Magnetic Measurements at Low Flux Densities Using the Alternating Current Bridge

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A resumé is given of the basic relations between the magnetic characteristics of the core of a coil and the inductance and resistance of the coil as measured on an alternating current bridge. Modifications of the simple relations to take account of the interactions of eddy currents and hysteresis in the core material are developed, and are seen to require a more complicated interpretation of the data in order to obtain an accurate separation of the eddy current, hysteresis, and "residual" losses. Means are described of minimizing or eliminating the disturbing effects of distributed capacitance, leakance and eddy current loss in the coil windings. Essential details of the alternating current bridge and associated apparatus, and of the core structure, are given.

THE modern alternating current bridge, with its high precision and sensitive balance, has almost completely superseded the ballistic galvanometer for determining the magnetic properties of core materials at the low flux densities employed in telephone and radio apparatus. The suitability of the alternating current bridge for this purpose has been recognized for some time,[1] but the continued improvements in magnetic materials, and the more exacting requirements of modern communication apparatus, have necessitated refinements in apparatus, in technique, and in interpretation of measurements. This paper considers the modified technique required to take account of eddy current shielding and hysteresis in the magnetic core, distributed capacitance and leakance in the coil winding, and the necessary details of the bridge and associated apparatus to realize the desired accuracy of measurements.

Fundamentally, the a-c. method involves measurements of the inductance and effective resistance of a winding on the test specimen, such measurements being made at several frequencies, and at several values of current.[2] From these measurements the magnetic properties of the test core can be computed for the low flux density range. The details of such calculations will be given below, beginning with approximate methods, and proceeding to successively more accurate computations.

[1] M. Wien, *Ann. d. Physik* [3] **66**, 859 (1898).

[2] The annular form of magnetic core, wound with a uniformly distributed test winding will here be treated, but the results will be found to be readily transferable to other forms of core.

Simple Analysis of Inductance; Hysteresis Neglected

The magnetizing force in a thin annular core of mean diameter d (cm.) due to current i (ampere) flowing in a uniform winding of N turns is

$$H = \frac{0.4Ni}{d} \text{ œrsted.} \tag{1}$$

In a core of appreciable radial thickness, the effective magnetic diameter rather than the arithmetical mean diameter must be used in this and following equations, as will be explained in eq. (61).

When the bridge is balanced with sinusoidal current of peak value i_m, the peak inductive voltage drop across the standard coil, $2\pi fLi_m$, must equal that across the test coil $2\pi fN\Phi_m \times 10^{-8}$, where Φ_m is the peak magnetic flux in the coil.[3] Whence, for an annular coil,

$$L = \frac{4N^2\Phi_m}{H_m d} \times 10^{-9} \text{ henry.} \tag{2}$$

The flux within an annular coil is composed of that in the core and that in the air space. The expression for inductance can therefore be separated into two terms, giving

$$L = \frac{4N^2}{d}(\mu_m A + A_a) \times 10^{-9} \text{ henry,} \tag{3}$$

where A and A_a are the cross-sectional areas of the core and residual air space, respectively, and μ_m is the magnetic permeability of the core, now assumed to be constant throughout the cycle.

The inductance due to the core alone is then

$$L_m = L - L_a' = \frac{4N^2\mu_m A}{d} \times 10^{-9}, \quad \text{where} \quad L_a' = \frac{4N^2 A_a}{d} \times 10^{-9}. \tag{4}$$

The permeability of the core material can be obtained from this inductance as

$$\mu_m = \frac{L_m d}{4N^2 A} \times 10^9. \tag{5}$$

The peak flux density in the core is derived from eqs. (5) and (1) as

$$B_m = \mu_m H_m = \frac{\sqrt{2} L_m I}{NA} \times 10^8 \text{ gauss,} \tag{6}$$

where I is the r.m.s. current in the winding.

[3] A list of most frequently used symbols will be found in the appendix.

Simple Analysis of Eddy Current Resistance

Again, at bridge balance, the resistance of the standard is equated to the resistance of the test coil, which is composed of the copper resistance R_c and a resistance which corresponds to the a-c. power P dissipated in the core. Thus

$$R = R_c + P/I^2. \tag{7}$$

Power is dissipated in the core through eddy currents and magnetic hysteresis. Although both types of magnetic loss occur simultaneously, they will first be considered as if occurring alone, after which the details of separating and identifying the two types will be discussed.

The resistance due to eddy current power loss depends upon the form of the magnetic core—whether of laminations, wire, or powder—upon the frequency, upon the permeability of the magnetic material, and upon the hysteresis loss, since this modifies the permeability. It is determined with sufficient accuracy for many practical purposes by calculating the eddy current power loss in a volume element consisting of a thin tube so drawn that neither magnetic flux nor eddy currents cross its surfaces, when the flux it encloses varies sinusoidally, and then integrating between proper limits to include the entire cross-section of the lamination. By this method [4] it can be shown that the power consumption per unit volume of sheet core material is

$$P_1 = \frac{\pi^2 t^2 f^2 B_m{}^2}{6\rho} \times 10^{-7} \text{ watt,} \tag{8}$$

where t is the sheet thickness in cm., f is the frequency, and ρ is the resistivity of the material in e.m.u.

This relation is derived on the assumption of a very extensive plane sheet with magnetizing force parallel to its surface, but it applies sufficiently well to any sheet material, flat or curved, provided that the magnetizing force is parallel to its surface, and provided that the width of the magnetic sheet is large in comparison to its thickness. These conditions can be fulfilled in a core built up of ring shaped laminations, and wound with an annular winding.

