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Cable Crosstalk—Effect of Non-Uniform Current Distribution in the Wires

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When wires are close to each other as they are in cable, the mutual inductance coupling between pairs is not a simple number, constant at all frequencies. Because of the non-uniform and non-symmetrical distribution of current over the cross-sections of the conductors the "effective mutual inductance" is of the form $M = M_a + jM_b$ where both M_a and M_b vary with frequency. This paper discusses the results of certain measurements which have been made of the effective mutual inductance between straight wires and between cable pairs over a wide range of frequencies extending up to a million cycles. This is of interest in connection with crosstalk problems in cable carrier telephone systems.

FOR many years it has been recognized that non-uniform distribution of current over the cross-section of a conductor reduces the efficiency of transmission in either power or communication circuits. With direct current or with alternating current of very low frequency the current is distributed almost uniformly. As the frequency increases, the current distribution becomes more and more non-uniform.

If the two conductors of a circuit are remote from each other the high-frequency current distribution in either conductor is practically symmetrical with respect to its center, the density of the current being lowest in the center of the conductor and highest near the surface of the conductor. If, however, the conductors are close together, the high-frequency current distribution in either conductor is unsymmetrical due to the proximity of the other conductor. This is known as the proximity effect.

It is probably not well known that this proximity effect may have an important bearing on the crosstalk between communication circuits.[1] While the effect is negligible in open-wire circuits, it is quite marked in cable circuits. This paper describes an investigation of the influence

[1] This effect was mentioned in the Carson-Hoyt paper on "Propagation of Periodic Currents Over a System of Parallel Wires," in the *Bell System Technical Journal* of July, 1927.

of the proximity effect on crosstalk between long non-loaded cable circuits which are being studied in connection with the development of high frequency carrier systems suitable for toll telephone cables. More specifically, the paper covers tests made to determine the influence of the proximity effect on the mutual inductance between circuits; data are given both for the case of two isolated non-twisted pairs and for the case of pairs in a quadded 19-gauge cable.

In cable carrier systems it is not practicable to operate like frequency bands in opposite directions on different pairs in the same cable without heavy shields between the pairs. The relatively large level differences that may exist between pairs transmitting in opposite directions would result in excessive crosstalk of the near-end type. Like carrier frequency bands are, therefore, transmitted in the same direction in a cable and the crosstalk between pairs used for carrier systems is of the far-end type.

It has been shown [2] that far-end crosstalk at carrier frequencies between long non-loaded cable pairs can be considerably reduced by the use of simple networks connected between the two pairs at one point in their length. The crosstalk balanced out by such networks is of the "transverse" [3] type. Crosstalk of the interaction type varies in a complicated way with frequency, and cannot, therefore, be annulled by a simple network. For any two similar circuits all the elements of transverse crosstalk, due to the unbalances occurring at various points along the line, arrive at the same time at the far end of the line. The crosstalk currents due to unbalances of the same type such as capacitance unbalances arrive in the same or opposite phase (if the circuits are perfectly smooth). It will be seen, therefore, that a properly designed network connected at one point in the line may be used to practically annul the far-end transverse crosstalk. In order to design the most effective type of network for balancing transverse crosstalk it is necessary to know the manner in which the crosstalk coupling in any elementary length varies with frequency.

The crosstalk coupling between two pairs in an elementary length may be represented by a mutual admittance and a mutual impedance. It can be considered that the voltage between the two wires of the disturbing circuit drives crosstalk currents into the disturbed circuit through the mutual admittance. The currents in the disturbing circuit acting through the mutual impedance also cause crosstalk

[2] As discussed in the Clark-Kendall paper on "Carrier in Cable" in the *Bell System Technical Journal* of July, 1933.

[3] The various types of crosstalk are discussed in the paper on "Open-Wire Crosstalk" by A. G. Chapman in the *Bell System Technical Journal* of January and April, 1934.

currents. The mutual admittance is due almost entirely to capacitive coupling, the leakance ordinarily being negligible in its effect on crosstalk coupling. This capacitive coupling varies but little with frequency and its effect on crosstalk may be balanced out by means of a simple condenser. If the proximity effect were negligible, the mutual impedance would be substantially that of a simple mutual inductance constant with frequency. The crosstalk due to this coupling would, therefore, be balanceable by means of a simple inductance coil. If, however, the proximity effect is not negligible the mutual impedance is due to a complex mutual inductance both of whose components vary considerably with frequency. This is the case in cable circuits and a complex balancing unit must be designed if the complex magnetic coupling is to be accurately simulated.

The mutual impedance, Z_M, between two circuits is by definition the negative ratio of the induced series voltage [4] (e) in the disturbed circuit to the current (I) in the disturbing circuit. Thus,

$$Z_M = -\frac{e}{I}.$$

Since the induced voltage is proportional to the time variation of the magnetic field set up by the disturbing current, it is important to visualize how this field may be altered by changes in the distribution of the current, I, over the cross section of the disturbing conductor. Four types of current distribution will be considered and the effects on Z_M noted.

