Circulating Currents and Singing on Two-Wire Cable Circuits

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One of the important factors limiting the working net losses of two-wire cable circuits is the possibility of excessive circulating currents or actual singing.

A theory is developed for the computation of the distribution of singing margins on groups of two-wire circuits from the known gains and losses and known functions of the deviations of loading coils, loading coil spacing, cable capacitance and office equipment. The distribution functions of circulating current margins, of active return losses and of active singing points are also derived.

The possible application of these methods to specific problems is discussed and an example of the computations involved is given.

The theory herein involves certain approximations and empiricisms in determining the singing limitations but it is believed to give an answer which approaches the exact answer rather closely.

Introduction

AMONG the considerations which limit the minimum working net losses [1] of two-wire cable circuits, one of the most important is the desirability of avoiding excessive circulating currents. These circulating currents may manifest themselves as a quality impairment due directly to frequency and phase distortion or as sustained oscillation (singing).

In a given two-wire cable circuit, if the exact location and nature of each irregularity were known, it would be possible to compute exactly whether sufficient singing margin is available. The practicable method, however, is to compute the singing margins which will be exceeded on various percentages of a large group of such circuits, from the information which is available about the irregularities on a distributional basis. This paper first derives theoretical distributions of circulating current margins and singing margins without regard to various practical considerations such as the effect of repeating coils and other apparatus. In the second main division are discussed various considerations which are involved in applying the theory. In the third main division detailed computation methods are illustrated. The attached appendices cover the mathematical derivation of certain quantities.

[1] "Certain Factors Limiting the Volume Efficiency of Repeatered Telephone Circuits," L. G. Abraham, *Bell Sys. Tech. Jour.*, October, 1933.

Theoretical Distributions

Circulating Current Margins

A two-wire loaded cable circuit will be considered which consists of a number of repeater sections with 22-type repeaters. As shown on Fig. 1 there are various circulating paths in such a two-wire circuit

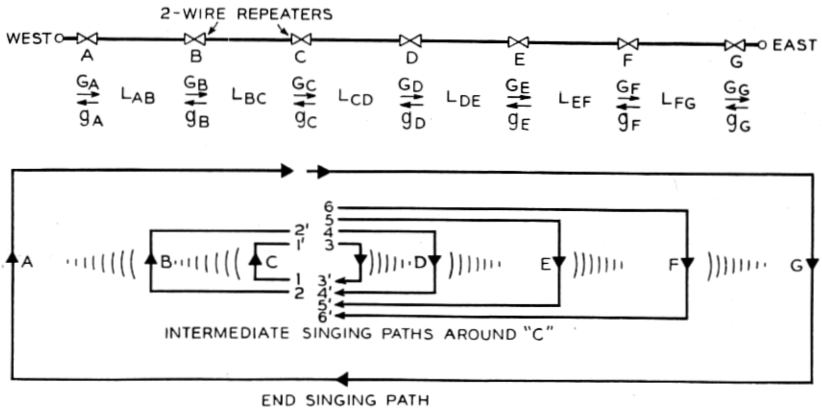

Fig. 1—Singing paths in a two-wire circuit.

which might cause objectionable circulating currents at any given frequency. Each repeater section consists of a large number of loading sections which have approximately the same capacitance and the same inductance per mile. Practically, however, there are deviations of the capacitance and inductance in a given loading section from the nominal average, each of which introduces an impedance irregularity which prevents the balancing network from exactly balancing the line. To determine the amount of current returned from each of these irregularities in a given case and the phase at which it is returned is, in general, impracticable. It is possible, however, to determine in what percentage of circuits the circulating current at any given frequency will exceed a certain percentage of the original current or, in other words, the percentage of cases in which the loss in the circulating current path will be less than a certain amount. The loss in the circulating current path at a given frequency is called the circulating current margin at that frequency.

The distribution of return losses at any given frequency in a cable section without repeater has been determined by G. Crisson in a paper entitled " Irregularities in Loaded Telephone Circuits," *Bell System Technical Journal*, October, 1925. This derives the distribution of the return losses which would be measured at a given frequency on a large number of such cable circuits in terms of certain functions of the capacitance and inductance deviations. From this paper the distribution function of the return losses may be expressed in decibels as follows:

$$S = S_H + S_w - S_A + S_F = S_1 + S_F$$

(See Appendix VI for Nomenclature).

In this equation S_F is the distribution function of the return losses and S_1 is the return loss at the frequency in question which is exceeded by

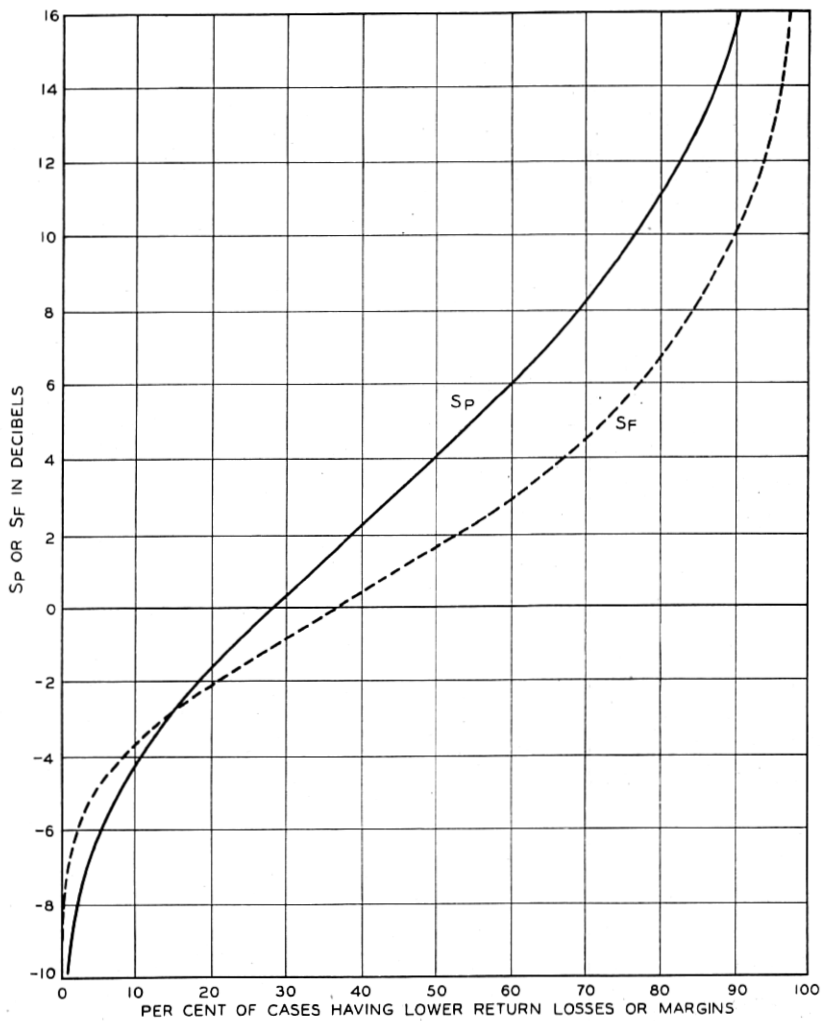

Fig. 2—Values of S_P and S_F.

63 per cent of the total return losses measured. S_H is determined entirely from the fractional deviations of the capacitances and inductances. S_w is determined by the proximity of the frequency in question to the cutoff frequency of the line facilities. S_A is determined by the attenuation of the cable circuit and is, in effect, the summation function of the different sources of irregularity.

Figure 2 shows the distribution curve S_F from the paper referred to above. In a similar manner the distribution curve of several repeater sections in tandem or in parallel [2] may be determined. This is known as an active return loss. Referring to Fig. 1 the return loss measured across the west hybrid coil of repeater C will be determined not only by the irregularities in the immediately adjacent repeater section but also by the irregularities in the other repeater section to the west of repeater C as seen through the intervening losses and gains of the circuit. Strictly speaking, the return losses on the east sides of repeaters A and B will also affect the active return loss because circulating paths around each repeater and around various combinations of repeaters will exist. However, these circulating paths have so much loss in any practical field circuit in which the other requirements are satisfied, that the return losses on the east side of A and B need not be considered.

Appendix I derives the formula for the distribution curve of an active return loss when the passive return losses of which it is made up are of the form given above, assuming no returned currents from beyond the terminal repeater. This distribution function is:

$$S_1 - F(N, T) + S_F.$$

In this case it is assumed that the value of S_1 is the same for the passive return losses of each repeater section, but from the appendix, the more general case where there is a different value of S_1 for each section may be determined.

Figure 3 gives values of $F(N, T)$ for the specific case where each repeater section has the same loss and each intermediate repeater has the same gain. The value of this function is also derived in Appendix I.

