Radio Propagation Over Spherical Earth *

By CHAS. R. BURROWS

The paper shows how Watson's solution for the propagation of electromagnetic waves over perfectly conducting spherical earth merges into the Abraham solution for propagation over a perfectly conducting plane for shorter distances.

The effects of refraction by the lower atmosphere and of the imperfect conductivity of the earth are taken into consideration. The magnitude of the former, which is appreciable, is obtained. The latter is relatively unimportant for ocean water and frequencies of the order of a megacycle and less.

The theoretical solution for radio propagation over perfectly conducting spherical earth with atmospheric refraction is in agreement with available experimental data for propagation over ocean water for frequencies below a few megacycles.

Eckersley's extension of Watson's solution to take into account the effect of the imperfect conductivity of the earth by the phase integral method is found to contain approximations which render its results questionable.

Theory

THE electrical disturbance at the surface of the earth due to a vertical dipole has been calculated by G. N. Watson [1] and others. The results for the case of a perfectly conducting spherical earth with transmitter and receiver both situated on the surface may be reduced to the form:

$$E = \frac{30(2\pi)^{5/3} HI}{a^{5/6}\lambda^{7/6}\sqrt{\sin\theta}} \sum_{n=1}^{\infty} \frac{e^{-\beta_n d\sqrt[3]{2\pi/a^2\lambda}}}{\rho_n}, \qquad (1)$$

where ρ_n and β_n are constants whose values have been calculated as follows:

$\rho_1 = 0.8083$

n	ρ_n/ρ_1	$\beta_n\sqrt[3]{2\pi/a^2}$
1	1.000	0.00376
2	3.188	0.01199
3	4.74	0.0178
4	6.047	0.0227
5	7.236	0.0272
6	8.336	0.0313

* Published in *I. R. E. Proc.*, May, 1935. Presented before U. R. S. I. meeting, Washington, D. C., April 26, 1935.

[1] G. N. Watson, "The Diffraction of Electric Waves by the Earth," *Proc. Roy. Soc.* (*London*) **A95**, 83–99, October 7, 1918.

and

H is the effective height of the transmitting antenna in kilometers,
I is the transmitting antenna current in amperes,
λ is the wave-length in kilometers,
a is the radius of the earth in kilometers (= 6370),
d is the distance between transmitter and receiver in kilometers,
θ is the angle at the center of the earth subtended by radii to transmitter and receiver (= d/a), and
E is the received field strength in volts per kilometer.

ρ_n and β_n were evaluated for $n = 1$, 2 and 3 by H. M. Macdonald,[2] while the remaining values have been calculated by the present author.

For distances for which this solution would be used (i.e., where the effect of the ionized region of the upper atmosphere may be neglected) $\sin\theta$ very nearly [3] equals θ so that the above formula reduces to the following:

$$E = \frac{30(2\pi)^{5/3}HI}{a^{1/3}\lambda^{7/6}d^{1/2}} \sum_{n=1}^{\infty} \frac{e^{-\beta_n d\sqrt[3]{2\pi/a^2\lambda}}}{\rho_n}. \tag{2}$$

This equation may be reduced to a form more readily comparable with the Abraham [4] solution for the field strength over a conducting plane,

$$E = \frac{120\pi HI}{\lambda d}. \tag{3}$$

Equation (2) then becomes

$$E = \frac{120\pi HI}{\lambda d} f(x), \tag{4}$$

where

$$f(x) = \frac{\sqrt[3]{\pi^2/2a}}{\rho_1} \sum_{n=1}^{\infty} \frac{\sqrt{x}}{\rho_n/\rho_1} e^{-\beta_n \sqrt[3]{2\pi a^2}\, x} \tag{5}$$

and

$$x = d/\sqrt[3]{\lambda}. \tag{6}$$

The constant before the summation sign is equal to 0.1136 when the earth is the sphere under consideration.

[2] H. M. Macdonald, "The Transmission of Electric Waves Around the Earth's Surface," *Proc. Roy. Soc.* (*London*) **A90**, 50–61, April 1, 1914.

