

Contemporary Advances in Physics, XXIX The Nucleus, Fourth Part

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The earlier parts of this series have dealt with the charge, the mass, the stability or instability, and the liability to transmutation, of the atom-nucleus; this one deals with the two remaining properties which are ascribed to nuclei, to wit, magnetic moment and angular momentum. Since these exhibit themselves chiefly by influencing the spectra of the atoms to which the nuclei belong, the bulk of the article is concerned with various laws of atomic spectra; advantage is taken of the opportunity to describe some features of the electron-systems surrounding the nuclei, and to explain how the concept of the spinning electron enters into atomic physics. There follows an account of various experiments in which streams of atoms are deflected by inhomogeneous magnetic fields, and the laws of the deflections indicate the magnetic moment or the angular momentum of the nucleus or both. Finally there is a summary and tabulation of existing knowledge of these quantities.

THE NUCLEUS AS A QUANTIZED VECTOR

UNDER this somewhat forbidding title I propose to discuss some phenomena—mostly spectroscopic, but in certain cases magnetic or even chemical—which are interpreted by supposing that the nuclei of atoms are endowed with two vectorial qualities, *magnetic moment* and *angular momentum*. One may say that these nuclei are to be visualized, no longer simply as particles possessed of mass and charge alone, but as bodies—usually, as congeries of particles both charged and uncharged—which are in incessant rotation: the spinning of the mass constitutes their angular momentum, the spinning of the charge a perpetual circular current-flow which is equivalent to a magnet. Why, then, should I not have entitled this section “The Spins and Magnetic Moments of Nuclei”? Chiefly because it might have suggested, at the very outset, that nuclear spins and magnetic moments are observed as clearly and measured as directly, as are nuclear masses and charges: which in the main is not so. With only a couple of exceptions (for magnetic moment) they are deduced from phenomena which certainly carry no obvious sign of their character. Indeed, what is called the nuclear angular momentum or the *spin* is distinguished by a feature, which is altogether strange and foreign to angular momentum of ordinary wheels and gyroscopes and the other spinning things of daily life; and it is by virtue of this foreign and paradoxical feature—and not any of the familiar qualities of spinning things—

that the concept of "spin" is a valuable addition to atom-models. This feature it is which is responsible for calling the nucleus a "quantized" vector. But I need not further justify at this point my suggestion that spin is a quality both strange and remote from immediate experience, for this will be only too evident in what is to follow.

It is desirable to make a longish excursion through some of those early-known and more-or-less familiar features of spectra, which are explained solely by calling into account the orbital or circum-nuclear electrons of the atom, without taking notice of any property of the nucleus excepting its charge which holds those electrons together. This excursion will have the incidental advantage of making us aware of the magnetic moment and the spin of the *electron*, though we cannot pause long enough to get an adequate appreciation of the scope and value of this concept of the "spinning electron." It is best to simplify these preliminary steps as much as is conveniently possible, and therefore I will speak of a single spectrum only; yet not the spectrum of hydrogen, for this would carry simplicity rather too far. By a queer paradox of which we are destined to see altogether too much, it often happens in quantum mechanics that the seemingly-simplest of all cases—the ones to which one would go by choice for a start—are liable to a singularly-confusing complexity quaintly known as a "degeneracy." Let us therefore take sodium for our example.

The sodium atom has a nuclear charge $+11e$, and eleven orbital electrons, of which the eleventh or "valence electron" is sharply contrasted with the other ten. Employing the original atom-model of Bohr, one visualizes the ten as describing a network of interlacing close-packed orbits enclosing the nucleus as in a sort of cage, while the eleventh darts about in some sort of a far-flung orbit which at one end may enter the cage, while the other end reaches far out into space ("penetrating orbits") or alternatively an entire orbit may encircle the cage completely, not entering it at all ("non-penetrating orbit").

We now consider the major feature of the sodium spectrum—which indeed is the only feature of the absorption-spectrum of tranquil sodium vapor—a beautiful converging series of lines. All of the lines correspond to transitions between one and the same state, *viz.* the normal state of the sodium atom, and the various members of a sequence of abnormal or excited states. These are distinguished by an index n to which consecutive integer values are attached; and all together they are known as the P sequence, to distinguish them from other sequences of states which a fuller study of the sodium spectrum (not confined to the absorption-spectrum of the tranquil vapor) discloses. The terminology, of course, is a detail. What is essential is,

that they are all discriminated from each other by something physical, and yet they all have something in common which distinguishes them as a whole from states of other sequences. As to the nature of these "somethings," the original theory of Bohr is perfectly explicit. All the states correspond to orbits of the valence electron; the different P states correspond to orbits of different but definite sizes and shapes; what all the P orbits have in common is, *a common value of the angular momentum of the electron revolving in its orbit.*

To repeat Bohr's argument here would be an unjustifiable use of time and space. Let me merely recall that when it was applied to the hydrogen atom, it led to several extraordinary agreements with the data of experiment, to which others have since been added. To some degree these are transferable to the sodium atom, especially since the electric field in which the valence-electron of the sodium atom mostly revolves is extremely like that in which the electron of the hydrogen atom always revolves (*i.e.*, beyond the "cage" to which I referred, the ten electrons of the cage effectively cancel the influence of the portion $+10e$ of the nuclear charge, leaving uncanceled only the field of a nuclear charge $+e$ which is the same as the charge of a hydrogen nucleus). These agreements were the primary cause of the enormous role which angular momentum has ever since been playing in all atom-models. They were responsible also for the numerical values now assigned to angular momenta which figure in atoms; for according to the original theory, the orbital angular momentum of the P states is $2(h/2\pi)$, and all the other values which it may take for states of other sequences are other small-integer-multiples of $(h/2\pi)$; and while these values have since been somewhat altered without impairing the numerical agreements on which they rested and on which now the new ones rest, it remains true that *all angular momenta occurring in atom-models are expressed as multiples of $\frac{1}{2}(h/2\pi)$* , the multiplying factors being integers usually smaller than ten.

As my words have already implied, there are various types of angular momentum nowadays fitted into atom-models, the one already described—hereafter to be called the "orbital angular momentum"—being only one and the first. We turn now back to the principal series of sodium to discover why another type is required.

Examined with a sufficiently good spectroscope, each "line" of that series is found to be actually a close doublet. These imply that each of what I have been calling the P states is actually a pair of states.¹ (The confusion introduced into language by referring to one and the

¹ That it is not the normal state which is resolved into a pair is proved by various facts which it is not necessary to mention here.

same spectroscopic object sometimes as a line and sometimes as a pair or group of lines, and the corresponding confusion about states, are very bad impediments to clarity of exposition, but there is simply no way of getting around them.) Such is in fact the case. The two members of a pair have a common value of the index n and a common value of the orbital angular momentum of the valence-electron, and yet there must be something physical which distinguishes them. To leap at once to the conclusion: in the atom-model, this something is *orientation*.

But, orientation of what with respect to what? We have as yet introduced only one outstanding direction, only one vector, into the atom-model. The outstanding direction is that of the normal to the orbital plane of the valence-electron; the vector is the orbital angular momentum. To these it is necessary to supply a second vector,—as it turns out, a second angular momentum.

Since it may occur to some reader that the natural place to seek this angular momentum is among the electrons of the cage—that we should begin by assigning a net or resultant angular momentum to the ten electrons which we have hitherto so much neglected—I will recall that such was actually the first suggestion. It prevailed during the early twenties, and was generally accepted; but it suffered from certain disadvantages, which now there is no particular reason for retelling at length. Yet it was of the greatest assistance in preparing the ground and the technique for the suggestion which superseded it in the middle twenties, that the second angular momentum is to be ascribed to the electron itself; the electron is to possess, like the earth, not only a motion of revolution but also a motion of rotation. (As for the ten electrons of the cage, their angular momenta both of rotation and of revolution are so oriented as to balance one another out, and we have made no error in neglecting them.)

Now there are two vectors and two directions in the model of the sodium atom: that of the axis of the rotating electron, and that of the normal to the orbit—the spin momentum and the orbital momentum. To speak of different orientations of the one with respect to the other is now sensible. But different orientations must correspond to different energies if they are to explain the data, since the two lines of a principal-series doublet are separate and distinguishable because and only because the two members of a pair of P -states differ in energy. Why should they?

This happens to be the easiest question of the lot, or at any rate the one which can be answered from classical physics. The rotating electron is a magnet, by virtue of its whirling charge. Also it behaves

as though it were moving through a magnetic field, and this is not so easy to grasp, since we have postulated merely that it is revolving in the electrostatic field surrounding the nucleus and the cage of inner electrons. Yet if we were to shift our frame of reference and imagine, not the nucleus standing still and the valence-electron revolving around it, but the electron standing still and the nucleus revolving around it—then we should have no difficulty in realizing that the revolving nucleus, being a charge describing a closed path, and being therefore equivalent to a current, would produce a magnetic field. Reverting now to our original frame of reference, we may bring the magnetic field back with us, and say that the *spinning* electron revolves both in the electrostatic field aforesaid and in the magnetic field, and its energy is influenced by both. This is not a very sophisticated way of looking at the matter, and there is a tricky little relativistic detail in the shifting of the frame of reference, which produces an error of a factor 2 if disregarded; but it serves to bring out the idea. The energy of the valence-electron in its orbit is affected by this quasi-magnetic interaction due ultimately to the fact that it is a magnet moving through an electrostatic field; and *the value of the energy depends on the orientation*—on the angle θ between the axis of the magnet-electron and the normal to the orbital plane. One now sees readily that there will be a minimum energy occurring when these two directions are parallel, and a maximum energy occurring when they are anti-parallel. But why then do we not find the individual *P*-state spread out into a continuous band of states corresponding to all the energy-values between these two extremes, and all the infinity of different orientations between the value 0° and the value 180° of the angle θ ?

This is no question which classical physics can solve. The fact that the individual *P*-state, or what would otherwise be the individual *P*-state, is split into two instead of into an infinity—this fact implies that only two orientations occur in nature, are “permitted,” as the phrase is; and this instance of *quantization of direction*, like all the other instances of quantization, is a consequence of the quantum-mechanical constitution of the world. More lucid instances occur when the atom with its electrons is immersed in an applied magnetic field of known intensity; that is, when sodium vapor is exposed to the measurable field of a large-sized magnet, and its spectrum is observed. We will take up some of these instances before beginning with the assignment of spin to the nucleus.