Referring again to Fig. 1 there will be an active return loss which may be measured on the west side of repeater C and also an active return loss which may be measured on the east side of repeater C. Assuming that the distribution function of the active return loss toward the west is $S_{11} + S_F$ (not including path A) while the dis-

[2] While this paper develops the theory specifically for the case of repeater sections in tandem, e.g., in a single two-wire circuit, it is generally applicable also to the case of repeater sections in parallel, e.g., in a toll conference connection.

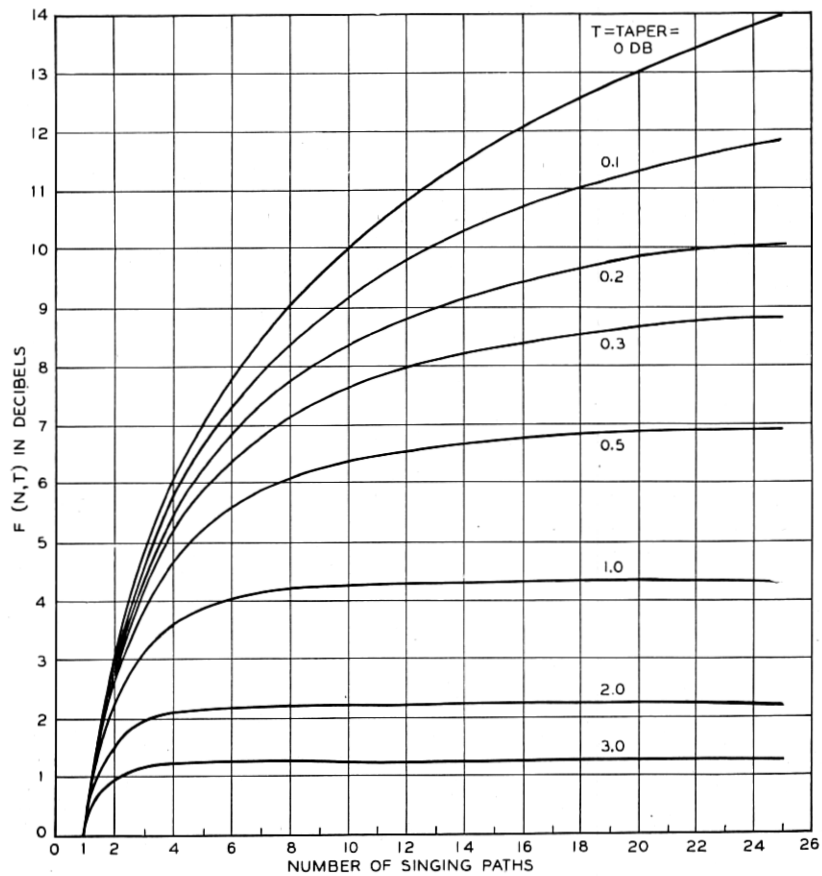

Fig. 3—$F(N, T)$ in perfectly regular circuit.

tribution function of the active return losses toward the east is $S_{12} + S_F$ (not including path G), the distribution function of the two return losses in series around the repeater will be $S_{11} + S_{12} + S_P$ as derived in Appendix II. In the general case, there will be gain in series with the active return losses, which may be considered as part of the return losses. For convenience, define S_{11} as follows:

$$S_{11} = S_1 - F(N_1, T) - G_C. \tag{1}$$

S_1 and $F(N_1, T)$ are as discussed above; N_1 is the number of repeater sections towards the west end of the circuit; and G_C is the gain of the repeater in the west to east direction, say. Similarly,

$$S_{12} = S_1 - F(N_2, T) - g_C. \tag{2}$$

The various quantities are like those enumerated above except that they are for the active return loss in the other direction and include the gain of the repeater in the east to west direction.

Figure 2 shows the value of S_P as derived in Appendix II plotted with S_F. By comparison of these curves, it may be seen that the spread of active return losses around a repeater will be somewhat larger than the spread of active return losses in one direction. The S_P curve approaches more nearly to a so-called normal law than the S_F curve does.

The previous discussion has been confined to the returned currents obtained from intermediate points in the circuit and also at a single pre-selected frequency. In addition to these currents, there is at each end of the circuit a current returned through the path called the terminal return loss. In a four-wire cable circuit this is the only path for circulating currents. From field data it appears that these terminal return losses approximately follow the S_F distribution curve also. The end path toward the west may be written as:

$$S_{21} + S_F = 7 + E_1 + S_F, \tag{3}$$

where 7 db is assumed as the terminal return loss for $S_F = 0$. In this case E_1 is the net loss at the particular frequency from the repeater in question (from the west side of repeater C) to the far end of the circuit and from that end of the circuit back to the repeater minus the gain of the receiving repeater on the latter side of the circuit (the W–E repeater). The end path toward the east is

$$S_{22} + S_F = 7 + E_2 + S_F. \tag{4}$$

If the losses of each cable section are equal and the gains of each intermediate repeater are equal, the repeater in question is at the center of the circuit, and $E_1 = E_2$, and at 1000 cycles E_1 would be equal to the nominal circuit net loss.

The active return loss at a given frequency toward the west which includes the end path (both including the west to east repeater gain) will be

$$S_{31} + S_F = (S_{11} \underset{p}{\mathrm{X}} S_{21}) + S_F \tag{5}$$

and toward the east will be

$$S_{32} + S_F = (S_{12} \underset{p}{\mathrm{X}} S_{22}) + S_F. \tag{6}$$

Where $\underset{p}{\mathrm{X}}$ means that the quantities so connected are combined as if their powers added directly; e.g., $S_{11} \underset{p}{X} S_{21} = S_{31}$ means that

$$10^{-S_{11}/10} + 10^{-S_{21}/10} = 10^{-S_{31}/10}.$$

The circulating current margin (M_c) around this repeater at a given frequency will therefore be $S_{31} + S_{32} + S_p$, as derived in Appendix II. Written in the more general form, the circulating current margin is

$$M_c = [S_1 - F(N_1, T) - G_C] \underset{p}{X} [E_1 + 7] + [S_1 - F(N_2, T) - g_C] \underset{p}{X} [E_2 + 7] + S_P. \quad (7)$$

Singing Margins

The objectionable effects of too low circulating current margin are to cause poor quality in transmission over the circuit and to cause the circuit to " ring " or sound " hollow " to the talker or listener. When a circuit oscillates or " sings," conversations over it become difficult or impossible, voice-operated devices on connecting circuits are locked up, parts common with other circuits such as common " C " batteries may be adversely affected, and other circuits are made noisy through crosstalk in the cable or in the repeater station. It is therefore important that the percentage of cases in which singing occurs shall be very small.

In general, the tendency will be for most of the loaded cable circuits to sing within a fairly limited frequency range which is usually near the upper frequency point at which the overall circuit loss begins to be appreciably greater than the loss at 1000 cycles. Figure 4 shows a

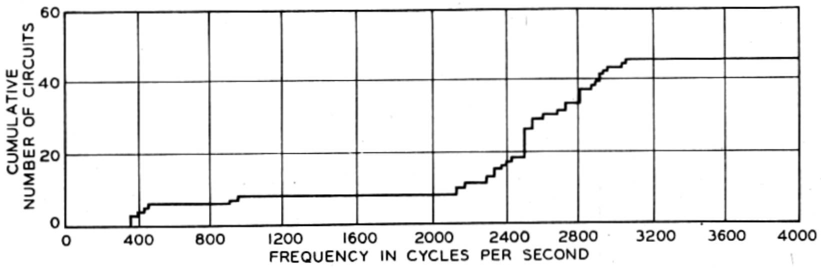

Fig. 4—Singing frequency in 22 tests on 46 19-gauge B & H–88–50 two-wire cable circuits.

cumulative plot of the singing frequencies during 22 tests [3] on 46 19-gauge B & H–88–50 two-wire cable circuits. It may be seen that

[3] A 22 test is a singing test made by increasing the gain of a normal working repeater in a two-wire circuit until singing begins.

over 80 per cent of these frequencies were between 2200 and 3100 cycles, while the nominal transmitted band on these facilities is from 250 to 3000 cycles.