[3] This approximation introduces an error of less than one-tenth of a decibel for distances less than 2250 km.

[4] M. Abraham, "Die Strahlung von Sendedrähten," *Theorie der Elecktrizität*, vol. 2, 2nd edition (1908), 283–294, and "Elektromagnetische Wellen," *Enc. der math. Wissen.*, vol. 5, pt. 2, 482–538, March 18, 1910.

Equation (4) states that the field strength at a point on the surface of a conducting sphere is less than that on the surface of a conducting plane by a factor which is a function of the quotient of the distance along the surface by the cube root of the wave-length.

This factor is plotted in Fig. 1. For small values of $x = d/\sqrt[3]{\lambda}$ it approaches unity so that the Watson solution for radio propagation over the surface of a perfectly conducting sphere merges into the Abraham solution for propagation over a perfectly conducting plane at short distances. In order to depict this graphically the curves that result from neglecting all terms except the first, first two, first three,

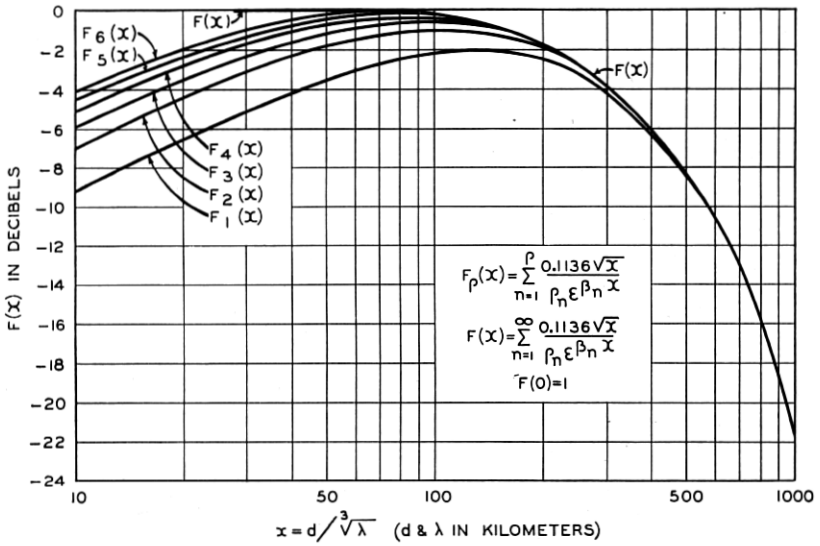

Fig. 1—Successive approximations to Watson's formula for the ratio of the field received over perfectly conducting spherical earth to that over perfectly conducting plane earth.

etc., have been plotted. It will be noted that as more terms of the complete Watson solution are added the resulting curves more nearly approach the Abraham solution for the shorter distances. When x equals 160 the curvature of the earth reduces the field strength one decibel. At this point the error in neglecting all of the terms except the first results in an error of a decibel. For larger values of x the first term approximates the complete series with increasing accuracy, as shown in the curves of Fig. 1. *In other words, no error greater than one decibel is incurred if the Abraham solution is used when* $d/\lambda^{1/3} < 160$ *and only the first term in the Watson solution is employed when* $d/\lambda^{1/3} > 160$.

In obtaining the solution for the propagation of radio waves over the surface of the earth, besides assuming the earth to be a perfect conductor, Watson assumed that the electromagnetic properties of the air were independent of the height above the earth's surface. Data to be presented later indicate that neglecting refraction in the lower atmosphere introduces the greater error for certain frequencies. Fortunately in such cases, it is simpler to extend the solution to take into account atmospheric refraction than the imperfect conductivity of the earth.