The total eddy current power loss in a core of volume πAd is then

$$P_e = \frac{\pi^3 t^2 f^2 B_m{}^2 Ad}{6\rho} \times 10^{-7}. \tag{9}$$

As already mentioned (eq. 7), such a power loss in the core of a coil

[4] C. P. Steinmetz, "A. C. Phenomena," p. 195 (1908).

appears in an a-c. bridge measurement as a resistance

$$R_e = \frac{P_e}{I^2} = \frac{\pi^3 t^2 f^2 B_m{}^2 A d}{6\rho I^2} \times 10^{-7}. \tag{10}$$

Substituting for B_m from eq. (6), and for L_m from eq. (4), gives

$$R_e = \frac{4\pi^3 t^2}{3\rho} \mu_m L_m f^2 \text{ ohm}. \tag{11}$$

Since this equation contains explicitly no geometrical details of the core, other than the sheet thickness t, it is applicable to any type of core in which the flux density is uniform, as it is in an annular core. If the resistivity is expressed in microhm-cm., the eddy current resistance becomes

$$R_e = \frac{0.0413 t^2}{\rho_1} \mu_m L_m f^2 \text{ ohm}. \tag{12}$$

A similar solution for the case of a core consisting of a hank or bundle of insulated magnetic wires of diameter t cm. gives

$$R_e = \frac{\pi^3 t^2}{2\rho} \mu_m L_m f^2, \tag{13}$$

or with ρ in microhm-cm.,

$$R_e = \frac{0.0155 t}{\rho_1} \mu_m L_m f^2. \tag{14}$$

A compressed magnetic dust core can be idealized as composed of closely packed insulated spheres. Although there is considerable concentration of flux at various points in any practical core, the power loss in a sphere of diameter t_1 can be calculated to a first approximation by assuming it permeated by a uniform flux density parallel to the direction of the magnetizing force. Computing the eddy current power loss in a cylindrical shell of such a sphere with shell axis parallel to H, and integrating to obtain the total loss gives

$$P = \frac{\pi^3 t_1{}^5 f^2 B_m{}^2}{120\rho} \times 10^{-7}. \tag{15}$$

The power expended in a cubic centimeter of such a core is then

$$P_1 = \frac{\pi^2 t^2 f^2 B_m{}^2 r}{20\rho} \times 10^{-7}, \tag{16}$$

where t^2 is the mean square sphere diameter, and r is the packing factor, i.e., the ratio of the volume occupied by metal to the total volume of the core.

The flux density in the spheres is larger than the apparent flux density by the factor $r^{-2/3}$. The eddy current resistance for such a structure then becomes

$$R_e = \frac{2\pi^3 t^2}{5\rho r^{\frac{1}{3}}} \mu_m L_m f^2, \tag{17}$$

where μ_m is the permeability of the core, as calculated from the inductance L_m.[5] With ρ in microhm-cm., this equation becomes

$$R_e = \frac{0.0124 t^2}{\rho_1 r^{\frac{1}{3}}} \mu_m L_m f^2. \tag{18}$$

In cases where merely comparative tests are to be made, or where ρ and t are not known, it is convenient to lump the coefficient of the eddy current resistance in the form

$$R_e = e\mu_m L_m f^2. \tag{19}$$

Simple Analysis of Hysteresis Resistance

In addition to the power loss due to eddy currents, there is a loss caused by magnetic hysteresis. The energy in ergs dissipated per cubic centimeter of core during one hysteresis cycle is

$$W = \frac{1}{4\pi} \oint H dB = \frac{a_1}{4\pi}, \tag{20}$$

where a_1 is the area of the hysteresis loop in gauss-œrsteds. The power consumption on this account in an annular core of volume $\pi A d$, carried through f cycles per second, is

$$P_h = W\pi d A f = \tfrac{1}{4} a_1 d A f \times 10^{-7} \text{ watt}. \tag{21}$$

In the same manner as above, this power is observed by an a-c. bridge balanced for the frequency f as a resistance

$$R_h = \frac{8\pi W}{B_m{}^2} \mu_m L_m f = \frac{2a_1}{B_m{}^2} \mu_m L_m f \text{ ohm}. \tag{22}$$

By this relation, the hysteresis resistance can be used to compute the energy loss per cycle W, or the hysteresis loop area a_1, which would be obtained by ballistic galvanometer measurements of sufficient sensitivity.

[5] Cf. R. Gans, *Phys. Zeit.* **24**, 232 (1923).

The Rayleigh Hysteresis Loop

The hysteresis loop area for magnetic cycles at low flux densities (i.e. for which μ_m is not more than 10–20 per cent higher than μ_0) can be calculated from the general shape of such loops. Rayleigh found experimentally [6] that the two branches of such loops are parabolas, that the permeability corresponding to the tips of the loop increases in proportion to the peak magnetizing force, thus,

$$\mu_m = \mu_0 + \alpha H_m, \tag{23}$$

and that the remanent flux is

$$B_r = \frac{\alpha}{2} H_m^2. \tag{24}$$

The loop equation which satisfies the above conditions is

$$B = (\mu_0 + \alpha H_m)H \pm \frac{\alpha}{2}(H_m^2 - H^2), \tag{25}$$

where the points on the upper branch are obtained by using the + sign and those on the lower branch by using the − sign. Recent ballistic galvanometer measurements of high precision on an iron dust core tend to confirm the reliability of the Rayleigh loop equation for low flux densities.[7]

Integrating HdB around the cycle gives an area $4\alpha H_m^3/3$, which can be used in equation (22) to obtain the hysteresis resistance, as

$$R_h = \frac{8\alpha H_m}{3\mu_m} L_m f. \tag{26}$$

Defining the permeability variation with flux as

$$\lambda = \frac{\mu_m - \mu_0}{\mu_0 B_m}, \tag{27}$$

the value of α will be

$$\alpha = \mu_0 \mu_m \lambda, \tag{28}$$

and the hysteresis resistance becomes

$$R_h = \frac{8}{3} \lambda H_m \mu_0 L_m f. \tag{29}$$

[6] *Phil. Mag.* [5] **23**, 225 (1887).
[7] W. B. Ellwood, *Physics* **6**, 215 (1935).

It thus appears that the Rayleigh hysteresis loop implies a definite relationship between the variation of permeability with H or B, as calculated from bridge measurements of inductance at various coil currents, and the observed hysteresis resistance. Efforts to judge as to the general applicability of the Rayleigh form of loop by means of such a-c. bridge comparisons have indicated fairly good agreement for most materials, but occasional deviations as high as 40 per cent.[8] Best agreement is generally found in well annealed and unstressed materials, while deviations are found in such materials as compressed dust cores. In such comparisons, the anomalous residual loss, variously termed magnetic viscosity, and after-effect, is excluded. This additional loss will be discussed below.