Consider the case of the return loss of one circuit in a group of cable sections. A typical curve of the passive return loss of such a circuit is shown on Fig. 5. It will be seen that it consists of a set of "wabbles"

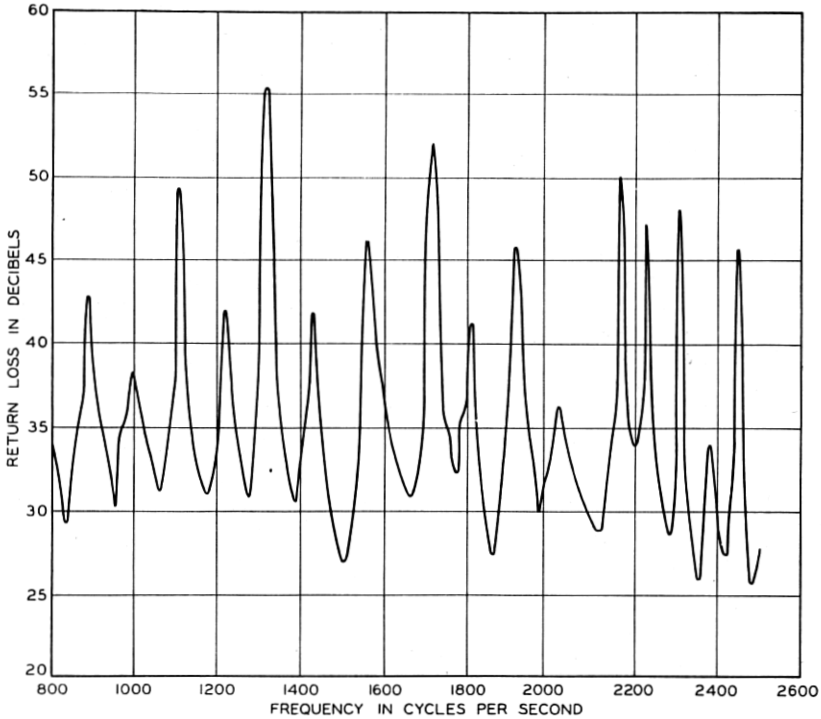

Fig. 5—Typical return-loss-frequency characteristic of a 19-gauge H–88–50 side circuit repeater section.

around a general trend line. When a measurement of return loss is made at a single pre-selected value, the return loss obtained is determined by what may be considered as two components, (1) the general quality of the repeater section as determined by the trend line, and (2) the particular part of the " wabble " on which the particular frequency measurement happens to fall (i.e., the bottom or top, or in between point of a given " wabble "). In measuring the return losses of a large group of lines at, say, 2900 cycles, the lower values of return loss will tend to be those which happened to be measured at the bottom of a " wabble." The higher the return loss of a given circuit, the more chance there is that the return loss of that line happened to be measured at the top of a " wabble."

When singing points [4] are measured, however, it may be said that when singing takes place above, say, 2000 cycles, very nearly the minimum value of return loss minus gain is obtained. The distribution of singing points on a large group of lines, therefore, will be lower (in decibels after correction for gain characteristic) for a given percentage than the distribution of single-frequency return losses even at the frequency which has the lowest computed values of any within the transmitted band (say, 2900 cycles for a repeater cutting off about there). Following the reasoning concerning the " wabble " given above, the amount by which the singing point will be lower will be small for the lower return losses and singing points of a given group of lines, and will tend to increase more and more as the value of return loss becomes higher.

Figure 6 shows the S_F curve plotted on a scale which makes all cumulative normal law distributions come out as straight lines. It may be seen that the differences just described will tend to make the singing point distribution more nearly a normal law than the S_F curve. Field measurements show that such singing point measurements do approximate a normal law much more closely than they approximate the S_F curve. Measurements of singing points on about 900 19-gauge H–172–63 side circuits at 18 places during completion tests gave standard deviations about the average of the group at each place which, when added together as the weighted root mean square, gave a general standard deviation of 2.02 db. About 400 similar measurements on 16-gauge H–44–25 side circuits at 16 points gave a standard deviation of 2.05 db. Similar measurements on 233 19-gauge H–88–50 side circuits and 77 19-gauge H–88–50 phantom circuits gave standard deviations of 2.13 and 2.03 db, respectively. It therefore seems reasonable to conclude that a standard deviation of about 2 db is substantially correct for the distribution curve of singing points at a given place. It should be realized that if singing points for a given type of facility from a large number of places are grouped together and a standard deviation of the entire group obtained around the average of the entire group, it may be considerably larger than 2 db due to the differences in average values of the different groups. Such computations on about 7500 measurements on one type of facilities showed a standard deviation of about 3 db for the entire group.

[4] The singing point of a given line is the gain which must be connected between the two sides of a hybrid coil to just cause singing, when the line terminals of the hybrid coil are connected to the line in question and the network terminals are connected to the normal balancing network circuit. Unless otherwise specified, the singing point is expressed in terms of the 1000-cycle gain.

The other important point in determining the singing point distribution is to fix the amount by which its average is less than say, the reference 63 per cent point on the S_F curve; i.e., if the distribution

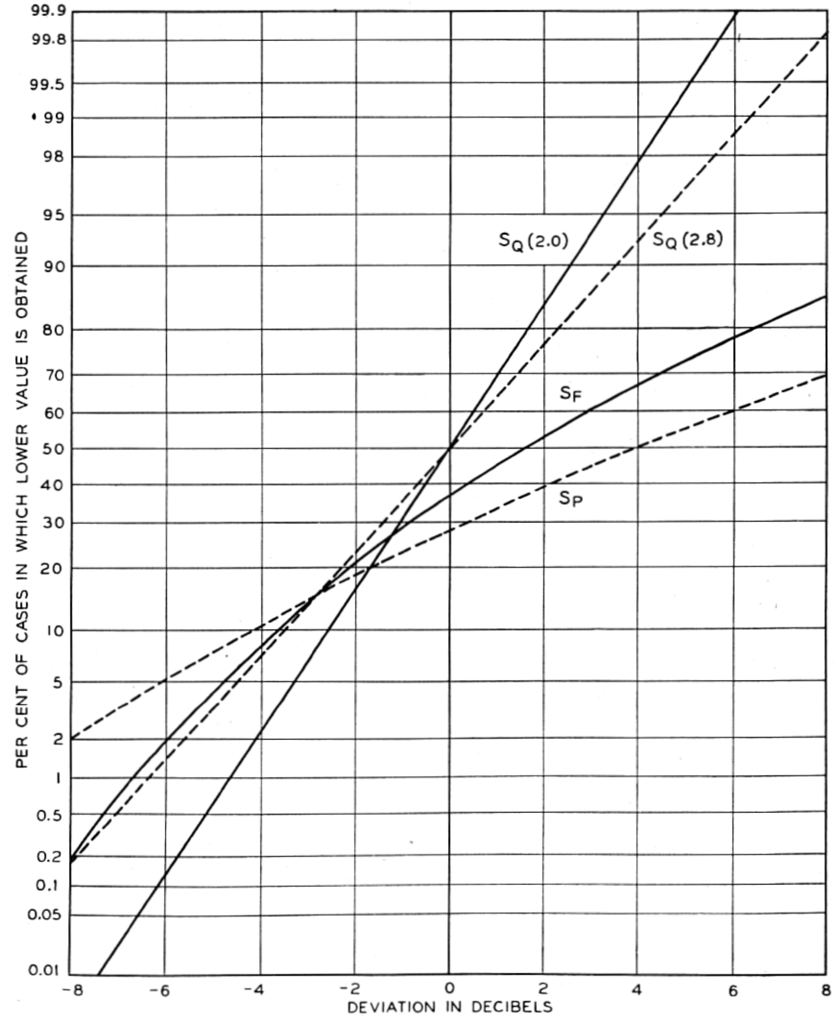

Fig. 6—Distribution functions for return losses and singing points.

curve of single frequency return losses for, say, the highest frequency having as much gain in the repeater as the gain at 1000 cycles, is $S_1 + S_F$, the distribution of singing points will be $S_1 - B + S_Q$ (2.0) where B is S_1 minus the average of the singing points, S_Q (2.0) is a normal law with a standard deviation of 2 db, and Q is the percentage of cases which will have lower singing point values. It was not possible to compute B from the measurements referred to above due to incomplete information.

From field measurements including a large group on 19-gauge B & H–88–50 facilities during a Newark-Philadelphia trial, however, it appears that B is about 2.5 db. As further evidence, the reasoning given above concerning the " wabble " in the return loss distribution when considered together with the standard deviation of 2 db leads to the conclusion that the value of B must lie within a range from about 2 to say, 3 or 4 db for all facilities.

It is accordingly considered reasonable to assume that the distribution of singing points on passive repeater sections at a given place will be a normal law of the form $S_1 - 2.5 + S_Q(2.0)$.[5] The S_Q (2.0) law is shown on Fig. 6.

When the singing points of several such repeater sections in tandem are measured, it seems reasonable to say that the distribution of singing points will be a normal law of the form $S_1 - F(N, T) - 2.5 + S_Q(2.0)$, by analogy with the distribution functions of single-frequency return losses.

