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Long-Wave Radio Transmission Phenomena Associated with a Cessation of the Sun's Rays

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The variations in long-wave radio field strength near the time of sunset on long transmission paths, which have been reported by many observers, were studied for the purpose of formulating rational methods of forecasting their time of occurrence. During some preliminary observations fair agreement was found between the time of minimum field and the sun's position relative to a particular point on the transatlantic path under observation. The more extended study of radio field variations during sunset periods and solar eclipses disclosed that in general no exact relationship could be established between the sun's position at any point and the occurrence of the minimum field.

Observations of field variations were made on radio signals at a number of different frequencies and over several paths. It was concluded that a characteristic sunset cycle of field variations is present on frequencies between 18 kc. and 68 kc. for transmission paths longer than 700 km. For paths less than 200 km. long, such variations are negligible. There is some evidence that the amplitude of these field variations is smaller at lower frequencies.

Analysis of the data presented indicates that long waves over long paths are transmitted predominately by "sky waves." From the data it was not possible to establish any satisfactory picture of the path followed. It was established, however, that empirical methods based on observations over a particular transmission path may be employed to forecast the approximate time of occurrence of field variations.

INTRODUCTION

COMMERCIAL transatlantic radio telephone service to Europe was inaugurated on January 7, 1927, using a long-wave circuit on a frequency of 60 kilocycles. The difficulties of maintaining a satisfactory long-wave circuit through that part of each day when sunset conditions existed along the transmission path had been recognized [1] * for several years prior to the opening of service. After short-wave facilities were added to the transatlantic service, it became important to coordinate operating times of the short-wave transmitters in relation to the long-wave channel to assure maximum reliability and efficiency of service. In order to do this, a more precise knowledge of the

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* Numbers refer to appended list of references.

behavior of the long-wave circuit during the sunset period was needed, and it was for this purpose that many of the observations reported in this paper were made.

An analysis of these observations indicated that, aside from their practical application, some rather fundamental information concerning the probable mechanism of transmission on long waves could be obtained. To these observations other data were added, and all of this available material was systematically studied to determine the effect of the cessation of the sun's active rays on radio transmission at long wave-lengths. Although the results alone are rather inconclusive, they do provide sufficient evidence to indicate that the mechanism of long-wave transmission is in some ways the same as that of short-wave transmission and for the longer paths depends primarily on waves returned to the earth by layers in the atmosphere.

No attempt will be made in this paper to review the present status of radio transmission theory or of related geophysical phenomena, except in special cases where required to show the significance of our results. A background of the theory has been provided by many investigators, among whom Smith,[2] Pedersen,[3] Anderson,[4] Appleton,[5] Green,[6] Namba,[7] Yokoyama and Tanimura,[8] Hollingworth [9] and Heising [10] should be mentioned.

The analysis of the data taken during the present investigation indicated that the connection between solar and radio phenomena is effected through the agency of electromagnetic radiation, and not by means of low velocity corpuscular solar emission. A partial corroboration of this conclusion was secured by Schafer and Goodall,[11, 12, 13] the U. S. Bureau of Standards,[14] and others during several solar eclipses. The rather complete analysis of data taken during eclipses which has been made by Appleton and Chapman [15] also confirms this view.

METHOD OF MEASUREMENT AND ESTIMATED PRECISION

The field strength of special 60-kc. test dashes transmitted from WNL, Rocky Point, New York, was measured at Houlton, Maine, Chatham, New Jersey, and Cupar, Scotland. The Houlton and Chatham measurements were made by means of meter comparisons with a calibrated local oscillator and it is believed that their relative accuracy is of the order of ± 0.1 db, although the absolute accuracy probably falls short of this figure by a considerable margin. Due to the comparatively slow rate of long-wave field variation, no effort was exerted to obtain better timing than ± 10 seconds. Since weak fields and high noise are common during the late afternoon hours on the transatlantic path, it is believed that the precision of the Cupar measurements necessarily falls short of that possible for local tests.

Measurements made at Houlton on telegraph traffic from Tuckerton, Nauen, Ongar and Rocky Point are believed to be subject to relative errors as great as or even greater than ± 2.5 db. Comparisons between a local oscillator and telegraph traffic ordinarily are made by means of a cathode ray tube, and errors are occasioned both by the difficulty of reading the tube scale and by the varying strength of telegraph signals whose intensity is a function of the keying speed probably because of sluggish antenna systems.

The measurements on special 60-kc. test signals are believed to be sufficiently precise to provide a satisfactory index of the phenomena. The measurements on telegraph traffic provide data for a qualitative estimate of the nature of the various phases of the phenomena but probably are of no great value in fixing the exact time of occurrence of field strength increments smaller than 2 db, which may represent the total variation of some phases of the diurnal cycle.

General Characteristics of Sunset Effect on Short Paths

The form of the average diurnal sunset cycle of received field strength, plotted as a function of the sun's angular altitude at some salient point on the great circle transmission path, is shown on Fig. 1 for several paths. With the exception of the shortest path of 122 km. between Rocky Point and Chatham, New Jersey, a well defined typical sunset cycle was observed in all cases. If we assume that the sunset dip is due directly or indirectly to the cessation of the active solar rays in the upper atmosphere, the absence of characteristic large field variations at sunset for short paths may be ascribed to a predominance of low elevation transmission which does not enter the layers ionized by the sun. If rectilinear this transmission would pass ½ km. above Chatham on the 122-km. path, due to tangency with the earth's surface. The possibility of transmission between two points 122 km. apart on the earth's surface without a "sky wave," [5] therefore requires that the ray be bent around the spherical contour of the earth by diffraction and atmospheric refraction.[3, 16]

