The Present Status of Ferromagnetic Theory *

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DISCOVERY of the loadstone and some of its magnetic properties is now reputed to be some three thousand years old. During these many years ferromagnetism has resisted very successfully the attack of theorists, and even at the present time theory lags far behind experiment. But advances in theory have been particularly rapid during the last five or ten years; the author describes in this paper what he regards as the high points of this progress.

Not until the last quarter of the last century was any considerable work done on magnetic materials. During this period data were gathered rapidly until, just before the close of the century, an excellent book [2] of four hundred pages, containing practically all of the important experimental and theoretical facts, was written by J. A. Ewing, later Sir James Ewing. The shape of the magnetization curves of iron, cobalt, and nickel, the existence of magnetic saturation and the magnetic transformation temperature, the existence and some of the laws of hysteresis, the simpler effects of stress and of magnetostriction, together with the important methods of measurement—all were known then, and silicon steel had just been invented.

Strangely enough, during the next fifteen years there was but little advance in knowledge of magnetic materials, but there were many applications of existing knowledge by engineers to electrical machinery, including those in electrical communication. During this period, also, the Heusler alloys (non-ferrous alloys exhibiting ferromagnetic properties) were invented; and although these served to stimulate those interested in the theoretical aspects of ferromagnetism, still there was little progress.

Beginning between 1915 and 1920 and extending to the present, there has been a rapid development on both the experimental and theoretical sides of ferromagnetism. To illustrate the progress that has been made in the improvement of magnetic materials, Table I has been prepared. The improvements made during the last 20 years have resulted from new methods of purification of the materials, new

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* This paper as herein published contains a few revisions and additions to the paper as it appeared in the November 1935 issue of *Electrical Engineering*. It is scheduled for presentation at the A. I. E. E. Winter Convention, New York, N. Y., January 28–31, 1936. A subsequent paper in the same field of endeavor is planned in which entirely new material will be presented.

TABLE I

SOME EXTREMES IN THE PROPERTIES OF MAGNETIC MATERIALS AVAILABLE IN 1915, AND IN 1935

Material	Property	Value 1915	Value 1935
Iron	Maximum permeability	45,000[11]	340,000[12]
	Initial permeability	300	20,000[12]
	Coercive force in œrsteds	0.3[11]	0.03[12]
Iron-nickel [13]	Maximum permeability	2,800[14]	600,000[15]
	Initial permeability	700[16]	12,000[17]
	Coercive force in œrsteds	1.5[14]	0.01[15]
Silicon-iron	Initial permeability	400	2,000[12]
Iron	Hysteresis at $B_M = 100$ gausses, in ergs per cm.^{3} per cycle	20	0.1[12]
Iron-cobalt-nickel "perminvar"	Hysteresis at $B_M = 100$ gausses, in ergs per cm.^{3} per cycle	....	0.000003[18]
Iron-cobalt	Saturation value in gausses [19]	25,800	25,800
	Permeability at $B = 16,000$ gausses	2,100	19,000[18, 12]
Tungsten steel	Coercive force [4] in œrsteds	80	80
New K. S. steel	Coercive force [21] in œrsteds	....	900

Superior numerals refer to references at end of paper.

compositions (alloys), and new methods of heat treatment. Some of these figures refer only to laboratory specimens, and not to materials available in commercial quantities.

But the chief topic of this paper is the theoretical side of ferromagnetism. How is one to explain the different values of magnetic permeability, ranging from 1 to 600,000 for various materials? Or, to consider first the more fundamental questions, what is the elementary magnetic particle, and why is ferromagnetism associated with so few elements?

ORIGIN OF FERROMAGNETISM

It was suggested by Ampère about one hundred years ago that molecules might behave as magnets because of the electric currents circulating in them. Today, with the advance in knowledge of atomic structure, the origin of ferromagnetism can be discussed in more specific terms. Strangely enough, the spectroscopists have supplied, so to speak, the elementary magnetic particle. It is the spinning electron. In order to explain their extensive observations on spectral lines, they found it necessary to revise the picture of the atom. For some time it has been supposed that an atom was made of a heavy nucleus with a positive charge and of electrons moving in circular or elliptical orbits around the nucleus. To this picture now must be added the idea that each electron itself is spinning about an axis that passes through its center. Thus, there is circulation of electricity in an atom, both around the nucleus and within each electron—and the latter motion is called the "electron spin" because of its similarity to a spinning ball. Each electron in an atom is then a small gyroscope, possessing a definite magnetic moment on account of its moving electrical charge and a definite angular momentum on account of its moving mass. The ratio of these two quantities is known from various independent lines of reasoning and evidence to possess a particular value. Electrons revolving *in orbits* also exhibit both magnetic moments and angular momenta due to their orbital motions, but for these the ratio is just half what it is for the spinning electron.

The Barnett experiment[22] shows in a very direct way the existence of these magnetic and mechanical moments of the electron and confirms the ratio between them in ferromagnetic materials (Fig. 1). A rod of iron is hung from a fine suspension and then is magnetized suddenly, whereupon the rod is observed to turn, twisting the suspending fiber a minute but measurable amount. The spinning electrons responsible for ferromagnetism have been turned by the applied field so that they are more nearly parallel to it; but the mechanical moment, which is also a property of those same electrons, causes the whole rod to rotate in just the way that a gyroscope would. Or, to put it differently: When the elementary magnets, pointing originally in all directions, are turned more nearly into parallelism with the axis of

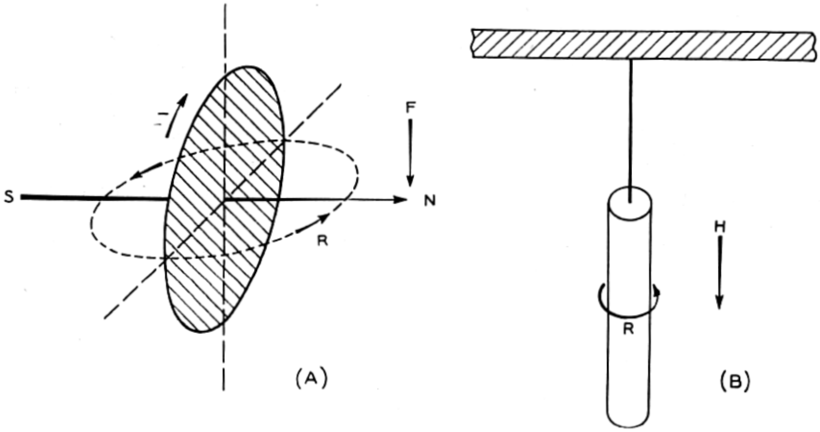
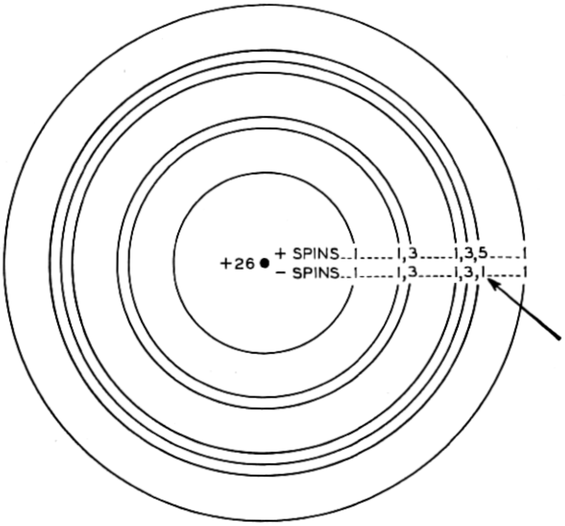


Fig. 1—Gyroscopic action (left); force F produces rotation R. Gyromagnetic effect (right); field H produces rotation R.

the rod by the applied field, they acquire a net angular momentum parallel to that axis. By the principle that action must be balanced by a corresponding reaction, the rod itself now must recoil with an equal and opposite momentum; it is this last that manifests itself by the sudden twist of the rod and may be calculated from the measured value of the twist. Its sign shows that the spinning magnetic particle is charged negatively, and its magnitude is what would be expected from the hypothesis that that particle is a spinning electron. Thus a change in magnetization is fundamentally a change in the direction of the spin of the electrons in the atom, and not a change in orientation of the whole electron orbit.

