, that of determining what shape and spacing for a pair of conductors with circular shield will result in minimum high-frequency attenuation. This problem is of particular interest inasmuch as the circular shape is ordinarily the most convenient and practical one for a shield.

In attacking this problem the fundamental principles which determine the high-frequency attenuation of a circuit comprising a pair of conductors surrounded by a shield may be briefly examined. At high frequencies, where the currents are crowded toward the surfaces of the conductors, the attenuation is proportional to the product of the resistance and capacitance of the circuit, both of which are functions of the flux density in the dielectric.

With a circular shield, and round conductors, the flux density is far from uniform around the surfaces of the conductors, being relatively high at points nearest the shield and also at points nearest the shield's center, and a minimum at points about half-way between. Accordingly, it appears that the high-frequency resistance of the conductors can be reduced by reshaping them so as to make the flux distribution more uniform. This can be accomplished by squeezing the conductors at regions of maximum flux density and bulging them at regions of minimum flux density, thereby producing a conductor of approximately elliptical cross-section.

The flux distribution around the shield is also far from uniform, being a maximum at points nearest the conductors and a minimum at points 90 degrees away. Making the enclosed conductors elliptical tends to reduce this non-uniformity, thereby reducing the circuit resistance due to loss in the shield.

This process of reshaping the conductors can not be carried very far, however, because it soon increases the circuit capacitance more than it decreases the resistance. It is difficult to treat this problem by rigorous mathematics, but an analysis can be made which yields an approximate solution.

For certain conductor shapes, the high-frequency attenuation of a pair with circular shield may be determined by a method involving the substitution of charged filaments for the conductors. Let any number of positively and negatively charged filaments be included in the shield, the net charge on the filaments being zero. The electrostatic potential at any point of this system can readily be determined by known methods. Thus, for example, Fig. 15 shows the location of the equipotential surfaces for the case of two oppositely charged filaments placed within a circular shield, the distance from each filament to the center of the shield being .46 times the shield radius.

In any such system, a conducting cylinder whose external surface corresponds to, and whose potential is equal to the potential of, a particular equipotential surface may be substituted for the part of the system contained within that surface without disturbing the flux distribution external to it. Consequently, the capacitance of a shielded circuit employing equal and oppositely charged conductors having the same shape as any two corresponding equipotential surfaces of the electrostatic system can be determined.

The flux density at any points on the conductors or on the shield is proportional to the rate of change of the potential with respect to the normal to the surface at that point. The high-frequency resistances of the conductors and shield, respectively, are proportional to the

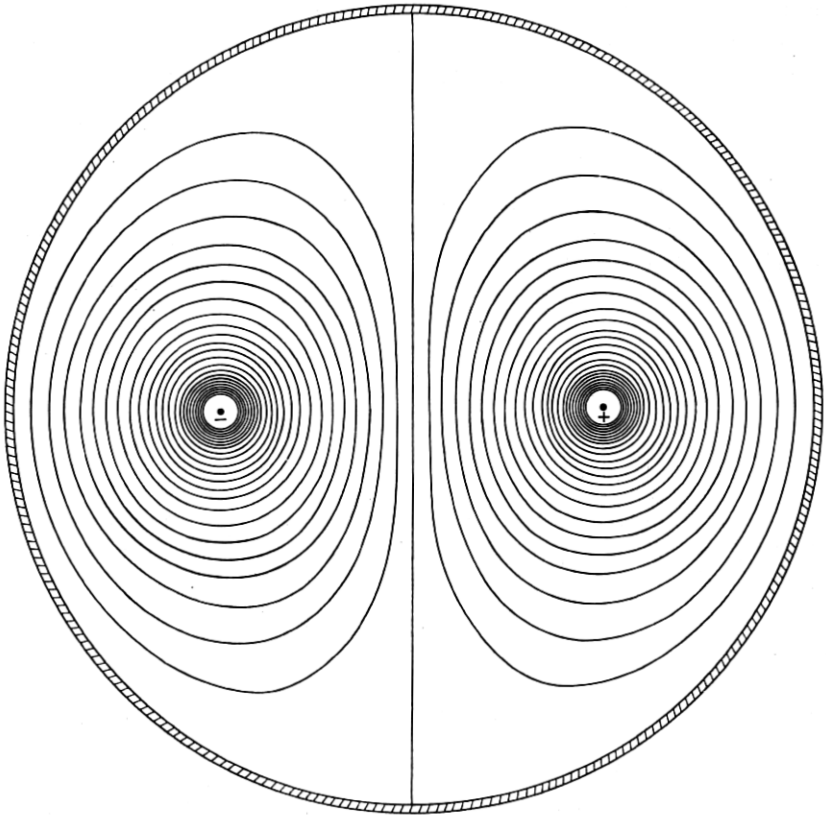

Fig. 15—Equipotential lines around shielded charged filaments.

integral of the square of the flux density around their periphery. Thus the high-frequency resistance of the circuit may be determined, and from this and the capacitance, the high-frequency attenuation.