The daily occurrence of a sunset dip on paths of 700 km. and longer may be explained in several ways, all of which, however, require the existence of a downcoming ray. For example, the sunset phenomena may be due to a "fault" in the reflecting ionized layer,[7] or to interference between two reflected rays or between a reflected ray and a ground wave.[9, 17, 18]

Briefly, in accordance with the "fault" hypothesis, the shadow cast upon the ionized reflecting layer by the sun's active rays tangent to the earth's surface, or to an opaque atmospheric layer concentric with the earth which hereafter will be called the occulting layer, produces a

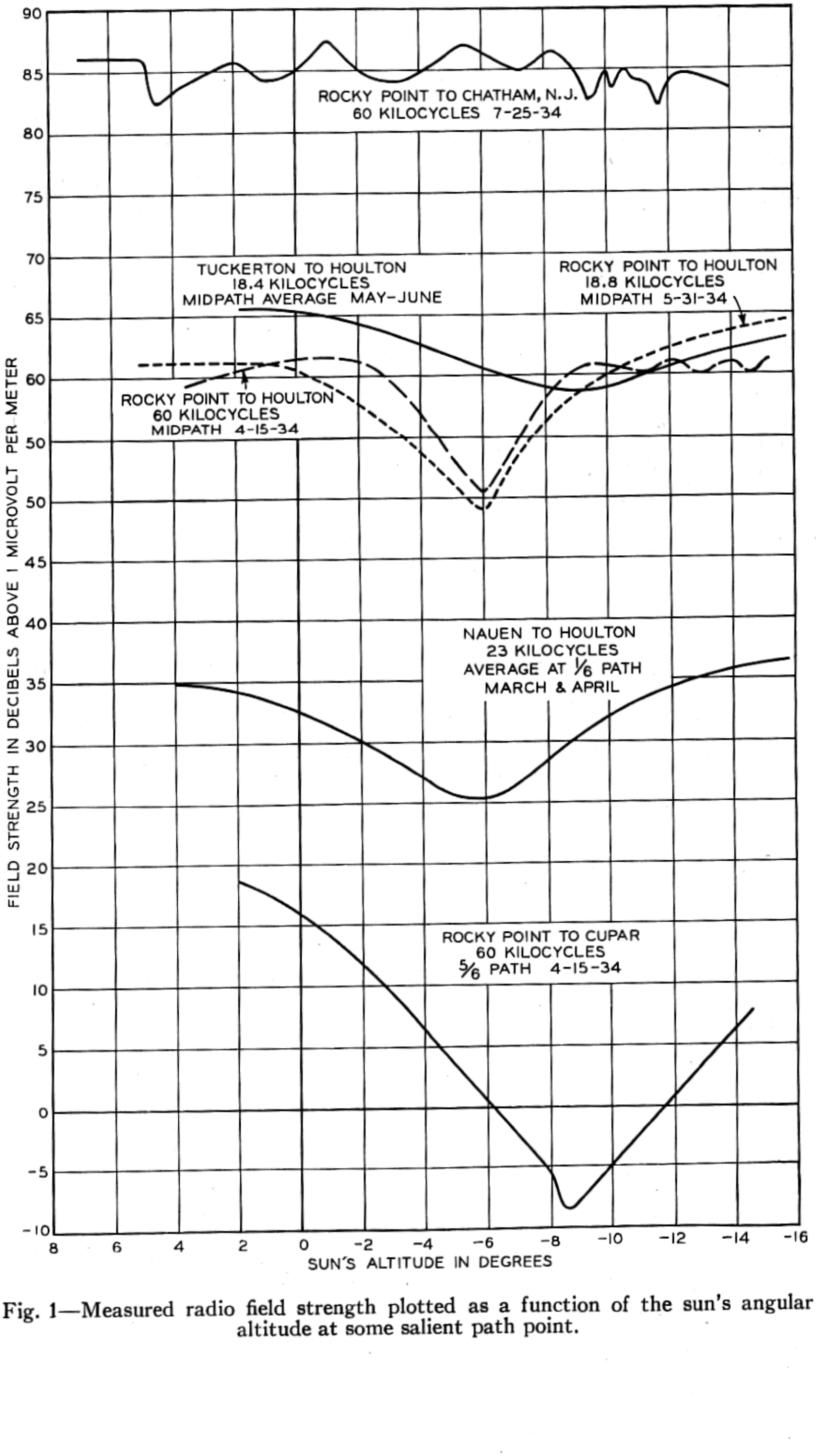

Fig. 1—Measured radio field strength plotted as a function of the sun's angular altitude at some salient path point.

"fault" in the otherwise uniform daylight ionized layer surface. The form of fault presumably is a ring generated by the intersection of the shadow cone of the occulting layer with the spherical reflecting layer surface. As the fault passes over the reflected radio ray apex, reception of the optimum ray may be impeded, either because of scattering due to an irregular reflecting surface,[3] or on account of the passage of the ionization density through a value defined by the Brewster angle, thereby changing the mode of transmission from daylight metallic reflection to night-time refraction.[7]

One of the objects of this analysis was to find the angular altitudes of the sun at some fixed point on the transmission path, coinciding with the phases of the sunset cycle of field variation at the receiving station. It was expected that with these data available over the period of a year, it would not be difficult to locate the salient point on the transmission path affected by the cessation of the sun's rays by means of Sumner lines of position [19, 20] provided, of course, that the physical structure of the layers showed negligible variation. From the above-mentioned data it would also be possible to compute the distance between the occulting and reflecting layers [21] and to accurately predict the time of occurrence of the sunset minimum. As shown in Tables I–IV, annual and fortuitous variations in the time of occurrence of the phases of the phenomenon prevent the satisfactory application of this method to long path effects, and when applied to the Rocky Point-Houlton path the point so located apparently is situated a considerable distance to the southwest of the most southerly path terminal. This presumably indicates that, if the phenomena take place at a fixed location on the transmission path, there must either be an annual variation of considerable magnitude in the effective distance between the reflecting and occulting layers, or the mechanism involved must be considerably more complex than that initially assumed.