The next question is: Why is not every substance ferromagnetic? The picture of the atom of iron as now envisioned by the experts in this field, is represented by the diagram in Fig. 2. The twenty-six electrons in iron are divided into four principal "shells," each shell a more or less well defined region in which the electrons move in their orbits, and some of these shells are subdivided. The first (inner) shell contains two electrons, the next shell eight, the next fourteen, and the last two. As the periodic system of the elements is built up from the lightest element, hydrogen, the formation of the innermost shells begins first. When completed, the number of electrons in the first four shells are two, eight, eighteen, and thirty-two, counting outward,



	+ SPINS	− SPINS	EXCESS SPINS
Cr	4	0	4
Mn	5	0	5
Fe	5	1	4
Co	5	2	3
Ni	5	3	2
Cu	5	5	0

Fig. 2—Electron shells in the iron atom.

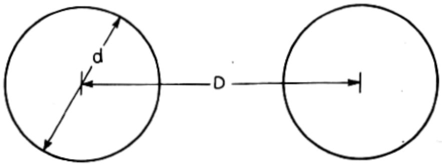
but the *maximum* number in each shell is not always reached before the next shell begins to be formed. For example, when formation of the fourth shell begins, the third shell contains only eight electrons instead of eighteen; it is the subsequent building up of this third shell that is intimately connected with ferromagnetism. In this shell some electrons will be spinning in one direction and others in the opposite, and these two senses of the spins may be conveniently referred to as positive and negative. The numbers inserted in Fig. 2 show how many electrons are present in each shell which have positive and negative spins, and it may be noticed that in the iron atom all of the shells except the third contain as many electrons spinning in one direction as in the opposite. The magnetic moments of the electrons in each of these shells mutually compensate one another so that the shell is magnetically neutral and cannot have magnetic polarization. In the third shell, however, which is not yet filled to this extent, there are five electrons with a positive spin and one with a negative so that four electron spins are (unbalanced or) uncompensated and there is a resultant polarization of the atom as a whole. If one more positive charge and its associated mass (a proton) be added to the nucleus and one more electron to an outer shell, the iron is transformed into cobalt; and by repeating the process, the cobalt is transformed into nickel. In iron these additional electrons and their spins are so oriented that there is what may be called an excess spin of four units in iron, three in cobalt, and two in nickel. In manganese, the element just preceding iron in the periodic table, there is an excess of five spins. Only in incomplete shells such as this, shells that are being filled as new and heavier atoms are made, is there such excess spin. The completed shells are magnetically neutral because the spins mutually compensate one another.

The outermost electrons are those responsible for the ordinary chemical properties, and they are influenced by chemical combination. They do not contribute to ferromagnetism for reasons that will appear later.

Exchange Forces

Only in certain parts of the periodic table are there found electrons being added to *inner* shells, and one of these places is in the iron group; but since there are other parts, notably those occupied by the palladium, platinum, and rare earth metals, where these inner groups are being filled, there arises the further question: Why are not these other elements also ferromagnetic?

For an element to be ferromagnetic, it is necessary not only that there be uncompensated spin in the electron orbits, but also that the resultant spins in neighboring atoms be parallel. Calculation of the energies of the electrons indicates that to align the spins in all the atoms in a small region, the diameter of an atom must bear the proper ratio to the diameter of the electron shell in which the electron spins are uncompensated [23] (Fig. 3). This proper ratio is required because



In ferromagnetic substances, D/d is greater than 1.5 (Slater)

Fig. 3—"Incomplete" shells in neighboring atoms.

the electron spins and charges influence each other to an amount depending upon the distance between them; and it is only when this influence, which is known technically as the "exchange," has the right value that the spins all can be aligned in the same direction,[24] that is, that the material can become ferromagnetic.[25]

The forces of "exchange," the existence of which has been realized only in the last few years, act to keep the spins parallel, while thermal agitation tends, obviously, to disturb this alignment. When the temperature is high enough, the temperature agitation prevails and the material ceases to be ferromagnetic. This temperature is the familiar Curie point, or magnetic transformation point, 770 degrees centigrade (or 1,043 degrees absolute) for iron. It is seen then that the height of the Curie point, θ (on the absolute temperature scale) is an indication of the strength of the forces of exchange, which cannot yet be calculated theoretically except as to order of magnitude. These Curie points are plotted in Fig. 4 for the elements near iron in the periodic table; if a continuous curve be drawn through the points, it has a maximum near cobalt. Now the saturation value of magnetization depends both on the exchange and on the number of effective electron spins, that is, upon the number of electrons that can be oriented parallel to the field and the strength of the forces that hold them parallel. In a very rough way, it may be said to depend on the product of the exchange and the number, S, of uncompensated spins in the atom. Adopting θ as a measure of the exchange forces and forming the product θS, the right-hand curve in Fig. 4 is obtained, which indicates that the highest saturation should be attained in an iron-cobalt alloy, and that under certain appropriate conditions manganese might be ferromagnetic. Both these indications are substantiated by the data: The only known alloys having a higher saturation value than pure iron are the iron-cobalt alloys; and compounds and alloys of manganese are more magnetic than any others that do not contain iron, cobalt, or nickel. The Heusler alloys, composed of manganese, aluminum, and copper, have a saturation almost as high as nickel, and numerous compounds of manganese are ferromagnetic in a less degree.

The forces of exchange are purely electrostatic in origin. But they are not electrostatic in the classical sense of the word; they are the result of electric charges distributed in space in a definite way. It does not seem to be possible to describe them easily in words, for it takes a great many mathematical equations to derive the result, which is a consequence of the assumptions of quantum mechanics. These forces account for the fact that it is easy to align the excess spins of large groups of atoms of some materials. In fact, when the forces of exchange are large as they are in ferromagnetic materials, the stable situation is one in which the spins are parallel, even when no magnetic field is applied. But the parallelism under such circumstances does not extend over the whole of a specimen of ordinary or even of visible size; for some reason not understood it is limited to smaller regions. On the average, those regions are found experimentally to have the volume of a cube about 0.001 inch on an edge. *An actual ferromagnetic body is composed of a great many such regions, called "domains," each domain being magnetized to saturation (i.e., electron spins parallel) in some direction.* When the material is said to be unmagnetized, the domains are oriented equally in all directions so that the magnetization of the specimen as a whole is zero.

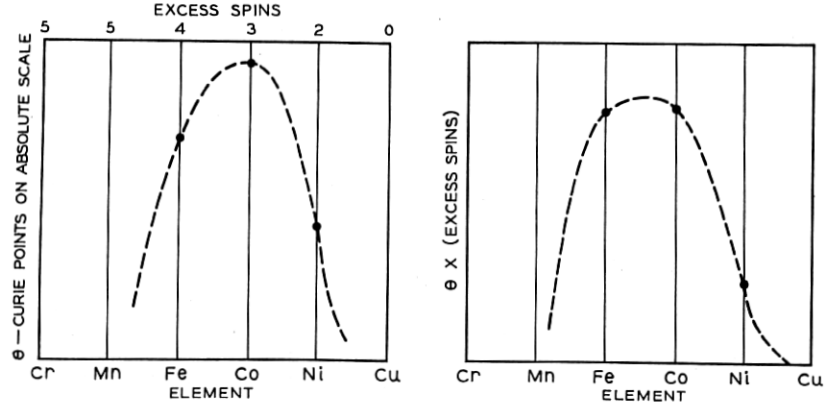


Fig. 4—Iron-cobalt alloys have the highest value of saturation magnetization.

Experimental evidence of the existence of these domains is supplied by the so-called "Barkhausen effect" (Fig. 5). If a small portion of a magnetization curve such as is shown in Fig. 5 could be magnified a billion times, it would be seen to be made up of steps, each a sudden change in magnetization as the field is increased, with no further change until the field reaches a certain higher value. No known apparatus can give such direct magnification, but these sudden jumps can be detected by winding a coil around the specimen and connecting its ends to an amplifier at the output of which is a pair of telephone receivers. When the field is slowly increased, a series of clicks, or "noise," is heard in the receivers; a more quantitative method shows that the average click corresponds to the reversal of magnetization in a region the size [26] of a cube 0.001 inch on an edge, containing 10^{15} atoms. Under favorable conditions this "Barkhausen noise" can be heard without an amplifier, with the receivers connected directly to the coil.