Mutual Effect of Rayleigh Hysteresis and Eddy Current Shielding in Sheet Material

Taking the above equation as the simplest general representation of hysteresis loops at low flux densities, it now becomes necessary to review the previous work with additional refinements to include the effects of hysteresis upon eddy currents, and of eddy currents upon themselves, and upon hysteresis. Thus the fact that B varies according to a hysteresis loop equation rather than directly with H modifies the eddy current loss somewhat. Also, eddy currents set up magnetizing forces within the magnetic material which more or less neutralize that applied by the coil winding, and thus effectively shield the inner parts of magnetic laminations of wires. Such eddy current shielding reduces the total flux in the core, thus decreasing the inductance and loss resistance observed at higher frequencies.

The fundamental differential equation giving the relation between B and H at a point x distant from the median plane of a magnetic sheet is[9]

$$\frac{\partial B}{\partial t} = \frac{\rho}{4\pi} \frac{\partial^2 H}{\partial x^2} . \tag{30}$$

For the simple case of constant permeability in which $B = \mu_0 H$, this equation has been solved by Heaviside, J. J. Thomson,[10] and others.

For the case in which B is given by Rayleigh's equation (25), the solution is very much involved. The variable permeability gives rise

[8] E. Peterson, *B. S. T. J.* **7**, 775 (1928).
[9] *E.g.*, Russell, "Alternating Currents," Vol. I, p. 487 (1914).
[10] *Electrician* **28**, 599 (1892).

to odd harmonic voltages which are important from the standpoint of modulation and noise. In a-c. bridge measurements where the balance is made so as to bring the voltages of fundamental frequency to equality, it suffices to carry through the mathematics for this frequency alone. This has been done for sheet and wire cores to an accuracy sufficient for most purposes by W. Cauer.[11] From his results for the inductance and power loss in a laminated core, the apparent permeability and loss resistance are calculated to be

$$\mu_{fm} = \frac{\mu_0}{\theta}\frac{\sinh\theta + \sin\theta}{\cosh\theta + \cos\theta} + \alpha H_m\left(1 - \frac{4\theta^2}{9\pi} - \frac{7\theta^4}{60} + \frac{2\theta^6}{45\pi} + \cdots\right) \quad (31)$$

$$= \mu_0\left(1 - \frac{\theta^4}{30} + \frac{\theta^8}{732} - \cdots\right) + \alpha H_m\left(1 - \frac{4\theta^2}{9\pi} - \frac{7\theta^4}{60} + \frac{2\theta^6}{45\pi}\cdots\right) \quad (32)$$

$$= \mu_0\left[1 + \lambda B_m - \frac{4\lambda B_m\theta^2}{9\pi} - \left(1 + \frac{7}{2}\lambda B_m\right)\frac{\theta^4}{30} + \frac{2\lambda B_m\theta^6}{45\pi}\cdots\right], \quad (33)$$

$$R_{fm} = \frac{2\pi f L_0}{\theta}\frac{\sinh\theta - \sin\theta}{\cosh\theta + \cos\theta} + \frac{8\alpha H_m f L_0}{3\mu_0}\left(1 + \frac{\pi\theta^2}{4} - \frac{7\theta^4}{60} - \frac{\pi\theta^6}{40} + \cdots\right) \quad (34)$$

$$= \frac{\pi f L_0\theta^2}{3}\left(1 - \frac{17\theta^4}{420} + \frac{\theta^8}{600} - \cdots\right) + \frac{8}{3}\lambda B_m L_0 f\left(1 + \frac{\pi\theta^2}{4} - \frac{7\theta^4}{60} - \frac{\pi\theta^6}{40}\cdots\right). \quad (35)$$

The quantity $\theta = 2\pi t\sqrt{\mu_0 f/\rho}$, where ρ is in e.m.u.; and $B_m = \mu_m H_m$, where μ_m is independent of f.

The hyperbolic function parts of these equations are valid at any frequency, but they give only those parts of μ and R which are due to the constant initial permeability μ_0. The series having α or λ as coefficients give the increases due to hysteresis.

The apparent permeability μ_{fm}, which is calculated from the measured inductance, decreases as the measuring frequency is increased. Furthermore, at higher frequencies, this permeability rises less rapidly with rise in measuring current than it does at low frequencies, and it will actually decline with increasing H at frequencies higher than that necessary to make $\theta > 1.6$, approximately. Thus, for the accurate determination of μ_0, μ_m, λ and α, it is necessary to make measurements at frequencies low enough to suppress these correction terms.

[11] W. Cauer, *Arch. f. Elektrotechnik* **15**, 308 (1925).

The equations for resistance are similarly complicated. The first series gives that part of the eddy current resistance which is due to the constant μ_0. The coefficient of the second series indicates that this series involves the hysteresis resistance. However, terms in the second series which contain the factor f^2 will be recognized as eddy current components introduced by the fact that the permeability has been increased from the value μ_0 by the factor λB_m.

The complicated form and slow convergence of the above equation (35) for resistance make it difficult for use in interpreting a-c. bridge measurements. Considerable simplification is effected by dividing the observed resistance (eq. 35) by the observed inductance (from eq. 33) for each measuring current and frequency. Performing this operation, and rejecting series terms in λB_m higher than the first power, gives

$$\frac{R_{fm}}{L_{fm}} = \frac{4\pi^3 t^2}{3\rho}\mu_m f^2\left[1 - \frac{\theta^4}{140}(1 + 5\lambda B_m) + \cdots\right] + \frac{8}{3}\lambda H_m \mu_0 f\left[1 - \frac{\theta^4}{36}(1 - 5\lambda B_m) + \cdots\right]. \quad (36)$$

The coefficient of the first series is identical with the eddy current expression previously derived (eq. 11), which neglected eddy current shielding and hysteresis. The series itself, which includes these other effects, converges rapidly for $\theta < 1$, provided that the value of λB_m is not carried too high.

The coefficient of the second series is identical with the hysteresis expression derived from Rayleigh's equation (29) in which eddy currents were neglected. The second term of this series gives the amount by which eddy current shielding reduces hysteresis resistance at higher frequencies. It appears to converge less rapidly than the series for the eddy current resistance, but this is partly offset by the decrease of its second term with increase of λB_m. Thus, the coefficients of θ^4 become equal to 1/88 in both series if $\lambda B_m = 13/110$. This value of λB_m is reached when the flux density in the material is large enough to raise the permeability some 10 per cent above μ_0. Evidently, this value of λB_m can be exceeded somewhat without making the coefficients excessively large. However, if the measurements are made at too high flux densities, the hysteresis loops diverge more and more from the simple Rayleigh loop, and the present analysis becomes inapplicable.