FREQUENCY RANGE OF PHENOMENA

The characteristic diurnal sunset cycle was found on all frequencies studied during this investigation, the scope of which was limited to frequencies between 18 and 68 kilocycles. Available published data taken by other investigators indicate that the pronounced sunset minimum characteristic of long-wave transmission is not observed at broadcast frequencies.[6, 22] This may be due to the failure of transmission to improve after the sunrise drop, giving an all day minimum, or, if the minimum is due principally to interference bands, the fineness of band pattern at high frequencies may prevent the phenomenon from being recognized.

Observations on GBR at 16 kilocycles at Houlton during the latter part of April, 1934, showed little evidence of a minimum. These measurements were made on telegraph traffic and if a minimum occurred its amplitude must have been less than the observational errors of this method of measurement. These results seem to be confirmed by the observations of Yokoyama and Tanimura [8] which show a pronounced decrease in the amplitude of the sunset cycle at frequencies below 17 kilocycles, while frequencies slightly above this value display the characteristic effects. This apparent difference in the behavior of 18 and 16-kc. transmission, seems rather unusual and if real may have some geophysical significance.

RESULTS

An examination of the data taken on the Rocky Point-Houlton 60-kc. transmission path as plotted on Figs. 2 and 3 discloses that of the ten cases plotted, nine show that a minimum in measured field occurs 22–26 minutes or an average of 23 minutes after surface sunset at Rocky Point, corresponding to an altitude of the sun of 4 to 5½ degrees below the horizon, and in a single case at 18 minutes after sunset on August 29, 1934. The results in the winter months seem less regular than in summer and the cases of 2/25/34 and 1/21/34 are especially noteworthy in that they show multiple minima, the 23-minute minimum being subsidiary to a minimum occurring about an hour after Rocky Point sunset on 1/21/34. The fact that minima occur at a nearly constant interval after Rocky Point surface sunset, which occurs at the same time that the sun's rays are tangent to concentric elevated layers above this point, rather than sunset at some other path point, is believed to be the result of a fortuitous combination of circumstances rather than a rational relationship between these times. This hypothesis is strengthened by data shown in Table I taken over the same path at 18 kilocycles and data in Table II taken at 18 kilocycles over the 900-km. Tuckerton-Houlton path which do not show this same constant relationship with the time of sunset at the transmitting station. Of three observations obtained over the Rocky Point-Houlton 18-kc. transmission path the minimum in field occurred 34 minutes after sunset at Rocky Point in two cases but on the other day it occurred 22 minutes after sunset at Rocky Point. On all three occasions, however, the minimum was between 35 and 37 minutes ($-6°$ to $-6.5°$ altitude of sun) after surface sunset at the mid-path point. As in the case of the 60-kc. path for which data are shown on Figs. 2 and 3, the field began to fall at mid-path surface sunset.