When an end path is measured by itself, the singing point in the 2000–3000-cycle range will generally be a little less than would be indicated by single-frequency return-loss measurements. However, the difference between the two sets of measurements will probably be less on the average than the 2.5 db indicated above, since the " wabbles " in measurements of terminal return losses are usually much less than in measurements of repeater section return loss, particularly for the lower terminal return losses which generally are on non-loaded loops. It is accordingly estimated that the distribution of singing points of the end paths is

$$S_{21} - 1 + S_Q(2.0) = 6 + E_1 + S_Q(2.0). \tag{8}$$

When an end path through a terminal return loss is added to the intermediate paths, tests have shown that the singing point of the resultant will in each case be approximately as if the currents of the singing points of the intermediate paths and the end path added directly. A group of such measurements is shown in Appendix III. The singing point of the resultant will therefore have a distribution of

[5] It is interesting to note here that in an article in *Electrical Communication* for July, 1934, entitled "The Prediction of Probable Singing Points on Loaded Cable Circuits," the law is given as effectively $S_1 - 2.5 + S_Q(1.75)$. The difference between these equations is generally within the limits of experimental error. It may be a real difference, however, since there are certain differences in adjustments of building-out condensers which might be expected to affect the standard deviation.

approximately

$$(S_{11} - 2.5) \underset{i}{\mathrm{X}} (S_{21} - 1) + S_Q(2.0), \tag{9}$$

where $\underset{i}{\mathrm{X}}$ means that quantities so connected are combined as if their currents added directly, e.g., $S_{11} \underset{i}{\mathrm{X}} S_{21} = S_{31}$ means that

$$10^{-S_{11}/20} + 10^{-S_{21}/20} = 10^{-S_{31}/20}.$$

When two end paths are connected in series, say around a four-wire cable circuit, it is believed that singing points in the 2000–3000-cycle range will add directly, because (1) due to the large amount of phase shift around the loop, there will be a large number of frequencies which will have the proper phase shift to permit singing and, (2) the return loss in the frequency range of interest on a given line will change slowly with frequency, which will cause the gain required to produce singing to change only slowly with frequency. On this assumption and assuming the normal law distribution given above, it follows directly from the mathematics of such functions that the distribution of the singing points of two such end paths in series is

$$12 + E_1 + E_2 + S_Q(2.8) = S_{21} - 1 + S_{22} - 1 + S_Q(2.8). \tag{10}$$

On the other hand, if only intermediate paths were present on the two sides of a two-wire repeater, the singing points in the two directions would not, in general, add directly, because they would generally be at different frequencies and when singing occurs at one of these frequencies (or at some new frequency) the sum of the return losses will generally be greater than would be indicated by the sum of the singing points. Tests show that the internal singing margin (the 22-test value corrected for frequency characteristic of the repeater, with only intermediate paths) is about 2.5 db higher on the average than would be indicated by the sum of singing points in the two directions. Similar tests show that when the end path is added so that it is very important compared with the intermediate paths, this average difference becomes about zero.

Also, the average one-way singing point on intermediate paths is about $B = 2.5$ db below the reference single-frequency return loss, as outlined above. Considering the internal singing margin as effectively a 21-test [6] measurement through two return losses in series,

[6] A 21-test is a singing test made by increasing the gain of a repeater until singing occurs, with a line and network connected to the line and network terminals, respectively, of one hybrid coil and a fixed known return loss connected to the other hybrid coil.

it seems reasonable to expect this singing point to be about 2.5 db on the average below the reference single-frequency return loss of the two in series.

From either or both of the above two paragraphs, it may be assumed that the internal singing margin is about

$$M_I = S_1 - F(N_1, T) - G_C + S_1 - F(N_2, T) - g_C - 2.5 + S_Q(2.8). \quad (11)$$

The distribution function $S_Q(2.8)$ is an assumption which seems reasonable from general considerations.

From the above discussion the singing margin of the entire circuit will be a function of all the intermediate and end paths which meets the following conditions:

1. When the end paths are unimportant compared to the intermediate paths, the singing margin is

$$M_I = S_{11} + S_{12} - 2.5 + S_Q(2.8). \quad (12)$$

2. When the intermediate paths are unimportant compared to the end paths, the singing margin is

$$M_E = S_{21} - 1 + S_{22} - 1 + S_Q(2.8). \quad (13)$$

3. When both kinds of paths are fairly important, the singing margin is some compromise between

$$(S_{11}) \underset{p}{\mathrm{X}} (S_{21}) + (S_{11}) \underset{p}{\mathrm{X}} (S_{21}) + S_Q(2.8),$$

which would have an average value equal to the reference value of the circulating current margin and about 2.5 db less than this. The singing margin is obviously greater than

$$(S_{11} - 2.5) \underset{i}{\mathrm{X}} (S_{21} - 1) + (S_{12} - 2.5) \underset{i}{\mathrm{X}} (S_{22} - 1) + S_Q(2.8)$$

because the active singing point in one direction is almost certain to be at a different frequency from that of the active singing point in the other direction.

A reasonable empirical compromise which satisfies all of these conditions and preserves the symmetry of the equation is to say that the total singing margin of the circuit is

$$M_s = (S_{11} - 1.25) \underset{p}{\mathrm{X}} (S_{21} - 1) + (S_{12} - 1.25) \underset{p}{\mathrm{X}} (S_{22} - 1) + S_Q(2.8). \quad (14)$$

This may be written in more general fashion as

$$M_s = (S_1 - F(N_1, T) - G_C - 1.25) \underset{p}{\text{X}} (6 + E_1) + (S_1 - F(N_2, T) - g_C - 1.25) \underset{p}{\text{X}} (6 + E_2) + S_Q(2.8) \quad (15)$$

or still more generally,

$$M_s = (S_H + S_w - S_A - L + T + q - F(N_1, T + q) - 1.25) \underset{p}{\text{X}} (6 + E_1' + 2N_1q) + (S_H + S_w - S_A - L + T + q - F(N_2, T + q) - 1.25) \underset{p}{\text{X}} (6 + E_2' + 2N_2q) + S_Q(2.8). \quad (16)$$

Criterion of Satisfactory Performance

The above discussion derives methods of determining the circulating current margin and the singing margin that will be obtained (on a distributional basis) for a cable circuit under a given set of conditions. As a practical matter, the overall net loss of the circuit will sometimes vary, regulating repeaters will change in gain setting, temporary troubles will occur and, in some instances, toll circuit terminations will be removed while connections are being set up. Each of these factors will reduce the singing margin that the circuit had under average conditions. In the Bell System toll circuits are usually designed so that a 10 db singing margin or more is obtained in 90 per cent of the loaded two-wire cable circuits under average conditions. This, of course, is the same thing as saying that 12 db singing margin should be exceeded under average conditions in 71 per cent of the cases, or 8 db in 97.7 per cent of the cases (see $S_Q(2.8)$ curve on Fig. 6). The following table shows the percentages of circuits which will have various lower singing margins under average conditions than the indicated values, for various different assumptions as to the design requirement (i.e., the singing margin which must be exceeded in 90 per cent of the cases).

Design Requirement in Db	Singing Margin in Db =	Percentages Having Lower Singing Margins					
		14	12	10	8	6	4
12		29	10	2.3	0.23	0.03	0.002
10		57	29	10	2.3	0.33	0.03
8		84	57	29	10	2.3	0.33
6		95	84	57	29	10	2.3

The following table shows the same information for circulating current margins, computed on the assumption that $S_{11} - 1.25 = S_{12} - 1.25 = (S_{21} - 1) = (S_{22} - 1)$. The design requirements in this case are also that 12 db, 10 db, 8 db or 6 db singing margin (not circulating current margin) shall be exceeded in 90 per cent of the cases under average conditions.

Design Requirement in Db	Circulating Current Margin in Db =	Percentages Having Lower Circulating Current Margins					
		14	12	10	8	6	4
12		11.3	5.55	2.25	0.78	—	—
10		19.4	11.3	5.55	2.25	0.78	—
8		29.4	19.4	11.3	5.55	2.25	0.78
6		40.2	29.4	19.4	11.3	5.55	2.25

It is estimated that the reduction in circulating current margin due to regulation, variation in circuit net loss, and temporary troubles will seldom exceed $4\frac{1}{2}$ db by very much but about this amount of reduction will occur fairly frequently. Similarly, the reduction in singing margin due to these causes and also to the removal of a termination while a circuit is being set up will seldom exceed $5\frac{1}{2}$ db. It is believed that about 4 db circulating current margin at the critical frequency is as small as can be obtained without very objectionable quality distortion.

When $5\frac{1}{2}$ db reduction in singing margin is obtained on the average, we may read from the first table above at $5\frac{1}{2}$ db margin, the percentage of cases in which actual singing will take place. For example, with a design requirement of 8 db, interpolating between the vertical columns headed 6 and 4 db, respectively, we find that about $1\frac{1}{2}$ per cent of cases will result in actual singing. Similarly, by reading at $4 + 4\frac{1}{2} = 8\frac{1}{2}$ db circulating current margin in the second table, we find the percentage of cases in which the quality will be objectionably distorted during the period in which a $4\frac{1}{2}$ db reduction in circulating current margin is obtained. The following table shows these two different percentages for the design limits given above. From this table it

Design Limit in Db	Percentage of Cases With Less Than 4 Db Circulating Current Margin Under Extreme Conditions	Percentage of Cases Which Will Sing Under Extreme Conditions
12	0.97	0.015
10	2.8	0.18
8	6.8	1.45
6	13.1	7.1

seems reasonable to require a 10 db singing margin under average conditions in order to keep the percentage of circuits which will have objectionable circulating currents and the percentage which will sing occasionally low enough that serious service reactions will not be obtained.