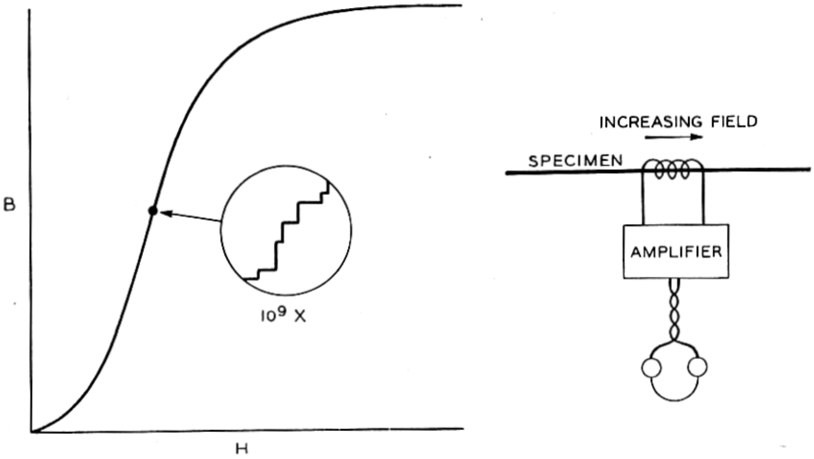


Fig. 5—Sudden changes in magnetization cause the Barkhausen effect.

It has been pointed out that the forces of exchange are opposed by the disordering forces of temperature agitation. As a result, the saturation value of magnetization decreases continuously as the temperature is increased, until at the Curie point the ferromagnetism disappears. Data for saturation at various temperatures are shown in Fig. 6, plotted in such units that the saturation is unity at the absolute zero of temperature, and the Curie point is unity on the temperature axis. On such a plot it is found that the data for iron, cobalt, and nickel fall close together. The lower curve is the theoretical one calculated thirty years ago on the assumption that the elementary magnets, when they are disturbed by temperature agitation, can assume any orientation. If it be assumed, on the contrary, that the spinning electrons responsible for ferromagnetism can assume only two orientations with respect to the other electrons in the atom, the upper curve is the result. If we assume that four orientations are possible the calculated curve lies close to the upper curve of Fig. 6, but somewhat below it, and as the number of possible orientations is increased the curve approaches the lower one shown in the same figure. The close agreement between the data and the upper calculated curve is of special interest because the spectroscopists and atomic structure experts have come independently, each group from its own data, to the conclusion that each electron in an atom can assume only a small number of orientations with respect to the rest of the atom.

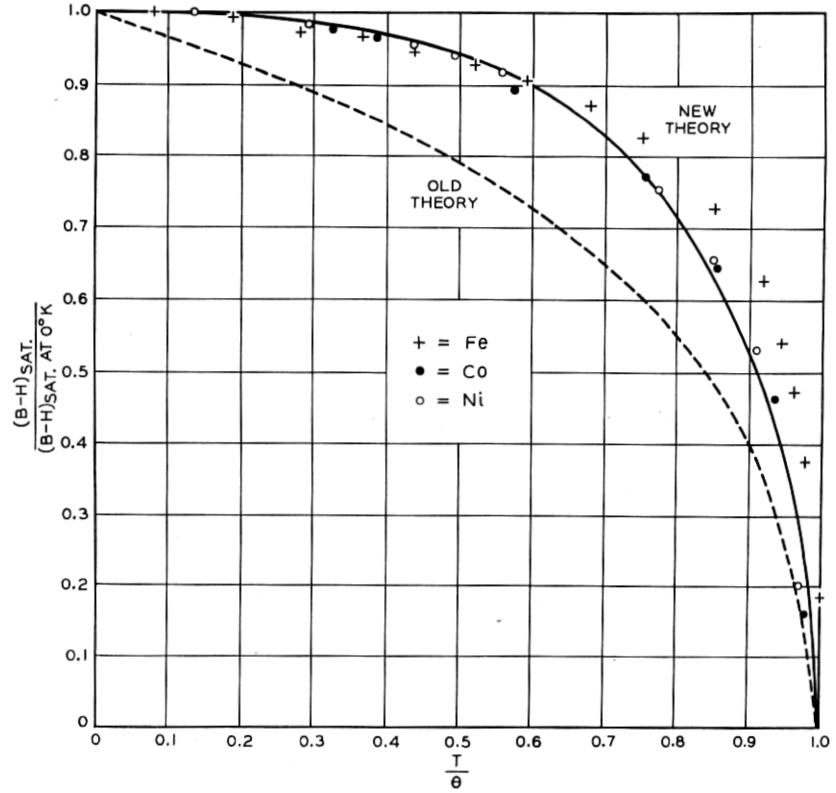


Fig. 6—Dependence on temperature of the saturation magnetization of iron, cobalt, and nickel.

Effect of Crystal Structure

There is another kind of force that must be postulated in order to explain the properties of a single crystal. Because of the spinning electrons which it contains, and also because of their orbital motions, each atom may be regarded as a small magnet. These magnets will influence each other in a purely magnetic way,[27] just as a group of bar magnets will; and in a crystal it may be readily appreciated that because of these magnetic forces between atoms arranged in a regular fashion, some directions of magnetization are more stable than others. In iron the most stable direction is observed to be that of the cube edge, one of the cubic axes of the crystal. In nickel it is the cube diagonal (Fig. 7).

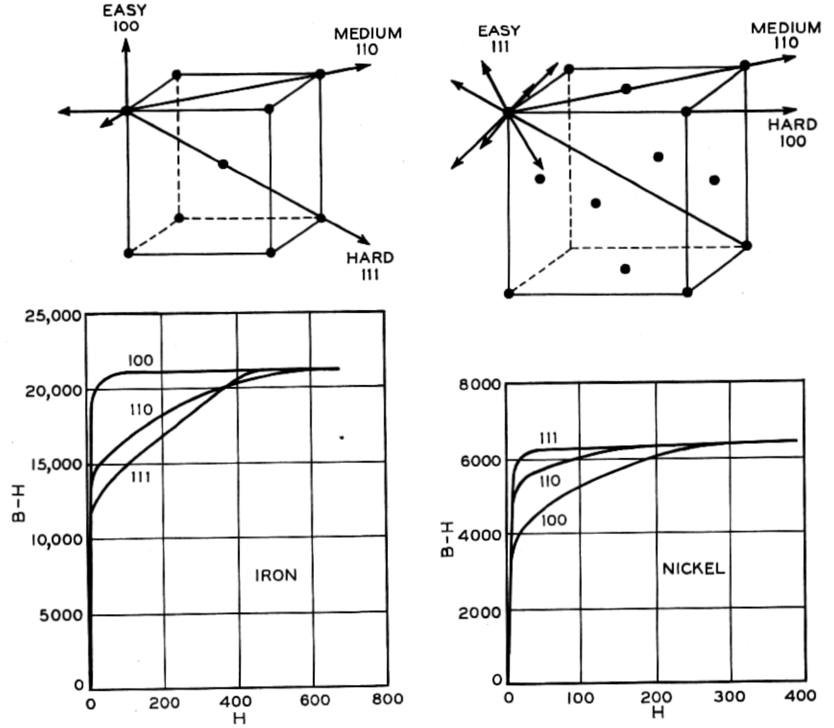


Fig. 7—Magnetic properties and crystal structures of single crystals of iron and nickel (Beck, Honda and Kaya, Webster).

Ordinarily a piece of iron is composed of crystal grains each one of which is too small to be detected by the naked eye. In recent years, however, means have been found to control the grain size of all the common metals, and single grains (i.e., single crystals) have been prepared which are so large that experiments may be performed and data collected on just one such crystal.

The structure of a single crystal of iron may be represented by a cube with an atom at each corner and one in the center, the whole crystal made up of such cubes packed together face to face. It is found experimentally that in the direction of an edge of this cube (called by the crystallographers a [100] direction) the magnetization curve labeled 100 in Fig. 7 is obtained.[28] In the two other principal directions, the direction of a face diagonal and that of a cube diagonal, the other magnetization curves are obtained, as shown. The difference in the initial parts of the magnetization curves is negligible, the effects being large only above half saturation.