When a magnetic field of moderate strength is applied to sodium vapor, each of the doublets of the principal series is split up into a pattern of several lines or “components.” I can no longer say that

the P -states themselves are split up into as many components as are the lines: that was a happy coincidence while it lasted, but it does not repeat itself henceforward (nor usually). From the subdivision of the lines it is necessary to deduce the subdivision of the initial and the final states of the corresponding transitions: that is a classical task of spectroscopy, which we may assume to have been achieved. It is found that the normal state of the sodium atom (not belonging to the P sequence) is resolved by the magnetic field into two components, while of each pair of states belonging to the P sequence, one is resolved into two and the other into four components. Incidentally, the separations of these components are proportional to the strength of the magnetic field. It appears, therefore, that the sodium atom possesses properties of a magnet, quantized in direction; or rather, that in different states it is equivalent to different magnets, since in certain states it has two permitted orientations in the field, while in others it has four.

As might be guessed, the magnet to which the atom-as-a-whole is equivalent is a sort of *resultant* of the two magnets, spinning electron and "orbital magnet," which have already been separately inserted into the model. This quantum-mechanical resultant, however, possesses a couple of peculiarities, into which we shall have to look rather carefully. To lead up to them, it is desirable to look at the two special cases in which (a) there is no spin and (b) there is no orbital angular momentum, so that the resultant reduces to a magnet of one of the two types. Strictly speaking, case (a) never occurs in sodium, but to work it out is useful, nevertheless.

I have spoken of the electron revolving in its orbit as being equivalent to an "orbital magnet": now is the time to fortify that statement by giving a cardinal relation between the magnetic moment of that orbital magnet (not now the magnetic moment due to the electron-spin!) and the angular momentum of that orbital motion. It is sufficient to work out the relation for a circular orbit. Let r stand for the radius of the circle, v for the speed of the electron, $n(= v/2\pi r)$ for the number of times per second that the electron runs around the circle; e for the charge, m for the mass, p for the angular momentum of the electron, M for the magnetic moment of the system. The revolving electron is equivalent to a current² ne/c flowing in a circuit enclosing the area πr^2 ; the magnetic moment of such an affair is equal to the current-strength times the enclosed area:

$$M = (ne/c)\pi r^2, \quad (1)$$

² The factor c enters in because e is commonly expressed in electrostatic units, whereas in equation (1) the current must be expressed in electromagnetic units.

while for the angular momentum, we have:

$$p = mvr, \quad (2)$$

and eliminating vr between (1) and (2), we get:

$$M/p = e/2mc. \quad (3)$$

Here we have on the left the *ratio of magnetic moment to angular momentum*, a very important quantity for all these categories of atomic and subatomic magnets. It is equated to $e/2mc$, a value which is correct whenever we are dealing with the magnet constituted by the orbital motion (not the spin!) of an electron; this equation in fact is valid for any sort of an orbit described in a central field, and is one of the few that have survived unamended all of the stages in the evolution of Bohr's original theory into quantum mechanics. I will rewrite the equation thus:

$$M/p = g(e/2mc), \quad g = 1, \quad (4)$$

and this is meant to imply that for other categories of atomic and subatomic magnets, the ratio of the moments is not always equal to $e/2mc$, which is true. In general it is the custom to characterize any one of these magnets by giving its value of g . *Orbital magnets, then, are characterized by the value unity for the g-factor.*

We next ascertain the energy which the orbital magnet possesses by virtue of being in the applied field H . Letting α stand for the angle between the direction of the field and the axis of the magnet, we find for the torque exerted on the magnet by the field,

$$T = -MH \sin \alpha \quad (5)$$

and integrating to obtain the energy in question,

$$U = \int T d\alpha = MH \cos \alpha. \quad (6)$$

This we now write as follows:

$$U = (MH/p)p \cos \alpha = (geH/2mc) p \cos \alpha, \quad (7)$$

and here is as good an opportunity as any to recall a well-known theorem of classical mechanics, fundamental in the theory of the gyroscope. When a torque is acting upon a rotating body, the body precesses around the direction of the field responsible for the torque; and if the torque be equal to a constant T_0 times the sine of the angle between the field-direction and the axis of the rotating body, then the angular velocity ω of the precession is equal to the ratio between T_0

and the angular momentum of the rotation.³ This ratio is the one enclosed in parentheses in equation (7): we may therefore write:

$$U = \omega p \cos \alpha. \quad (8)$$

The precession of which ω is the angular velocity is known as the *Larmor precession*. To keep this precession in mind as a feature of the atom-model is usually desirable, though not constantly necessary. It must be supposed to occur not only when an atom is immersed in an extraneous magnetic field, but also when two of the subatomic magnets within a single atom are influencing one another.

We have treated the (non-existent) extreme case in which there is nothing but the angular momentum and the magnetic moment of the orbital motion of a single electron to be taken account of; we turn now to the other extreme in which there is nothing but the spin of a single electron to be taken into account. Were we still confined to the original atom-model of Bohr, this case would be equally non-existent; for no electron-orbit could have a vanishing angular momentum unless it were a straight line passing to and fro through the nucleus, and this was formerly excluded as unthinkable. Quantum mechanics, however, assigns the value zero to the angular momentum of a valence-electron in a state of the *S* sequence (to which the normal state of the sodium atom belongs). Whether the student prefers to visualize a straight-line "orbit" for such a case, or a spherical cloud of charge or of "probability-of-charge," is to some extent a matter of taste, though usually the latter is the better policy. For such a state, there is no angular momentum and there is no magnetic moment save those of the electron-spin itself.

To this spin of the electron—whether isolated as in this extreme example, or compounded with an orbital motion into a resultant—we are compelled by various reasons to assign the value $2(e/2mc)$ for that important ratio of magnetic moment to angular momentum. Otherwise expressed: *the spin of the electron is characterized by the value 2 for the g-factor.*

There is a classical argument for this assertion, based on an evaluation of the ratio in question for a sphere of homogeneous charge rotating about an axis passing through its centre. There is a more powerful quantum-mechanical argument, based on the fact that when Schroedinger's fundamental equation of wave-mechanics was amended by Dirac to be conformable with relativity, there appeared in it a term attributable to a whirling charge with a *g-factor* of 2. Apart from

³ Slater and Frank, "Introduction to Theoretical Physics," Chapter X.

this last, the strongest argument is furnished by the validity of the verifiable formula for the g -factors of various atoms in various states, which we shall presently be deriving. For, inasmuch as in nearly every state of an atom there are both electron-spins and electron-orbits, and the net magnetic moment and net angular momentum of the atom are sorts of resultants of these, the g -factor of the atom-as-a-whole varies from state to state and from one kind of atom to another in a remarkable fashion, which imposes a stringent test on the contemporary atom-model.

In explaining how this test is satisfied, I can no longer postpone specific numerical statements about the angular momenta of electron-orbits and electron-spins and the atoms into which these enter. These statements will be made confusing by the fact that to each one of these angular momenta it will be necessary to assign two different numbers. This is one of the impediments which are unavoidable in fitting visualizable atom-models to the results of quantum-mechanical theory, and which to avoid, some theorists would be willing to forego models altogether.

Were it not for this impediment, I could say quite simply that in every P -state of the sodium atom, the valence-electron has a spin, with angular momentum $\frac{1}{2}(h/2\pi)$, and is moving in an orbit with angular momentum $(h/2\pi)$; that as regards the two members of each of the aforesaid pairs of P -states, these two angular momenta are oriented parallel for one member and anti-parallel for the other, so that the net angular momentum of the atom-as-a-whole is $\frac{1}{2}(h/2\pi)$ in one case and $\frac{3}{2}(h/2\pi)$ in the other; that when an atom with a net angular momentum of $\frac{1}{2}(h/2\pi)$ is exposed to an applied magnetic field, it orients itself either parallel or anti-parallel to the field, so that the projection of its angular momentum upon the field-direction is either $+\frac{1}{2}(h/2\pi)$ or $-\frac{1}{2}(h/2\pi)$; that when an atom with a net angular momentum of $\frac{3}{2}(h/2\pi)$ is exposed to an applied magnetic field, it orients itself in one or another of four permitted ways so that the projection of its angular momentum upon the field-direction is either $+3/2$ or $+1/2$ or $-1/2$ or $-3/2$ times $h/2\pi$.

This sort of thing is frequently said in the literature, and one must realize its limitations. The trouble is, that quantum mechanics prescribes for these angular momenta (but not for their projections on the field-direction!) magnitudes which differ from those which I have been giving. For the spin s of the electron, it substitutes $\sqrt{1/2 \cdot 3/2}(h/2\pi)$ for $\frac{1}{2}(h/2\pi)$; for the orbital motion l in the present case, it substitutes $\sqrt{1 \cdot 2}$ for the factor unity whereby $(h/2\pi)$ was multiplied; in the two values j' and j'' of angular momentum of the atom-as-a-

whole, it substitutes $\sqrt{1/2 \cdot 3/2}$ and $\sqrt{3/2 \cdot 5/2}$ for the factors $1/2$ and $3/2$. In order that these new values of j' and j'' may be resultants of s and l , it is necessary that the angular momenta of orbital motion and of spin should be neither quite parallel nor quite anti-parallel to one another—the two permitted values of the angle between them must differ from 0° and 180° ; and this also is admitted by quantum mechanics. The last two clauses of the foregoing paragraph remain, however, unchanged.

Then why not say from the outset that the spin of the electron has the magnitude $\sqrt{1/2 \cdot 3/2}(h/2\pi)$, and that all the other angular momenta occurring inside an atom have the magnitudes assigned to them by quantum mechanics? Inertia of habit, sanctified by years of earlier theories, is itself a mighty obstacle: after so long a time of saying that the electron-spin is $1/2$, the world of physics could scarcely get accustomed to saying that really it is $\sqrt{1/2 \cdot 3/2}$ (in terms of $h/2\pi$ as unit). There is also a difficulty of printing to be considered: these numbers have often to be used as subscripts; it is bad enough to print $1/2$ or $3/2$ as a subscript, without resorting to their quantum-mechanical substitutes. The most serious reason, however, is, that the original $1/2$ and $3/2$ (and, of course, their analogues in the many other kinds of atomic states) lend themselves uniquely well to stating how many permitted orientations there are. Thus in the present example, I quoted 2 and 4 respectively as the number of permitted orientations in a magnetic field, of certain P states for which the angular momenta were designated as $j(h/2\pi)$ and j had the values $1/2$ and $3/2$ respectively. I was reminded of those numbers by the rule that they are equal to $(2j + 1)$. Had I kept in mind only the magnitudes $\sqrt{3/4}$ and $\sqrt{15/4}$ assigned by quantum mechanics to these angular momenta, the rule would not have been available.