Practical Considerations

Equipment and Terminating Effects

When actual cable sections are investigated, the effect of the equipment must be considered. The effect of adding, say, repeating coils in the line and network sides of the repeater, is to introduce some reflected current and to add a certain loss in the line which must be made up by increasing the repeater gain. The cable return loss is increased by twice the loss introduced, so that the only effect on circulating current paths is due to the differences in the line and network equipment.

Appendix IV derives an expression for the distribution curve of an active or passive single-frequency return loss of a line with equipment, on certain assumptions. The complication of using this method to determine circulating current margins or singing margins is large, however, and an approximate method is illustrated below.

Figure 7 shows an example of the combination of the equipment-return loss with the return loss of a cable section without equipment. It is assumed in this case that the equipment-return loss is 13 db higher than the value of cable-return loss for which $S_F = 0$. For example, the equipment return loss might be 40 db while the line return loss was $27 + S_F$ db. A generally similar curve would be obtained from the combination of the repeater section terminating effect with the cable return losses.

Since the principal interest in connection with these return losses is with the lower values, it seems reasonable to say that the effect may be approximated generally by shifting the S_F curve a certain part of a decibel, depending upon the difference between S_E and the cable-return losses. Strictly speaking, of course, it not only shifts the curve but also changes the standard deviation, but the complication of taking account of this does not seem worth while with present lines and equipment.

Since we are interested in the singing margin which is exceeded 90 per cent of the time under average conditions, it would be possible to select a value of cable return loss from the distribution curve which, if it occurred in each repeater section of a particular circuit, would give the indicated singing margin when the various intermediate singing paths are combined together as the sum of their power ratios.

This would be roughly where $S_F = -2$ db. The percentage of cable section return losses which would exceed this value, however, would be about 80 per cent, so a line which gave such a low singing margin would, in general, tend to be made up of a few very low-return losses

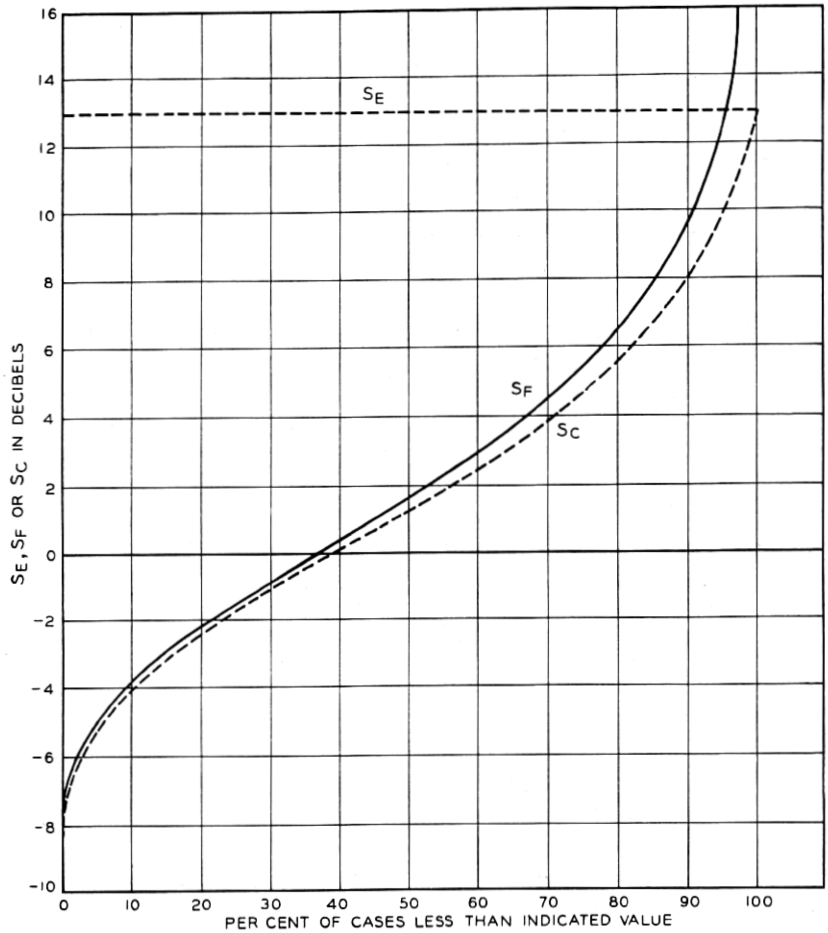

Fig. 7—Return loss distributions of cable pairs (S_F), equipment (S_E), and the combination (S_C).

and a considerable percentage of higher-return losses. It, therefore, seems that this value of cable-return loss, if combined with the equipment and terminating return loss, would not be sufficiently degraded to consider that degradation as a fair average degradation for the entire curve. On the other hand, if the cable return loss which is exceeded in 50 per cent of the cases is taken (i.e., $S_F = + 1.8$ db), the degradation would evidently be too large due to the equipment since the lines in which the singing margin might be expected to be low, would generally be those which happen to have more than the usual number of low line return losses.

As an approximation in the usual case, it seems reasonable to combine the line equipment return losses with the cable return losses as the sum of their power ratios, with a value of S_F of about -1 db (i.e., where 28 per cent of the return losses are lower than the value considered). One db added to the resultant will give the value of the resultant for which $S_F = 0$. The same thing may also be accomplished by combining the cable return loss for $S_F = 0$ with the near and far-end equipment return losses each increased by 1 db. The example which is given later shows the method used.

Taper

The taper of a regular two-wire circuit is the amount of decrease in repeater output level at a given frequency which occurs between succeeding repeaters from the transmitting to the receiving end of the circuit. For example, starting at the transmitting repeater, if the repeater output levels at 1000 cycles were, respectively, 3.0, 2.5, 2.0, 1.5, etc., the 1000-cycle taper would be 0.5 db.

An optimum taper from the standpoint of singing will be reached when the singing margin around the most critical intermediate repeater is equal to the singing margin around the terminal repeater. This is certain to occur for some taper under the present method of distributing gain since an increase of the taper causes the gain of the receiving terminal repeater to be increased by a multiple of the increase of taper. However, the optimum taper from the standpoint of singing is not generally used in the field, because the optimum taper from the standpoints of crosstalk and noise is zero db. A compromise is generally made on two-wire cable circuits; e.g., 19-gauge B and H–88–50 facilities use 0.5 db and 16-gauge H–44–25 facilities use 0.2 db taper.

Active Balances

In some cases, it may be more convenient to measure active return losses or active singing points rather than singing margins. The expected distribution of active return losses and active singing points including the end paths when the end paths are the normal working terminations, have been discussed above.

In practice, however, the termination when such an active singing point test is made will normally be a fixed one, generally consisting of resistances with a return loss substantially the same at all frequencies. Appendix V derives the distribution of active singing points for these cases.

For exact results, the modification of the return loss due to the near-end and far-end equipment should be added before combining the end path. In the practical case, however, only the lower values of return loss are generally significant, and the approximate method outlined above may generally be used without serious error.

Minimum Value of a Group of Singing Points

When a large number of independent measurements are taken, say of singing points on different pairs at a given repeater station, it is often of interest to know whether the maximum or minimum in that group represents some special condition, say, definite circuit trouble, or is merely a case where the various components happen to have added up or subtracted by pure chance.

In the case of singing points, the question of interest may usually be stated as follows: In " n " similar measurements at a given place, what is the probability that the smallest value will be more than x db below the true average measurement, assumed known?

If the fraction of cases which will be greater than a value y db below the average is P, the probability that " n " values will all be greater than this value is P^n. In other words $P_m(n, y) = P_m = P^n$ is the probability that no one in " n " values will be as much as y db below the true average.

For example, if the singing point distribution is $25 + S_Q(2)$, the chance that any given value selected at random will be as low as $25 - 4.6 = 20.4$ db is one in a hundred. However, the chance that the lowest one of 20 measurements will be that low is $1 - (0.99)^{20} = 0.1819$ or almost one in five. If we substitute $V_m = 1 - P_m$ and $V = 1 - P$, we may write that $1 - V_m = (1 - V)^n$.

$$V_m = 1 - (1 - V)^n = 1 - 1 + nV - \frac{n(n-1)}{2!} V^2 + \cdots$$

$$= nV - \frac{n(n-1)V^2}{2!} + \frac{n(n-1)(n-2)}{3!} V^3 - \cdots .$$

And for small values of V, $V_m = nV$. In the above case, therefore, if we wanted to compute a value of singing point which would be lower than all the 20 measurements nine times out of ten, we may compute $V = (0.1/20) = 0.005$, and read on the normal distribution $S_Q(2)$ that this corresponds to about 5 db below the average; i.e., to $25 - 5 = 20$ db. The more exact formula would be that $0.9 = P^{20}$

or $20 \log P = \log 0.9$ or $\log P = -0.002288$ or $P = 0.99476$ or $V = 0.00524$. Reading on the normal law distribution for this value changes the 20 db limit as computed above to about 20.03 db, the difference being negligible compared to the usual accuracy of such measurements.