The structure of nickel may be represented also by an assemblage of cubes, but the atoms are arranged in a different manner, being at the corners of the cubes and the centers of the cube faces (Fig. 7). The magnetization curves for nickel corresponding to the same three principal directions are shown also in Fig. 7, and it may be seen that the curves are reversed in order from those of iron. In iron the [100] direction is said to be the direction of easy magnetization and the [111] the direction of most difficult magnetization, whereas the reverse is true in nickel. It might be said that the electrostatic exchange forces align the spins parallel to each other and that the crystal forces determine the particular crystal direction along which they shall be aligned. The forces of exchange are so powerful that they are able to align the spins of a group of atoms, a situation that in the absence of such exchange forces could be accomplished at room temperature only by an applied field of 10,000,000 œrsteds. On the other hand, the crystal forces are so feeble that it takes only 1,000 œrsteds to redirect the spins of an entire group of atoms from any direction to any other direction. The ratio between these two equivalent fields is thus 10^7 divided by 10^3, or 10^4.

As a result of the forces of exchange and the magnetic crystal forces in a single crystal of iron, for example, the situation is as represented in Fig. 8. Even when the crystal is apparently unmagnetized, or demagnetized, there are small regions, called domains, that are magnetized to saturation in one of the six equivalent directions of the crystal axes. Actually, the domains vary considerably in size and shape, but are represented conveniently as squares. Each of the six directions is equally stable and equally probable when no field is applied. The initial effect of applying a magnetic field is to change the direction of magnetization from one stable position to another, thus

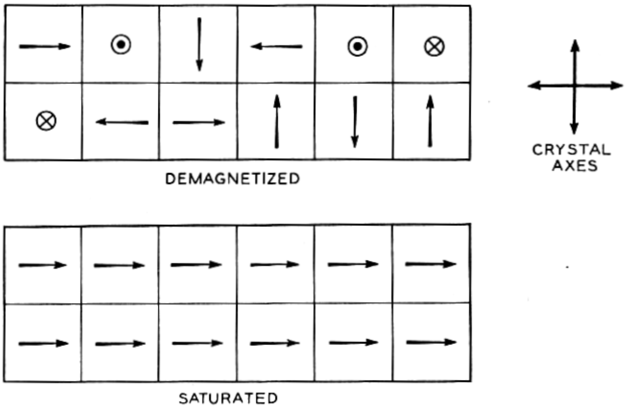
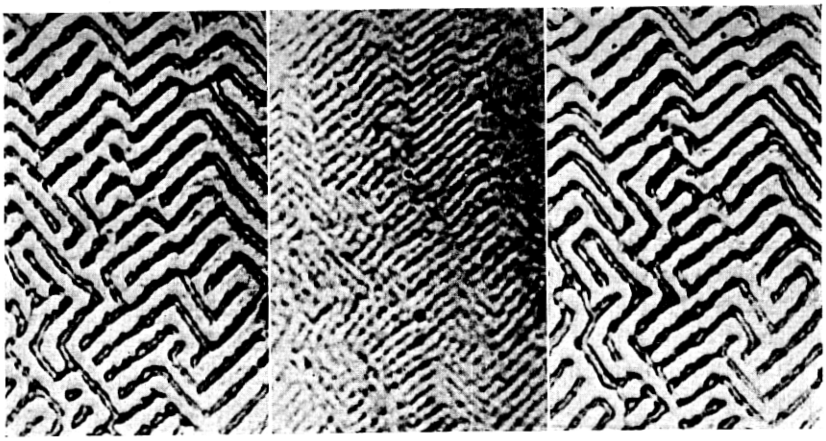


Fig. 8—Domains in a single crystal of iron.

increasing the resultant magnetization in the direction of the field. These changes take place suddenly, and they are the cause of the Barkhausen effect; each sudden change in orientation of a domain accounts for one step in the magnified magnetization curve shown in Fig. 5, or for one click heard in the telephone receiver when listening to the Barkhausen effect.

There is even more direct evidence of the existence of domains in a piece of iron. The iron is placed under a microscope with a magnification of 500, and is covered with a colloidal suspension of iron oxide.[29] It is found (Fig. 9) that the colloidal particles are concentrated along



(a) (b) (c)

Fig. 9—Powder-patterns for iron (McKeehan and Ellmore): (left) field outward; (middle) demagnetized; (right) field inward.

lines determined by the crystal axes, indicating that stray magnetic fields go in and out of the surface just as if some sections were magnetized differently from their neighbors. This occurs even when the iron is unmagnetized, but never occurs with materials that are not ferromagnetic.

Now consider in more detail by what processes changes in magnetization occur. Most changes are attributable to the reorientation of electron spins in domains, from one direction of easy magnetization to another (Fig. 10). These are the changes that take place over the large central portion of the magnetization curve. In general, however, it is obvious that this process is complete before the material is saturated. When all the domains are magnetized parallel to that direction of easy magnetization which is nearest to the direction of the applied field, the only way in which the magnetization can be increased further is by rotating the electron spins in each domain out of the stable position toward the field direction. Such a process is described loosely as the "rotation of the domain." This is the process that

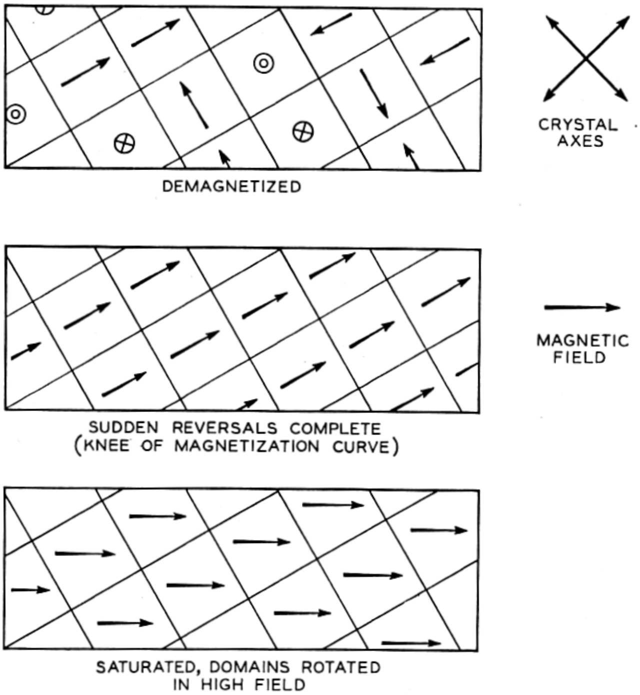


Fig. 10—As the magnetic field strength increases, domains first change direction suddenly, then rotate smoothly.

occurs in high fields, of the order of 10 to 100 œrsteds; as may be seen in Fig. 7, its beginning corresponds to the place where the curves suddenly bend over, away from the almost vertical section. It is only when the field is applied to a single crystal in the direction of easiest magnetization that this last process is avoided. When the field is applied in the direction of most difficult magnetization, the rotational process begins at a field-strength lower than in any other case.

One other important property of single crystals is accounted for by this picture. This property is evident when a field is applied to a single crystal in a direction not parallel to a principal axis. For example, let the field be applied 30 degrees from a cubic axis of an iron crystal, as indicated in Fig. 11 by the longest arrow. As this field is increased from zero, the magnetization will correspond in magnitude and direction to the other arrows shown. First it is parallel to the field, but as the field increases it deviates toward a direction of easy magnetization until finally it is saturated in that direction. As the field is increased further, the magnetization approaches again the direction of the field and finally is saturated in this direction. Theory

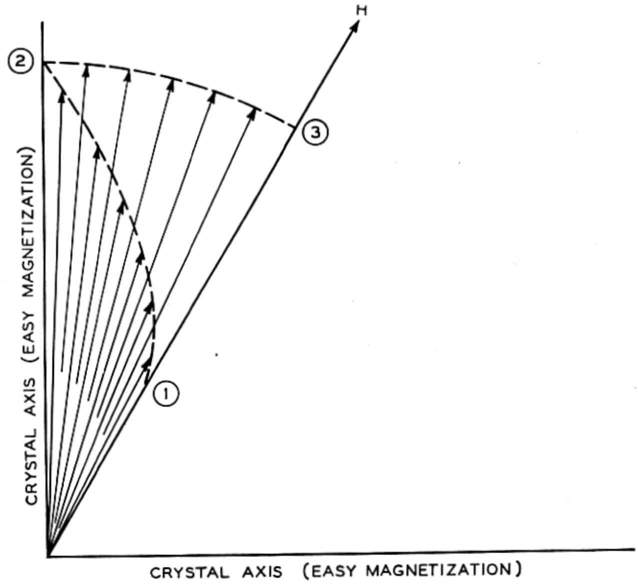


Fig. 11—Vectors represent B-H in iron, increasing in magnitude as the magnetic field (H) increases. First B-H is parallel to H (1); then as B-H increases it deviates in direction from H (2); and finally in high fields is again parallel to H (3).

agrees with experiment in that it predicts [30] the direction and amount of the deviation of B from H for any given value of B.