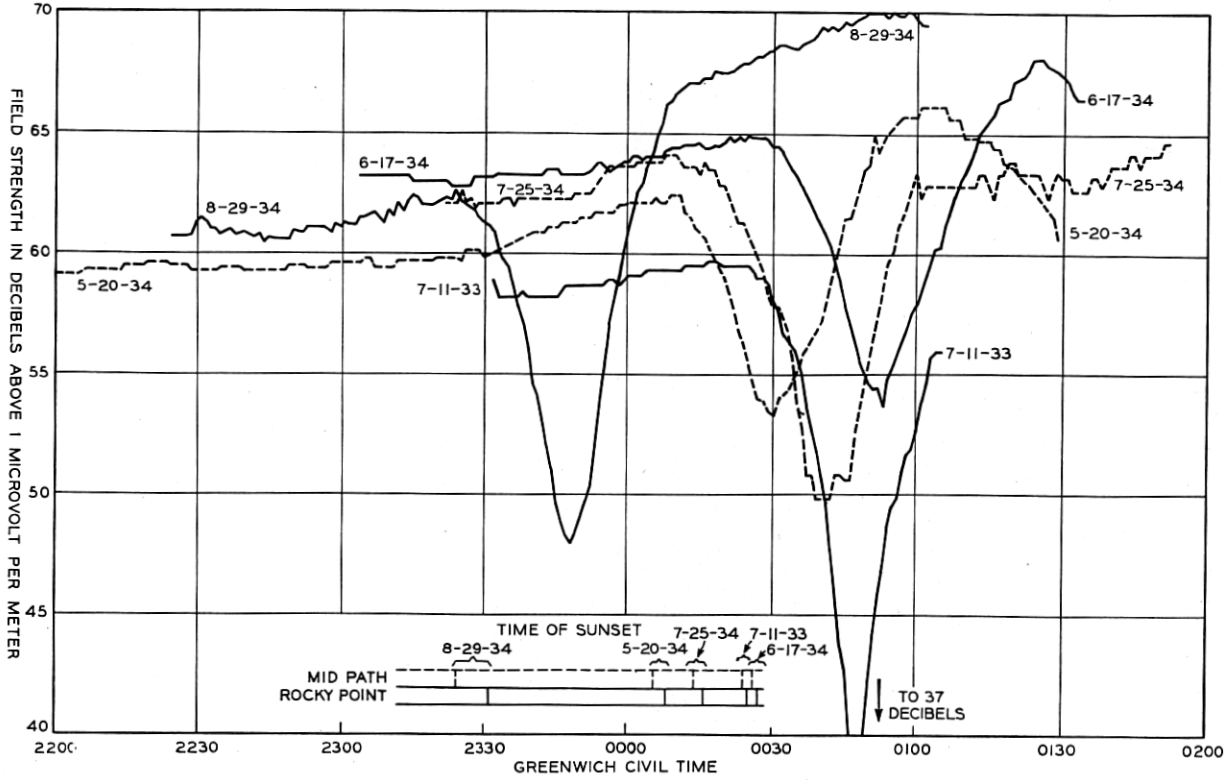

Fig. 2—Field strength variations on the Rocky Point-Houlton 60 kc. transmission path during the summer months.

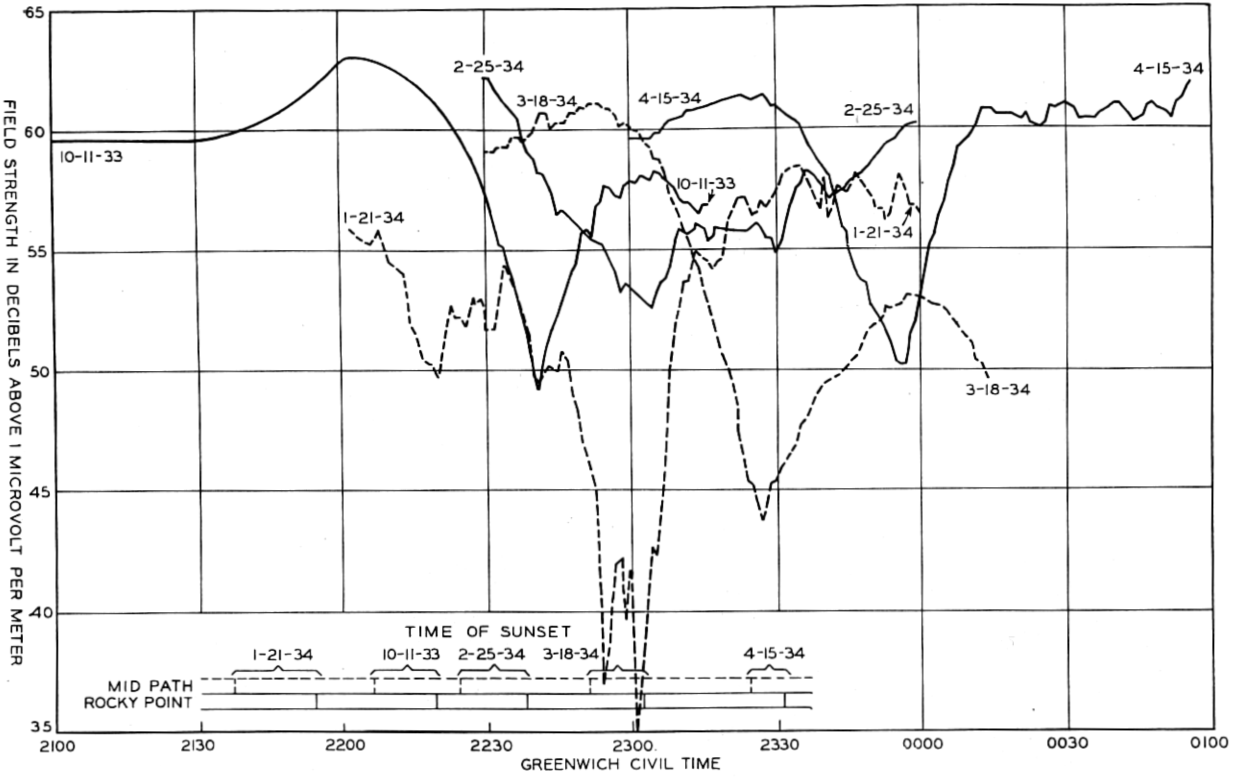

Fig. 3—Field strength variations on the Rocky Point-Houlton 60 kc. transmission path during the winter months.

TABLE I

ROCKY POINT-HOULTON—18.8 KC.

Distance 700 Km. Average Minimum 13 Db Below Field Immediately Preceding Drop

Date	Time of Sunset, GCT		Minimum Field		Interval Between Sunset and Minimum		
	Rocky Point	Houlton	Time GCT	Sun's Altitude at Midpath Degrees	Mid-path	Rocky Point	Houlton
6/ 1/34....	0016	0014	0050	−6.0	0:35	0:34	0:36
8/ 2/34....	0009	0003	0043	−6.7	0:37	0:34	0:40
10/21/34....	2204	2137	2226	−7.4	0:35	0:22	0:49
			Average	−6.7	0:36	0:30	0:42

TABLE II

TUCKERTON-HOULTON—18.4 KC.