One should therefore keep in mind both the "marker" or "quantum-number" of an angular momentum, of which the foregoing $1/2$ and 1 and $3/2$ are examples; and the numerical value assigned by quantum-mechanics to the magnitude of that angular momentum. Fortunately this is rendered easy by the fact that there is a general formula for the latter in terms of the former, with which we can now make acquaintance in the course of taking a deeper plunge into the lush notation of spectroscopy; as follows:

The angular momentum of electron-spin has the quantum-number s and the magnitude $\sqrt{s(s + 1)}$, and s is always equal to $1/2$.

The angular momentum of the orbital motion of an electron has the quantum-number l and the magnitude $\sqrt{l(l + 1)}$, and l may have the various values $0, 1, 2, 3 \dots$.

The angular momentum of the atom-as-a-whole has the quantum-

number j and the magnitude $\sqrt{j(j+1)}$; for the sodium atom j may have the various values $1/2, 3/2, 5/2 \dots$.

I next give the rule for compounding the last of these three angular momenta out of the first two, it being as I earlier said a "sort of resultant" thereof.

Rule for compounding l and s into j . If l is greater than s , start with the numerical value of $(l+s)$ and write down all the sequence of numbers spaced at unit intervals from $(l+s)$ to $(l-s)$ inclusive: to wit $(l+s), (l+s-1), (l+s-2), \dots (l-s)$. These $(2s+1)$ numbers are the permitted values of the quantum-number j . If on the other hand s is greater than l , start with the numerical value of $(s+l)$ and write down all the sequence of numbers spaced at unit intervals from $(s+l)$ to $(s-l)$ inclusive: to wit, $(s+l), (s+l-1), (s+l-2) \dots (s-l)$. These $(2l-1)$ numbers are the permitted values of the quantum-number j .

(It will be noticed that this rule is much more generally phrased than is required for the case of sodium, where $s = 1/2$, and it suffices to say that $j = 1/2$ for $l = 0$ and $j = l \pm 1/2$ for $l > 0$. If, however, we were dealing with an atom having more than one valence-electron, s might be replaced by a quantum-number different from $1/2$ —not because the individual electrons would have new values of spin, but because the spins of two or more of them would be compounded—and the general phrasing of the rule would then be required). And now, to close (temporarily) the sequence of quantum-numbers and of rules:

Suppose the atom immersed in a magnetic field of strength H , parallel to the z -direction. It may then take any of several distinct permitted orientations, these being denoted by various values of a quantum-number m_j . In any such orientation the projection of the angular momentum of the atom-as-a-whole upon the z -direction or field-direction is equal to $m_j(h/2\pi)$. To ascertain how many permitted orientations there are and what are the corresponding projections, start with the numerical value of j and write down all the sequence of numbers spaced at unit intervals from $+j$ to $-j$, inclusive: to wit, $j, j-1, j-2, \dots -j$. These $(2j+1)$ numbers are the permitted values of the quantum-number m_j .

We now start out upon a train of reasoning which leads to the remarkable verifiable formula already once alluded to, the successfulness of which speaks more powerfully than any other single test for the rightness of this elaborate hypothetical structure which has been devised for the atom.

Note, in the first place, that the rule given above for the permitted orientations of the atom in the applied magnetic field is in accordance

with a major fact of experience: the energy-values of the atom in its various "magnetic levels," as I shall hereafter call them, are known from spectroscopy to follow upon one another in a uniform evenly-spaced sequence. This is just what the rule requires: for if M denotes the magnetic moment of the atom and θ , the angle which it makes with the z -direction, the energy due to the field is equal to $MH \cos \theta$, which is $MH(m_j/\sqrt{j(j+1)})$, and this changes by uniform steps as m is changed from each of its permitted values to the next.

Now recalling the importance of the ratio of magnetic moment to angular momentum, and noticing that $M/\sqrt{j(j+1)}$ is none other than this ratio, and introducing the g -factor of equation (7), and the precession ω of equation (8), we may write for ΔU the energy-difference between one magnetic level and the next:

$$\Delta U = g(e/2mc)H = \omega, \quad (9)$$

so that a measurement of the energy-difference or separation of two magnetic levels of an atom gives immediately the value of g for that atom. One sees at once how to make a special test of the value 2 assigned to the g -factor of the electron-spin; for when the sodium atom is in any state for which $l = 0$, it is the spinning valence-electron which contributes the whole of the magnetic moment and the angular momentum of the atom; and when from the spectrum of sodium vapor in the magnetic field the value of ΔU is determined for these states, it is precisely the value $2(e/2mc)H$ which is found.

To get a notion of what actually happens in the general case, it is best to take a sheet of paper and make a graphic composition of the angular momenta. These three— s , l , and j , to denote them by their quantum-numbers—are to be laid down as a triangle having sides of the lengths $\sqrt{s(s+1)}$, $\sqrt{l(l+1)}$, and $\sqrt{j(j+1)}$; for convenience I drop out the common factor $h/2\pi$ for the next few lines. The cosines of the three angles are obtained in terms of the sides by applying the well-known trigonometric formula and getting three equations of which here is one,

$$s(s+1) = l(l+1) + j(j+1) - 2\sqrt{l(l+1)}\sqrt{j(j+1)} \cos \theta_{l, j}. \quad (10)$$

The magnetic moment of the orbital electron-motion is a vector parallel to l and of the length $g(e/2mc)\sqrt{l(l+1)}$, with *unity* put as the value of g . The magnetic moment of the electron-spin is a vector parallel to s and of the length $g(e/2mc)\sqrt{s(s+1)}$, with *two* put as the value of g . Owing to the inequality of these g factors, the resultant of the magnetic moments is not parallel to j . We could easily calculate its magnitude and direction, but they are not relevant. It is

to be presumed that s and l are constantly revolving or precessing around the direction of j ,—the triangle aforesaid is constantly revolving around its side j as a fixed axis, while retaining its size and shape unchanged. If we resolve the resultant angular momentum into a component parallel to j and another component perpendicular to j , the latter will be forever changing in direction and its average will vanish, leaving only the former as a perpetual constant. But this former component we can evaluate by projecting separately upon the j -direction the magnetic moments associated with orbital motion and spin, and adding the two projections. Thus for the magnetic moment of the atom-as-a-whole we get an effective average which is a vector parallel to the angular momentum of the atom-as-a-whole, and it is of the magnitude:

$$M_a = [\sqrt{l(l+1)} \cos \theta_{l, j} + 2\sqrt{s(s+1)} \cos \theta_{s, j}] \frac{e}{2mc} \frac{h}{2\pi} \quad (11)$$

(where I have restored the factor $h/2\pi$). If we work this out with the aid of (10) and the similar equation for $\cos \theta_{s, j}$, and then divide it by the angular momentum $(h/2\pi)\sqrt{j(j+1)}$ and by $e/2mc$, we get the *g-factor for the atom-as-a-whole*—commonly denoted by g_j —in terms of the quantum-numbers s , l and j :

$$g_j = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}, \quad (12)$$

and this is the celebrated *g-formula*, which is tested by applying magnetic fields H to atoms, splitting their stationary states into clusters of levels, measuring the separation between successive levels of a cluster, equating it to $\omega = g(e/2mc)H$, evaluating g and comparing it with the value which the right-hand member of (12) assumes when in it s , l and j are given the values appropriate to the state from which the cluster of levels was formed. So great is the variety of atomic states, so great the number of different triads of values of s , l , j represented among them, that the study of even a single element like sodium produces many different checks of the validity of (12); and since many different elements have been studied, the total of the available verifications of the *g-formula*, and therefore of the intricate network of its underlying ideas, is considerably impressive.

The temptation of going onward and onward into the details of these properties of the extranuclear electrons and their orbits is difficult to resist, but it must be overcome, for the field is practically endless. I must add only, that when an atom possesses two or more valence-

electrons, their orbital momenta $l_1, l_2 \dots$ are likely⁴ so to orient themselves as to form a fixed resultant L , and their spin-momenta $s_1, s_2 \dots$ are likely to orient themselves so as to form a fixed resultant S ; then L and S are likely so to orient themselves as to form a fixed resultant J , following in so doing the rule of page 295. Considered as quantum-numbers, J and S may have half-integer values only ($1/2, 3/2 \dots$) or full-integer values only ($0, 1, 2 \dots$) according as the number of valence-electrons is odd or even, while L for any atom may have full-integer values only. The three resultants are vectors of magnitudes $\sqrt{J(J+1)}$, $\sqrt{S(S+1)}$ and $\sqrt{L(L+1)}$. The g -factor associated with L is unity and the g -factor associated with S is two, and the value of g for the atom-as-a-whole is given by (12) with capital letters replacing the small ones; so that the g -formula is verifiable with atoms of all kinds, as I intimated before. A final point: one might expect the complexity to go on increasing tremendously from one end to the other of the Periodic Table, but there is a counteraction. In all atoms excepting the lightest, most of the electrons have oriented their orbits and their spins in such a way that they have interlocked themselves into groups or "closed shells" for which L is zero and S is zero and J is zero and the magnetic moment is zero, as have the ten electrons of the "cage" of the sodium atom to which I alluded. The so-called valence-electrons are those few which have not been locked into any such a cage. It is this quality which makes the Periodic Table periodic; but this must be left for some other place.

We arrive at last at the nuclear moment.

Suppose that even with these spins and these orbital motions of all the extra-nuclear electrons, we have not yet exhausted the internal angular momenta of the atom, and that the nucleus itself possesses one. Suppose, to be specific, that the nucleus has an angular momentum with a quantum-number I and a magnitude $\sqrt{I(I+1)}(h/2\pi)$, and a propensity for orienting itself in distinct permitted directions with respect to the other angular momenta of the atom. How shall we detect this, and how shall we determine I ?

It is practically necessary to be yet more specific. One could probably not tell *a priori* whether the nuclear angular momentum would tend to orient itself with special respect to individual electron-momenta, or with special respect to some resultant or in particular to that grand resultant of all electron-momenta which we have denoted by J . However, in the cases which have been successfully analyzed, it

⁴This is the description of what is known as "Russell-Saunders coupling" or "LS coupling"; in certain states of certain atoms, the mutual orientations of the vectors conform to different schemes.

turns out that the last condition is the one which prevails. Suppose then that I compounds itself with J according to the following rule, repeated word-for-word with appropriate changes of symbol from the rule for compounding s and l into j :

Rule for compounding I and J into F . If I is greater than J , start with the numerical value of $(I + J)$ and write down the sequence of numbers spaced at unit intervals from $(I + J)$ to $(I - J)$ inclusive: to wit, $(I + J)$, $(I + J - 1)$, $(I + J - 2)$, \dots $(I - J)$. These $(2J + 1)$ numbers are the permitted values of the quantum-number F . If J is greater than I , start with the numerical value of $(J + I)$ and write down the sequence of numbers spaced at unit intervals from $(J + I)$ to $(J - I)$ inclusive. These $(2I + 1)$ numbers are the permitted values of the quantum-number F .