In order to simplify the following qualitative explanation of the effect of current distribution on mutual impedance it will be assumed that in all cases the disturbed wire is a filament. When the disturbed wire is finite in cross section the effect is generally similar, but more complicated.

Case I—Current Concentrated in a Filamentary Disturbing Wire

In the case of a wire of infinitely small cross section the magnetic field due to a sinusoidal current, I, induces a voltage in another filamentary wire located in this field as expressed by the familiar equation

$$e = -j\omega MI.$$

The mutual impedance is a pure reactance equal to $j\omega M$, where M, the coefficient of mutual inductance, is a pure number and independent of frequency.

[4] This voltage is defined as being the negative of the value of an inserted electromotive force such as to bring the total current in the disturbed circuit to zero.

Case II—Current Uniformly Distributed in a Solid Cylindrical Disturbing Wire

Consider next the case where the total disturbing current is uniformly distributed over the cross section of a solid cylindrical wire. Such a distribution exists exactly with direct current only, but is closely approximated at very low frequencies. Since the magnetic field outside of a conductor carrying a uniformly distributed current is the same as would exist if the total current were concentrated in the center filament, the total induced voltage in a filamentary wire located in this field is again equal to $-j\omega MI$, where M is the same as in the case of two filaments similarly located in space.

Case III—Current Symmetrically Distributed in a Solid Cylindrical Disturbing Wire

The a.-c. distribution in a solid cylindrical wire is not uniform. However, when the wire is at a considerable distance from its return, the current distribution is practically symmetrical about the axis of the wire although its density varies from a minimum value at the center to a maximum value at the surface. Such a distribution is caused by the fact that the counter-electromotive force induced in a filament near the center of the wire due to the current in all of the other filaments is greater than that induced in a filament at the surface. This is the well-known skin effect.

In this case the total current, I, may be considered as distributed in infinitely thin concentric rings in any one of which the current is the same in phase and magnitude at all points. Since the field outside of one such ring is the same as would exist if all of the ring current were concentrated in a filament at the center, the total field due to the sum of the currents in all the concentric rings is the same as would exist if the total current were concentrated at the center of the wire. Thus, the total voltage induced in a filamentary wire by the field set up by a symmetrically distributed current in the disturbing wire is again expressed by $-j\omega MI$, where M is again a pure number as in the case of two filaments.

Case IV—Current Unsymmetrically Distributed in a Solid Cylindrical Disturbing Wire

If a solid wire and its return are placed close together, as in a cable pair, the a.-c. distribution is neither uniform nor symmetrical about the axis of the wire. In this case the magnetic field set up by the current in the return wire contributes to the counter-electromotive force acting in each filament of the other conductor and causes a further redistribution of the current in that conductor over and above that due to the above mentioned skin effect. This additional alteration in current distribution is known as the proximity effect.

The resultant current distribution can no longer be symmetrical about the axis of either wire. The current in the return wire sets up greater back-electromotive forces in the filaments of the other wire which are close to it than in the more remote filaments. These back-electromotive forces tend to act in opposition to those set up by the current in the wire itself since the current in the return wire is opposite in sign. Hence, the proximity of the return wire reduces the counter-electromotive force acting in the filaments closest to it in the other wire more than it does in the filaments farther away. This results in higher current density in the sides of the wires adjacent to each other.

The current distribution due to the combined action of skin and proximity effects is shown for a pair of round copper wires in space in Figs. 1-A and 1-B.[5] The wires are No. 19 A.W.G. and are separated a distance equivalent to that between wires in 19-gauge cable pairs. The current distribution at 56 kilocycles is shown in Fig. 1-A and at 112 kilocycles in Fig. 1-B. It is seen that the tendency at the higher frequencies is for the current to concentrate on the sides of the wires adjacent to each other. With perfect conductors the current would all be on the surface of the wires and for this wire spacing would be distributed as shown in Fig. 1-C.[6] With actual conductors this distribution is approached as the frequency increases toward the highest conceivable wire communication frequency.

In addition to this unsymmetrical distribution of current with respect to magnitude the currents in various filaments in the conductor may be considerably out of phase with the current at the center. This phase shift may be quite unsymmetrical as indicated for three wire diameters in Figs. 2-A and 2-B. While similar phase shifts occur when only skin effect is present, such shifts are symmetrical about the center of the wire so that the currents at all points in a thin concentric ring have the same phase. Figure 2-A shows the phase shift at 56 kilocycles and Fig. 2-B the phase shift at 112 kilocycles. It is seen that the tendency at the higher frequencies is for the currents at different points on the surface to become in phase with each other. At infinite frequency the surface currents would be in phase.

[5] The current distribution and phase change at 56 and 112 kilocycles were computed from formulas given by Harvey L. Curtis in Bureau of Standards Scientific Paper No. 374, entitled, "An Integration Method of Deriving the Alternating Current Resistance and Inductance of Conductors."