Distance 900 Km. Average Minimum 7.9 Db Below Field Immediately Preceding Drop

Date	Time of Sunset, GCT		Minimum Field		Interval Between Sunset and Mininum		
	Tuckerton	Houlton	Time GCT	Sun's Altitude at Midpath Degrees	Mid-path	Tuckerton	Houlton
4/28–29/34 .	2349	2335	0030	− 9.0	0:48	0:41	0:55
4/30–5/1/34.	2351	2337	0033	− 9.0	0:49	0:42	0:56
5/ 1– 2/34 .	2352	2338	0040	−10.0	0:55	0:48	1:02
5/ 3– 4/34 .	2354	2342	0045	− 9.6	0:57	0:51	1:03
5/ 9–10/34 .	2359	2349	0053	−10.2	1:00	0:54	1:04
5/10–11/34 .	0000	2350	0047	− 9.4	0:52	0:47	0:57
5/11–12/34 .	0001	2351	0027	− 5.9	0:31	0:26	0:36
5/17–18/34 .	0006	2359	0048	− 8.0	0:46	0:42	0:49
5/21–22/34 .	0010	0004	0046	− 6.9	0:39	0:36	0:42
5/24–25/34 .	0013	0007	0050	− 7.0	0:40	0:37	0:43
5/28–29/34 .	0016	0011	0115	− 9.8	1:02	0:59	1:04
6/ 4– 5/34 .	0021	0017	0117	− 9.1	0:58	0:56	1:00
6/ 7– 8/34 .	0023	0019	0115	−11.0	0:54	0:52	0:56
6/11–12/34 .	0025	0022	0120	− 9.0	0:57	0:55	0:58
6/14–15/34 .	0027	0023	0122	− 9.3	0:57	0:55	0:59
			Average	− 9.5	0:51	0:47	0:55

The 18-kc. Tuckerton-Houlton 900-km. path of nearly the same azimuth as the Rocky Point-Houlton path, showed a minimum occurring irregularly at from 31 to 62 minutes, or an average of 51 minutes after mid-path sunset, corresponding to an average sun's altitude of − 9.5°. Since the data of Table II are subject to the errors inherent to measurements made on telegraph traffic, a portion of the large variation in time may be due to experimental errors.

The wave-like variations in field intensity observed on the Rocky Point-Houlton 60-kc. path on 1/21/34 and 4/15/34 have the appearance of interference fringes,[9, 17] and if explained on this basis the question at once arises whether or not interference is the principal cause of the sunset cycle. If daylight communication is accomplished either by means of a single reflected ray in combination with a ground wave, or by the resultant of a number of reflected rays of nearly constant complex propagation constants, at sunset the occultation of the ionizing rays from the transmission medium by tangency with an occulting layer, might reasonably be expected to produce variations in both the real and imaginary portions of the propagation constant of the medium, thereby causing interference fringes through the variation of the relative phase of different rays.

As an example, we may assume that on the Rocky Point-Houlton transmission path the ground ray provides the principal agency of daylight communication by means of atmospheric refraction and diffraction. At the approach of sunset, reduced ionization between the earth and the reflecting layer reduces the attenuation in the path of the reflected ray, producing a resultant received field which is a function of the relative intensity and phase of the two waves. As the effect of the sun's active rays becomes still less, the decreased ionization of the layers produces variations in the phase of arrival of the reflected wave, either through changes in the propagation constant, or on account of greater path length occasioned by an increased virtual height of the reflecting layer.

Now since the resultant received field is the vector sum of the two rays, when one ray is much smaller than the other, variations in their relative phase will produce small amplitude fluctuations in the resultant, with maxima and minima equal to the sum and difference of the two components. As the two components approach equality, however, the maxima will approach twice the intensity of one component ray, while the minima will approach zero, thereby producing a very deep "dip" in the received field.

THE "D" LAYER

It has been suggested by Heising,[10] Appleton [23] and Chapman [21] that passage through a low-elevation layer of ozone produced by solar ionization, is one of the principal causes for the daylight attenuation of the reflected ray.[2] Radio transmission measurements at broadcast frequencies, reported by the U. S. Bureau of Standards [22] and by the Australian Council for Scientific and Industrial Research,[6] show a rapid decrease in sky-wave attenuation beginning shortly before tangential sunset and continuing for from one-half to two hours after sunset. On the 60-kc. long-distance path the evidence of the initial phases of this phenomenon is not so well defined, due probably to the gradual nature of the change, and to the characteristic sunset minimum which occurs shortly after sunset. Recent measurements made on the transatlantic radio telephone channels, however, indicate a presunset rise in field of about 2 db, beginning when the sun's altitude becomes low enough to cause an appreciable lengthening of the atmospheric ray path, thereby decreasing the intensity of the active solar rays. A presunset increase is very apparent on the Rocky Point-Houlton 60-kc. measurements, beginning at about 40 minutes before, and continuing to a maximum at the instant of mid-path sunset. There is a possibility, however, that this phase may be due to interference phenomena.

More conclusive evidence of the effect of a reduction in the intensity of the sun's ionizing rays is provided by long-wave field strength measurements made during the solar eclipses of January 24, 1925, and August 31, 1932. The data taken in 1925 were secured by means of an automatic recorder on the Rocky Point-Belfast 57-kc. path and manual measurements were made at the European receiving stations. In 1932, automatic recorders were used on both the Rocky Point-Houlton 60-kc. path and the Rugby-Houlton 68-kc. path. In all cases where automatic recorders were used the data were abstracted from the record and replotted for reproduction.

Figures 4 and 5 show the eclipse circumstances and the concurrent variations in the measured radio field strength. The Chedzoy, New Southgate, and somewhat less clearly the Aberdeen measurements in 1925, show the results of reduced attenuation almost immediately after the first darkening of the transmission path. For the 1932 data this effect is clearly evident on the Rocky Point-Houlton measurements.

The sudden increase in field to be expected at the beginning of the eclipse at Belfast in 1925 is not apparent from the data, and its absence may be due either to improper recorder operation or to some fortuitous phenomenon peculiar to that particular eclipse, such as, for example, the possibility that the phases of ground and sky waves were in quadrature. The Rugby-Houlton observations in 1932 likewise are ambiguous because the true eclipse effect is complicated by superposed sunset effects originating at the eastern path terminal, and the observed increase in field may be due to these rather than to the eclipse.