The quantum-number F refers to a vector of magnitude $\sqrt{F(F + 1)}$ ($h/2\pi$), which has taken over from J the role of the angular momentum of the atom-as-a-whole, being the resultant of J and of the nuclear angular momentum I .

Now if all these suppositions are correct, we may expect to find not individual states, but whole serried clusters of states, corresponding to individual values of J . If out of the manifold term-system of an atom we select a state for which $J = 1/2$, one for which $J = 3/2$, one for which $J = 5/2$ and so on as far upward as our knowledge extends, we may expect on close scrutiny to find that these apparent states are actually clusters, each cluster comprising a number of states which for one or two or more of the lowest values of J may be equal to $(2J + 1)$, but for higher values reaches and remains at a limit which we identify as $(2I + 1)$.

What is observed with the spectroscope, though, is not the individual state, but the line which reveals a transition between two different states. What in a feeble spectroscope appears as a single line, and is attributed to a transition between two states with resultant electronic angular momenta (I fear no shorter term will serve henceforth) j' and j'' , should in an excellent spectroscope appear as a cluster of lines due to transitions between the several members of two clusters of states.

This again is exemplified by the principal series of sodium. As I said earlier, this appears in a feeble spectroscope as a series of single lines, each of which is resolved by a good spectroscope into a doublet. This structure, by the way, is called the "fine structure" of the lines; and this it is which indicates that the P -states of sodium are close pairs, and which thus invites and requires the introduction of the quantum-numbers j and s and the spin-momentum of the electron.

With a very good spectroscope indeed, each member of each principal-series doublet is in its turn resolved into a pair. This structure is called the "hyperfine structure" of the lines; and this it is which indicates that a still further subdivision of the states is necessary, and invites and requires the introduction of the quantum-numbers F and I and the angular momentum of the nucleus. Indeed, the concept of the nucleus as a quantized vector was invented or discovered (whichever word the reader may prefer) during the interpretation of hyperfine structure of spectrum lines.

Hyperfine structure of lines or states (for the name is applied to both) is usually more crowded and compact than fine structure, and yet there are exceptions: the fine structure of hydrogen is much harder to resolve than the hyperfine of (say) the familiar mercury lines 2537 and 5461, which itself was called fine before the theory was developed. These structures, however, are generally near and often, it is to be suspected, beyond the utmost capacities of the most refined of optical instruments; whence, in many cases, extraordinary difficulties in measuring or even estimating the separations, the relative intensities, actually the mere number of the distinct lines forming a hyperfine pattern; observers of great skill will often disagree with one another, and judgment will often depend on a photograph taken with a spectroscopic instrument such as an echelon or an etalon, which looks totally different from the pictures obtained with gratings or prisms. Perhaps this last is an advantage after all, as it discourages attempts by the inexpert to interpret published photographs. Often several different isotopes of an element produce different patterns which signify different values of I , and are so nearly superposed on one another as to make analysis superlatively hard. Hyperfine structure is for the present, and quite probably will be forever, the "last frontier" of spectroscopy.

The task of deriving, from the hyperfine line-pattern connecting two states or (better) state-clusters, the hyperfine subdivision of the state-clusters or "hyperfine multiplets" themselves, is again an example of the classical function of spectroscopy, which we shall take as having been achieved. Actually it involves, of course, the use of selection-principles, themselves connected with the atom-model, but omitted from this article in order not to complicate it still more. Some confusion may be prevented if I state that in our favorite case of sodium, where the fine-structure splitting of the principal-series lines implies a splitting of the P -states only, the hyperfine splitting implies something more complex: it is due jointly to hyperfine structures of both the P -states and the normal S -state, the latter being predominant.

We now consider briefly the methods of determining the quantum-number I of the nuclear angular momentum.

(a) The *ideal method* is the one already described: investigate line-clusters connecting state-clusters of as many different values of J as there are; ascertain thus the number N of states per cluster; verify that N is equal to $(2J + 1)$ whenever J is less than or equal to some particular value J_m (say), and that it is equal to $(2J_m + 1)$ whenever J is equal to or greater than J_m . All this being verified, the value of I must be J_m .

One seldom if ever finds such a programme as this worked out very fully. The difficulties seem to be that, at best, it is a lot of work to analyze the hyperfine-structure of even one line, let alone a great number; while at worst, lines connected with states of certain J -values, high ones especially, may be quite unobservable; also there is the striking fact that in a given spectrum a very few lines or even one alone may have their hyperfine-structures spread out so much more broadly than all the rest, that research is practically concentrated on them alone. (One hears so much about 4722 of bismuth as almost to have it blotted from mind that bismuth has other lines!) But of course if one is willing to accept without test the rule for compounding the vectors I and J , then it suffices to discover and analyze a state-cluster for which N is less than $(2J + 1)$.

(b) *The intervals between the members of a state-cluster* may give a clue to the value of I . These intervals are of course energy-differences, and the fact that they exist shows that there are forces between the spinning nucleus and the system of revolving and spinning electrons which surrounds it. If these forces are magnetic, then they may reasonably be expected to vary as the sine of the angle $\theta_{I, J}$ between the angular momenta of nucleus and extranuclear electron system; for either the magnetic moments of these two will be parallel to the respective angular momenta, or else (by the reasoning of page 297) their non-parallel components will presumably change so rapidly as to be ineffective, leaving only the parallel components to be detectable. Comparing the different orientations of I and J which correspond to the several values of F and thus to the states of the cluster, one sees that their energies—or rather, the parts W_F thereof which are due to the interaction—should then vary as $\cos \theta_{I, J}$: putting for which the formula based on (10)

$$W_F = \text{const.} \frac{F(F + 1) - I(I + 1) - J(J + 1)}{2\sqrt{I(I + 1)}\sqrt{J(J + 1)}} \quad (13)$$

Insert in (13) the permitted values of F , which are $(2I + 1)$ or $(2J + 1)$ in number according as J is or is not greater than I , and are spaced at unit intervals from $(I + J)$ downwards (page 299); call them, in order of descending magnitude, $F_m, F_{m-1}, F_{m-2} \dots$. Form the $2I$ or $2J$ consecutive first differences between the so-computed permitted values of W_F ; call them $\Delta W_m, \Delta W_{m-1}, \Delta W_{m-2}, \dots$. Then as is readily worked out,

$$\begin{aligned} \Delta W_m : \Delta W_{m-1} : \Delta W_{m-2} \dots &:: F_m : F_{m-1} : F_{m-2} \dots \\ &:: (I + J) : (I + J - 1) : (I + J - 2) \dots \end{aligned}$$

The successive energy-differences or *intervals* should stand to one another as the successive members of the chain of integers (or half-integers, as the case may be) stepped off at unit intervals and stretching from $(I + J)$ downwards.⁵

This is an *interval-rule* based on a specific notion of the intra-atomic forces (the sine-law aforesaid), and having analogues in the parts of atomic theory having to do with the interactions between electrons the extra-nuclear electrons only. If verified, it enables one to determine $(I + J)$ and therefore I from the analysis of a single cluster of states with a single value of J , even when J is smaller than I and the preceding method would fail. Much use has been made of this method, and there are a few cases in which a fairly accurate measurement of a chain of intervals has shown that it closely agrees with a chain of consecutive integers or half-integers, though more usually the intervals are small and the measurements rough and it is merely assumed that there is perfect agreement with that particular succession of half-integers or integers with which there is the nearest apparent agreement.

(c) *The relative intensities of the members of a line-cluster* are capable of giving information about the quantum-numbers of the states which they connect, provided one adapts quantum-mechanical formulae developed for transitions into which the nuclear angular momentum does not enter. The formulae are of appalling complexity, while intensity-measurements, especially when one is working so near the limits of the possible as when hyperfine-structure is being measured, are notoriously liable to error. This method is probably to be classified as by far the least reliable, for the present at any rate.

⁵ The biggest interval may be that between the highest and the next-to-highest energy-value, or that between the lowest and the next-to-lowest; whichever case is realized gives a clue to the "sign" (page 318) of the magnetic moment; usually the former corresponds to a positive, the latter to a negative moment, but features of the extra-nuclear electron-system may cause this statement to be reversed. Incidentally, relative intensities of lines also have a bearing on the sign of the moment.

(d) *The phenomenon of alternating intensities in band-spectra* serves to reveal the spins of a few kinds of nuclei, and in a very interesting and reliable way, but must be left for another occasion.

There remain the methods which involve the use of a magnetic field in one way or another, and some of which incidentally tell most of what we know about the magnetic moments of nuclei, though nowhere near so amply or so exactly as we should like.

First it must be said that the analogy between the vectors I and J on the one hand, L and S on the other, which thus far has been so full and helpful, breaks down completely when the atom is exposed to a magnetic field of ordinary strength. Were the analogy perfect, an atom in a state distinguished by the quantum-number F for its total angular momentum would act as a rigid spinning body and would be able to assume $(2F + 1)$ discrete orientations in the magnetic field, corresponding to $(2F + 1)$ magnetic levels. This would be true of each of the $(2I + 1)$ or $(2J + 1)$ states comprised in what I have been calling a "cluster" with a common value of J , though the value of F and hence of $(2F + 1)$ would differ from one state to the next. The magnetic levels would be distributed in groups, each corresponding to a different value of F . The numbers in the different groups would be unequal. The total number for all the groups or states of the cluster would amount, as the reader can figure out, to the product $(2I + 1) \times (2J + 1)$.

It is altogether probable that this is precisely what does happen in magnetic fields so weak as not to separate the magnetic levels perceptibly (their separation being then, it will be recalled, proportional to the field-strength). Yet in fields strong enough to produce a measurable effect, the disposition of the magnetic levels has only one thing in common with this hypothetical distribution. Their total number is precisely $(2I + 1)(2J + 1)$. They are, however, distributed in $(2J + 1)$ groups, each consisting of $(2I + 1)$ levels; as though first of all the atoms were to forget their nuclear angular momentum and remember only their electronic angular momentum, and were to orient themselves in the field in the $(2J + 1)$ different ways which were prescribed for them (page 295) while the nucleus was still being neglected; and as though then they were to remember the nuclear angular momentum, and were to allow for it by adopting, in place of each separate one of the $(2J + 1)$ very different orientations, a group of $(2I + 1)$ orientations differing only a little from it and from each other.