In a-c. bridge measurements it is seldom desirable to measure at flux densities which will carry the permeability more than 10 per cent above its initial value. If measurements at higher flux densities are desired, sufficient sensitivity can generally be obtained by wattmeter or ballistic galvanometer methods.

Graphical Separation of Losses

Since it is generally important to distinguish between types of magnetic losses, methods of analyzing the measurements have been devised to accord with the degree of refinement desired. A fairly simple graphical loss separation method is suitable if magnetic shielding can be ignored. However, it will be seen to lead to the inclusion of an additional term to account for the residual loss. If the effect of eddy current shielding is also to be considered, a more complicated analytical method of separation will be found necessary.

With magnetic shielding negligible, eq. (36) reduces to the form

$$\frac{R_m}{fL_m} = \frac{8}{3}\lambda H_m \mu_0 + \frac{4\pi^3 t^2}{3\rho}\mu_m f \tag{37}$$

$$= \frac{8}{3}\lambda B_m \frac{\mu_0}{\mu_m} + \frac{4\pi^3 t^2}{3\rho}\mu_m f \tag{38}$$

or

$$\frac{R_m}{\mu_m f L_m} = aB_m + ef. \tag{39}$$

The last form of the expression is most suitable for routine testing and design purposes. The hysteresis area constant a will be seen to be intimately related to the hysteresis loop area a_1 previously discussed; thus

$$a = \frac{2a_1}{B_m^3}. \tag{40}$$

Within the limits of applicability of Rayleigh's equation, the following relations also apply:

$$a = \frac{8\lambda\mu_0}{3\mu_m^2} = \frac{8\alpha}{3\mu_m^3}. \tag{41}$$

The losses observed on any test core can be separated graphically[12] by calculating the values of $R_m/\mu_m f L_m$ for a fixed value of H_m at all measuring frequencies, and plotting such values against frequency. The slope of the resulting straight line should then give e, and the intercept aB_m. When this process is repeated for other values of H_m, a series of intercepts will be obtained, all of which would be expected to yield a constant value for the loop area constant a. However, this is frequently found not to be true, but if the several intercepts so

[12] B. Speed and G. W. Elmen, *Trans. A. I. E. E.* **40**, 596 (1921).

obtained be plotted against B_m, they generally fall upon a fairly straight line whose slope is a, and whose intercept on the line $B_m = 0$ is c. The value of a so obtained agrees fairly well in most cases with the value calculated from the permeability variation coefficient λ, which is the justification cited above for the Rayleigh equation. The presence of residual loss necessitates rewriting the loss equation with an additional term—

$$\frac{R_m}{\mu_m f L_m} = aB_m + c + ef. \tag{42}$$

The value of the intercept c, however, has no counterpart in the Rayleigh equation. It indicates the presence of a power loss proportional to the frequency, and thus similar to hysteresis, but contrarily proportional to the square of the magnetizing force, instead of to the cube. It is found not to contribute to harmonics or modulation generated by a core material, and might thus be represented by an elliptical increment to the Rayleigh loop.[13]

Residual loss has been ascribed to viscosity or "after-effect" in the core material.[14] The chief obstacle to this explanation is the observed constancy of c over a wide range of frequencies, in contrast to the variation to be expected from ordinary viscosity losses. Residual loss has been ascribed to inhomogeneities in the magnetic material[15] which lead to higher a-c. power losses than expected from the area of the hysteresis loop. This explanation seems promising, but the work to date has been chiefly qualitative, and it has not been shown to yield the required additional loss proportional to H^2. The parallel between this loss and eddy current loss, which is also proportional to H^2, is alluring, but the dependence of eddy current loss upon f^2 has remained a stumbling block. The mechanical dissipation of power through magnetostrictional motions seems also a possible explanation.[16]

Somewhat analogous to the residual loss is the excess eddy current loss generally observed. When the observed value of e is used to calculate the resistivity of a magnetic material, it generally gives a value somewhat smaller than the true resistivity, which indicates that the observed eddy current losses are correspondingly too large. The apparent resistivity so obtained approaches the true resistivity quite closely for well insulated laminations of pure, well annealed materials. It is interesting to note that the residual loss for such well

[13] H. Jordan, *Ann. d. Physik* [5] **21**, 405 (1934).
[14] H. Jordan, *E. N. T.* **1**, 7 (1924); F. Preisach, *Zeit. f. Phys.* **94**, 277 (1935).
[15] L. W. McKeehan and R. M. Bozorth, *Phys. Rev.* [2] **46**, 527 (1934).
[16] For a more thorough discussion see W. B. Ellwood, *Physics* **6**, 215 (1935).

annealed materials is also practically absent. For many materials, however, the apparent resistivity falls to 50–75 per cent of the true resistivity. The increase in eddy current loss thus observed is technically very undesirable since it necessitates rolling laminations considerably thinner than otherwise required, in order to suppress eddy current losses sufficiently.

The cause of extra eddy current losses in laminated material is definitely chargeable to the hard, low permeability surface of the material. The eddy current losses are determined largely by the high interior permeability, and the laminar thickness. The material near the surface conducts large eddy currents induced by interior material of high permeability, but it contributes very little to the average permeability for the entire sheet. Removal of low permeability surface material by etching [17] lowers the eddy current losses and increases the average permeability of the core, so that the apparent resistivity approaches more closely the true d-c. value. Of course, selection of material and proper mechanical working and heat treating technique are most desirable in avoiding at the outset such inhomogeneities, with their resulting excessive losses.

Analytical Separation of Losses

For special investigations where the accuracy of the graphical method of loss separation is not sufficient, it is necessary to return to the unabridged form of eq. (36), and employ an analytical method. For example with sheet material,

$$e = \frac{4\pi^2 t^2}{3\rho} \quad \text{and} \quad \theta^2 = \frac{3}{\pi} e\mu_0 f.$$

Rewriting eq. (36) with these substitutions and with an additional term to provide for the residual loss,

$$\frac{R_{fm}}{fL_{fm}} = aB_m\mu_m\left[1 - \frac{9e^2\mu_0{}^2f^2}{36\pi^2}(1 - 5\lambda B_m) + \cdots\right] + c\mu_m + e\mu_m f\left[1 - \frac{9e^2\mu_0{}^2f^2}{140\pi^2}(1 + 5\lambda B_m) + \cdots\right]. \quad (43)$$

It should be recalled that R_{fm} and L_{fm} are the core resistance and inductance measured at a definite current and frequency, while μ_m is the permeability of the core measured at the same current, but at a frequency low enough to make eddy current shielding negligible.