In general, measurements outside of the limits so determined are probably cases of trouble. However, in the case of active singing points, certain systematic changes in the measured values from the computed values must be allowed for before trouble is proved.

Example

Following is an example of the application of these various methods.

Given: The following losses at the critical frequency for various successive repeater sections shown in Fig. 1.

Repeater Section	Loss in Db
A to *B*	17
B to *C*	17
C to *D*	18
D to *E*	15
E to *F*	15
F to *G*	16
Total	98

The following gains and return losses at the critical frequency are assumed. The gains give a 9 db overall net loss at the critical frequency.

At Point	Repeater Gain at the Critical Frequency—db			Return Losses at the Critical Frequency for $S_F = 0$, i.e., Values Exceeded in 63 Per Cent of Cases—db							
				Bare Line **		Near-End Apparatus *		Far-End Apparatus †		Combination ‡	
	A to *G*	*G* to *A*	Sum	Toward *G*	Toward *A*	Toward *G*	Toward *A*	Toward *G*	Toward *A*	Toward *G*	Toward *A*
A	2.4	12.0	14.4	27.6	—	41	—	46	—	27.35	—
B	15.0	15.0	30.0	27.6	27.6	41	41	46	46	27.35	27.35
C	16.8	16.9	33.7	27.6	27.6	41	41	48	46	27.37	27.35
D	16.9	13.9	30.8	27.6	27.6	41	41	42	48	27.25	27.37
E	13.9	13.9	27.8	27.6	27.6	41	41	42	42	27.25	27.25
F	13.9	14.9	28.8	27.6	27.6	41	41	44	42	27.32	27.25
G	10.1	2.4	12.5	—	27.6	—	41	—	44	—	27.32
Total	89.0	89.0	178.0								

* This is 40 db nominal apparatus return loss plus 1 db allowance as described above.

† This is 11 db far-end equipment terminating return loss plus 1 db allowance as described above plus twice the loss from the measuring point to the far-end equipment.

‡ These are the combinations of the three preceding return losses as the sums of their power ratios.

** Twice the loss of the near-end equipment is included.

Considering C as the critical repeater:

Path

Toward A	1	27.35
	2	$27.35 + 34 - 30 = 31.35$
	Total	25.90 Items 1 and 2 combined as the sum of their power ratios.
Toward G	1	27.37
	2	$27.25 + 36 - 30.8 = 32.45$
	3	$27.25 + 38 - 31.8 + 30 - 27.8 = 35.65$
	4	$27.32 + 38 - 31.8 + 30 - 27.8 + 30 - 28.8 = 36.92$
	Total	25.42. Items 1, 2, 3 and 4 combined as the sum of their power ratios.

The internal circulating current margin then is $25.90 + 25.42 - 33.7 = 17.62$ db, or more in 72 per cent of the cases and the internal singing margin is $25.90 + 25.42 - 2.5 - 33.7 = 15.12$ db or more in 50 per cent of the cases, both under the average conditions specified.

Now 10 db internal circulating current margin would be exceeded under average conditions in the percentage of cases for which S_P is $10 - 17.62 = -7.62$ db or greater; i.e., in about 97.5 per cent of the cases, reading from the S_p curve on Fig. 6. Or, similarly, 10 db internal singing margin will be exceeded in about 97 per cent of the cases, reading at $S_Q(2.8) = 10 - 15.12 = -5.12$ from the $S_Q(2.8)$ curve on Fig. 6.

The single-frequency loss in the end path toward A, before the gain of repeater C is introduced is, for a net loss at the critical frequency of 9 db, $34 + 34 - 30.0 - 14.4 + 7 + S_F = 30.6 + S_F$. In the other direction the corresponding function is $36.1 + S_F$. When the net loss of the circuit is expressed as E rather than 9 db, these functions become respectively $21.6 + E + S_F$ and $27.1 + E + S_F$. The corresponding singing point distributions are $20.6 + E + S_Q(2)$ and $26.1 + E + S_Q(2)$, respectively.

The desired value of 10 db or more singing margin obtained in 90 per cent of the cases under average conditions will be obtained when the following equation is satisfied:

$$10 = (25.90 - 1.25) \underset{p}{\text{X}} (20.6 + E) + (25.42 - 1.25) \underset{p}{\text{X}} (26.1 + E) - 33.7 + S_Q(2.8), \quad (17)$$

where $Q = 10$ per cent, $S_Q(2.8) = -3.5$ db. Substituting in the equation and simplifying,

$$-1.62 = 0\underset{p}{\mathrm{X}}(E - 4.05) + 0\underset{p}{\mathrm{X}}(E + 1.93), \qquad (18)$$

which is satisfied if $E = 8.8$ db.

The active balance of the circuit with a 9 db net loss at the critical frequency measured from the " A " end may be computed as follows: The successive paths from the line side of repeater " A " are (using values for $S_F = 0$):

1. 27.35
2. $27.35 + 34 - 30 = 31.35$
3. $27.37 + 34 - 30 + 34 - 33.7 = 31.67$
4. $27.25 + 34 - 30 + 34 - 33.7 + 36 - 30.8 = 36.75$
5. $27.25 + 34 - 30 + 34 - 33.7 + 36 - 30.8 + 30 - 27.8 = 38.95$
6. $27.32 + 34 - 30 + 34 - 33.7 + 36 - 30.8 + 30 - 27.8 + 30 - 28.8 = 40.22.$

Combining these paths as the sum of their power ratios gives 24.33 db, which is the computed single-frequency return loss without the end path from the line side of the terminal repeater for $S_F = 0$ (i.e., which will be exceeded in 63 per cent of the cases). The active singing point from this point will be $24.33 - 2.5 + S_Q(2) = 21.83 + S_Q(2)$. The corresponding active singing point referred to the drop side of the repeater will be $21.83 - 14.4 + S_Q(2) = 7.43 + S_Q(2)$ db. With a fixed termination giving a return loss of 5 db at the circuit terminal, the end path from the drop side of the repeater will be $18 + 5 = 23$ db. From Appendix V, for $d = 23 - 7.43 = 15.57$ db, the expected active singing point including the end path is the distribution curve $7.43 + S_0(15.57)$ under average conditions. For example, by interpolation in the table in Appendix V, two per cent of such circuits will have lower active singing points from the drop than about $7.43 - 5 = 2.43$ db and from the line side of the repeater than $2.43 + 14.4 = 16.83$ db, both under average conditions.

This answer may be computed by reading from the $S_Q(2.0)$ at two per cent which gives $S_2(2.0) = -4.1$ db. At this percentage, therefore, the active singing point without the end path is $7.43 - 4.1 = 3.33$ db. Combining this with the fixed and path of 23 db gives $3.33 - 0.9 = 2.43$ db.

APPENDIX I

Active Return Loss Of Intermediate Paths

Let L_k = the loss from the west side of repeater C to the output of repeater X which is the k th repeater to the west, plus the loss from the input of the W–E side of repeater X back to the input of repeater C (Fig. 1).

$$\text{Let } L_k = 20 \log_{10} t_k^{2k}. \tag{19}$$

Now from Equations (26) and (27) from " Irregularities in Loaded Telephone Circuits ":[7]

$$I_{1k} = \frac{I_0 R_L}{t_k^{2k}} \sqrt{\sum_{j=1}^{m} A_k^{4(j-1)} \cos^2 \theta_{jk}}, \tag{20}$$

$$I_{2k} = \frac{I_0 R_L}{t_k^{2k}} \sqrt{\sum_{j=1}^{m} A_k^{4(j-1)} \sin^2 \theta_{jk}}, \tag{21}$$

where I_{1k} is I' (from the paper) for the currents from the $(k+1)$ th repeater section from C (counting the adjacent repeater section as the first) and I_2k is I'' for the same section.

The total components will be

$$I_1 = I_0 R_L \sqrt{\sum_{j=1}^{m_k} \sum_{k=0}^{N-1} \frac{A_k^{4(j-1)}}{t_k^{4k}} \cos^2 \theta_{jk}}, \tag{22}$$

$$I_2 = I_0 R_L \sqrt{\sum_{j=1}^{m_k} \sum_{k=0}^{N-1} \frac{A_k^{4(j-1)}}{t_k^{4k}} \sin^2 \theta_{jk}}, \tag{23}$$

where N = the total number of repeater sections so considered.