It is known [5] that for electromagnetic waves propagated along the surface of the earth, the optical effect of the existing changes in refractivity with height in the lower atmosphere is the same as the effect that would be produced if the earth's radius were increased. If this " effective radius " is substituted for the actual radius in equation (5) the resulting equation for the ratio of the field to that received over a perfectly conducting plane becomes

$$f(y) = 0.1136\sqrt{y}\sum_{n=1}^{\infty}\frac{1}{\rho_n/\rho_1}e^{-\beta_n\sqrt[3]{2\pi/a^2}\,y}, \tag{7}$$

where

$$y = x/\sqrt[3]{K^2} = d/\sqrt[3]{\lambda K^2} \tag{8}$$

and K is the ratio of the effective radius of the earth to the actual radius.

From this it can be seen that the effect of refraction is to multiply the distance at which a given reduction in the field due to the earth's curvature occurs by a factor which is equal to the two-thirds power of the ratio of effective to actual radius of the earth. The analysis of the available meteorological data in the aforementioned article [5] indicates that this radius ratio is about 4/3 on the average. This results in an increase of 1.21 times in the distance at which the reductions in fields occur.[6]

The ratio of the field received over perfectly conducting spherical earth with refraction by the lower atmosphere to that which would be received over a perfectly conducting plane is shown in Fig. 2.

Watson [7] has pointed out the relation of the empirical Austin-Cohen

[5] J. C. Schelleng, C. R. Burrows and E. B. Ferrell, "Ultra-Short-Wave Propagation," *Proc. I.R.E.* **21**, 427–463, March, 1933 and *Bell Sys. Tech. Jour.* **12**, 125–161, April, 1933.

[6] The increase in range is not as great as this due to the inverse distance factor. This advantage would not be realized for waves greater than a certain length. This limit occurs when that part of the atmosphere for which the refractive index no longer decreases at the assumed rate becomes important in the propagation of the waves.

[7] G. N. Watson, "The Transmission of Electric Waves Around the Earth," *Proc. Roy. Soc.* (*London*) **A95**, 546–563, July 15, 1919.

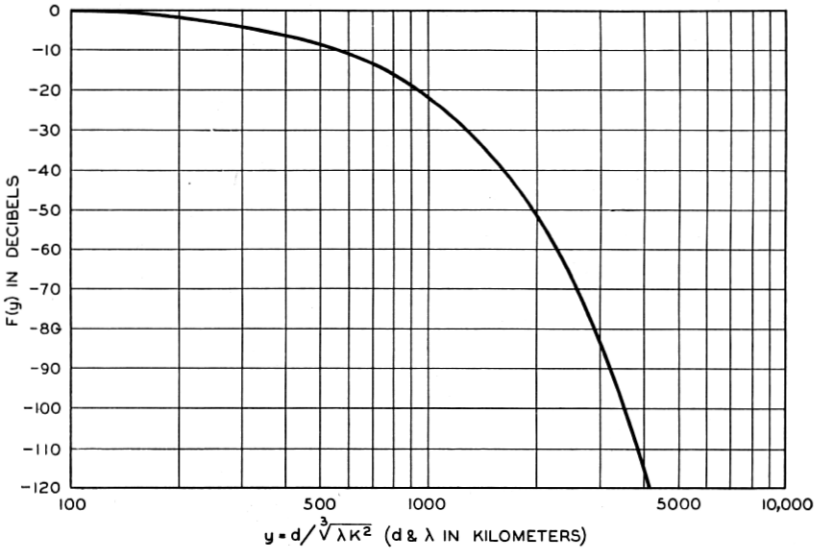

Fig. 2—Ratio of the field received over perfectly conducting spherical earth with refraction by the lower atmosphere to that over perfectly conducting plane earth.

formula for long-distance long-wave communication,

$$E = \frac{120\pi HI}{\lambda d} e^{-0.0015d/\sqrt{\lambda}}, \tag{9}$$

to the above diffraction formulas. He showed that this formula, (9), could be obtained by considering the earth surrounded by a conducting shell some 100 km. above the earth's surface. He also showed that the factor $\lambda^{-1/2}$ instead of $\lambda^{-1/3}$ occurs only when the effect of the upper atmosphere becomes important. Equations (1), (2) and (4) apply only for distances in which the effect of the upper atmosphere may be neglected.