[17] Legg, Peterson and Wrathall, U. S. Patent 1,998,840 (1934).

Subtracting the value of R_{fm}/fL_{fm} for frequency f_1 from the corresponding value for frequency f_2, and dividing by the frequency interval $\Delta f = f_2 - f_1$, gives

$$\frac{\Delta\left(\dfrac{R_{fm}}{fL_{fm}}\right)}{\Delta f} = e\mu_m\left[1 - \frac{9e^2\mu_0^2}{140\pi^2}(1 + 5\lambda B_m)(f_1^2 + f_2^2 + f_1 f_2) - \frac{2}{3\pi^2}e\mu_0\lambda B_m(1 - 7\lambda B_m)(f_1 + f_2)\cdots\right]. \quad (44)$$

An approximate value for e is sufficient in obtaining the correction terms in this equation. With the precise value of $e\mu_m$ thus obtained, the eddy current term in eq. (43) can be calculated for any frequency and permeability. Subtracting the proper eddy current term for each value of R_{fm}/fL_{fm} gives the hysteresis terms as remainders, which can be further analyzed in their relation to magnetizing force, as in the previous graphical loss separation. Loss separations, made thus precisely, reveal frequency variations of apparent resistivity and of the residual loss constant.

Capacitance, Leakance, and Eddy Current Loss of the Winding

In the discussion thus far, it has been assumed that the measured inductance and resistance of a test coil depend solely upon the core permeability and losses. This assumption must be modified under some conditions, for it is found that the distributed capacitance and leakance of the coil winding act as shunt impedances, which may diminish sufficiently at high frequencies to mask the actual inductance and resistance of the coil. Furthermore, the resistance of the test coil includes an amount corresponding to the power expended by eddy currents in the copper winding itself. It will be shown that such disturbing factors can generally be eliminated, either by modifications in the method of core loss separation, for materials in which eddy current shielding is negligible; or by winding the test core to give an inductance low enough to suppress such disturbing factors, for materials in which eddy current shielding is not negligible.

If the distributed capacitance and leakance can be considered as single lumps, C, and G, in parallel with the coil of inductance L and resistance R, the observed inductance at a frequency corresponding to $\omega = 2\pi f$ is found to be

$$L_{\text{obs.}} = \frac{L(1 - \omega^2 LC) - CR^2}{(1 - \omega^2 LC)^2 + 2GR + G^2(R^2 + \omega^2 L^2) + \omega^2 C^2 R^2}.$$

This simplifies at frequencies well below resonance to

$$L_{\text{obs.}} = L(1 + \omega^2 LC). \tag{45}$$

Thus the observed inductance tends to increase at higher frequencies on account of distributed capacitance, in contrast to its tendency to decrease on account of magnetic shielding in the core according to eq. (33). If the inductance L is known from low frequency measurements, and if computations from eq. (33) show that it does not decline appreciably because of eddy current shielding at the measuring frequency, the capacitance can be calculated from the relation.

$$\omega^2 LC = \frac{L_{\text{obs.}} - L}{L}. \tag{46}$$

Similar complications arise in measuring the resistance of a coil at high frequencies. Under the same assumptions as above, the observed resistance is

$$R_{\text{obs.}} = \frac{R + G(R^2 + \omega^2 L^2)}{(1 - \omega^2 LC)^2 + 2GR + G^2(R^2 + \omega^2 L^2) + \omega^2 C^2 R^2}.$$

At moderate frequencies, this reduces to

$$R_{\text{obs.}} = (R + G\omega^2 L^2)(1 + 2\omega^2 LC), \tag{47}$$

from which it appears that leakance enters as an important part, and that the capacitance gives twice as large an increment for the resistance as for the inductance. The effect of distributed capacitance can be eliminated by dividing the observed value of resistance by $(1 + 2\omega^2 LC)$, where the correction factor is obtained from eq. (46). Thus

$$\frac{R_{\text{obs.}}}{1 + 2\dfrac{L_{\text{obs.}} - L}{L}} = R + G\omega^2 L^2. \tag{48}$$

The leakance term $G\omega^2 L^2$ can be eliminated as will be shown below.

The resistance term R includes the desired magnetic core resistance, but it also contains the resistance of the copper coil, which may have a considerable eddy current loss of its own. The copper eddy current loss occurs principally in the lower layers of the winding, which are cut by the alternating magnetic flux set up by the current in the winding. It is similar to the eddy current loss in the core material itself, varying with the square of the frequency, to a first approximation.[18] This loss must, therefore, be eliminated before accurate

[18] Cf. M. Wien, *Ann. d. Phys.* [4] **14**, 1 (1904); S. Butterworth, *Exp. Wireless* **6**, 13 (1929).

determination of the core loss is possible. The eddy current resistance of the copper winding is of the form

$$R_{ce} = e_c L_a f^2, \tag{49}$$

where L_a is the total air inductance of the winding, and e_c is the copper eddy current coefficient.