In all cases except the two mentioned above, the increase in field was followed by a rapid drop, with a minimum occurring at the approximate time the totality shadow crossed the transmission path.

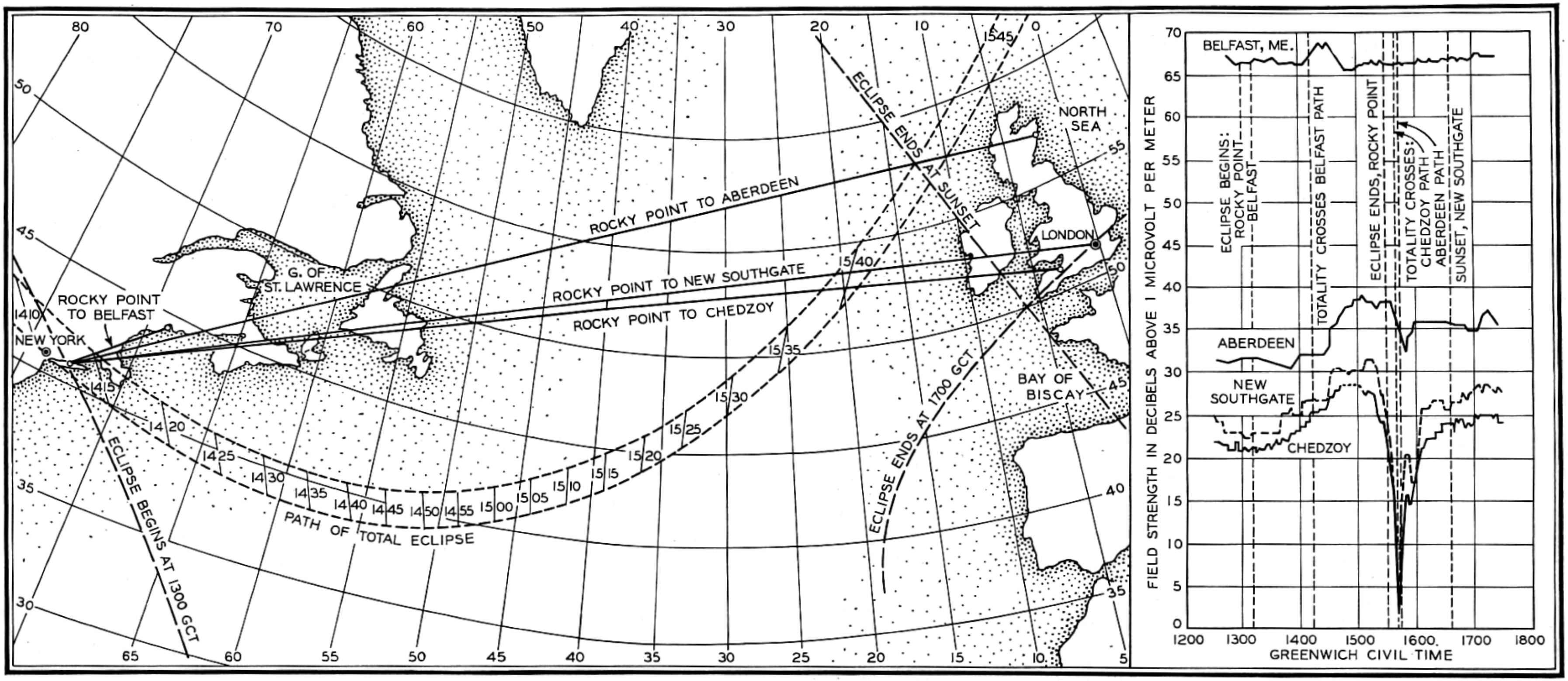

Fig. 4—Relation between 57 kc. radio transmission and circumstances of the solar eclipse of January 25, 1925.