This rather animistic idea is not very far from the model commonly conceived. It is supposed that in the strong magnetic field the nucleus

is somehow broken away from its interlocking with the system of extra-nuclear electrons; not in the sense that it is torn out of the system or that its electrostatic attraction for the electrons is suspended, but in the sense that somehow or other the rule for the compounding of I and J into various values of a resultant vector F is done away with. The system of electrons with its angular momentum J chooses among $(2J + 1)$ orientations with respect to the field, just as if the nucleus were not there; and the nucleus chooses among $(2I + 1)$ orientations with respect to the field, just as if the electronic system were not there. Nucleus and electrons, I and J , are said to be "decoupled" from each other; it is supposed that their angular momenta precess each at its own rate separately around the direction of the field.⁶

This account suggests that the energy-values of the magnetic levels would be given by the various values of the expression

$$M_e H \cos \theta_{J, H} + M_n H \cos \theta_{I, H}, \quad (14)$$

where $\theta_{J, H}$ and $\theta_{I, H}$ stand for the inclinations of the angular momenta J and I with respect to the field-direction, while M_e and M_n signify the magnetic moments of the extra-nuclear electron-system and of the nucleus, or, if these magnetic moments be not parallel to J and I respectively, then their projections upon the directions of J and I . The different groups of levels would then correspond to different permitted values of $\cos \theta_{J, H}$, the different levels of any one group to different permitted values of $\cos \theta_{I, H}$. The first term, one might say, would determine the $(2J + 1)$ separate energy-values which would occur if there were no angular momentum of the nucleus, while the second term would subdivide each of these into a group of $(2I + 1)$ levels.

It is found, however (as we shall later see) that the magnetic moments M_n of nuclei are always so very small by comparison with those of extranuclear electron-systems, that the second term of (14) is quite negligible. We have therefore to look for some other cause for the observable subdivision. This cause is thought to be the force between the moments of the nucleus and the electron-system. We assumed it to be overborne by the strong field in so far as its ability to control the quantized directions of the angular momenta is concerned,

⁶ The analogy of I and J with L and S is restored when the impressed field is very strong, for then L and S are similarly decoupled from one another—"Paschen-Back effect" as distinguished from the "Zeeman effect" which we have hitherto been considering. Thus it is roughly correct to say that hyperfine structure reacts to a weak magnetic field as fine structure does to a strong one, though this statement should be carefully qualified if use were to be made of it.

but now must suppose it still to be potent enough to affect the energy of the atom. It adds a third term to (14) which is supposed to be of the form *const.* $M_e M_n \cos \theta_{I, H} \cos \theta_{J, H}$, but in any event will have $(2I + 1)$ distinct values for every value of J , and is not necessarily (or anyhow is not known to be) too small to account for the observed separations between the members of each group. It therefore allows us to estimate I by counting the members of such groups and equating their number to $(2I + 1)$, and this (when available) is one of the most acceptable ways of determining the angular momentum of the nucleus.

With a spectroscope one counts, as always, not the number of levels but the number of lines connecting them with some other family of levels, and expects that the two numbers will not be the same. By a rare if not unique coincidence, however, they *are* the same: the selection-principle which is involved is such, that each group of levels produces a group of an equal number of lines, or (in other words) if the influence of the nucleus resolves every state of the atom in a magnetic field into a group of $(2I + 1)$ different levels, then it also resolves every line connecting two such states into a group of $(2I + 1)$ different lines. There are in the literature magnificent photographs of the spectrum lines of bismuth exposed to a magnetic field, each line under high resolution exhibiting ten components and proving the value $9/2$ for I .

It is, however, sometimes possible to count the levels directly, by sending a beam of fast-moving atoms through an inhomogeneous magnetic field which spreads it out into a diverging fan of smaller beams or pencils, each consisting exclusively of atoms having a certain distinctive value for the projection of the magnetic moment upon the field-direction. This requires a great refinement of the celebrated method of Gerlach and Stern, a refinement which has been achieved by Rabi and his school.

We take, as usual, sodium for our example. Consider a narrow beam of sodium atoms, moving with uniform speed along the x -direction into a region pervaded by a magnetic field which is parallel to the z -axis, and of which the magnitude H varies as rapidly as possible with z . Were it not for this variation of H with z , nothing would happen to the beam, for (to make the crudest possible picture) each atomic magnet would have both its north and its south pole exposed to the same field-strength, and one would be pushed as hard as the other was pulled, resulting in no net force upon the magnet and no deflection. But when the field varies with z and the atomic magnet is oriented otherwise than at right angles to the z -axis, the north and the south pole will be exposed to different field strengths, there will be a resulting



force and a resulting deflection of the flying atom. Force and deflection will increase with the z -component of the magnetic moment of the atom, and atoms with different values of this component will go in different directions.

First we disregard the influence of the sodium nucleus. The sodium atoms in experiments of this type are always in their normal state, for which I recall that $l = 0$ and $j = s$: the angular momentum and the magnetic moment are exclusively those of the spin of the valence-electron, the former having quantum-number $1/2$ and magnitude $\sqrt{\frac{1}{2}(\frac{1}{2} + 1)} (h/2\pi)$, the latter being parallel to the former and having magnitude $g(e/2mc)$ times the magnitude of the angular momentum, with 2 for the value of g . Having this value, the angular momentum of the atom may orient itself in either of the two ways which are crudely (page 292) called "parallel" and "antiparallel" to the field, though it is better (page 293) to think of the two permitted inclinations to the field-direction as being $\text{arc cos } \frac{1}{2}/\sqrt{\frac{1}{2}(\frac{1}{2} + 1)}$ and $\text{arc cos } (-\frac{1}{2}/\sqrt{\frac{1}{2}(\frac{1}{2} + 1)})$. Half of the atoms are so oriented that the z -component of their magnetic moment is $\frac{1}{2}g(e/2mc)(h/2\pi)$, the other half so that the z -component has the negative of this value: the beam is split into two, diverging oppositely and symmetrically from the axis of x . The detection of this splitting is the Gerlach-Stern experiment.

Now we suppose that the sodium nucleus has an angular momentum of quantum-number I , as a result of which the two orientations aforesaid are not really two, but actually are two groups of $(2I + 1)$ not-very-different orientations apiece. The problem is, to refine the method sufficiently to bring out the fact (if it is a fact) that each of the two apparent beams aforesaid is actually a close group of several, and to count the several.

It is necessary to lengthen out the path of the atoms in the inhomogeneous magnetic field, since the longer their exposure to the deflecting agent lasts, the farther the separate beams are drawn apart; this means magnifying the scale of the apparatus and the volume which has to be kept evacuated, and carrying to a very high pitch the geometrical accuracy of its design, since the initial not-yet-separated beam must be exceedingly narrow and must be shot forth in a very-exactly-adjusted direction from its source into the field. It is essential also to reduce the broad distribution-in-velocity which the atoms owe to the fact that they come out of a furnace (in which sodium is being vaporized) with the random velocities of thermal agitation appropriate to the temperature of the furnace, and which would more than suffice to merge the beams which it is now desired to separate. One gathers that even at present it would not be possible to make the wished-for separation,



were it not for one more feature of the laws of the behavior of atoms and their internal angular momenta in the magnetic field.

I have described at length how, in magnetic fields of the field-strengths customary in spectroscopy, where the angular momenta I and J of nucleus and electron-system are "decoupled" and orient themselves independently in the field, each state with a given value of J is converted into $(2J + 1)$ groups of $(2I + 1)$ levels apiece. But in no field at all, I and J are "coupled" into a resultant F , or rather into one or another of several such resultants; and I mentioned that there is reason to suppose that, in fields very much weaker than the customary ones, this coupling subsists and the atom orients itself as a single entity in the field. I emphasized then (page 303) that different as are these two extremes, they have one feature in common: the total number of the levels corresponding to a single value of J , which at both extremes is $(2I + 1)(2J + 1)$. The practical usefulness of this theorem is diminished by the fact that some of these levels may have identical values of energy, but in the atom-model they are nevertheless distinct.

One naturally guesses that as the field-strength is increased from "very weak" to "customary," each level of the one extreme passes over into a level of the other extreme, so that for any field-strength low, intermediate or high there are always just $(2I + 1)(2J + 1)$ of them. There arise then the lesser problem of ascertaining the "correlation," i.e. which level of the one extreme goes over into which of the other; and the greater problem of ascertaining just how, for each of these continuously-definite levels, the energy-value and the component of the magnetic moment along the field-direction—which latter determines the deflection, and which let us call M_z —vary with the field-strength H . Formidable theoretical articles have been written on both of these problems, culminating in rules for the former and formulae for the latter. They were worked out originally for the behavior of the vectors L and S in applied magnetic fields, but are translated into rules and formulae available for our present interests by simply replacing these vectors with I and J and making corresponding changes in the g -factors.⁷ For such an atom as hydrogen or sodium in its normal state, for which $J = \frac{1}{2}$, I will quote the formula from Breit and Rabi.

⁷ Strictly one should take into account the influence of the magnetic field on the interrelations between L and S and on those between I and J simultaneously, but it usually happens that when H is increased to a magnitude which already suffices to decouple I and J pretty thoroughly, it is not yet great enough to do much to the coupling between L and S . I have spoken of this range of magnitudes as "customary," on the ground that it is usual in experiments on the Zeeman effect; but there is no good single word for qualifying it, inasmuch as it is simultaneously weak with respect to the (L, S) coupling and strong with respect to the (I, J) coupling.

We have seen that in the absence of magnetic field, the normal state is subdivided into two by the influence of the nuclear angular momentum I and its coupling with J . In one of these—call it N' —the vectors I and J are nearly parallel, and their resultant, the angular momentum F of the atom-as-a-whole, has the quantum number $(I + J)$ which is $(I + \frac{1}{2})$; in the weak field where the coupling is not broken, and the state N' maintains its identity, the number of permitted orientations is $(2F + 1)$ which is $(2I + 2)$. In the other—call it N'' —the vectors I and J are nearly anti-parallel, and F has the quantum number $(I - \frac{1}{2})$; in the weak field the number of permitted orientations is $(2F + 1)$ which is $2I$. Adding, we get for the total number of magnetic levels the value $(4I + 2)$, which is equal to $(2I + 1)(2J + 1)$ as I stated. In the weak field, the different levels are distinguished by their values of the magnetic quantum-number m , which is defined by saying that the projection F_z of the angular momentum (of the atom-as-a-whole) on the field-direction is equal to $m(h/2\pi)$. The permitted values of m are $(I + \frac{1}{2})$, $(I - \frac{1}{2})$, $(I - \frac{3}{2}) \dots$, $-(I - \frac{1}{2})$, $-(I + \frac{1}{2})$. The first and the last of these values are attached each to a single level, belonging (in the weak field) to the state N' ; each of the others is attached to a pair of levels, one belonging to the state N' and the other to the state N'' .