This eddy current coefficient is inversely proportional to the number of strands in the wire, so that it can be minimized by using wire consisting of many insulated strands. It increases somewhat with the number of layers in the winding. The coefficient may be determined for any type of winding by resistance measurements on an air core coil of dimensions and winding details similar to those of the magnetic core to be tested. Subtracting the eddy current resistance R_{ce} so computed, and the d-c. copper resistance R_c, from eq. (48) gives as the residual resistance

$$\Delta R = \frac{R_{\text{obs.}}}{1 + 2\dfrac{L_{\text{obs.}} - L}{L}} - R_c - e_c L_a f^2$$

$$= \mu_m L_m[(aB_m + c)f + ef^2] + G\omega^2 L^2. \tag{50}$$

This residual resistance consists of the core loss resistance, and an increment due to leakance. The latter can be largely suppressed by the use of low leakance insulating materials, by insuring that the winding is free from moisture, and by making the distributed capacitance as small as possible. Furthermore, it is known from experiments on the electrical conductance of insulating materials at elevated frequencies that the "quality" $Q = \omega C/G$ is practically a constant (C is the capacitance associated with G, — in this case the distributed capacitance). Inserting this value of G in eq. (50) gives

$$\Delta R = \mu_m L_m[(aB_m + c)f + ef^2] + 8\pi^3 CL^2 f^3/Q. \tag{51}$$

Theoretically, the coefficients in this equation can be obtained from resistance measurements taken at three different frequencies. Unavoidable errors in the measurements render such an analysis unreliable, so that it is generally preferable to obtain a larger number of observations, and to determine the coefficients graphically. Dividing by $\mu_m L_m f$, and neglecting the air inductance of the coil, the equation becomes

$$\frac{\Delta R}{\mu_m L_m f} = (aB_m + c) + ef + \frac{8\pi^3 CLf^2}{\mu_m Q}. \tag{52}$$

If the data at lower frequencies are sufficiently reliable, this parabola can be extrapolated to give the zero intercept $(aB_m + c)$ which contains the sought for hysteresis constants of the core. Subtracting the intercept so found, and dividing again by f gives

$$\frac{1}{f}\left\{\frac{\Delta R}{\mu_m L_m f} - (aB_m + c)\right\} = e + \frac{8\pi^3 CLf}{\mu_m Q}. \tag{53}$$

This is the equation of a straight line, when plotted against f. The intercept e is the desired eddy current coefficient for the core material. The slope of this line, S, yields the dielectric quality

$$Q = \frac{8\pi^3 CL}{\mu_m S}. \tag{54}$$

Here C is the distributed capacitance, which can be obtained from eq. (46). This relation is useful in comparing the qualities of various insulating and spacing materials, and in calculating the total losses to be expected in any proposed coil.

Accurate Separation by Limiting Inductance

It appears from the above discussion that magnetic loss separations can be made in spite of interference by distributed capacitance, leakance and eddy current resistance of the coil windings, provided that the interference is not too large, and provided that eddy current shielding in the test core is negligible. When the latter condition is not fulfilled, it becomes necessary to suppress the interference due to capacitance, etc., to negligibly small quantities. This is facilitated by proper technique in applying the windings, but any degree of suppression can be secured by sufficient limitation of the coil inductance, as will appear by reference to eq. (47). Although reduction of the coil inductance by using a winding with few turns is desirable in thus suppressing errors, it is undesirable in that it reduces the core loss resistance (cf. eq. 50) to a value which may be difficult to measure accurately on any available bridge. It is thus necessary to wind the test core to an inductance which will yield the largest possible loss resistance, without exceeding the allowable error from capacitance, leakance, and copper eddy current loss. The value of this maximum allowable inductance is obtained by calculating the inductance required to make the errors due to capacitance, leakance, and copper eddy currents at the highest measuring frequency equal to some tolerable small fraction of the core loss resistance.

A good separation of losses requires measurements at four or more frequencies, up to a point where the eddy current resistance is several times the hysteresis resistance, and at four or more values of measuring current in the useful range. The maximum frequency necessary to make the eddy current resistance mount to p times the hysteresis resistance is

$$f_m = \frac{ph}{e}, \qquad \text{where} \qquad h = (aB_m + c), \tag{55}$$

and the total core loss resistance at this frequency is

$$R_m = \frac{\mu_m L_m h^2 p(p + 1)}{e}. \tag{56}$$

At this maximum frequency, the observed resistance is

$$R_{\text{obs.}} = (R_c + e_c L_a f^2 + R_m + 8\pi^3 CL^2 f_m{}^3/Q)(1 + 8\pi^2 LCf_m{}^2). \tag{57}$$

Since it is desired that the observed resistance indicate directly the d-c. copper and core loss resistance, all other terms in eq. (57) may be considered as errors, to be suppressed to a small fraction q of the core resistance. Setting the total error equal to qR_m, and rejecting errors of higher orders gives

$$qR_m = e_c L_a f_m{}^2 + 8\pi^2 LCf_m{}^2(R_c + R_m + \pi Lf_m/Q). \tag{58}$$

Substituting the above values for f_m and R_m at the maximum measuring frequency gives a quadratic equation in L (neglecting air inductance), which solves quite accurately to require,

$$L = \frac{e[eq\mu_m(p + 1) - 8\pi^2 CR_c p]}{8\pi^2 Cp^2 h[\pi/Q + \mu_m h(p + 1)]} - \frac{e_c L_a p}{eq\mu_m(p + 1) - 8\pi^2 CR_c p}. \tag{59}$$

Since it is desirable to use a large inductance for ease in resistance determination, it appears that the copper resistance R_c, the copper eddy current coefficient e_c, and the distributed capacitance C should be made as small as possible, while Q should be large.

As an illustrative example, assume a core material of permeability $\mu_m = 100$, to be measured up to a frequency such that $p = 5$, with an error of not more than 1 per cent at the maximum frequency, i.e., $q = 0.01$. Assume also, $e = 25 \times 10^{-9}$, $h = 0.5 \times 10^{-4}$, $C = 25 \times 10^{-12}$, $Q = 20$, $e_c L_a = 10^{-11}$, $R_c = 2$.

The maximum frequency for measurement will then be $f_m = 10{,}000 \sim$. The core should be wound to give an inductance of 5.30 mh. At the maximum measuring frequency the eddy current resistance will be 1.325 ohms and the hysteresis resistance 0.265 ohm. At the lower end of the frequency range, say at 1000~ these figures become $R_e = 0.0132$ ohm and $R_h = 0.0265$ ohm. This indicates that a bridge will be required for such measurements capable of measuring increments of resistance to an accuracy of about 0.0002 ohm at 1000~, and about 0.002 at 10,000~.

Computations of this sort show the high quality of a-c. bridge generally demanded for core loss measurements. Such measurements require equipment and a bridge with a maximum of sensitivity of both a-c. and d-c. balances, and a minimum of losses in standards, pick-up, unbalanced impedance to ground, and variable contact.