Assuming (as in the paper) that $I_1 = I_2$

$$I_1 = I_2 = \frac{I_0 R_L}{\sqrt{2}} \sqrt{\sum_{k=0}^{N-1} \frac{(1 - A_k^{4m_k})}{t_k^{4k}(1 - A_k^4)}}. \tag{24}$$

By analogy, Equation (34) from the paper referred to may be used for the distribution of the total current I_F, with $I_1 = I'$; i.e.,

$$F = e^{-(I_F^2/2I_1^2)}. \tag{25}$$

Equation (42) from the paper may therefore be written

$$S = S_H + S_W + S_F - S_A - F(N, T) \tag{26}$$

where

$$S_A + F(N, T) = 10 \log_{10} \left(\sum_{k=0}^{N-1} \frac{(1 - A_k^{4m})}{t_k^{4k}(1 - A_k^4)} \right). \tag{27}$$

[7] G. Crisson, October 1925, *Bell Sys. Tech. Jour.*

Where all values of A_k are the same, and equal to A,

$$S_A = 10 \log_{10}\left(\frac{1 - A^{4m}}{1 - A^4}\right) \tag{28}$$

$$F(N, T) = 10 \log_{10}\left(\sum_{k=0}^{N-1} t_k^{-4k}\right). \tag{29}$$

Where all values of " t_k " are the same and equal to " t,"

$$F(N, T) = 10 \log_{10}\left(\frac{1 - t^{-4N}}{1 - t^{-4}}\right). \tag{30}$$

This is the case where all the gains are equal and all the losses are equal at all points.

In the case where $t = 1$, i.e., $T = 0$ db,

$$F(N, O) = 10_{10} \log N. \tag{31}$$

APPENDIX II

Distribution of Two Active Return Losses in Series

Assume two return losses in series, the two being selected at random from groups of return losses (active or passive) having distributions represented by $S_{11} + S_F$ and $S_{12} + S_F$, respectively. What is the distribution curve of the two in series?

Let

$$p_1 = \frac{i_1}{k_1^2} e^{-i_1^2/2k_1^2}, \tag{32}$$

$$p_2 = \frac{i_2}{k_2^2} e^{-i_2^2/2k_2^2}, \tag{33}$$

where p_1 is the relative probability of obtaining a returned current i_1 from the first line $(S_{11} + S_F)$ and k_1 corresponds to I_1 in Appendix I. The quantities with subscript 2 are the same for the second line $(S_{12} + S_F)$. Now if $i_1 i_2 = i_3$, the probability that i_3 exceeds a certain value I is

$$P(i_3 > I) = \int_{i_2=0}^{\infty} \int_{i_1=I/i_2}^{\infty} p_1 di_1 p_2 di_2. \tag{34}$$

Substituting from Equations (32) and (33) and letting

$$x = \frac{i_2^2}{2k_2^2}, \qquad a = \frac{I}{2k_1k_2}, \tag{35}$$

$$P(1_3 > I) = \int_0^{\infty} e^{-(x+(a^2/x))} dx, \tag{36}$$

$$P(1_3 > I) = 2aK_1(2a), \tag{37}$$

where K_1 is a Bessel's function of the second kind (see " Theory of Bessel Functions " by G. N. Watson).

Now

$$a = \frac{I}{2k_1k_2} = \text{anti-log}_{10}\left(\frac{S_{11} + S_{12} - S_I}{20}\right), \tag{38}$$

where S_I is the return loss obtained from the two in series.

Let

$$S_p = -\ 20 \log_{10} a = -\ S_{11} - S_{12} + S_I, \tag{39}$$

$$S_I = S_{11} + S_{12} + S_p. \tag{40}$$

This last expression is the distribution curve of the two return losses in series.

APPENDIX III

Addition of End Path to Active Singing Points
(16 Ca. H-44-25 2-Wire Cable Side Circuits)

(1) Number of Repeater Sections	(2) Active Singing Point to Terminal Repeater on Zero—db	(3) Loss of End Path—db	(4) Active Singing Point with End Path Added—db	(5) (2) and (3) Added as Sum of Current Ratios	(6) (2) and (3) Added as Sum of Power Ratios
4	29.8	20.4	17.1	17.9	19.0
4	31.2	20.4	18.5	18.2	20.2
4	32.3	23.8	21.3	21.0	23.2
4	31.2	23.8	21.2	20.7	23.1
4	33.1	23.8	20.9	21.2	23.3
4	29.3	20.8	17.8	18.0	20.2
4	35.7	20.8	18.7	19.3	20.7
4	27.0	20.8	18.0	17.3	19.9
8	25.6	22.2	19.0	17.7	21.6
8	24.2	19.9	16.1	15.8	18.5
18	17.9	19.1	12.7	12.4	15.4
18	17.7	19.1	13.5	12.4	15.3
18	20.1	25.2	15.5	16.2	18.9
18	18.7	25.2	15.1	15.3	17.8
18	20.0	19.0	14.5	13.5	16.5
18	16.9	19.0	12.0	11.9	14.8
18	19.1	25.1	16.2	15.6	18.1
18	19.9	25.1	17.2	16.1	18.8
18	21.0	20.9	14.0	14.9	17.9
18	20.1	24.3	14.1	14.5	17.5
18	19.0	24.3	15.7	15.2	17.9
18	17.8	24.3	15.0	14.4	16.9
		Average =	16.45	16.34	18.88

The different tests using the same number of repeaters are different circuits and different testing repeaters. Columns (2) and (4) are measured singing points. Column (3) is the sum of the measured losses from the measuring point to the drop side of the terminal repeater and back, plus the return loss of resistance termination applied at the drop side of the repeater.

Columns (5) and (6) show computed values corresponding to column (4), on different assumptions as to the method of addition of the intermediate and end paths. The values in column (5) are generally much closer to the values in column (4) than are those in column (6).

APPENDIX IV

Distribution of Active Return Losses with Intermediate Paths Including Equipment

From a consideration of the paper, "Irregularities in Loaded Telephone Circuits," by G. Crisson,[7] it appears that it is reasonable to say that the total current returned from a cable section with near-end equipment will be

$$i_t = \sqrt{i_1^2 + i_e^2},$$

where i_1 is the current returned from the line without equipment and i_e is the current returned from the equipment. (It is assumed here that i_t and i_e are referred to the line side of the coil and not the drop side.)

Then the probability that the current returned from the cable alone will exceed I is

$$F(I) = \frac{1}{I_1^2}\int_I^\infty i_1 e - \frac{i_1^2}{2I_1^2} di_1 = e - \frac{I^2}{2I_1^2}.$$

The probability that the total current will exceed $I_3 = \sqrt{I^2 + i_e^2}$ is

$$P(i_t > I_3) = e - \frac{(I_3^2 - i_e^2)}{2I_1^2} = e \frac{i_e^2}{2I_1^2} F(I_3)$$

or

$$P(i_t > I_3) = \text{antilog}_{10}\left(.434 \, \text{antilog}_{10}\left(\frac{S_1 - S_E}{10}\right)\right) F(I_3) = F',$$

where

$$S_1 = -10 \log_{10} 2I_1^2 \qquad \text{and} \qquad S_E = -20 \log_{10} i_e.$$

This equation says that the probability of getting more than a given value of return current I_3 from the combined cable section and near end equipment is equal to the product of the probability of getting I_3 from the cable section alone by the factor $e(i_e^2/2I_1^2)$. Or by analogy with the method used in the paper referred to above, letting the distribution curve of the combination return loss be $S_1 + S_{F'}$,

$$S_{F'} = -\log_{10}\left(\log_e \frac{1}{F} + \frac{i_e^2}{2I_1^2}\right)$$

$$= -\log_{10}\left(\log_e \frac{1}{F} + \text{antilog}_{10} \frac{S_1 - S_E}{10}\right)$$

$$S_{F'} = S_F \underset{p}{\text{X}} (S_E - S_1).$$

[7] Loc. cit.

Or the distribution curve of the return losses from the repeater section with equipment is

$$S_1 + S_{F'} = (S_1 + S_F) \underset{p}{X} (S_1 + S_E - S_1)$$
$$= (S_1 + S_F) \underset{p}{X} (S_E)$$
$$S_{F'} = (S_F) \underset{p}{X} (S_E - S_1).$$

Curves of $S_{F'}$ could be drawn for different values of $S_E - S_1$, since S_F is known for a given value of F. These would apply either for passive return losses of a single repeater section or for active return losses of several repeater sections in tandem.

For example, suppose $S_E = S_1$. For $S_F = 0$, the value of F is about 0.368 and the value of $S_{F'} = 0 \underset{p}{X} 0 = -3.01$ db for $F' = 0.368$. Or the same point could be derived from the earlier formulas as follows: When $S_F = -3.01$ db, $F = 0.1353$. The factor antilog $\left(0.434 \text{ antilog}\left(\frac{0}{10}\right)\right)$ equals 2.718 and the value of F' for $S_{F'} = -3.01$ db is $F' = (0.1353)(2.718) = 0.368$.