[6] This distribution was calculated by Ray S. Hoyt.

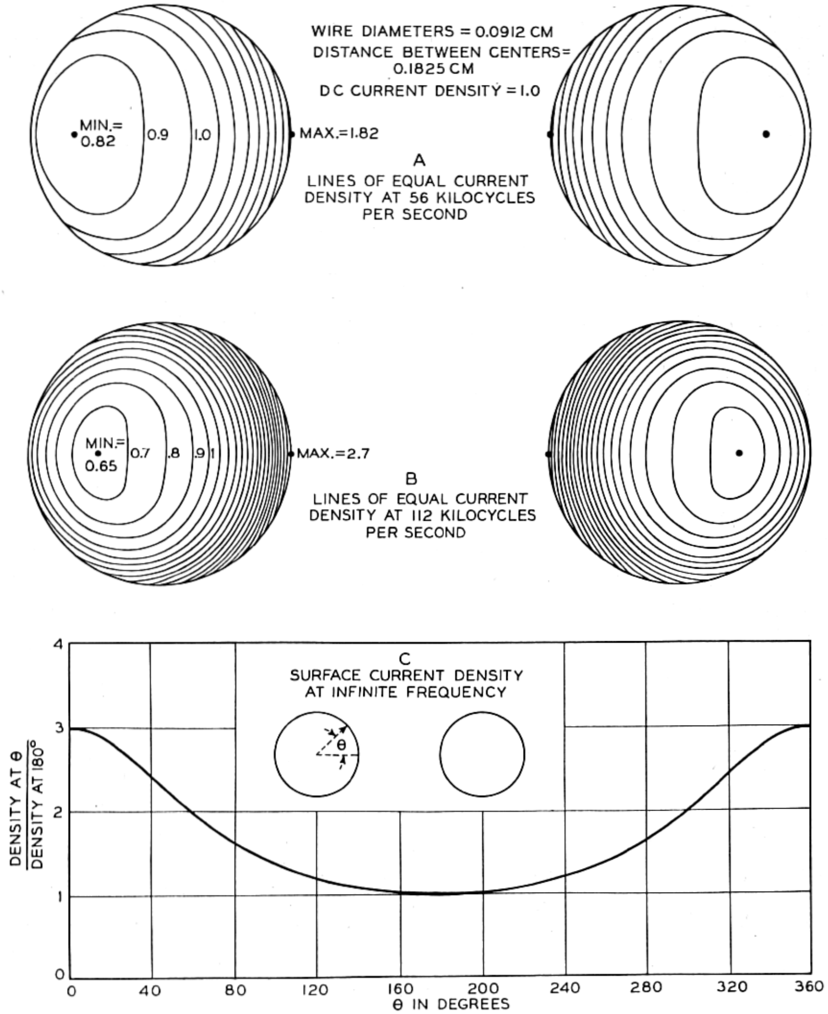

Fig. 1—Current distribution in parallel 19-ga. wires.

If a wire is carrying a total current, I, distributed unsymmetrically in phase and magnitude as in Figs. 1 and 2, the magnetic field surrounding the wire can no longer be the same as would be produced by the same current flowing in the center filament of the wire. The voltage induced in a disturbed filamentary wire located in this field must therefore be different from that induced by the field set up by any of the preceding types of current distribution in the disturbing wire. In each of those cases the induced voltage, e, was exactly in