Bridges, and Test Procedure

The essential features of bridges suitable for core loss measurements will now be mentioned in general terms. It will be appreciated from the above discussion that the specific range of frequencies, and the required resistance sensitivity must be adapted to the loss characteristics of the magnetic core to be measured.

Although certain types of resonance bridges [19] have advantages for measurements at high frequencies, the most suitable bridge for the usual measurement of magnetic core coils is an equal arm inductance comparison bridge,[20] on which inductances can be measured directly, and on which a-c. and d-c. resistance measurements can be made in prompt succession, to eliminate the effect of gradual temperature changes on the resistances of the bridge and test coil. A suitable circuit is shown in Fig. 1.

Inductance coils for bridge standards should be as stable as possible against frequency and current. Although low effective resistance per unit of inductance is desirable, it is more important for core loss measurements to design such standards for a minimum increase of resistance with frequency, so as to keep calibration corrections small in comparison to the resistance increments to be measured. A satisfactory type of standard coil consists of an air core toroidal form, with a bank winding of finely stranded wire. The bank winding minimizes capacitance effects on the observed inductance and resistance of the coil, and the fine stranding minimizes eddy current losses in the copper; cf. eq. (49). The small residual corrections must finally be included as calibrations when making measurements with the aid of standard coils.

[19] W. J. Shackelton and J. G. Ferguson, *B. S. T. J.* **7**, 82 (1928).
[20] W. J. Shackelton, *B. S. T. J.* **6**, 142 (1927).

The effect of contact resistance is minimized by changing few, or preferably no, contacts between a-c. and d-c. readings. This is facilitated by supplying current to one corner of the bridge through the sliding contact of the slide wire resistance used for fine balancing. This excludes contact resistance errors from resistance determinations in which both a-c. and d-c. balances fall within the range of the slide wire, and thus increases the bridge accuracy for small values of effective resistance. Usual precautions as to clean and positive contacts are sufficient for larger resistance measurements.

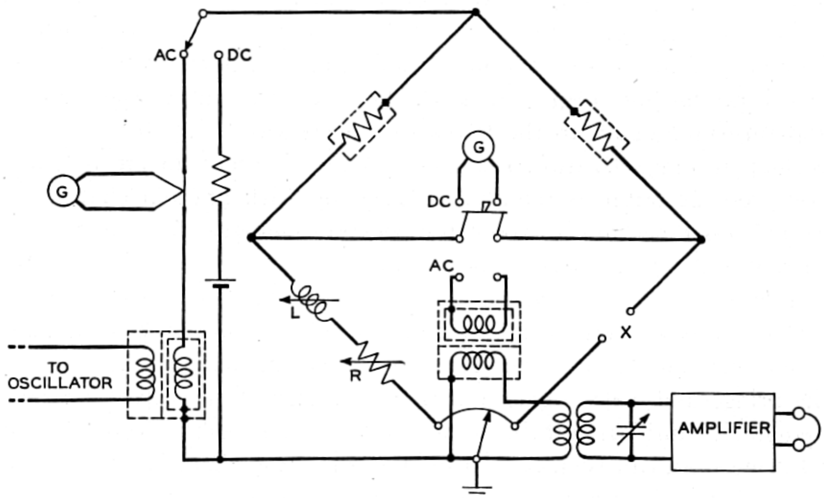

Fig. 1—Diagram of inductance comparison bridge suitable for measurement of magnetic core coils.

The a-c. supply to the bridge should, of course, be a sine wave, and the bridge transformers should be designed for minimum distortion. The frequency should be known accurately, and the voltage should be constant during any set of measurements. Rheostats are required to permit accurate adjustment of bridge current, and they must be designed and shielded to avoid stray coupling with the bridge. A suitable thermocouple is provided for measuring the current into the bridge, from which measurement the current through the test coil can be readily determined, since the bridge has equal ratio arms.

A distortion-free amplifier and a filter circuit tuned to the measuring frequency, are essential for magnifying the bridge unbalance current so as to permit precise measurements. Such unbalances may be detected by a vibration galvanometer at frequencies below say 200∼

and by head phones at frequencies within the audible range. For measurements at higher frequencies, a heterodyne detector is needed. Detection with a galvanometer is feasible when used in connection with a rectifier and filter, the filter being required so as to eliminate errors due to currents of extraneous frequencies.

The galvanometer used for d-c. bridge balancing should be sensitive enough to secure resistance readings of precision equal to that of the a-c. balance. The d-c. supply to the bridge should be limited to a current of the same order of magnitude as the a-c. supply, to guard against permanent magnetization of the magnetic material under test.

The usual test procedure is to set the oscillator at the lowest desired frequency, and measure the inductance and resistance of the test coil at several currents beginning at the lowest, increasing to the highest, and returning again to the lowest, so as to detect any tendency for permanent magnetization or magnetic aging. Direct current balances are taken as often as required to keep up with gradual changes of circuit resistance due to room temperature changes, the direction of current through the bridge being reversed each time to detect and eliminate stray currents and thermal e.m.f.'s. The differences between the observed a-c. and d-c. resistances gives the a-c. increment resistance of the test coil, except for corrections on account of the calibration of the bridge and coils. This process is repeated at successively higher frequencies until a suitable range of data has been covered. The resulting data can then be analyzed to show the characteristic of the magnetic core by the appropriate method as described above.

The Test Core

The design of the test core depends upon the physical and magnetic characteristics of the material to be tested. In general the radial thickness should be small in comparison with the diameter. Strain sensitive materials must be protected from mechanical stresses of handling and winding. Types of insulation and winding depend upon the loss characteristics of the core.

In any practical core, the diameter ranges between an inside value d_i and an outside value d_0. Since the diameter enters into the denominator of the expression for H and certain other magnetic quantities, the effective diameter must be calculated and used in such expression rather than the simple mean diameter. The effective magnetic diameter of a core having a rectangular cross-section is the reciprocal of the value obtained by averaging $1/d$, namely

$$d = \frac{d_0 - d_i}{\log_e \dfrac{d_0}{d_i}}. \tag{60}$$

This expression can be converted to the following convenient series

$$d = d_m \left[1 - \frac{1}{12} \left(\frac{\Delta d}{d_m} \right)^2 - \frac{1}{180} \left(\frac{\Delta d}{d_m} \right)^4 - \cdots \right], \tag{61}$$

which indicates that the effective diameter is smaller than the arithmetical mean diameter d_m, by an amount depending upon Δd, the difference between inside and outside diameters. This series converges so rapidly that terms beyond the second may be neglected for all practical purposes.