The two ways of changing magnetization that have been described for single crystals, namely sudden changes to new directions of easy magnetization, and continuous rotation of domains, apply equally well to ordinary polycrystalline material, the properties of the latter being those of the former averaged for all orientations. One result of this averaging, of course, is that the specimen is now isotropic and B is parallel to H.

These last remarks must be qualified, for the magnetic materials used by engineers are not always isotropic, that is, the crystal axes are not always distributed equally in all directions. It has been known for many years that when a metal sheet is rolled, the crystals composing it tend to be oriented in special ways with respect to the direction of rolling and to the rolling plane. Even after the sheet has been annealed and recrystallized, these special orientations exist, in some metals all the way up to the melting point. Since the magnetic properties depend on the crystal direction in a single crystal, it follows that sheets composed of crystals having special orientations will not have the same magnetic properties in all directions. This was observed some years ago in iron, nickel, and iron-nickel alloys.[31] More recently, there has appeared on the market a silicon-iron [32] alloy for which the permeabilities in different directions are markedly different. Measured parallel to the direction of rolling this material has a permeability in high fields (B = 15,000 gausses) of 4,000, while measured at right angles to the direction of rolling the permeability is only 400. X-ray analysis shows [33] that the crystals in this material are aligned so that most of them have a cubic axis lying within a few degrees of the direction of rolling. Thus the direction of rolling coincides with the direction of easy magnetization.

In considering the properties of single crystals, the properties in very low fields have not been considered, chiefly because precise data for single crystals are very difficult to obtain. The process that occurs in this region in single crystals and polycrystals must be different from either of the two so far considered, because in ordinary polycrystalline material there are no discontinuities in magnetization, i.e., no Barkhausen effect, and also the fields are not strong enough to rotate the domains to any significant extent against the crystal forces, out of a direction of easy magnetization. Knowing the relation between magnetic force and angular displacement in high fields, it is calculated that if this same mechanism applied to changes in magnetization in very low fields the highest value of initial permeability in iron would be about 20 instead of many thousands. In the past the process occurring in low fields has been the cause of much speculation, but recently a satisfactory explanation seems to have been found.[5] The changes that take place here are visualized as displacements of the boundaries of domains (Fig. 12); the transition region of a few atom diameters (calculated from the forces of exchange to be about 30 atom-diameters [7]) moves so as to enlarge a domain magnetized in the direction of the field at the expense of a domain pointing in a less favorable direction. Such a movement can progress for only a short distance compared with the linear dimensions of a domain, and is limited by the strains present in any actual material.

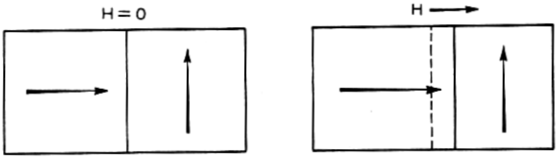


Fig. 12—Magnetization in very low fields progresses by slight displacement of domain boundaries (Becker).

Thus in the magnetization of an ordinary well-annealed ferromagnetic material three processes occur, corresponding to the three well known sections of the magnetization curve (Fig. 13): growth of one

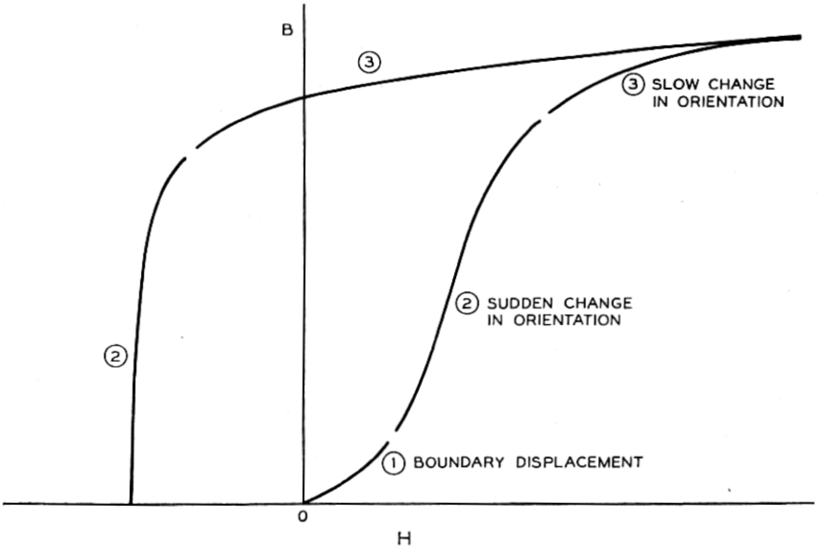


Fig. 13—Illustrating the three kinds of change in magnetization.

domain at the expense of a neighboring one in the initial portion of the curve, sudden changes of direction of domains (with resulting large energy losses) in the middle portion, and continuous or smooth rotation of the domains in the upper portion. The latter two processes occur during the traversal of a large hysteresis loop with tips at high flux densities; the first process is important only in low fields after demagnetization.

EFFECT OF STRAIN

This picture of the changes in magnetization has been made for materials that are free from any considerable strain. As a matter of fact, strain can affect magnetization in an important way, and under certain circumstances a tensile stress of 5,000 pounds per square inch may change the flux density B as much as 10,000 gausses [34]—almost from zero magnetization to saturation (Fig. 14). The effect is illustrated well by data for 65 and 85 permalloy (iron-nickel alloys containing, respectively, 65 and 85 per cent nickel). For 65 permalloy the effect of tension is to increase the magnetization in all fields; for 85 permalloy the effect is the opposite; and in each case the effect of compression is opposite to that of tension. For ordinary iron the effect of tension is to increase the magnetization in small fields, but to decrease it in high fields.

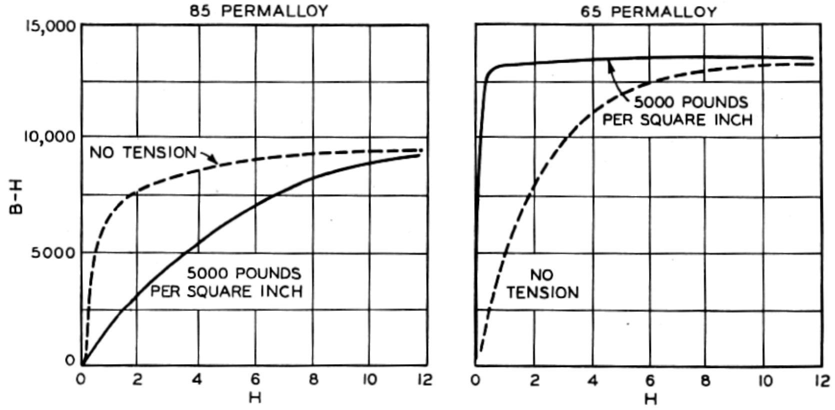


Fig. 14—Effect of tension on magnetization (Buckley and McKeehan).

The effect of strain on magnetization has its counterpart in an effect of magnetization on the length of a piece of ferromagnetic material. When a rod of iron is magnetized its length increases by a small amount. This is but one example of a large class of effects exhibited by all ferromagnetic bodies and known collectively as "magnetostriction." Figure 15 shows the data for change in length of rods of nickel, iron, and two alloys, plotted against the field *H* on the one hand and against relative *B-H* on the other. When saturation of magnetization is reached, the limiting value of magnetostriction, called "saturation magnetostriction," also is attained. Its values for some iron-nickel alloys are shown in Fig. 16. Note here that the change in length is an extension in the alloys containing less than 81 per cent nickel, a contraction otherwise. There is a close relation between magnetostriction and the effect of strain on magnetization, it being a general rule that when the magnetostriction is positive (increase in length with magnetization) the effect of tension is to increase magnetization, and vice versa (Figs. 14, 15, and 16).