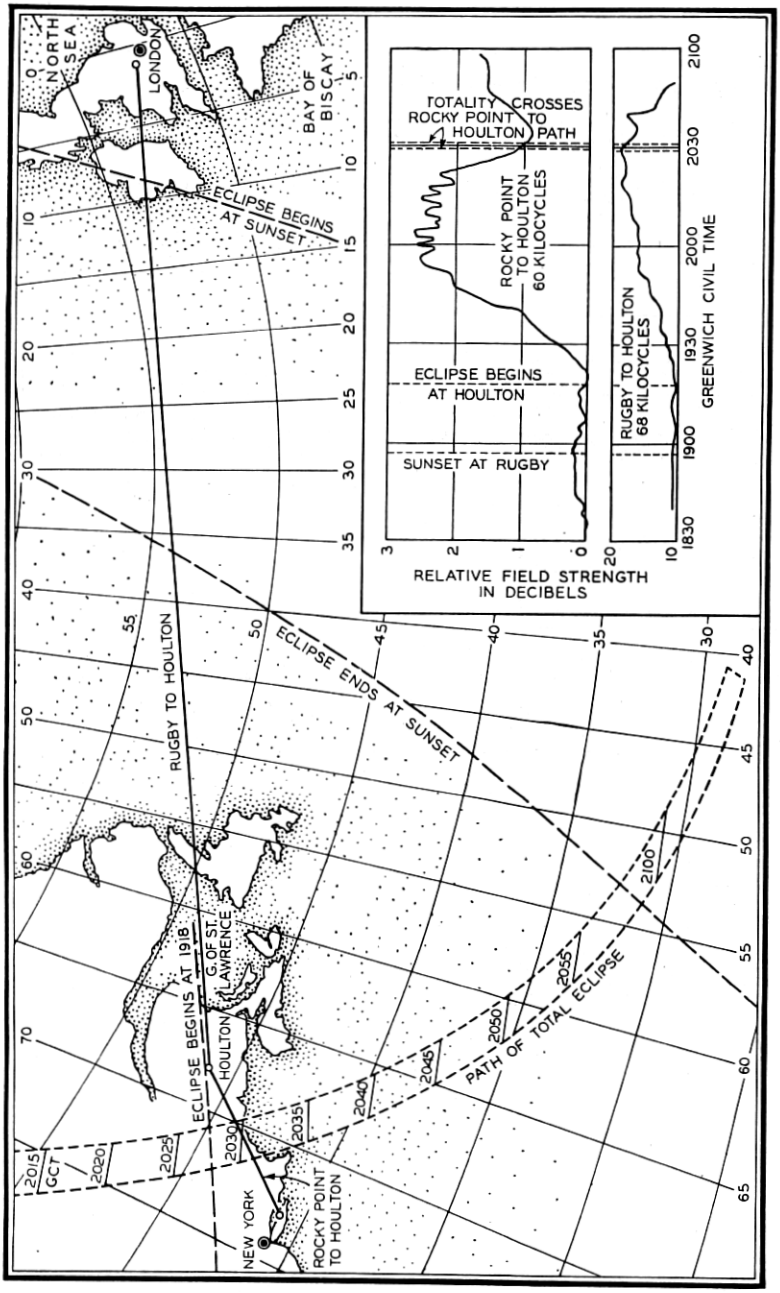

Fig. 5—Relation between 60 kc. radio transmission and circumstances of solar eclipse of August 31, 1932.

Long Transmission Paths

An examination of long-distance data of Tables III and IV discloses the following facts regarding these phenomena.

TABLE III

Nauen-Houlton—23 Kc.

Distance 5600 Km. Average Minimum 13 Db Below Field Immediately Preceding Drop

Date	Time of Sunset, GCT		Minimum Field		Interval Between Sunset and Mininum	
	Nauen	1/6 Path Point	Time GCT	Sun's Altitude at 1/6 Path Point Degree	Nauen	1/6 Path Point
3/12/34.....	1706	1756	1838	−6.2	1:32	0:42
3/19/34.....	1718	1810	1836	−4.2	1:18	0:26
3/22/34.....	1723	1816	1900	−5.5	1:37	0:44
3/26/34.....	1730	1824	1850	−4.2	1:20	0:26
3/29/34.....	1735	1830	1903	−5.5	1:28	0:33
4/ 2/34.....	1742	1839	1925	−6.7	1:43	0:46
4/ 5/34.....	1747	1845	1857	−2.5	1:10	0:12
4/ 9/34.....	1754	1853	1940	−7.0	1:46	0:47
4/10/34.....	1756	1855	1944	−7.0	1:48	0:49
4/12/34.....	1759	1859	1950	−7.2	1:51	0:51
			Average	−5.6	1:33	0:37

TABLE IV

Rocky Point-Cupar—60 Kc.

Distance 5172 Km. Average Minimum 25 Db Below Field Immediately Preceding Drop

Date	Time of Sunset, GCT		Minimum Field		Interval Between Sunset and Minimum	
	Cupar	5/6 Path Point	Time GCT	Sun's Altitude at 5/6 Path Point Degrees	Cupar	5/6 Path Point
10/11/33....	1724	1817	1910	− 8.0	1:46	0:53
11/22/33....	1554	1644	1807	−10.4	2:13	1:23
12/20/33....	1548	1626	1756	−10.6	2:08	1:30
2/25/34....	1736	1830	1958	−12.5	2:22	1:28
3/18/34....	1820	1916	2029	−10.7	2:09	1:13
4/15/34....	1917	2015	2117	− 8.4	2:00	1:02
6/17/34....	2101	2206	2322	− 6.6	2:21	1:16
			Average	− 9.6	2:08	1:15

Rocky Point-Cupar 60-Kc. Path, Table IV

1. Minimum field does not follow surface sunset at any point on the transmission path by a constant time interval independent of season.

2. The altitude of the sun as measured from the most easterly path apex of an hypothetical three-reflection path varies from approximately − 6° in summer to − 13° in winter at the instant of minimum field, and computation shows that there is no point on the path at which the altitude remains constant.

3. Daily irregularities of considerable magnitude are often noted.

ONGAR-HOULTON PATH

1. Multiple minima of irregular distribution seem characteristic of this path.

2. Averages of field strength as a function of the sun's altitude at three path apices show separate minima when the sun is at about − 6° during May and June.

NAUEN-HOULTON, TABLE III

1. The time of minimum field varies as much as 40 minutes within a few days.

2. An average of the March-April data indicates that the minimum takes place when the sun's altitude is approximately − 5.6° at the first of three path apices, or 37 minutes after sunset at the earth's surface under this point.

CONCLUSIONS

Based solely upon the rather meagre data gathered during this study, and therefore subject to further confirmation before being considered generally applicable to all long-wave transmission paths, our conclusions may be recapitulated as follows:

1. A characteristic cycle of events accompanies the cessation of the sun's active rays, consisting of an initial increase in field during the reduction of solar intensity, followed by a minimum received field after the sun's rays are cut off at the earth's surface. This cycle, occurring both at sunset and during total solar eclipses, is typical of long-wave transmission in the 18–70-kc. band over paths in excess of a few hundred kilometers.

2. The time interval between sunset at any point on the transmission path and the instant of minimum field has an annual and an apparently fortuitous daily variation. As a result of these variations the minimum does not occur when the sun is at a fixed angular altitude at any point on the transmission path.