This method makes it possible to determine and compare the high-frequency attenuations of conductors having shapes corresponding to the equipotential surfaces for various assumed arrangements and numbers of charged filaments. If, however, the problem be that of determining the attenuation for a given shape of conductor, there may be great difficulty in finding the arrangement and number of filaments which will produce an equipotential surface to coincide with the given shape.

By applying this method to a series of approximately elliptical conductors previously shown to be of the shape that would be expected to have lower attenuation than circular conductors, what is considered a close approximation to the optimum shape of conductor for a pair with circular shield has been arrived at. This is approximately an ellipse whose major axis is about 5 per cent longer than its minor axis, the latter being in line with the center of the shield. The high-frequency attenuation of a circuit with circular shield and conductors of this shape is approximately 2 per cent lower than that for the same shield with round conductors. This reduction does not appear enough to offset the practical difficulties involved with conductors of such shape.

Shielded Quad

The number of conductors enclosed within a shield, instead of being one, as in the coaxial, or two, as in the shielded pair, may be more. By placing four conductors within a common shield, two separate balanced-to-ground circuits may be obtained. If sufficiently good balance can be obtained between these circuits, the total frequency band which can be transmitted within a given cross-sectional area may be increased. To obtain balance, the plane of the conductors of one circuit needs to be at right angles to that of the other circuit and all conductors should be equidistant from the axis of the shield. The pairs may be twisted or spiralled about the axis of the shield.

An arrangement of this kind is shown in Fig. 16, where four round conductors are placed within a circular shield to form a shielded quad, or, as it is frequently described when the conductors are twisted, a "shielded spiral four." Diagonally opposite conductors are used as the sides of a circuit.

Approximate formulas for the high-frequency attenuation of either circuit of Fig. 16, when the enclosed conductors are solid, have been derived in unpublished work of Mrs. S. P. Mead and S. A. Schelkunoff. The optimum high-frequency proportioning of the system, assuming the same conductivity for both enclosed conductors and assuming gaseous dielectric, has been determined by Mrs. Mead. The results are shown in Figs. 17 and 18, where the optimum diameter ratio and spacing ratio are plotted as functions of the ratio of the conductivity of the enclosed conductors to that of the shield. For the case of equal conductivities of conductors and shield the optimum values are

$$\rho = \frac{c}{b} = 6.8; \qquad \sigma = \frac{h}{c} = .49. \tag{59), (60}$$

These values may be compared with 5.4 and .46, respectively, for the pair of round conductors with circular shield. The high-frequency attenuation of each shielded quad circuit with optimum design is,

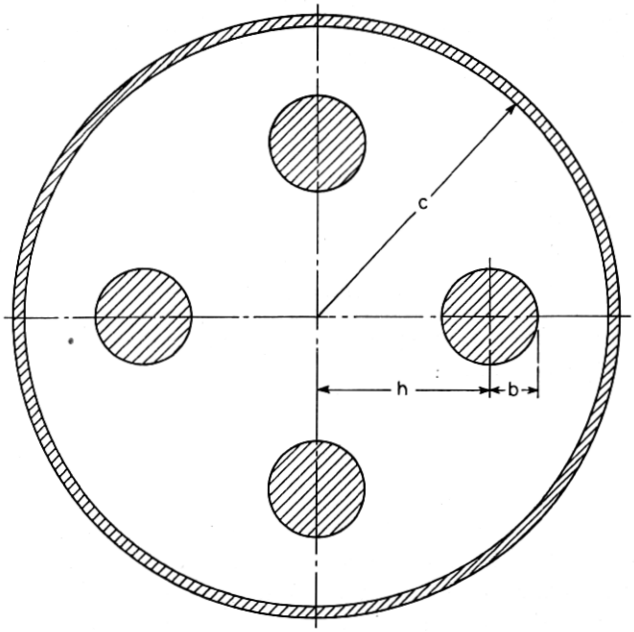

Fig. 16—Shielded quad.