We know that each of these levels maintains its identity as the field-strength is increased, even when the coupling of I and J into F is broken down and the separate states N' and N'' lose their identities. We wish to know how the value of M_z for each level is varying as the field increases. Let a stand for $2m/(2I + 1)$; let b stand for the energy-difference between N' and N'' ; let g stand for the g -factor associated with the extra-nuclear electron-system and with the angular momentum J ; let x stand for $(g/b)(eH/2mc)(h/2\pi)$. The formulae of Breit and Rabi are as follows⁸:

$$M_z = \pm \frac{a + x}{2(1 + 2ax + x^2)^{1/2}} g(e/2mc)(h/2\pi) \quad (16)$$

For the levels characterized by the extreme values of m (*viz.*, $\pm (I + \frac{1}{2})$) and initially belonging exclusively to N' , the first factor is equal to one half and the two levels are distinguished by the two choices of sign, and M_z is independent of field-strength. With respect to the other values of m , the situation is more complex and curious. A single value of m , say $(I - \frac{1}{2})$, corresponds to two different values of M_z which are equal in magnitude and opposite in sign; the opposite value of m ,

⁸ Perhaps it is not superfluous to remark that in the factor $(e/2mc)$, the symbol m always stands for electron-mass, never for magnetic quantum-number.

in this case $-(I - \frac{1}{2})$, also corresponds to two different values of M_z which are equal and opposite, and each of which at zero field-strength (but not for $H = 0$) coincides with one of the previous two. Now consider any two of these values of M_z which correspond to equal and opposite values of m and coincide with one another at $H = 0$. As H is increased from zero, these two are altered in opposite senses, and one of them actually passes through zero and then reverses at its sign at a certain value of field-strength (m and hence a are negative, and M_z vanishes when $x = a$) while the other is shifted in the opposite sense and never vanishes (m and a are positive).⁹ We shall presently see (page 310) that this behavior is the basis of one of the methods of evaluating I . One more peculiarity of these equations must be stressed: the magnetic moment of the nucleus nowhere appears in them! This becomes evident when the field-strength is put equal to zero and x vanishes, for then the several values of the right-hand member of (16) become simply the projections, upon the field-direction, of the magnetic moment of the extra-nuclear electron-system. Thus we have the paradox that in these experiments the magnetic field gives us information about the nucleus by virtue of the force which it exerts upon the atom, and yet this force is exerted practically upon the electrons alone, and not to any perceptible extent upon the nucleus.

The laws expressed in equation (16) have thus far assisted in three ways in the study of the nucleus:

First, in respect to the experiment which I was describing (page 306) when I began on this detour: as H is decreased from what I called the "customary" magnitude, the $(2I + 1)$ levels constituting each of the there-mentioned groups draw gradually apart—i.e. they differ more and more in respect of the value of the component of the magnetic moment along the field-direction, which is what controls the deflection. The experiment must therefore be performed with field-strengths H which are sufficiently low, much lower than those customarily employed in the Gerlach-Stern experiment or in spectroscopy; and this is one of the distinctive features of the technique of Rabi and his school. Narrowness of the beam is all the more required, since dH/dz must be large enough to produce considerable deflection, and if both its value and the breadth of the beam in the z -direction were large, H could not be small in every part of the beam. The beam must also be made nearly homogeneous in speed, and this is done by a

⁹ Exception must be made for pairs of values of M_z , both members of which correspond to $m = 0$ and vanish at $H = 0$; each member of such a pair departs farther and farther from zero, to equal extents in opposite senses, as H is increased from zero.

clever utilization of the law embodied in (16), for which I must refer to the original papers.—When the experiment was performed on sodium, it was found that the beam is split into eight components instead of merely two, proving the value 4 for the factor $(2I + 1)$ and the value $3/2$ for the quantum-number of the nuclear angular momentum.

Second, suppose an experiment done by measuring the number of atoms which go through the deflecting field entirely undeflected. These are the atoms for which $M_z = 0$, and in ideal conditions there would never be any such atoms, excepting at one of the particular values of field-strength for which the M_z of one (or rather, two at a time) of the levels mentioned on page 309 is passing through zero; in actual conditions one would expect the curve of the number-of-undeflected-atoms *versus* the field-strength to exhibit peaks. On examining equation (16) one may see that there would be one peak for $I = 1$ or $3/2$, two for $I = 2$ or $5/2$, three for $I = 3$ or $7/2$, and so on.¹⁰ The number of peaks by itself thus gives a partly ambiguous indication, but the ambiguity can be resolved by another theorem deducible from (16): if we denote by H_1, H_2, \dots the abscissæ of the consecutive peaks, then all the intervals $(H_i - H_{i-1})$ are equal in any case, but the value of H_1 is equal to the half or to the whole of their common value, according as I is a full integer or a half-integer.—The curve for caesium was found to display three peaks, and the second criterion showed that the value of I is a half-integer, therefore $7/2$.

Third, when M_z is found by measuring the deflected beams in an apparatus where field and field-gradient are accurately known (not a stringent requirement in either of the two previous cases), one may use equation (16) to compute b : thus determining the "hyperfine-structure" separation between two states without an optical measurement! This has been done with both varieties of hydrogen, the heavy isotope and the light, because for these very important atoms the separation in question is far too small to be detected by any optical device: the method of magnetic deflection has proved itself the superior of the long-established arts of spectroscopy, hitherto regarded as the *ne plus ultra* of subtlety and refinement.¹¹ The results of these experiments are mostly quoted for their bearing on the magnetic moment

¹⁰ Not counting the peak at $H = 0$ which (it is obvious) must always appear but has no bearing on the value of I .

¹¹ I should, however, perhaps make exception for the most delicate of these, which is the derivation of hyperfine structure from observations on the resonance radiation produced by polarized light acting on atoms of gases in magnetic fields, and is practiced by Ellett and his school at the University of Iowa. The complexity of the theory forbids a description of the method in this place, but several values of I have been obtained by it.

of the nucleus, that vector quality of which till now next to nothing has been said. Little indeed can be said about it with assurance, but we must consider at least that little.

I recall that the magnetic moment of the extra-nuclear electron-system is very simply ascertained by measuring the magnetic splitting-up of the stationary states, i.e. the energy-differences between the different orientations of an atom in an applied magnetic field, because it enters directly into the formula for those energy-differences; but although the nucleus itself produces a further splitting-up which in its turn is measured, the situation is so much more complicated that these measurements have no interpretable bearing on the value of the nuclear magnetic moment. For protium and for deuterium, the two isotopes of hydrogen, the magnetic moment of the nucleus has been directly measured by a magnetic-deflection method. For all the other kinds of atoms we are obliged to infer it by theory from the measured values of the energy-differences between the states of what I called a cluster, which are alike in respect of I and J and differ in respect of the mutual inclination of these vectors.

The theory can at least be illustrated by a quasi-classical derivation, though the differences between this and the quantum-mechanical method are not slight. One first visualizes the valence-electron as a charged particle running around and around its orbit, equivalent therefore to a steady current running around the orbit and producing a magnetic field at all points within the orbit and in particular at the point occupied by the nucleus; the nuclear magnetic moment is subjected to this field, and when it is shifted from one to another of its permitted orientations a certain amount of work must be done (or received) and constitutes the energy-difference in question. Supposing a circular orbit with radius r and angular momentum p , the argument commences like that of page 290; we have $pe/2\pi mr^2c$ for the strength of the equivalent current, pe/mr^3c for the field-strength which it produces at the centre of the circle where the nucleus is; we conceive the nucleus as having a magnetic moment M parallel to its angular momentum; we assign the quantum-number I to this angular momentum and the quantum-number l to that of the orbital motion of the electron, thus conceiving these as vectors having the magnitudes $\sqrt{I(I+1)}(h/2\pi)$ and $\sqrt{l(l+1)}(h/2\pi)$, which last is what I have been calling p . If we could ignore the spin of the electron, l could be replaced by J , and the torque exerted by the field upon the nucleus would be $M(pe/mr^3c) \sin \theta_{I, J}$. There would be two or more permitted values of $\theta_{I, J}$ corresponding to the various states of the cluster, and we should get the corresponding energy-values U by writing:

$$\begin{aligned}
 U &= M(pe/mr^3c) \cos \theta_{I, J} \\
 &= M(e/mc)(r^{-3})\sqrt{J(J+1)}h/2\pi \cos \theta_{I, J},
 \end{aligned}
 \tag{17}$$

and using expressions based on equation (13) for the cosine. Finally we should form the differences between the right-hand members of these equations, and equate them to the observed energy-differences between the states of the cluster, and solve for M .

Even this formula, when applied to the data, gives values of M of the same order of magnitude as do the more elaborate ones; otherwise there would be no point in quoting it. There is, however, very much to be done to improve it. There is the magnetic field produced at the nucleus by the spin of the electron. There is the alteration required by relativity. There is the task of applying quantum-mechanical rather than quasi-classical reasoning to the postulates. The procedure is strongly supported by the fact that it is copied from the argument which, in the theory of the interaction between the spin and the orbital angular momentum of the valence-electron, leads to a wonderful explanation of the fine-structure of the hydrogen spectrum. It is, however, certainly not perfect, since when applied to different states of a particular kind of atom it is likely to lead to different values of the nuclear magnetic moment, a result which either shows some of the mathematical methods of approximation to be faulty or else is a *reductio ad absurdum* of one or more of the postulates. The problem is in fact one of the great unmastered problems of atomic physics, and some believe that it is wrong to postulate that the nucleus can be regarded, in its interactions with the extra-nuclear electrons, as nothing but a simple magnet attached to a body having mass and charge. I shall therefore say nothing further about it, except for quoting the formula oftenest used in cases such as those of sodium and hydrogen, where the energy-difference b in question (to follow the notation of page 308) is that between the two members of a pair of states for both of which $L = 0$ and $J = S = 1/2$, while for one of them $F = I - J$ and for the other $F = I + J$:

$$b = (8\pi/3) \left(\frac{2I + 1}{I} \right) M(eh/4\pi mc)\Psi^2(0),
 \tag{18}$$

the last symbol standing for the square of the value which the Schrodinger wave-function has at the nucleus, which is known exactly for hydrogen and approximately for other one-valence-electron atoms; the formula is due to Fermi. Applying this formula to the values of b for light and heavy hydrogen which they had ascertained by the magnetic-deflection method (page 310), Rabi Kellogg and Zacharias

got values for the magnetic moments of proton and deuteron¹² which are of the order of one one-thousandth of those associated with electrons. These values are stated (for a reason which may be obvious but will be set forth presently) as 3.25 and as 0.75 times the quantity $(eh/4\pi m_p c)$, where m_p stands for the mass of the proton; the uncertainty is given as 10 percent for the former, about 25 percent for the latter.