3. The time interval between sunset at some fractional path point and the instant of minimum field, and likewise the angular altitude of the sun referred to the plane of the horizon at such a point, increases with both the path length and the wave-length, and is maximum during the winter months.

4. The amplitude of variation of the sunset cycle apparently is reduced greatly at frequencies below 17 kilocycles and on paths shorter than 200 km.

5. Evidence of interference fringes on some of the observations suggests the possibility that the main sunset minimum may be the result of interference phenomena.

6. The fact that the phases of the radio transmission cycle closely follow the optical eclipse circumstances indicates that the radio phenomena must be related to solar radiation of velocity similar to that of light.

7. On transatlantic paths during the spring and summer months the average sunset minimum occurred when the sun was approximately 6° below the horizon at one-sixth of the total path length from the eastern terminal. Since these paths varied considerably in latitude and length, the phenomena may be related to effects occurring at the most easterly apex of a three-reflection path. The data obtained in this investigation, however, are not sufficient to definitely establish the generality of this hypothesis.

8. Empirical rules may be formulated for the prediction of the time of occurrence of various phases of the sunset cycle on short transmission paths. For example, the beginning of the drop in field on the 60-kc. Rocky Point-Houlton path occurs at mid-path surface sunset, and the minimum occurs at approximately 23 minutes after surface sunset at Rocky Point. On longer paths larger fortuitous variations occur and available data fail to connect the time of the minimum with sunset at any point on the transmission path. Representative curves drawn from the data, although subject to random errors, provide an empirical method for the prediction of the approximate time of occurrence of the phenomena and are of service in traffic and power scheduling.

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APPENDIX I

POSITION OF INTERMEDIATE POINTS ON THE TRANSMISSION PATH

The method of determining the transmission path parameters and the position of intermediate path points spaced a given distance from a path terminal is given in detail below.

Let φ = Latitude,

L_o = Longitude,

D = Distance, nautical miles between subscript points,

C = Path azimuth at subscript point,

Subscript a denotes sending station,

Subscript b denotes receiving station,
Subscript x denotes intermediate point,
$L_{o_{ab}}$ = Difference in longitude between a and b.

By the law of cosines, in a spherical triangle of sides abc and angles ABC, if we are given two sides and the included angle, the side opposite the given angle may be computed from (1) below.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \tag{1}$$

or substituting geographical coordinates

$$\cos D_{ab} = \sin \varphi_a \sin \varphi_b + \cos \varphi_a \cos \varphi_b \cos L_{o_{ab}}. \tag{2}$$

This may be made more convenient for logarithmic computation by writing it in the following form:

$$\text{hav } D_{ab} = \text{hav } (\varphi_a - \varphi_b) + \cos \varphi_a \cos \varphi_b \text{ hav } L_{o_{ab}}. \tag{3}$$

Now by the law of sines

$$\left\{\begin{aligned} \sin C_a &= \frac{\cos \varphi_b \sin L_{o_{ab}}}{\sin D_{ab}} = \cos \varphi_b \sin L_{o_{ab}} \csc D_{ab}, \\ \sin C_b &= \frac{\cos \varphi_a \sin L_{o_{ab}}}{\sin D_{ab}} = \cos \varphi_a \sin L_{o_{ab}} \csc D_{ab}. \end{aligned}\right. \tag{4}$$

Equations (3) and (4) above determine the great circle distance between "a" and "b," the azimuth of "a" from "b," and "b" from "a." To find the position of a point "x" located a fraction of the total distance between "a" and "b" we again substitute in (1), obtaining

$$\left\{\begin{aligned} \sin \varphi_x &= \sin \varphi_b \cos D_{xb} + \cos \varphi_b \sin D_{xb} \cos C_b, \\ \sin \varphi_x &= \sin \varphi_a \cos D_{xa} + \cos \varphi_a \sin D_{xa} \cos C_a, \end{aligned}\right. \tag{5}$$

and by the law of sines

$$\left\{\begin{aligned} \sin L_{o_{ax}} &= \frac{\sin C_a \sin D_{xa}}{\cos \varphi_x} = \sin C_a \sin D_{xa} \sec \varphi_x, \\ \sin L_{o_{bx}} &= \frac{\sin C_b \sin D_{xb}}{\cos \varphi_x} = \sin C_b \sin D_{xb} \sec \varphi_x. \end{aligned}\right. \tag{6}$$

The latitude and longitude of intermediate point "x" are therefore determined by equations (5) and (6) above.

APPENDIX II

DETERMINATION OF SUN'S ALTITUDE AND AZIMUTH

The angle of the sun to the horizon and to the meridian plane at any hour may be computed by methods similar to the above. For this case we have a celestial triangle whose sides are a meridian through the observer's zenith, a meridian through the sun, and a great circle through the sun and the zenith. The arc subtended by the pole and zenith is the complement of the observer's latitude "φ," the arc subtended by the pole and the sun is the complement of the sun's declination "d" (celestial latitude), and the angle "t" at the pole between these two arcs is the sun's hour angle. With these two sides and included angle we may compute the arc between the sun and the zenith (complement of the altitude "h") and the sun's azimuth "z" which is the angle between the meridian containing the zenith and the great circle passing through the zenith and the sun.

By the law of cosines (1) above

$$\sin h = \sin d \sin \varphi + \cos d \cos \varphi \cos t \tag{8}$$

and by the law of sines

$$\sin Z = \frac{\sin t \cos d}{\cos h}. \tag{9}$$

Values of h and z as a function of φ, d and t are tabulated in convenient form in *hydrographic office publication H.O.* No. 203. The sun's declination and the computed times of sunset may be obtained from the *American Nautical Almanac.*