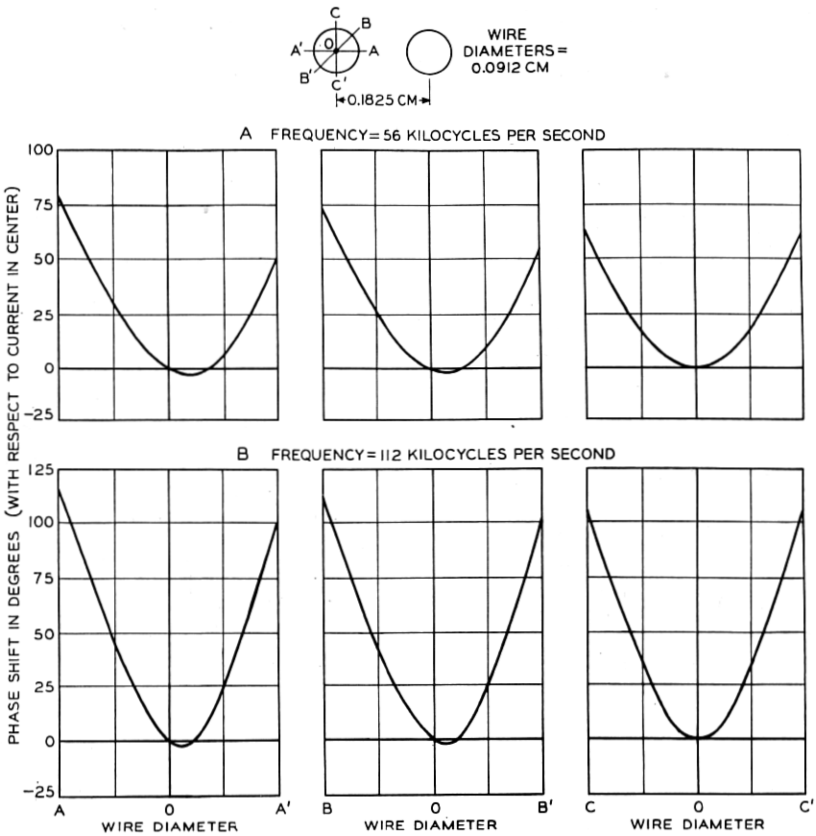

Fig. 2—Phase shift in parallel 19-ga. wires.

phase quadrature with the total disturbing current, I, and the mutual impedance was equal to $j\omega M$, a pure imaginary. For unsymmetrical current distribution in the disturbing conductor, the following discussion shows that the induced voltage can no longer be exactly in phase quadrature with the disturbing current and that the mutual impedance is complex.

The total voltage induced in a disturbed filamentary wire by the total current flowing in a solid wire is the vector sum of the induced voltages due to all of the currents in the various filaments of the disturbing wire. Thus, if $i_1, i_2, i_3, \cdots i_n$ are the vector currents in the various filaments of the disturbing wire and if $m_1, m_2, m_3, \cdots m_n$ are the corresponding coefficients of mutual inductance between each of these filaments and the disturbed filamentary wire, the total induced voltage in the disturbed circuit is

$$e = -j\omega(m_1 i_1 + m_2 i_2 + m_3 i_3 + \cdots m_n i_n).$$

The mutual impedance, Z_M, may therefore be written

$$Z_M = -\frac{e}{I} = \frac{j\omega(m_1 i_1 + m_2 i_2 + m_3 i_3 + \cdots m_n i_n)}{I},$$

where $I = i_1 + i_2 + i_3 + \cdots i_n$. This is a general expression and holds for any type of current distribution in the disturbing conductor.

In the case of symmetrical current distribution (Case III) all filamentary currents having the value i_1 lie in a ring concentric with the center of the wire. The voltage induced in a disturbed filamentary wire due to all the currents in one such ring is the same as if their total value, I_1, was concentrated in the center of the ring. This voltage is equal to $-j\omega M_1 I_1$ where M_1 is the coefficient of mutual inductance between the center filament of the disturbing wire and the disturbed filamentary wire. The same reasoning holds for currents having values i_2, i_3, $\cdots$ i_n and the mutual impedance may be written

$$Z_M = -\frac{e}{I} = \frac{j\omega(M_1 I_1 + M_2 I_2 + M_3 I_3 + \cdots M_n I_n)}{I}.$$

But $M_1 = M_2 = M_3 = \cdots M_n = M$ since all are computed from the center filament in the disturbing wire to the disturbed filamentary wire. Then

$$\begin{aligned} Z_M &= j\omega M \frac{(I_1 + I_2 + I_3 + \cdots I_n)}{I} \\ &= j\omega M \end{aligned}$$

since $I_1 + I_2 + I_3 + \cdots I_n = I$. This is the same expression for Z_M as given in the discussion on symmetrical current distribution.

However, when the current distribution in the disturbing wire is unsymmetrical in phase and magnitude it is impossible to make the above simplifications. In the general expression

$$Z_M = -\frac{e}{I} = \frac{j\omega(m_1 i_1 + m_2 i_2 + m_3 i_3 + \cdots m_n i_n)}{I}$$

there is no correspondingly simple way to separate the m's from the i's in the complex expression in brackets and the phase angle of the expression may be quite different from that of I. Therefore, e cannot be in phase quadrature with respect to I. In order to put the equation for Z_M in the same form as in preceding cases the bracketed expression may be arbitrarily rewritten as

$$m_1 i_1 + m_2 i_2 + m_3 i_3 + \cdots m_n i_n = I(M_a + jM_b).$$

Then,

$$Z_M = j\omega(M_a + jM_b) = -\omega M_b + j\omega M_a,$$

where the mutual inductance is now considered complex and as having two components such that

$$M = M_a + jM_b.$$

The total current in the disturbing circuit acting through the component M_b of the mutual inductance sets up an induced voltage in the disturbed circuit in quadrature with the induced voltage due to M_a, and in the same or opposite phase as the total current in the disturbing circuit. Ordinarily the phase will be opposite and the actual values of M_b will be negative with respect to M_a.