There remain the experiments whereby M was directly determined, for protons and for deuterons, from the force exerted by an inhomogeneous magnetic field on the nuclear moment itself. One cannot use the bare proton for such an experiment, since owing to its charge it would suffer, as it flies through the magnetic field, an electrodynamic force and a deflection by comparison with which the others would be trivial. One cannot use the isolated hydrogen atom, since just as in the case of sodium which we considered at such length, the force exerted by the field upon the magnetic moment of its electron would far outweigh that exerted on the nucleus. There remains the hydrogen molecule, which in its normal state has the convenient feature that the spins of its two electrons are oriented anti-parallel (in the loose sense of the term) and cancel one another out, while the angular momenta and the magnetic moments of their orbital motions likewise vanish. This seems to remove all the possible competitors to the nuclear moments, but there arises another which does not occur in individual atoms: the rotation of the molecule-as-a-whole, which has an angular momentum and a magnetic moment. This magnetic moment, however, is of the same order as those of the nuclei, and its contribution to the net magnetic moment of the nuclei can be estimated and subtracted from theirs. As for the nuclei, they may set themselves with their spins either parallel or antiparallel;¹³ in the latter case their magnetic moments cancel one another, and observations on such molecules teach us only about the rotation, knowledge which is useful; in the other case their magnetic moments add, and the data of the experiment yield a value which is double the moment of the individual proton—or of the individual deuteron, according as the molecule is formed of two light atoms or two heavy atoms of hydrogen.

¹² This substitute for the names *deuton* and *dipton*, by which the nucleus of the H^2 atom (deuterium or "heavy hydrogen") has usually been known in America and England respectively, was recommended at a recent meeting of the American Physical Society by Dr. Urey, the discoverer of deuterium.

¹³ This implies that nuclei conform to rules of quantization in direction relative to one another similar to those for electrons. This is true, and is superbly demonstrated by observations on band-spectra and by the chemical separation of the two kinds of hydrogen molecules ("ortho-hydrogen" and "para-hydrogen") here mentioned; but the story is much too long for this place.

The experiments were performed by Stern and his school. They are extremely delicate, owing to the excessively small value of the magnetic moment and hence of the deflection. Indeed, they might have been beyond instead of within the limits of the possible, were it not for two distinctive features of hydrogen: the just-mentioned doubling of the moment due to the parallelism of the spins of two nuclei, and the fact that a beam of slow-moving molecules can be utilized, because hydrogen does not have to be heated in a furnace in order to be vaporized, but will emerge as a molecular beam through a hole in the wall of a chamber no warmer, indeed very much colder, than the temperature of the room itself! For the magnetic moment of the proton, Frisch and Stern give $2.5 (eh/4\pi m_p c)$ with an uncertainty of "at most 10 per cent"; for that of the deuteron, Estermann and Stern say that the value is between 0.5 and 1.0 times $(eh/4\pi m_p c)$.

We now turn to the ensemble of the estimates of nuclear angular momenta and of magnetic moments, and the laws and rules which they seem to obey.

Estimates of I have by now been made for some fifty-five elements; but this is not an adequate statement to make, for the nuclear angular momentum is one of those qualities—as I shall presently stress—which may vary from one isotope to another of a single element, and there are already several cases in which values of I have been reliably assigned to two or more different isotopes. The great majority are derived from optical analysis of hyperfine-structure; four or five from magnetic-deflection experiments; about ten from alternating intensities in band-spectra, most of these last being checked from the hyperfine structure of line-spectra.

The values are of very unequal merit, some being derived independently and concordantly from several different properties of hyperfine structure (pages 301–302), some being further sustained by deductions from band-spectra, while others are guesses based on a few rough observations of intensities or intervals. The unreliability of these last is of a type not to be described by giving a most-probable-value coupled with a probable error. A critical and analytical review of the lot by a neutral expert is badly needed.

The first thing which strikes the eye on viewing a tabulation is the immense preponderance of half-integer values of spin; but this is only one sign of the most important of the rules of nuclear momenta, which is one of the most important of the rules of nature. I recall that the "mass-number" A of an atom is the integer nearest to the value of its mass expressed in terms of one sixteenth the mass of the commonest oxygen atom as unit. The rule, then, is:

Atoms of odd mass-number have half-integer or "odd" spins; while among the atoms of even mass-number, two of the lightest have the full-integer or "even" spin of unity, while all the rest display no hyperfine-structure at all and thus probably have the "even" spin-value zero (though we must never forget the possibility that the lack of observable hyperfine-structure means that the magnetic moment is very small, the clusters of states therefore so compact that the spectroscopist cannot resolve the hyperfine pattern, and the value of I therefore unascertainable).

Although not every kind of atom has yet been studied, the number of cases on which this rule is based is already so considerable that the discovery of an exception would be a sensation of the first order. Among the most striking exemplifications of the rule are those afforded by elements of many isotopes. Mercury, the outstanding example, displays a hyperfine-structure of wondrous complexity, which has been very successfully interpreted by assigning the value $3/2$ of I to its odd isotope of mass-number 201 (Hg^{201}), the value $1/2$ to its odd isotope Hg^{199} , and the value zero to its four principal even isotopes Hg^{198} , Hg^{200} , Hg^{202} and Hg^{204} ; the lines attributed to these isotopes stand to one another in the intensity-ratios deduced from the relative abundances of the isotopes as measured by Aston. Cadmium has two odd isotopes for both of which $I = 1/2$, and several even ones exhibiting no hyperfine structure at all. Several elements have two isotopes, both of odd mass-number; in some of these cases both have the same value of I (e.g. gallium $3/2$, rhenium $5/2$), in other cases they differ (e.g. rubidium $3/2$ and $5/2$). The outstanding and very-certainly-known case of hydrogen is distinguished by the values $1/2$ for the light and 1 for the heavy isotope.

(It may occur to the reader that if, say, four isotopes of mercury display no hyperfine-structure, then everything that I have heretofore said implies that their spectrum-lines and levels ought to coincide absolutely and be indistinguishable. These are, however, slightly separated, owing, it is presumed, to very slight differences in the fields surrounding nuclei of different isotopes even when they all belong to the same element and have the same nuclear charge and indistinguishable moments. The phenomenon, which is known as "isotope shift," is likely to be much studied in the theoretical physics of the near future).

As we go along the list of the atoms of odd mass-number from the lightest to the heaviest, the value $1/2$ for I appears at the start; the value $3/2$ at mass-number 6; $5/2$ at $A = 35$; $7/2$ at $A = 83$; $9/2$ (the highest yet inferred) at $A = 93$. Further observations may change

these numbers somewhat, but it seems fairly assured that the higher values first appear later than the lower. Yet the lower and even the lowest appear repeatedly all through the list; for the five most massive atoms for which I is known, its values are $1/2$ for Tl^{203} , Tl^{205} , and Pb^{207} , $9/2$ for Bi^{209} and $3/2$ for Pa^{231} . There is no sign of a periodic variation; the random fashion in which the display or the lack of detectable hyperfine structure are sprinkled among the elements would be sufficient proof that it is of nuclear origin, even were there no theory.

We recognize thus that I is a quality of nuclei which depends upon nuclear *mass* (not charge!) Moreover, everything connected with the concept of nuclear angular momentum—the behavior of the extranuclear momenta L and S and J of which it is an analogue, the nature of the phenomena which it is contrived to explain, the values of I deduced from these phenomena—all these things imply that I is a quantum-vector compounded out of the individual angular momenta of individual particles composing the nucleus, the quantum-numbers of these individual momenta being, like that of their resultant, integer multiples of $1/2$. The very simplest model would consist, of course, of particles all having identical angular momenta of quantum-number $1/2$.

Now there are two leading schemes for imagining a nucleus—let us say, of atomic number Z and mass-number A —as systems of minuter particles. According to the one, it consists of A protons and $(A - Z)$ negative electrons; according to the other, of Z protons and $(A - Z)$ neutrons.¹⁴ It would be correct to say that the former was the leading scheme until about two years ago, the latter now; but as the facts about nuclear momenta and nuclear magnetic moments have had much to do with bringing on this change of favor, I will not take it for granted in advance.

If we take the nucleus to be a congeries of protons and negative electrons, then we *are* postulating a system of which all the particles are known to have spins of quantum-number $1/2$ —the simplest conceivable system, as I said; and we should still have in reserve the possibility of assigning orbital motions with orbital angular momenta to some or all of the particles, should it ever seem desirable. But if we take the nucleus to be a congeries of protons and neutrons, we are introducing particles of a kind for which the spin is unknown, and must be fixed by assumption. Since we can put $1/2$ for the spin of the neutron, this affords little basis for choice.

¹⁴ The fact that alpha-particles are often mentioned as constituents of nuclei is not in contradiction with this statement, since an alpha-particle is interpreted as 4 protons and 2 electrons by the one scheme, 2 protons and 2 neutrons by the other.

It is, however, an important point—a very important point—that the different schemes imply different numbers of particles for a given nucleus: $(2A - Z)$ by the former, and simply A by the latter. I have spoken of a notable rule contrasting the values of I for even mass-number with those for odd mass-number. Notice now that if we adopt the latter or proton-neutron scheme, the rule becomes: *The quantum-number of the nuclear angular momentum is a half-integer if the number of particles in the nucleus is odd, and a full-integer (if ascertainable at all) if that number is even.* With the proton-electron scheme, this could not be said.

This difference would give an advantage to the latter scheme, even if the model were no more specific. However, the rules for quantizing orientations of vectors of quantum-number $1/2$ require that these shall set themselves either parallel or anti-parallel (in the loose sense) to one another. If I be built up out of such vectors, then necessarily an even number thereof implies a full-integer value and an odd number a half-integer value, and *vice versa*.

This argument for the proton-neutron scheme is therefore strong, though perhaps not so strong as it would be, were not the basis for the test narrowed down by one of the curious empirical rules of the world of atoms: it is found that mostly $(2A - Z)$ is even when A is even, and odd when A is odd. Fortunately there are some exceptions; the famous one is N^{14} , the chief isotope of nitrogen, for which it is certain (from alternating intensities in the band-spectrum) that I is full-integer (unity), whereas $(2A - Z)$ is odd but A is even. This is the only case of its kind, but there are something like ten in which $(2A - Z)$ is even but A is odd, and there is a hyperfine-structure believed to correspond to a half-integer value of I ; this seems especially well established for two isotopes of tin and two of mercury. Another and very powerful argument from alternating intensities in band-spectra, unfortunately too long to be expounded here, supports the belief that the nucleus of N^{14} has an even and not an odd number of constituent particles. On the whole it is pretty likely that any nucleus-model providing an even number of particles for an even mass-number and an odd for an odd will always be preferred to any model not having this feature.