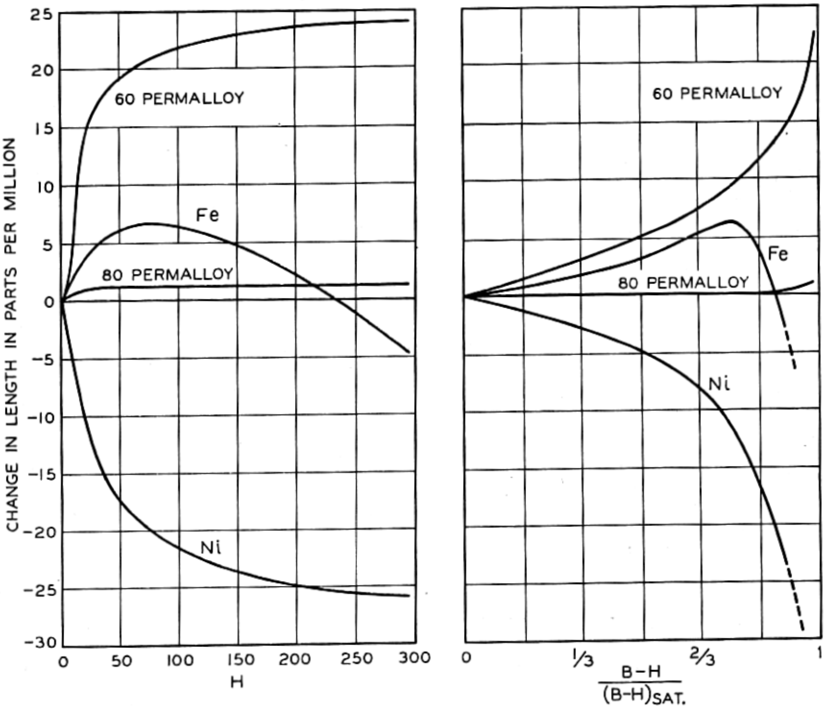


Fig. 15—Magnetostriction in iron, nickel, and two iron-nickel alloys (permalloys).

How much can theory say of magnetostriction and the effect of strain on magnetic properties? Figure 17 shows how the atoms are arranged in an iron crystal; each atom here is supposed to have a

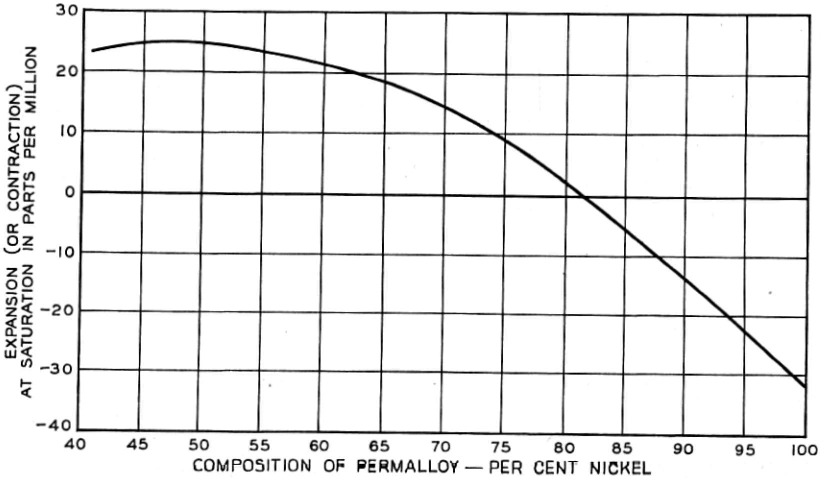


Fig. 16—Saturation magnetostriction of the permalloys (McKeehan and Cioffi, Schulze).

definite magnetic moment as a result of the spin and orbital motion of the electrons. This supposition makes it possible to calculate the magnitude of the mutual magnetic forces which are opposed by the elastic forces holding the crystal together. For iron, the calculations[5] indicate that equilibrium is reached when there has been a slight increase in length in the direction of magnetization and a decrease in length at right angles to this direction such that the volume remains practically unchanged. This calculated magnetostriction is in agreement with experiment as to sign and order of magnitude. With nickel the agreement is not so satisfactory. But in each case the theory is clear in predicting the proper qualitative relationship between magnetostriction and change in magnetization caused by strain.

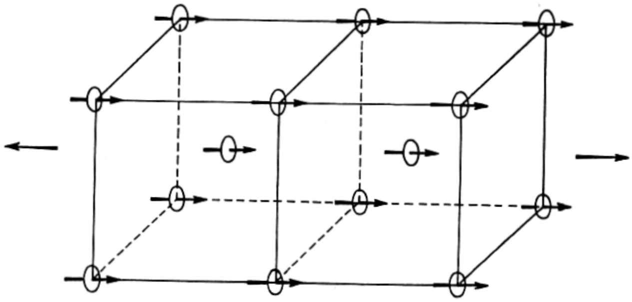


Fig. 17—The magnetic forces between atoms cause a slight elongation in iron (magnetostriction).

Thus magnetostriction and the magnetic effects of strain are reciprocal properties, and result from the same kind of magnetic forces between atoms as those that account for the variation in magnetic properties in different directions in a crystal. Just as the crystal structure determines a direction of easy magnetization in a strain-free crystal, so the strain controls the direction of easy magnetization when the strain is sufficiently great. Figure 18 shows how the domains are magnetized parallel to the crystal axes in unstrained iron, and how a sufficiently large tension will orient the magnetization parallel to the direction of tension in iron and at right angles to the direction of tension in nickel. When the stress is as large as 10,000 to 30,000 pounds per square inch, the strain effect begins to predominate over the crystal effect and the direction of magnetization is determined mainly by the strain. The calculations show also that in a material having positive magnetostriction the magnetization is increased by tension. In a qualitative way these considerations explain the increase in permeability of 65 permalloy (having positive magnetostriction) and the decrease in 85 permalloy (with negative magnetostriction). But so far the theory is quite inadequate to predict the magnitude of the effect.

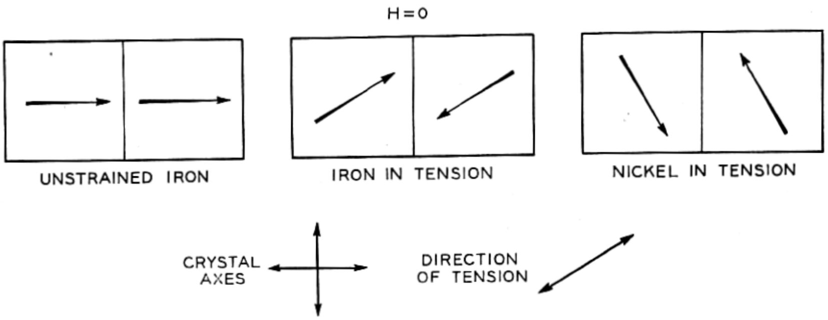


Fig. 18—Domains are oriented by crystal forces and by strain; $H = 0$.

In addition to uniaxial homogeneous strains, such as those produced by stretching a wire in the direction of its length, random (heterogeneous) strains are often found that vary in magnitude, sign, and direction from point to point throughout a material. Such strains are produced by cold working, phase transformations, and the like. In such materials the direction of magnetization in a domain is determined by the local strain, and is more stable the larger the strain. So it can be appreciated that it is harder to change the magnetization of a material that is more severely hard worked. These internal strains are the same ones that contribute to the hardness of a metal—hence the parallelism between magnetic hardness and mechanical hardness, which is so well known.

This relation between internal strain and permeability is illustrated by the data[35] shown in Fig. 19. The permeabilities of a series of specimens of 70 permalloy tape, originally cold rolled, increase as the annealing temperature is raised. X-ray data (the angular width of the reflected X-ray beam) on these same specimens indicate the magnitude of the internal strains existing, and show that they become progressively less as the annealing temperature is increased, the most rapid change taking place in each case between 400 and 600 degrees centigrade, in which region the microscope shows recrystallization has occurred.