Both M_a and M_b vary with frequency. While the total current in the disturbing wire is assumed constant, the unsymmetrically distributed currents in the various filaments change in relative magnitude and phase as the frequency changes. At very low frequencies the current is distributed nearly uniformly in phase and magnitude over the cross-section of the wire. The mutual inductance between this wire and the disturbed filamentary wire is nearly the same as the d.-c. value since M_a cannot be appreciably changed from the d.-c. value and M_b must be very nearly zero. At very high frequencies the major part of the current flows unsymmetrically on the surface of the disturbing wire but the filamentary surface currents are practically in phase with each other. This results again in a low value of M_b because the total induced voltage will be practically in phase quadrature with the total disturbing current. However, due to the unsymmetrical current distribution, the value of M_a is considerably altered from its d.-c. value. At intermediate frequencies the current distribution lies between these two extremes and produces corresponding values of M_a and M_b. Since M_b is zero for both zero and infinite frequency it is evident that a maximum value must be reached at some intermediate frequency.

As noted at the outset of this discussion, the disturbed circuit is assumed to be a filament. In all practical cases the wires involved are finite in cross section, and the reasoning outlined above must be applied to each filament of the disturbed conductor in order to get the total effect.

Discussion of Test Results

In order to study the variation with frequency of the mutual inductance between cable circuits, measurements were made on various combinations of pairs in a 55-foot length of No. 19 A.W.G. toll cable. To obtain information on the performance of the measuring apparatus, measurements were also made on the calculable case of two non-twisted pairs, six feet in length (approximately). Various separations between the two wires of a pair and three different wire gauges were used to show the change in mutual inductance for various degrees of proximity effect.

The measurements were made with a "crosstalk bridge" or admittance unbalance measuring set which permits the measurement of crosstalk in both phase and magnitude. Although the mutual inductance between two pairs may be determined from either near-end or far-end crosstalk tests, it was found that greater accuracy in M_b could be obtained from far-end tests. The computation of M_b from near-end tests involves two terms of opposite sign and of nearly the same magnitude. Consequently a small error in the reading of the crosstalk bridge may result in a considerably greater error in M_b.

The results of the tests on the six-foot non-twisted pairs are shown in Figs. 3 to 9. The data cover a range of 1 to 1000 kilocycles.

In Figs. 3 and 4 the variation with frequency of M_a and M_b is shown

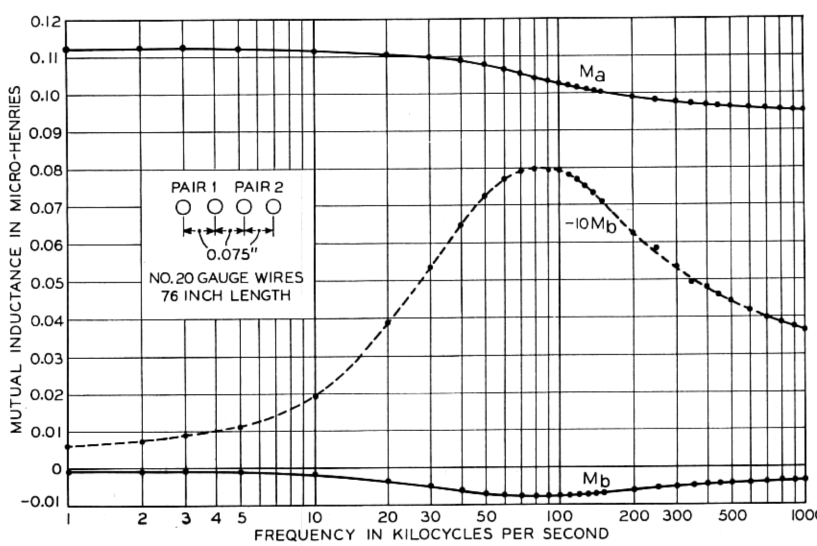

Fig. 3—Mutual inductance between pairs of parallel wires.

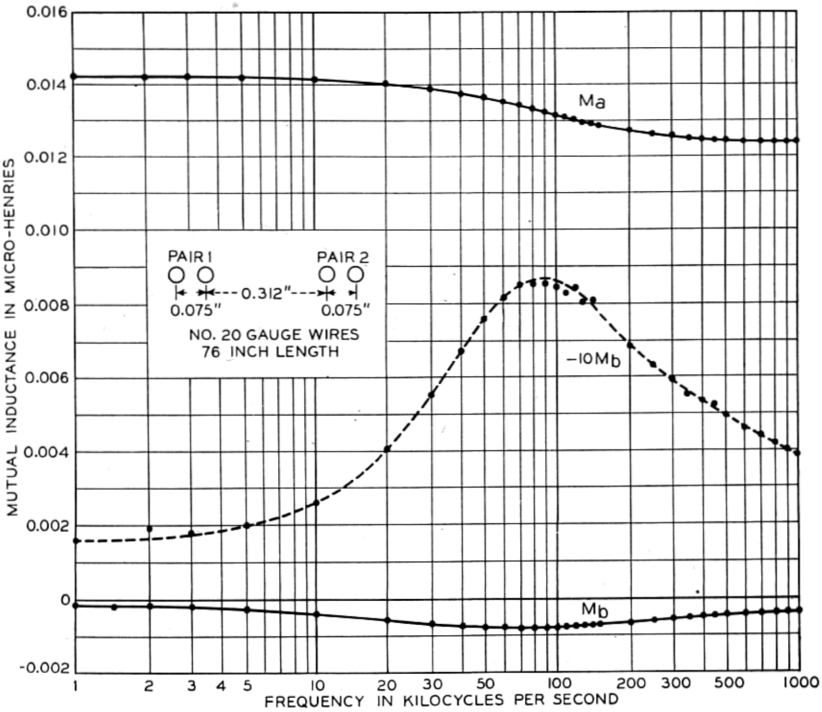

Fig. 4—Mutual inductance between pairs of parallel wires.