Experiment

In Figs. 3 and 4 the theoretical curve of Fig. 2 has been superimposed upon experimental data[8] obtained for 0.8 and 4 mc. transmission respectively. Theoretical curves are shown for radius ratios of 1, 4/3 and 1.45. The latter gives the best fit with the experimental data. The curve for a ratio of 4/3 estimated from available meteorological data is in fair agreement with the data, but since this is only an estimate of the average value of the ratio it is possible that 1.45 is a better value for the conditions of the experiment. It is doubtful,

[8] All experimental points that represented transmission affected by the ionosphere have been excluded.

however, whether the precision of the experiment would justify distinguishing between these two values. It will be noted that the effect of refraction is appreciable and that the agreement between experiment and theory is greatly improved by taking the effect of refraction into account.

As an indication of the effect of the finite conductivity of ocean water, the theoretical curve for propagation over imperfectly conducting plane earth has been added in each case. Curve 4 for imperfectly conducting plane earth is substantially the same as that for a perfectly conducting plane for 0.8 mc. (Fig. 3), indicating that the effect of the

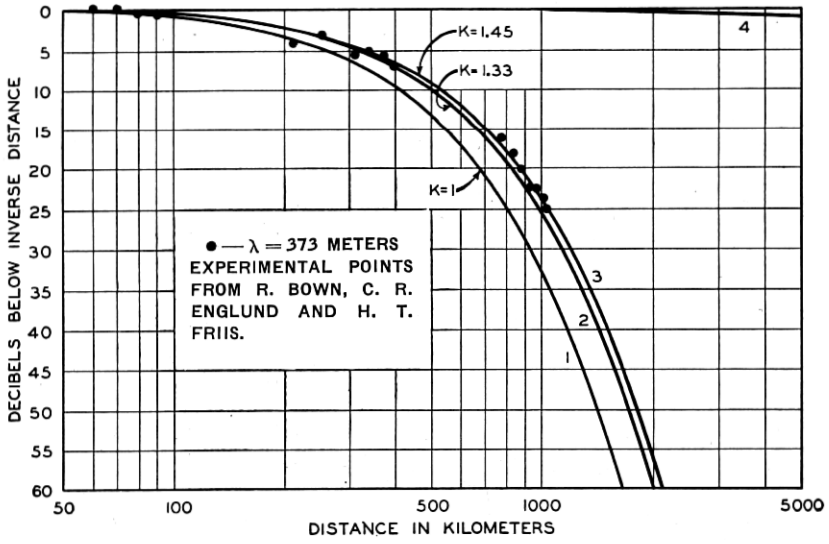

Fig. 3—Comparison between *theory* and *experiment* on 0.8 mc. Experimental points show received field strength from S. S. America, March 1922, taken from Fig. 16 of "Radio Transmission Measurements" by R. Bown, C. R. Englund and H. T. Friis, *Proc. I.R.E.* **11**, 115–152, April 1923.
Curve 1—Theoretical neglecting refraction.
Curve 2—Theoretical assuming average refraction from meteorological data.
Curve 3—Theoretical assuming refraction to give best fit with experimental points.
Curve 4—Theoretical for plane earth taking finite conductivity into account.

imperfect conductivity is negligible on this frequency. For 4 mc., Fig. 4, curve 4, the effect of the imperfect conductivity while not negligible is small compared to the effect of the earth's curvature.

If an attempt be made to take into account the imperfect conductivity of the earth by applying Eckersley's extension of Watson's solution, curve 3 for $K = 1.45$ of Fig. 4 would be moved almost back to curve 1 for $K = 1$. There are, however, several reasons for questioning this extension of Watson's work that will be discussed later.