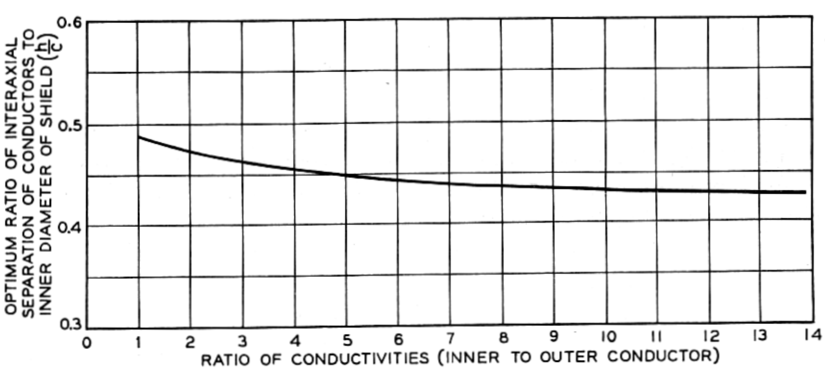

Fig. 17—Variation of optimum spacing ratio of shielded quad with conductivity ratio.

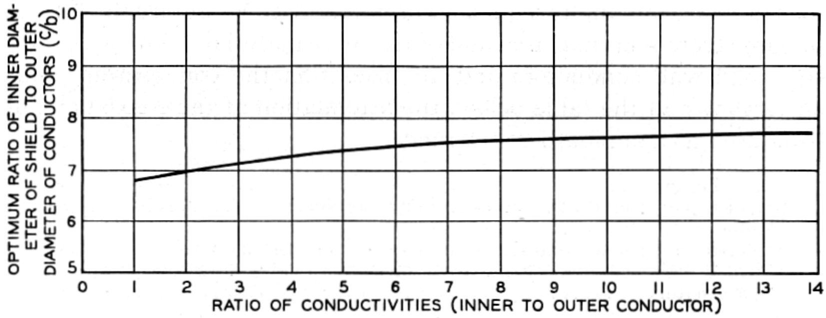

Fig. 18—Variation of optimum diameter ratio of shielded quad with conductivity ratio.

for the same diameter of shield, about 10 per cent higher than that of a shielded pair for its optimum design.

Conclusion

There have been discussed a number of different types of individually shielded circuits, both balanced and unbalanced, and the proportioning of these circuits for minimum high-frequency attenuation has been determined. The following table summarizes the optimum proportions for the more important circuits treated above. The values given are for the case where all the conductors are of the same material.

Circuit	Diameter Ratio (ρ)	Spacing Ratio (σ)
Simple coaxial	3.59	—
Double coaxial	3.59	—
Shielded pair, round conductors and circular shields	5.4	0.46
Shielded pair, round conductors and oval shield	3.7	0.47
Shielded quad	6.8	0.49

Of the transmission characteristics of these circuits, a property of particular interest is the attenuation, since, assuming adequate shielding, it is this which determines either the required repeater spacing for a given transmitted frequency band or the width of frequency band obtainable with a given repeater spacing. For each type of circuit considered there has been determined the ideal proportioning whereby the high-frequency attenuation of the circuit may be minimized. In addition a variety of methods for the solution of problems in optimum proportioning have been outlined.

It is, of course, feasible by adjustment of size to obtain the same high-frequency attenuation for all these different types of circuits. However, the size of a structure is usually reflected in its cost. An interesting picture can therefore be drawn by comparing the attenuations, at the same high frequency, of different types of circuits having the same cross-sectional area and of the same material. For structures with solid wall conductors and air insulation the comparison works out as shown in the table below, the attenuation of the coaxial circuit being used as a standard of reference.

Coaxial circuit	1.00
Shielded pair, round conductors and circular shield	1.50
Double coaxial circuit	2.00
Shielded pair, round conductors and oval shield, approximately	1.3
Shielded pair, circular shield with quasi-elliptical conductors, approximately	1.47

In each case the cross-sectional area is taken as that enclosed within the shield. This neglects any differences in the thickness of shield that may be required.

A specific comparison of considerable interest is that between an unbalanced coaxial circuit and a shielded pair, the latter being taken as representative of shielded balanced circuits. The table shows that, for the same attenuation, the cross-sectional area included within the shield is larger for the shielded pair than for the coaxial circuit. On the other hand, the use of balance in addition to shielding is advantageous in that it reduces the amount of shielding needed. The shielded pair makes possible the utilization of the entire frequency range, if desired, whereas with a coaxial circuit it is necessary to discard the lower frequencies where it is uneconomical to provide adequate shielding.

A thorough-going comparison of the relative advantages and fields of application of the various types of circuits which have been discussed would extend to great length. Clearly a large number of factors enter into the choice of the configuration of shielded high-frequency circuit to be used in any given instance. These factors include the width of frequency band to be transmitted, the degree of shielding required, the relative economy of manufacture of different structures, etc. While a complete exposition of these factors has not been attempted, the principles of optimum proportioning which have been discussed should be helpful in selecting the best configuration to meet given requirements, and the particular configuration chosen should be made to conform reasonably closely to the optimum.

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