for two arrangements of pairs in a horizontal plane. In both cases the axial separation of the two wires of a pair was about 0.075 inch, but in Fig. 3 the axial separation between the nearest wires of the two pairs was 0.075 inch and in Fig. 4 it was 0.312 inch. The wires were No. 20 A.W.G. cotton-covered and were pulled taut to maintain accurate spacing. Two plots are shown for M_b, one actual and the other after multiplying by -10 to show the values more clearly.

The above data are replotted in Fig. 5 to show the frequency variation of M_a and M_b in terms of the values of M_a at one kilocycle. The fact that the frequency characteristics for the two cases are so nearly alike despite the difference in the magnitude of the coupling indicates that the effect depends primarily on the spacing between the wires of a pair and not so much on the relative positions of the pairs.

The proximity effect may be reduced by separating the wires of each pair as shown in Fig. 6. In this case the frequency characteristic of M_a is nearly flat and M_b is so small that it could not be plotted on the same scale as M_a; the curves shown are M_a and $100M_b$. A comparison

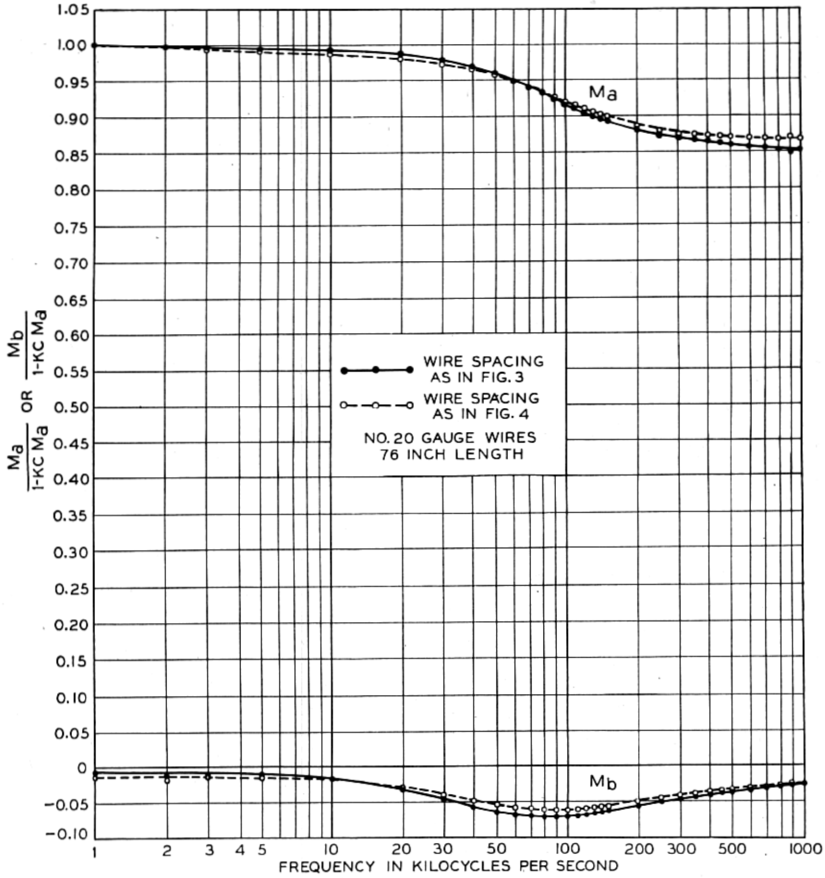

Fig. 5—Mutual inductance between pairs of parallel wires in terms of value for M_a at one kilocycle.

of the curves of M_a and M_b in Figs. 3 and 4 with the curves of Fig. 6 illustrates the relative importance of the proximity effect on magnetic crosstalk in cable pairs and in open-wire pairs. In cable pairs the wires are close together as in Figs. 3 and 4, while in open wire the separation is much greater than that shown in Fig. 6.