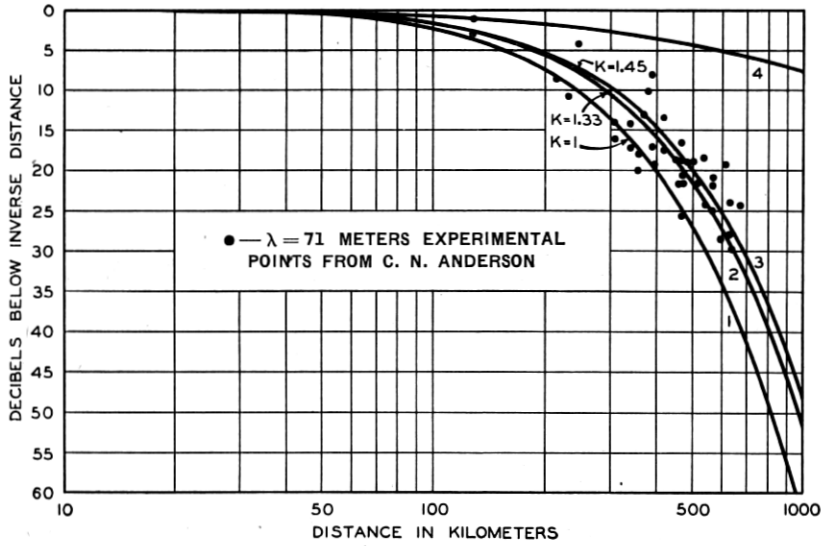

Fig. 4—Comparison between *theory* and *experiment* on 4.2 mc. Experimental points from Fig. 1 of "North Atlantic Ship-Shore Radio Telephone Transmission During 1930 and 1931" by C. N. Anderson, *Proc. I.R.E.* **21**, 81–101, January 1933.
Curve 1—Theoretical neglecting refraction.
Curve 2—Theoretical assuming average refraction from meteorological data.
Curve 3—Theoretical assuming same refraction as curve 3 of Fig. 3.
Curve 4—Theoretical for plane earth taking finite conductivity into account.

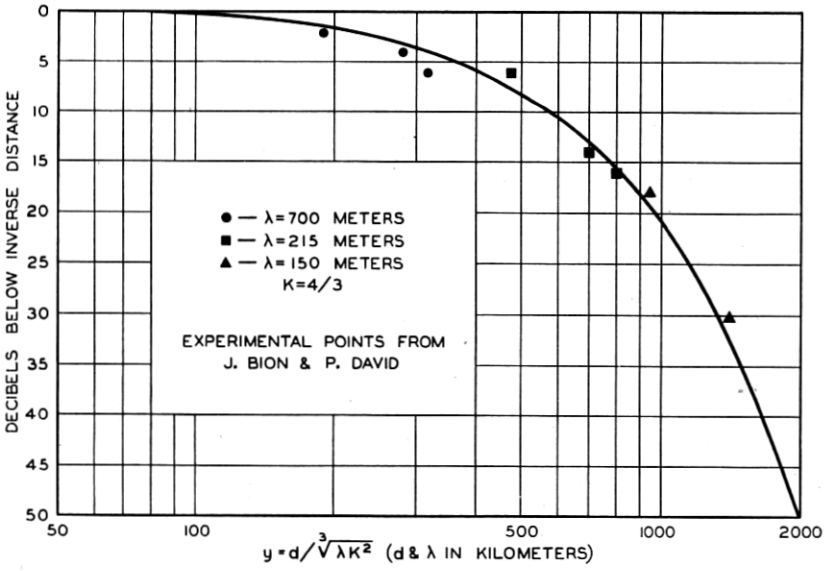

Fig. 5—Comparison between *theory* and *experiment* on 2, 1.4, and 0.43 mc. Experimental points from Bion and David; theoretical curve from equation (7).