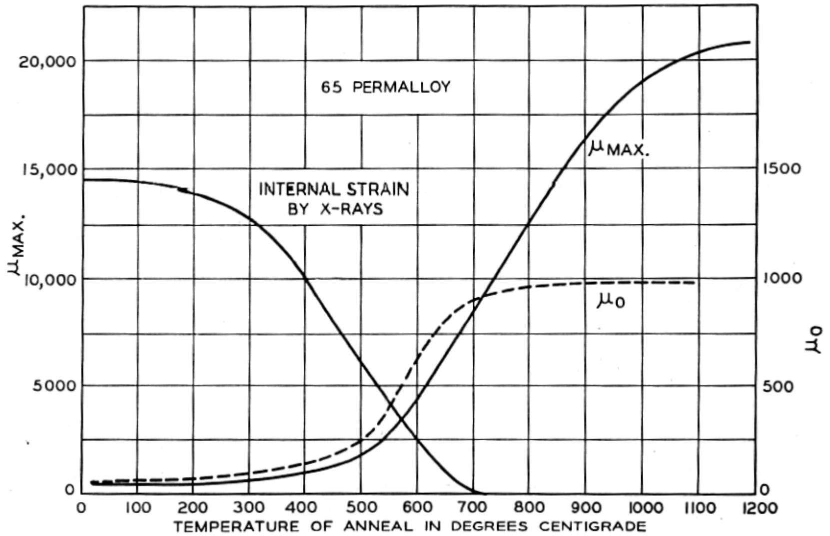


Fig. 19—Magnetic permeability rises as internal strain is diminished by annealing (Dillinger and Haworth).

Following out this same idea, it may be surmised that to make good material for a permanent magnet something with very intense internal strains is required. The direct determination by X-rays of internal strain in a good permanent magnet, confirms this view (Fig. 20). Here the widths of the reflected X-ray beams directly measure the internal strains. For comparison with the permanent magnet material, curves are shown for other materials with less internal strain. The magnet material in this case was an iron-nickel-aluminum alloy that was precipitation-hardened, a method used more and more extensively during the last three or four years for such materials. This method is often applicable when the alloy[36] in the stable condition consists of two phases at room temperature (Fig. 21), but when at a

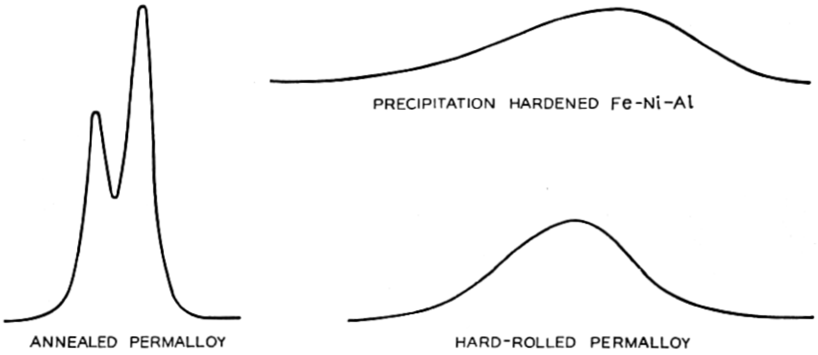


Fig. 20—The width of X-ray reflections indicates the amount of internal strain. Ordinates, intensity of X-rays reflected from metal surface; abscissas, angle of reflection.

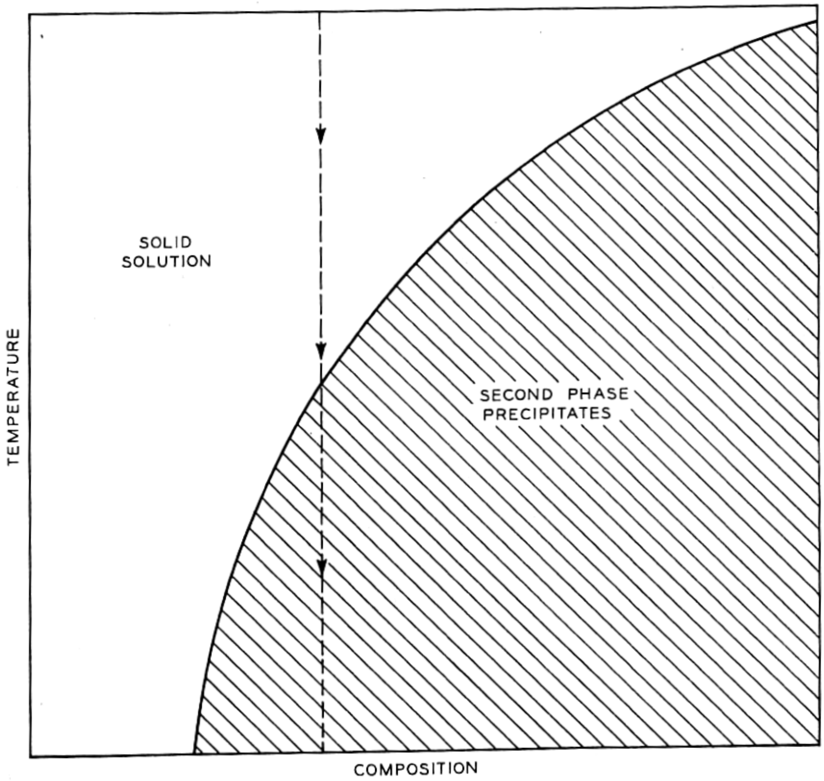


Fig. 21—Precipitation hardening of an alloy for a permanent magnet, such as an alloy of iron, nickel, and aluminum.

higher temperature the one phase dissolves completely in the other to form a solid solution. In making the material, it is quenched rapidly from a high temperature and then reheated to 700 degrees centigrade, at which point the second phase precipitates slowly in very finely divided form. When the optimum amount has precipitated, the material is cooled to room temperature, no more changes occurring. Each submicroscopic precipitated particle is a center of strain, and it is the presence of these unusually large internal strains that is responsible for the good quality of the permanent magnet.

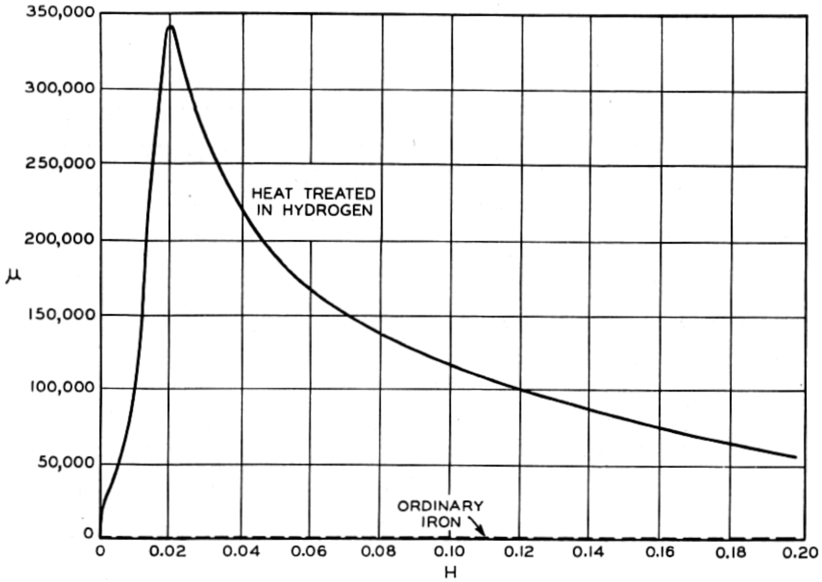


Fig. 22—Permeability curves of ordinary iron and of iron purified by heat treatment in hydrogen at 1,500 degrees centigrade (Cioffi).

Going now to the other extreme, where ease of magnetization is required, it is known, of course, that thorough annealing and a homogeneous structure are beneficial. Still there are at least two sorts of strains that annealing will not relieve. One is that attributable to the non-metallic chemical impurities that do not fit into the regular arrangement of atoms in a pure metal or alloy. It has been found recently that by heat treating iron in hydrogen at about 1,500 degrees centigrade the non-metallic impurities are largely removed, and that what are called "chemical strains" are much reduced. As a result it is found (Fig. 22) that the maximum permeability increases from 10,000 to 340,000,[12] and a large reduction in mechanical hardness occurs simultaneously.

After the chemical strains and the strains resulting from cold working have been removed, there is still another kind of residual strain—that attributable to magnetostriction. These are ordinarily random in direction because they are associated with randomly oriented domains, but by a suitable trick they all can be oriented so as to favor magnetization in a single desired direction at the expense of ease of magnetization at right angles. This trick is heat treatment in the presence of a magnetic field. Without going into a more detailed explanation, the experimental results obtained [15] about two years ago will be given.

When an annealed specimen of 65 permalloy is heated for a few minutes at 650 degrees centigrade while it is subjected to a magnetic field of 10 œrsteds, the maximum permeability is increased from about 20,000 to over 600,000 as shown in Fig. 23. This material holds the records for the highest maximum permeability, the lowest coercive force, and the lowest hysteresis loss at high flux densities. It may be compared with the most permeable material known in 1900, iron with a maximum permeability of less than 3,000.