The effect of wire gauge on the variation of M_a and M_b is shown in Figs. 7, 8, 9-A and 9-B. Two gauges of wire (No. 10 and No. 18 A.W.G.) were used with centers located at the corners of a 0.14-inch square. The mutual impedance between the vertically adjacent pairs was measured. The corresponding values of M_a and M_b are shown in

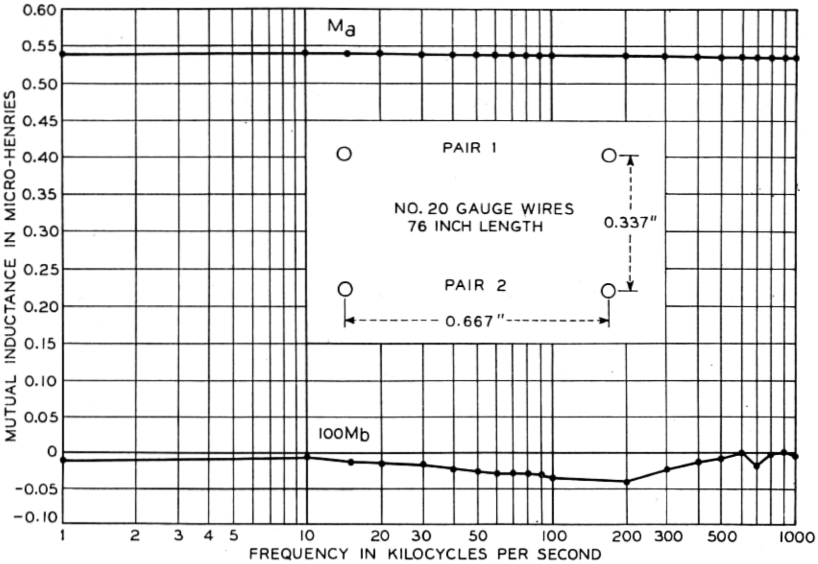

Fig. 6—Mutual inductance between pairs of parallel wires.

Fig. 7 for the 10-gauge wires and in Fig. 8 for the 18-gauge wires.[7] The values of $-10M_b$ are also plotted in Fig. 7 and $-100M_b$ in

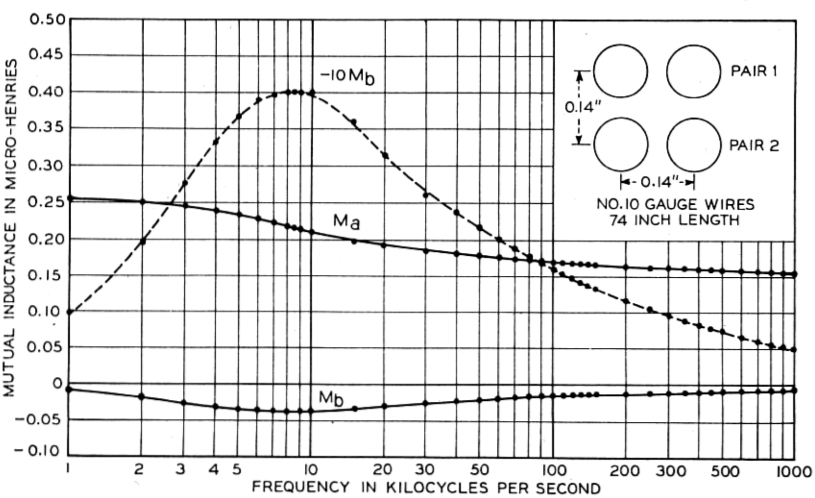

Fig. 7—Mutual inductance between pairs of parallel wires.

[7] These measured values are checked very closely by values calculated by Sallie Pero Mead from the complicated theoretical considerations involved in even the simple case of straight wires.

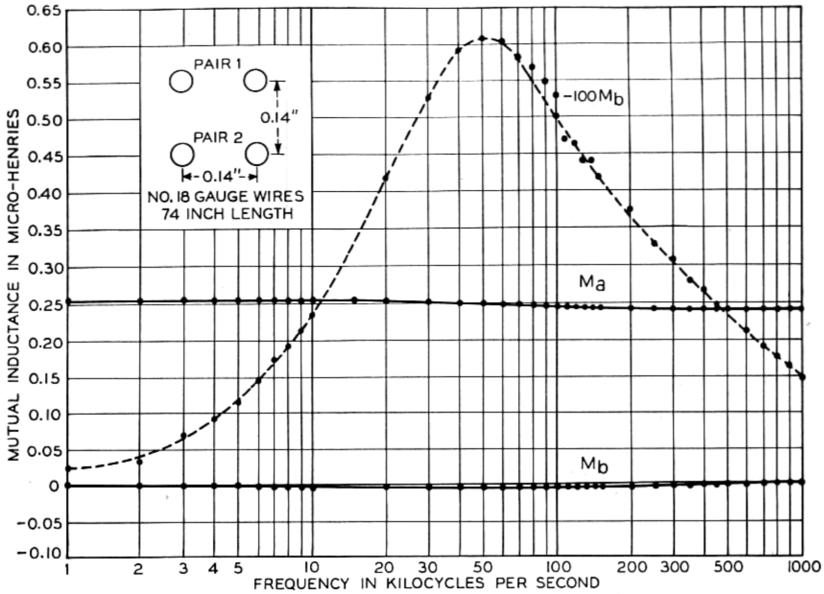

Fig. 8—Mutual inductance between pairs of parallel wires.