Data published by Bion and David [9] to show the inadequacy of Sommerfeld's solution for the propagation over sea water in the wavelength range 150 to 700 meters, have been plotted in Fig. 5. While only eight points are shown they represent data taken at regular intervals on a ship whose distance from the transmitter was continuously increased up to 1050 km. so that their precision is far superior to that possible with single measurements. The points have been plotted against the parameter $y = d/\sqrt[3]{\lambda K^2}$ using 4/3 for the value of K. The points lie close to the theoretical curve, substantiating the theoretical curve and indicating that atmospheric refraction was sufficient to increase the effective radius of the earth by the factor 4/3 for radio propagation (in this frequency range) over the Mediterranean Sea in January and February, 1932.

Effect of Imperfect Conductivity

Due to the complications introduced into the problem of the propagation of electromagnetic energy around the surface of the earth by the effect of imperfect conductivity, no rigorous solution has been made to date. The approximate solution due to T. L. Eckersley,[10, 11] however, has been used [12] to calculate the field strength of the ground wave at distances beyond those for which the solution for transmission over an imperfectly conducting plane applies. The results of this solution will be compared with the rigorous solutions of special cases, leaving a discussion of some of the approximations made and the uncertainties introduced thereby for the appendix.

Theoretical curves obtained by various methods for propagation over the surface of the earth are presented in Fig. 6 for comparison. Curve A is for perfectly conducting spherical earth based on Watson's solution. Curve B is based on Eckersley's solution for a spherical earth whose conductivity is small enough so that its magnitude is unimportant but large enough so that it is essentially a conductor rather than a dielectric.[13] Curves C and D result from using the coefficients given by Eckersley corresponding to the values of $\sigma^{1/2}\lambda^{5/6}$ indicated on the curves. Curves E, F, G and H are for imperfectly

[9] J. Bion and P. David, "Sur L'Affaiblissement des Ondes Moyennes et Intermediaires se Propageant de Jour sur Mer," *Comptes Rendus* **194**, 1723–1724, May 17, 1932.

[10] T. L. Eckersley, "Radio Transmission Problems Treated by Phase Integral Method," *Proc. Roy. Soc.* (*London*) **A136**, 499–527, June 1, 1932.

[11] T. L. Eckersley, "Direct Ray Broadcast Transmission," *Proc. I.R.E.* 20, 1555–1579, October, 1932.

[12] See for example, "Report of Committee on Radio Propagation Data," *Proc. I.R.E.* **21**, 1419–1438, October, 1933.

[13] For detailed explanation of this curve see the appendix.

conducting plane earth based on the solution by Sommerfeld,[14] Weyl,[15] Wise [16] and others and evaluated by Rolf,[17] for the corresponding values of $\sigma^{1/2}\lambda^{5/6}$ indicated on the curves.[18] The part of curve *H* shown also coincides with the solution for perfectly conducting plane earth as determined by Abraham.[4] This indicates that for conductivities greater than those for which $\sigma^{1/2}\lambda^{5/6} = 10^{-5}$ the earth may be regarded as a perfectly conducting sphere.

The fact that the plane earth solution for values of the parameter of the order of 10^{-7} and less (curves *E* and *F*) gives lower fields than Eckersley's solution for spherical earth indicates that the approximations made introduce large errors in these regions of the solution. This inconsistency between the Eckersley solution and the rigorous solution for plane earth in itself would indicate that the solution is