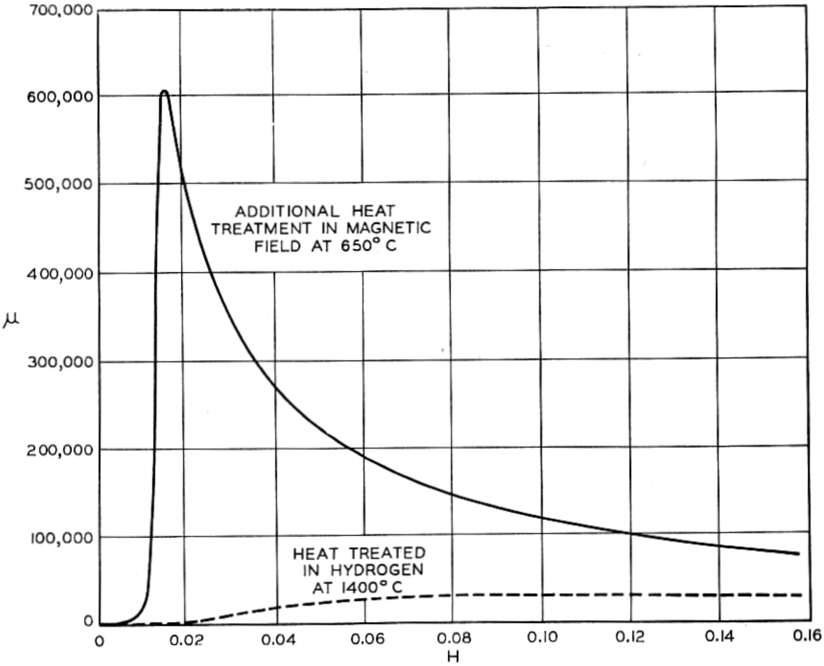


Fig. 23—Permeability curves of 65 permalloy after heat treatment in hydrogen and additional heat treatment in a magnetic field (Dillinger and Bozorth).

So far only the effects of stress on the orientation of domains in medium and high fields have been considered. But stress has an effect on the initial permeability also. It has been said already that in very weak fields a change in magnetization is attributed to a movement of the boundaries between domains, the domains oriented nearly parallel to the field growing at the expense of adjacent domains oriented in less favorable directions. Such a growth obviously may be hindered by strain. A relation has been derived[37] connecting the initial permeability with the internal stress and other magnetic quantities:

$$\mu_0 = \frac{0.018(B - H)^2_{\text{sat.}}}{(\Delta l/l)_{\text{sat.}}\,\sigma_i},$$

where μ_0 is the initial permeability, $(B - H)_{\text{sat.}}$ and $(\Delta l/l)_{\text{sat.}}$ are the (ferric) induction and magnetostriction at saturation, and σ_i is the average value of the internal stress in dynes per square centimeter.

Even when there are no internal strains caused by impurities, insufficient annealing, etc., there generally will be the strains of magnetostriction itself, and these will hinder the growth of one domain at the expense of another (Fig. 24). In this case the stress in the foregoing equation is equal to Young's modulus, E, multiplied by the magnetostrictive strain,

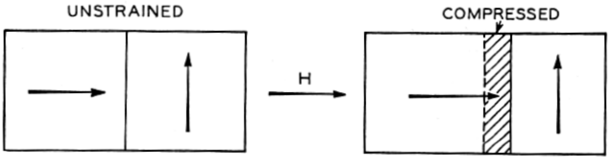


Fig. 24—Magnetostriction in the shaded region acts as a barrier to further change in magnetization.

$$\sigma_i = E(\Delta l/l)_{\text{sat.}}$$

and the former equation becomes

$$\mu_0 = \frac{0.018(B - H)^2_{\text{sat.}}}{(\Delta l/l)^2_{\text{sat.}}\,E},$$

This equation really gives a theoretical upper limit for μ_0. These theoretical limits and the highest observed values for iron-nickel alloys are shown in Fig. 25. This indicates why the composition of the "permalloy" having the highest initial permeability is very nearly the same as that for which the magnetostriction is zero.

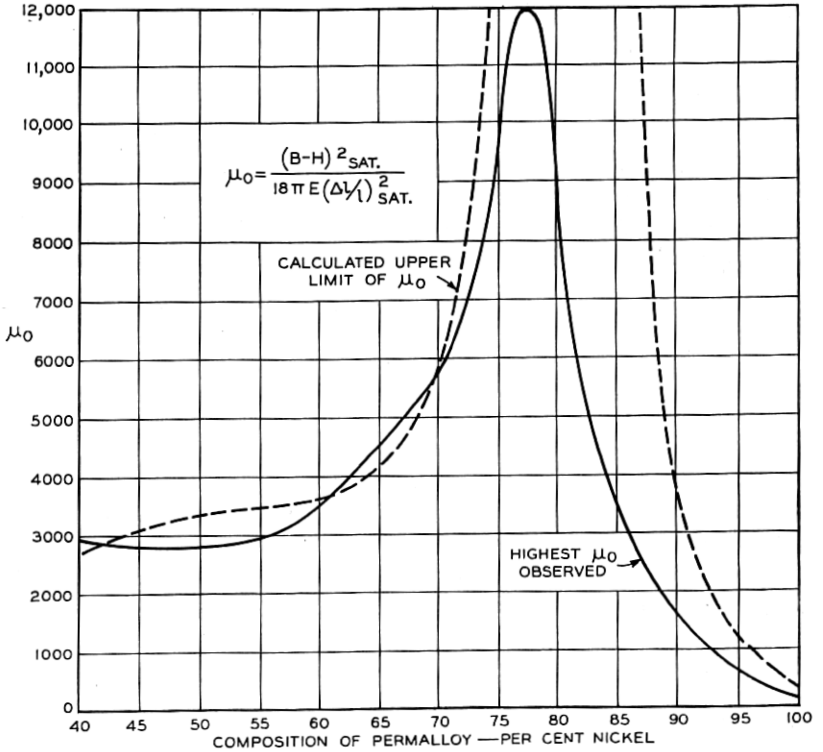


Fig. 25—Comparison of the theoretical upper limit of initial permeability (Kersten) with the highest initial permeabilities observed for iron-nickel alloys (Arnold and Elmen, Schulze).

The effects of strain will now be summarized briefly. The origin of the effects lies in the *magnetic* action between neighboring atoms. The magnetic action is balanced by the elastic (electrostatic) forces between atoms. The balance of these forces results in a change in shape of the magnetic body when it is magnetized (magnetostriction), and also a change in magnetization resulting from strain (strain-sensitivity). Magnetization may be either aided or hindered by a homogeneous uniaxial strain, the effect depending on the magnetostriction in a way that can be estimated qualitatively but not quantitatively. But material in which local strains are directed at random is more difficult to magnetize because the strains prevent a change in magnetization; and the more intense such strains are, the harder the material is to magnetize or demagnetize. The effect of local strains upon the initial permeability can be calculated with fair success, but other magnetic quantities, such as maximum permeability, can as yet be estimated in a qualitative way only.

SUMMARY

In concluding the author wishes to go back from here to summarize what is known about the origin of the forces responsible for the various magnetic properties and about the sizes of the various units. This information is summarized in Table II.

TABLE II

SUMMARY OF DATA REGARDING ORIGIN OF FORCES RESPONSIBLE FOR VARIOUS MAGNETIC PROPERTIES

Unit Concerned	Property	Origin of Property	Size of Magnetic Unit
Electron	Magnetic moment	Electron spin	One unit of spin per electron
Paramagnetic atom	Magnetic moment	Uncompensated spins and orbital motions of electrons	4, 3, and 2 uncompensated spins per atom in Iron, Cobalt and Nickel, respectively
Domain	Ferromagnetism. Change in properties at Curie point	"Exchange" between electrons in neighboring atoms	Volume of domain is about (0.001 inch)^3
Single crystal or region of homogeneous strain	Crystal anisotropy. Magnetostriction. Strain sensitivity	Magnetic forces between atoms	10^8 domains per cubic centimeter
Polycrystal	Orientation-average of single crystals and strain units	Sum of effects of single crystals and strains	Size of specimen

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