A test core of large radial thickness is to be avoided, when accurate measurements are desired, because of the considerable change of flux density from the inside to the outside diameter, with its accompanying modification of the core permeability. Such variations complicate eddy current and hysteresis behavior, particularly through reaction on the magnetic permeability. If the permeability at every point in the core bears a straight line relationship to the flux density, eq. (27) gives

$$\mu = \mu_0(1 + \lambda B). \tag{62}$$

The flux density at diameter y is

$$B = \mu H = \frac{0.4 N i \mu_0}{y} (1 + \lambda B). \tag{63}$$

Solving this equation for B, and integrating from d_i to d_0 gives the total flux in a unit height of core, from which the mean flux density can be calculated. This gives for the mean permeability, approximately

$$\mu_m \doteq \mu_0 \left(1 + \lambda B_m \frac{d^2}{d_i d_0} \right), \tag{64}$$

where B_m is the mean peak flux density in the core, and d is the effective magnetic diameter. Comparison of (64) with (62) shows that the increase of mean permeability in a core of considerable radial thickness is not precisely equal to the ideal increase of permeability for a given flux density.

The effect of radial thickness of core on losses can be attacked in a similar manner. Inserting the value of H at diameter y in Cauer's expression for power loss,[21] computing the loss in a ring of thickness $dy/2$, and integrating from d_i to d_0 yields an expression for core resistance identical with (33), (34) and (35) except that the terms containing α or λ must be multiplied by the factor d^2/d_id_0. It appears that radial thickness of the core affects only that part of the loss which depends upon the variation of permeability with H or B.

In preparing a test core, a compromise must be struck between the radial thickness, diameter, and axial height, so as to secure the desired cross-sectional area without excessive length of copper winding. The laminations of the core must be well insulated from each other. They should be of uniform thickness, which should be known accurately. A good technique used with material in the form of ribbon or tape, consists of winding it tightly in several layers upon a cylindrical mandrel and providing insulation against eddy current straying by dusting the strip with finely powdered alumina while winding. Such insulation is found to withstand the high temperatures ordinarily used in heat treating the core. In order to eliminate the airgap in this type of core, the inside end of the tape may be brought out, folded over, and welded to the outside end, before annealing the core. The use of 50 turns of tape or more in a spiral core reduces the airgap effect to a negligible amount so that welding the tape ends is not necessary. A mandrel diameter should be selected large enough to make the ratio $\Delta d/d_m$ quite small. Thus, a 9 cm. mandrel, wound to a depth of 1 cm., gives a core in which the magnetizing force decreases about 20 per cent from the inside diameter to the outside, while the correction term decreases the effective diameter about 0.4 per cent below the mean diameter. The correction to the permeability variation term is $d^2/d_id_0 = 1.002$.

The completed core, if strain sensitive, can be protected from the mechanical stresses of handling and winding by mounting it in a loose fitting toroidal box upon which the test windings are applied. Such spacing helps to decrease distributed capacitance, but even more important is sectionalizing, or bank winding of the coil. Types of insulation and windings depend upon the loss characteristics of the core. In general, lower loss characteristics in the core require higher quality windings, to permit measurements at higher frequencies.

SYMBOLS

a Hysteresis resistance coefficient.

a_1 Hysteresis loop area; $= \frac{1}{2}aB_m{}^3 = 4\pi W$.

[21] W. Cauer, *Arch. f. Elektrotechnik* **15**, 308 (1925).

A Cross-sectional area of magnetic core; $cm.^2$

α Permeability-magnetizing force coefficient; $= \mu_0 \mu_m \lambda$.

B Instantaneous flux density in core; gauss.

B_m Maximum flux density in core subject to alternating magnetizing force.

c Residual resistance coefficient.

C Distributed capacitance of coil winding; farad.

d Effective magnetic diameter of annular core; cm.

e Eddy current resistance coefficient of core.

e_c Eddy current resistance coefficient of coil winding.

f Frequency of alternating current.

G Distributed leakance of coil winding; mho.

h $= (aB_m + c)$.

H Instantaneous magnetizing force in core; œrsted.

H_m Maximum magnetizing force in core subject to alternating magnetizing force.

i_m Maximum of alternating current wave in coil; ampere.

I Effective or r.m.s. current in coil.

L Inductance, due to core and residual air space only; henry.

L_a' Inductance due to residual air space.

L_a Inductance due to coil with air core.

L_m Inductance with current of maximum value i_m due to core only.

L_{fm} Inductance due to core only, with current of maximum value i_m, at frequency f (i.e., subject to magnetic shielding).

$L_{obs.}$ Total inductance observed at frequency high enough to give increases due to distributed capacitance and leakance of the coil winding.

λ Permeability-flux density coefficient; $= \dfrac{\mu_m - \mu_0}{\mu_0 B_m}$.

μ_0 Initial permeability of core.

μ_m Permeability corresponding to L_m.

μ_{fm} Permeability corresponding to L_{fm}.

p Ratio of eddy current to hysteresis resistance at maximum frequency.

P Total power dissipated in core; watt.

P_e Eddy current power dissipated in core.

P_h Hysteresis power dissipated in core.

q Fraction of core loss tolerated as error due to capacitance, leakance, and copper eddy current loss in winding.

Q Insulation quality factor $= \omega C/G$.

R Resistance due to core and winding only; ohm.

R_c Direct current resistance of copper winding.

R_{ce} Eddy current resistance of copper winding.
R_e Eddy current resistance due to core.
R_h Hysteresis resistance due to core.
R_{fm} Resistance due to core only, corresponding to L_{fm}.
$R_{\text{obs.}}$ Total resistance corresponding to $L_{\text{obs.}}$.
ΔR Resistance due to core and leakance only.
ρ Resistivity of core material; abohm-cm.
ρ_1 Ditto; microhm-cm.
t Thickness of sheet, diameter of wire, or r.m.s. sphere diameter of magnetic material; cm.
θ Eddy current parameter; $= 2\pi t\sqrt{\mu_0 f/\rho}$ for sheet material.
W Hysteresis energy dissipated per cycle per cubic centimeter of core; erg.