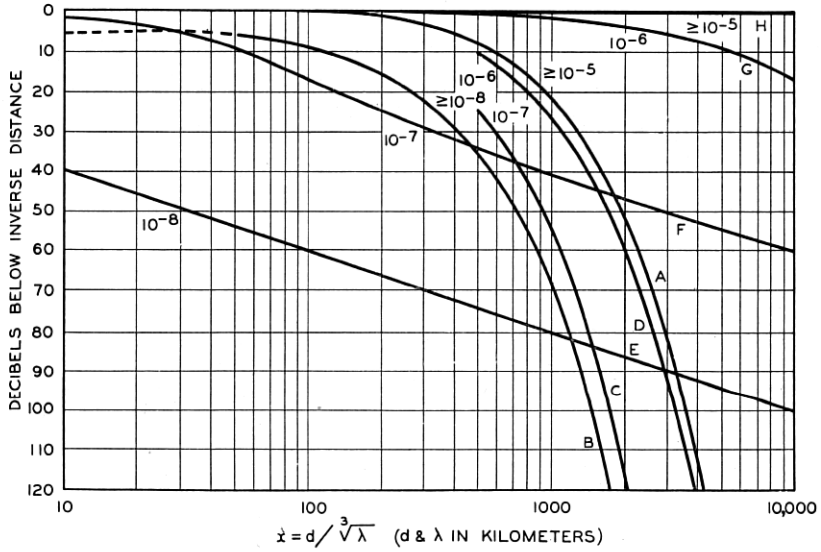

Fig. 6—Comparison of theoretical curves for radio propagation. The numbers on the curves indicate the value of $\sigma^{1/2}\lambda^{5/6}$ for the case represented by the curve in question (conductivity in electromagnetic units and wave-length in kilometers).

[14] A. Sommerfeld, "Über die Aursbreitung der Wellen in der drahtlosen Telegraphie," *Ann. der Phys.* **4**, 28, 665–736, March 16, 1909 and "Ausbreitung der Wellen in der drahtlosen Telegraphie. Einfluss der Bodenbeschaffenheit auf gerichtete und ungerichtete wellenzuge." Jahr. d. drahtlosen T. 4, 157–176, December, 1910.

[15] H. Weyl, "Ausbreitung elektromagnetischen Wellen über einem ebenen Leiter," *Ann. der Phys.* **4**, 60, 481–500, November 20, 1919.

[16] W. Howard Wise, "The Grounded Condenser Antenna Radiation Formula," *Proc. I.R.E.* **19**, 1684–1689, September, 1933.

[17] B. Rolf, "Graphs to Prof. Sommerfeld's Attenuation Formula for Radio Waves," *Proc. I.R.E.* **18**, 391–402, March, 1930.

[18] The usual parameter, $\sigma\lambda^2$, that occurs when the field over imperfectly conducting plane earth is plotted against distance, becomes $\sigma\lambda^{5/3}$ or $(\sigma^{1/2}\,\sigma^{5/6})^2$ when the field is plotted against $d\lambda^{-1/3}$.

not valid for values of $\sigma^{1/2}\lambda^{5/6}$ less than 10^{-7}. While no glaring inconsistencies are evident from Fig. 6 for values of the parameter somewhat greater than this it is the writer's opinion that implicit faith should not be placed in the results without experimental verification due to the nature of the approximations made in obtaining the solution.[19] Comparison of curves *D* and *A* shows that the Eckersley modification of the Watson solution for values of the parameter of the order of 10^{-6} is small which is consistent with the results for plane earth, curve *G*. It is in this region that Eckersley presents experimental data to substantiate his solution.