APPENDIX V

Distribution of Active Return Loss Made up of Intermediate Paths and a Fixed End Path

In a practical case, when an active singing point toward a circuit terminal is measured for maintenance purposes, the termination at the circuit drop will be a fixed known value of return loss rather than one selected at random from a distribution curve. The distribution curve of active singing points toward the drop will then be a curve obtained at each percentage, by the current sum addition of the active singing point with no current returned from beyond the terminal repeater and the end path singing point. If the active intermediate path singing point has a distribution curve $S_{11} - 2.5 + S_Q(2) = S_4 + S_Q(2)$ and the end path has a fixed loss S_5, the following table may be used to find the distribution curve of the combination $S_4 + S_g(d)$, for different values of $d = S_5 - S_4$. The table was derived from the equation

$$(S_4 + S_Q(2)) \underset{p}{X} S_5 = S_4 + S_g(d)$$

Percentage of Cases with Lower Singing Points	0.003	0.13	2.2	15.7	50.0	84.3	97.8	99.87	99.997	99.99997
$S_g(\infty)$ in db..	−8	−6	−4	−2	0	2	4	6	8	10
$S_g(15)$ in db..	−8.6	−6.7	−4.9	−3.2	−1.4	+0.2	+1.8	+3.4	+4.8	+6.1
$S_g(10)$ in db..	−9.0	−7.3	−5.6	−4.0	−2.4	−0.9	+0.5	+1.1	+2.9	+4.0
$S_g(6)$ in db...	−9.6	−8.0	−6.4	−4.9	−3.5	−2.2	−1.1	0	+0.9	+1.8
$S_g(3)$ in db...	−10.2	−8.6	−7.2	−5.9	−4.6	−3.5	−2.5	−1.6	−0.9	−0.2
$S_g(0)$ in db...	−10.9	−9.5	−8.2	−7.1	−6.0	−5.1	−4.2	−3.5	−2.9	−2.4
$S_g(-3)$ in db.	−11.9	−10.6	−9.5	−8.5	−7.6	−6.9	−6.2	−5.6	−5.2	−4.8
$S_g(-6)$ in db.	−13.1	−12.0	−11.1	−10.2	−9.5	−8.9	−8.4	−8.0	−7.6	−7.3
$S_g(-10)$ in db	−15.1	−14.2	−13.5	−12.9	−12.4	−12.0	−11.5	−11.3	−11.0	−10.8

or

$$(S_Q(2)) \underset{p}{\mathrm{X}} (S_5 - S_4) = S_g(d).$$

It will be noted that $S_g(\infty) = S_Q(2.0)$.

It should be noted that the values of $S_g(d)$ corresponding to the higher percentages may be considerably modified by the effect of the equipment return losses, particularly when d is large. However, this is usually of no very great importance because it is the lower percentages and the smaller values of d which are of the most practical importance.

APPENDIX VI

Nomenclature

A = attenuation factor per loading section (current ratio).

A_k = attenuation factor per loading section (current ratio) of k th repeater section from critical repeater.

a = $\log_{10}^{-1}\left(\dfrac{S_{11} + S_{12} - S_I}{20}\right)$.

B = S_1 minus the average singing point on a group of lines.

b = the standard deviation of a group of singing point measurements.

d = $S_5 - S_4$.

E = net loss of circuit in db.

E_1 = loss in end path toward West for zero terminal return loss. When primed, it is the loss at 1000 cycles.

E_2 = same as E_1 but toward East.

$F(I_F) = F$ = probability of obtaining a greater returned current then I_F from a randomly selected one of a group of lines.

F' = probability of obtaining a greater returned current than I_3 from a cable section with near end equipment.

$F(N, T)$ = $10 \log_{10}\left(\sum_{k=0}^{N-1} t_k^{-4k}\right)$.

G_C and g_C = gains as defined herein.

I = current obtained in a particular case.

I_0 = current sent into the line at the point of measurement.

I_1 = $\text{anti} \log_{10}\left(\frac{-S_1 - 3}{20}\right)$ = current for active return loss corresponding to I' from " Irregularities in Loaded Telephone Circuits " by G. Crisson in the October 1925 *Bell System Technical Journal.*

I_2 = $\text{anti} \log_{10}\left(\frac{-S_1 - 3}{10}\right)$ = current for active return loss corresponding to I'' from paper referred to under I_1.

I_3 = current defined above under F'.

I_F = current defined under $F(I_F)$ above.

I_{1k} = I' (from paper referred to above) for the currents from the k th repeater section from the critical repeater.

I_{2k} = I'' (from paper referred to above) for the currents from the k th repeater section from the critical repeater.

i_1 and i_2 = currents returned from particular active or passive return losses.

i_3 = current returned from a particular pair of active or passive return losses in series.

i_e = $\text{anti} \log_{10}\left(\frac{-S_E}{20}\right)$.

i_t = total current returned from line with equipment.

j = number of the loading section measured from the repeater at which the return loss is of interest.

$K_1(x)$ = Bessel function of the second kind and order unity with argument x.

k = the number of the repeater or repeater section from the critical repeater, e.g., for the adjacent repeater section $k = 1$, for the next $k = 2$, etc.

k_1 $= \text{anti} \log_{10}\left(\frac{-S_{11}-3}{20}\right)$ = current corresponding to S_{11} in same way as I_1 corresponds to S_1.

k_2 $= \text{anti} \log_{10}\left(\frac{-S_{12}-3}{20}\right)$ = current corresponding to S_{12} in same way as I_2 corresponds to S_1.

L = loss of a cable section.

L_k = the loss in db from the West side of the critical repeater to the output of the k th repeater to the West, plus the loss from the input of the k th repeater W–E to the input of the critical repeater W–E.

M_E = the end path singing margin, i.e., the singing margin with no currents returned from intermediate paths (e.g., with a four-wire circuit).

M_I = the internal singing margin on an active line, i.e., the singing margin without any currents returned from the end paths (e.g., with the terminal repeaters turned down).

M_S = the singing margin of the circuit; i.e., the singing margin with currents returned both from intermediate paths and from end paths.

m_k = total number of loading sections in the k th repeater section.

m = value of m_k when all repeater sections have the same number of loading sections.

N = number of repeater sections in the part of the circuit for which the active return loss is to be computed.

n = number of cases considered as a group.

p_x = relative probability of obtaining a returned current i_x from one active or passive return loss.

$P(x > y)$ = probability that the value of x selected at random from a distribution curve is greater than a pre-selected value y.

P = probability that a singing point will have a lower value than a pre-selected value y.

$P_m(n, y) = P_m$ = probability that the minimum of n values will be as high or higher than a pre-selected value y db below the true average.

Q = probability that a lower value of singing margin will be obtained.

q = frequency taper in db per repeater section.

R_L = the representative reflection coefficient (see paper by G. Crisson referred to above).

S_H = the irregularity function (see paper by G. Crisson referred to above).

S_W = the frequency function (see paper by G. Crisson referred to above).

S_A = the attenuation function (see paper by G. Crisson referred to above).

S_F = the distribution function (see paper by G. Crisson referred to above).

S = $S_H + S_W - S_A + S_F$.

S_1 = $S - S_F = S_H + S_W - S_A = -10 \log_{10} 2I_1^2$.

S_2 = Same as S_1 but for terminal return loss.

S_4 = $S_{11} - 2.5$.

S_5 = end path loss with fixed termination.

S_{11} and S_{12} = values, respectively, toward the West and the East of active or passive single frequency return loss for $S_F = 0$, made up of intermediate paths only.

S_{21} and S_{22} = same as S_{11} and S_{12} but for end paths.

S_{31} and S_{32} = same as S_{11} and S_{12} but for combination of intermediate and end paths.

S_I = return loss which is actually obtained on a particular case.

S_C = return loss in a particular case with equipment.

S_E = equipment return loss.

S_P = $-20 \log a$ = distribution function of two return losses in series, where each such return loss has a distribution curve of the form $S + S_F$.

S_F' = function corresponding to S_F after equipment has been added.

$S_Q(b)$ = distribution function of singing points or singing margins; i.e., a normal law with an average of zero and a standard deviation of b.

$S_g(d)$ = distribution functions of active return losses with fixed terminations.

t = $10^{-(T/20)}$.

T = taper in db per repeater section.

t_k = $10^{-(L_k/40k)}$.

V = $1 - P$.

V_m = $1 - P_m$.

$\underset{i}{\mathrm{X}}$ = a symbol indicating that the two quantities in decibels are combined as if their currents added directly.

E.g.,

$$S_{11} \underset{i}{\mathrm{X}} S_{21} = S_{31} \text{ means that } 10^{-(S_{11}/20)} + 10^{-(S_{21}/20)} = 10^{-(S_{31}/20)}.$$

$\underset{p}{\mathrm{X}}$ = a symbol indicating that the two quantities in decibels so connected are combined as if their powers added directly.

E.g.,

$$S_{11} \underset{p}{\mathrm{X}} S_{21} = S_{31} \text{ means that } 10^{-(S_{11}/10)} + 10^{-(S_{21}/10)} = 10^{-(S_{31}/10)}.$$

x, y = variables as used.

θ_{jk} = phase angle of current returned from the j th loading section in the k th repeater section.