The field is now open for interpreting the observed values of I by compounding proton-spins and neutron-spins (or proton-spins and electron-spins), and trying to find reasons for the resultants which are observed. It seems natural enough to have unity for the deuteron (proton and neutron spins parallel), zero for He^4 and C^{12} and O^{16} (spins cancelling each other two by two); but farther along the list

of atoms there are curiosities enough to keep theorists busy, one guesses, for years. I leave this for the future, and close by speaking briefly of the magnetic moments, which by themselves suffice to show that the task will not be easy.

For the magnetic moments attributable to extra-nuclear electrons (or rather to their projections upon the direction of the angular momentum corresponding), we found by theory and by experiment values which we wrote as $g(e/2mc) \sqrt{n(n+1)} h/2\pi$; here m stands for the mass of the electron; g is 1 for the orbital motion of a single electron, is 2 for the spin of a single electron, and has calculable values not greater than a very few units for the extra-nuclear electron-system as a whole; n is the quantum-number of the associated angular momentum, and is $1/2$ for the spin of a single electron and has various values for the other cases.

If now a proton be a simple particle with a spin $1/2$, one would expect for its magnetic moment a value differing from that of the electron-spin only by the substitution of m_p the proton-mass for m , therefore about 1840 times smaller. The actual value (page 313) is about three times as great as the expected one, so that the analogy with the electron is good enough to predict the order of magnitude, but is not perfect. This is as surprising a discovery as any that has cropped up in the last several years in the field of atomic physics, and shows that even the supposedly elementary particles still have mysteries for us. The value for the deuteron is a good deal smaller, which with the proton-neutron scheme implies that the neutron has a magnetic moment pointing in the opposite direction to that of the proton. The spins of proton and neutron must, however, be parallel, else their resultant could not be unity, which is the value of I for the deuteron. This brings up a new point: I have not yet said whether magnetic moment and angular momentum should be visualized as parallel or anti-parallel, or better, whether the component of the former along the direction of the latter should be taken as positive or negative. The qualities of the deuteron indicate that whichever is the case with the proton, the opposite is the case with the neutron. Actually it is often possible to tell, from features of hyperfine-structure of which I have said little (cf. footnote 5, p. 302), which of the cases is realized for the nucleus as a whole. For the proton and the deuteron these features are unhappily either absent or inaccessible, and the question remains open. For most of the others which have been analyzed, the magnetic moment is said to be positive, meaning that it is of the sign which one expects, considering the nucleus as a whirling positive charge. For a few nuclei, however, it appears to be negative

TABLE OF NUCLEAR SPINS

This tabulation is presented *sous toutes réserves*. The values are of very different degrees of reliability, but it would be foolhardy for a non-specialist to grade them. They are collected from many sources, including tables by H. E. White (*Introduction to Atomic Spectra*, McGraw-Hill, 1934), by H. Schüler (*ZS. f. Physik*, **88**, 323-335, 1934), by G. Beck (unpublished) and by R. F. Bacher (unpublished), and a number of papers which have appeared in the last fourteen months in *Die Naturwissenschaften*, the *Physical Review*, and other journals.

Values obtained by magnetic-deflection methods are marked R; those deduced from alternating intensities in band-spectra are labelled B; those marked H or not marked at all are inferred from optical observations on hyperfine-structure. Two values for a single mass-number signify either data compatible with both, or discordance between two authorities. X signifies that hyperfine-structure has been sought but not found, which may mean that the spin is zero, but on the other hand may mean that the magnetic moment is too small to produce an observable spread of the hyperfine pattern. When a value of zero for spin is deduced from band-spectra, it is not subject to that doubt.

A	Z	Element	I	A	Z	Element	I
1	1	H	1/2 B	119	50	Sn	1/2
2	1	H	1 B	121	51	Sb	5/2
4	2	He	0 B	123	51	Sb	5/2, 7/2
6	3	Li	X	127	53	I	5/2, 9/2
7	3	Li	3/2 B, H, R	129	54	Xe	1/2
9	4	Be	X	131	54	Xe	3/2
12	6	C	0 B	133	55	Cs	7/2 R, H
14	7	N	1 B	136	56	Ba	X
16	8	O	0 B	137	56	Ba	5/2
19	9	F	1/2	138	56	Ba	X
20	10	Ne	X	139	57	La	5/2, 7/2
22	10	Ne	X	141	59	Pr	5/2
23	11	Na	3/2 R, H	144	62	Sa	X
27	13	Al	1/2	147	62	Sa	≠ 0
31	15	P	1/2 B	148	62	Sa	X
32	16	S	X	149	62	Sa	≠ 0
35	17	Cl	5/2 B	150	62	Sa	X
39	19	K	3/2 B, R	151	63	Eu	≠ 0
41	19	K	> 1/2 R	152	62	Sa	≠ 0
40	20	Ca	X	153	63	Eu	≠ 0
45	21	Sc	7/2	159	65	Tb	3/2, ≥ 5/2
51	23	V	≥ 5/2	165	67	Ho	7/2, ≥ 5/2
55	25	Mn	5/2	169	69	Tu	1/2
59	27	Co	7/2	175	71	Lu	5/2, ≥ 3/2
63	29	Cu	3/2	177	72	Hf	1/2, 3/2
65	29	Cu	3/2	178	72	Hf	X
67	30	Zn	3/2	179	72	Hf	1/2, 3/2
69	31	Ga	3/2	180	72	Hf	X
71	31	Ga	3/2	181	73	Ta	7/2
75	33	As	3/2	185	75	Re	5/2
79	35	Br	3/2, 5/2 B	187	75	Re	5/2
81	35	Br	3/2, 5/2 B	197	79	Au	3/2
83	36	Kr	7/2, 9/2	198	80	Hg	X
85	37	Rb	5/2	199	80	Hg	1/2
87	37	Rb	3/2	200	80	Hg	X
87	38	Sr	3/2?	201	80	Hg	3/2
89	39	Y	1/2	202	80	Hg	X
93	41	Cb	9/2	203	81	Tl	1/2
103	45	Rh	X	204	80	Hg	X
110	47	Cd	X	204	82	Pb	X
111	47	Cd	1/2	205	81	Tl	1/2
112	47	Cd	X	206	82	Pb	X
113	47	Cd	1/2	207	82	Pb	1/2
114	47	Cd	X	208	82	Pb	X
116	47	Cd	X	209	83	Bi	9/2
115	49	In	9/2	231	91	Pa	3/2
117	50	Sn	1/2				

TABLE OF NUCLEAR MAGNETIC MOMENTS

The values are expressed in terms of the unit ($eh/4\pi m_p c$) customary in such tables (though for no very good reason) and sometimes called the "nuclear magneton." They are collected from the same sources as those in the Table of Nuclear Spins, and reduced to at most two significant figures; in cases of discrepancy, Schüler's value is the one usually taken. Those marked S are the ones obtained by Stern and his school; those marked R are obtained by Rabi and his school; those marked H or not at all are deduced from optical observations on hyperfine structure. I have heard the uncertainty of these last put at twenty per cent by one expert in the field, but some authorities would set it even higher. For the isotopes of hydrogen, the sign is unknown.

A	Z	Element		A	Z	Element	
1	1	H	2.5 S	93	41	Cb	3.5
			3.2 R	111	48	Cd	- 0.5
2	1	H	0.5-1.0 S	113	48	Cd	- 0.5
			0.77 R	115	49	In	5.3
7	3	Li	3.3 H, R	117	50	Sn	- 0.9
19	9	F	2.4	119	50	Sn	- 0.9
23	11	Na	2.1	121	51	Sb	2.7
27	13	Al	1.9	123	51	Sb	2.1
39	19	K	0.4 H, R	129	54	Xe	- 1
45	21	Sc	3.5	133	55	Cs	2.5
59	27	Co	2.8	137	56	Ba	1.0
63	29	Cu	2.7	197	79	Au	0.2
65	29	Cu	2.7	199	80	Hg	0.5
69	31	Ga	2.1	201	80	Hg	- 0.6
71	31	Ga	2.7	203	81	Tl	1.5
75	33	As	0.9	205	81	Tl	1.5
83	36	Kr	- 1.0	207	82	Pb	0.5
85	37	Rb	1.5	209	83	Bi	3.6
87	37	Rb	3.1				

The ratio of the magnetic moments of two isotopes of a single element may sometimes be calculated from hyperfine-structure with less uncertainty of theory than the value of either moment separately. For such ratios the following values are available:

$\text{Cu}^{63}/\text{Cu}^{65} = 1.00$; $\text{Ga}^{69}/\text{Ga}^{71} = 1.27$; $\text{Rb}^{85}/\text{Rb}^{87} = 2.04$; $\text{Cd}^{111}/\text{Cd}^{113} = 1$;
 $\text{Sn}^{117}/\text{Sn}^{119} = 1$; $\text{Sb}^{121}/\text{Sb}^{123} = 1.36$; $\text{Hg}^{199}/\text{Hg}^{201} = - 1.11$; $\text{Tl}^{203}/\text{Tl}^{205} = 1.02$.

The experiments of Rabi's school and those of Stern's indicate a value of about *four* for the ratio of the moments of proton and deuteron; this is substantiated by L. and A. Farkas (*Nature*, 135, 372; 9 March, 1935) who measure the rates of the reactions whereby ortho-hydrogen and ortho-deuterium transform themselves into the para-forms (and reversely) in the presence of oxygen; from measurements made at three different temperatures they get for the ratio the three values 3.85, 4.03, 4.07, which agree (they say) "within the limits of the experimental error, which is less than five per cent."

(it is perhaps significant that all of these have even atomic numbers and odd mass-numbers).¹⁵ As for its numerical magnitude, I recall that the insufficiencies of the theory render the recorded values—of which there are some twenty-five—subject to much doubt. Such as they are, it is a striking fact that none of them is more than a couple of times as great as that of the proton, and some are a good deal smaller. This injures still more the scheme of constructing nucleus-models out of electrons and protons, for the magnetic moment of a single electron not compensated by some other should of itself make the nuclear moment enormously larger. Yet if we adopt the proton and the neutron as the sole constituents of nuclei for the sake of banishing this difficulty, it will return to haunt us so soon as we attempt to reduce the number of elementary particles by conceiving either the neutron as a proton united with a negative electron, or the proton as a neutron with which a positive electron is combined.

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¹⁵ Two isotopes of Cd ($A = 111$ and 113), two of Sn (117 and 119), one of Hg (201), one of Kr (83), one of Xe (129).