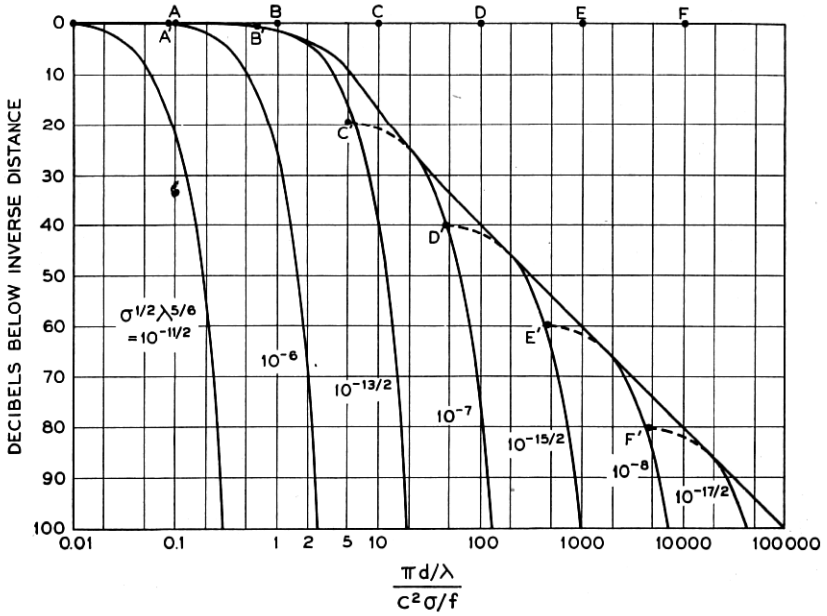

Fig. 7—Revised Eckersley curves for imperfectly conducting spherical earth. (The conductivity in e.m.u. is represented by σ and in e.s.u. by $c^2\sigma$, $c = 3 \times 10^{10}$.)

Recently Eckersley [20] has made the plausible suggestion that his curves should be shifted vertically until they are tangent or nearly tangent to the Sommerfeld curve. This results in the curves of Fig. 7.

[19] Since the writing of this paper an article by Jean Marique entitled "Note sur Quelques Measures due Rayonnement des Stations de Navires," *L'Onde Electrique* **13**, 149–156, March, 1934 has come to the attention of the author. Experimental data are presented from which he concludes that Eckersley's results do not apply for distances of the order 400 to 500 km. at a wave-length of 600 meters.

[20] T. L. Eckersley, "Study of curves of propagation of waves," Document A. G. (1934), No. 11, comm. II; International Scientific Radio Union, Vth Assembly.

Here the abscissa is chosen so that all of the Sommerfeld-Rolf curves coincide. If the effect of imperfect conductivity were unimportant, the curves for spherical earth would begin to depart from unity at the points A, B, C, etc. The effect of imperfect conductivity is to move these points to A', B', C', etc., on the present Eckersley theory. The horizontal motion was calculated by the approximate phase integral method, while the vertical motion is the result of this recent assumption. It can be seen that for the poorer conductivities the recent assumption causes a greater change in the value of the field strength than that calculated by the phase integral method. While this assumption has removed the most obvious inconsistency in the results, the writer believes that they still require experimental verification before reliance should be placed in them.

APPENDIX

The rigorous solution for the perfect conductivity case, equation (1), may be expressed in the form,

$$E = \sum_{u=1}^{\infty} A_n \cos \lambda_n, \tag{10}$$

where A_n and λ_n are functions of ρ_n. By his approximate phase integral method Eckersley was able to evaluate λ_n in the above expression. He found the same relationship between λ_n and ρ_n as Watson. The ρ_n's he obtained, however, differed from those obtained by Watson. The values of ρ_n as determined by Eckersley may be expressed

$$\rho_n = \frac{[3\pi(n - a_n + \eta)]^{2/3}}{2}, \tag{11}$$

where η depends upon the ground constants, being zero for perfect conductivity. a_n is a constant independent of n whose value Eckersley found by comparison with Watson's results to be 3/4. Herein is one of the inaccuracies introduced by the approximate method, for to obtain the correct values of ρ_n, a_n must be allowed to vary with n. While the necessary variation is small [21] for the case of perfect conductivity, without further proof we have no assurance that it is not much larger for the more general case.

Eckersley's method does not tell us anything about the magnitude of A_n in equation (10). He tacitly assumed A_n to be independent of

[21] $a_1 = 0.7819$, $a_2 = 0.7577$, $a_3 = 0.7544$, $a_4 = 0.7530$, $a_5 = 0.7523$, and $a_6 = 0.7519$. For larger values of n, a_n approaches 0.75 more closely.

the conductivity. An equally logical assumption leading to a different result would be that the functional relationship between A_n and ρ_n be independent of the conductivity. Both are undoubtedly incorrect but the error introduced may not be large for the better conductivities.

In obtaining curve B of Fig. 6 the values of a_n given in footnote 21 were used so that Eckersley's solution would be consistent with Watson's solution for the perfect conductivity case. The values of A_n were calculated on the assumption that the functional relationship between A_n and ρ_n be independent of the conductivity. If the magnitude of A_n were assumed independent of the conductivity, curve B would be raised approximately 7 db.