Fig. 8 to show the shapes more clearly. A comparison of the frequency characteristics of M_a and M_b for the two wire gauges is shown in Figs. 9-A and 9-B in terms of the one-kilocycle value of M_a for each case. In Fig. 9-A the frequency scale is logarithmic and in Fig. 9-B is linear. As would be expected, the use of smaller wires (No. 18 gauge) decreases the effect of proximity on the magnetic coupling and shifts the frequency at which M_b reaches a maximum value.

The results of tests on the 55-foot length of No. 19 A.W.G. quadded cable are shown on Figs. 10-A and 10-B. The data cover a range of 10 to 480 kilocycles. These figures show the variation with frequency of the average values of M_a and M_b in terms of the ten-kilocycle average value of M_a which is taken as unity. In Fig. 10-A the frequency scale is logarithmic and in Fig. 10-B it is linear. It will be seen that M_b is negative with respect to M_a as in the case of two pairs in space. As in Figs. 3, 4 and 7, curves are given both for M_b and for $-10M_b$, the purpose of the latter curve being to show the shape of M_b more clearly. The value of M_a decreases with frequency, becoming nearly constant above 300 kilocycles at a value 22 per cent less than the value at 10 kilocycles. The component M_b is of negative sign and at 56 kilocycles reaches a maximum value which is 13.4 per cent of M_a at this frequency.

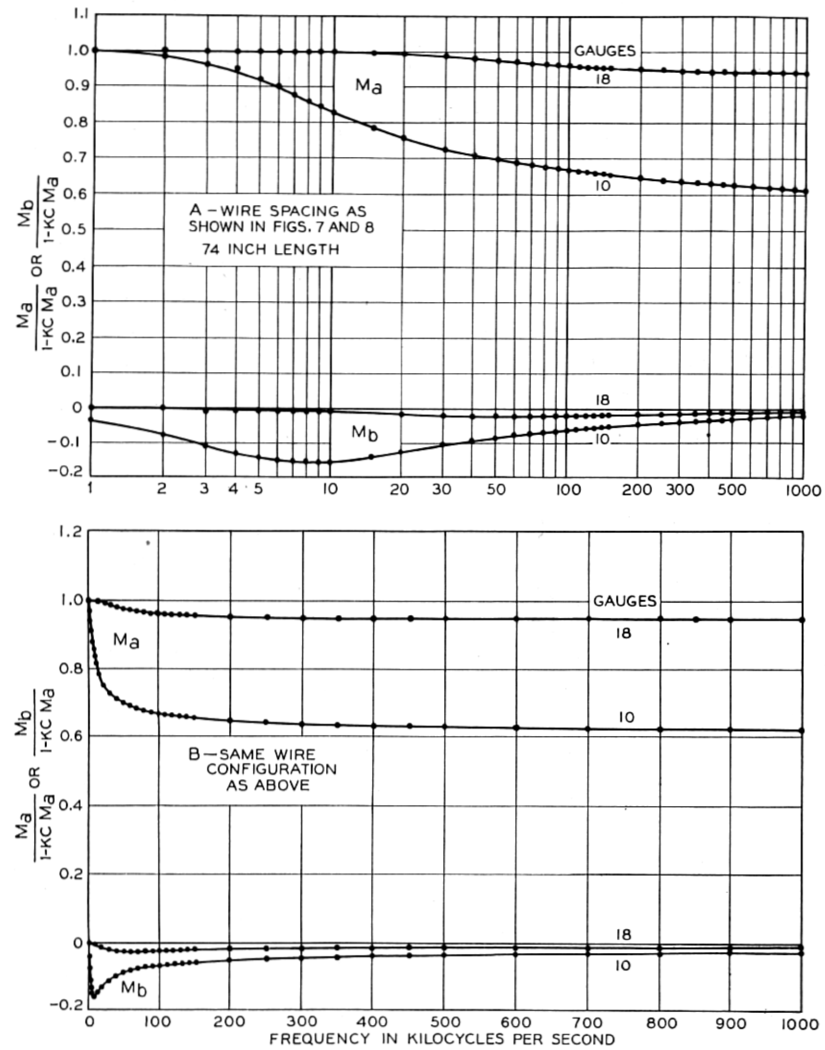

Fig. 9—Mutual inductance between pairs of parallel wires in terms of value for M_a at one kilocycle.

The frequency characteristics of M_a and M_b for individual pair combinations are about the same as shown on Fig. 10, although there are occasional differences such as a positive value of M_b or a change in sign in M_a or M_b at some frequency. Values for pairs in the outside layer did not appear to be much affected by eddy currents in the sheath. As in the case of measurements on parallel wires in space the values of M_a and M_b are very small. For example, between two pairs

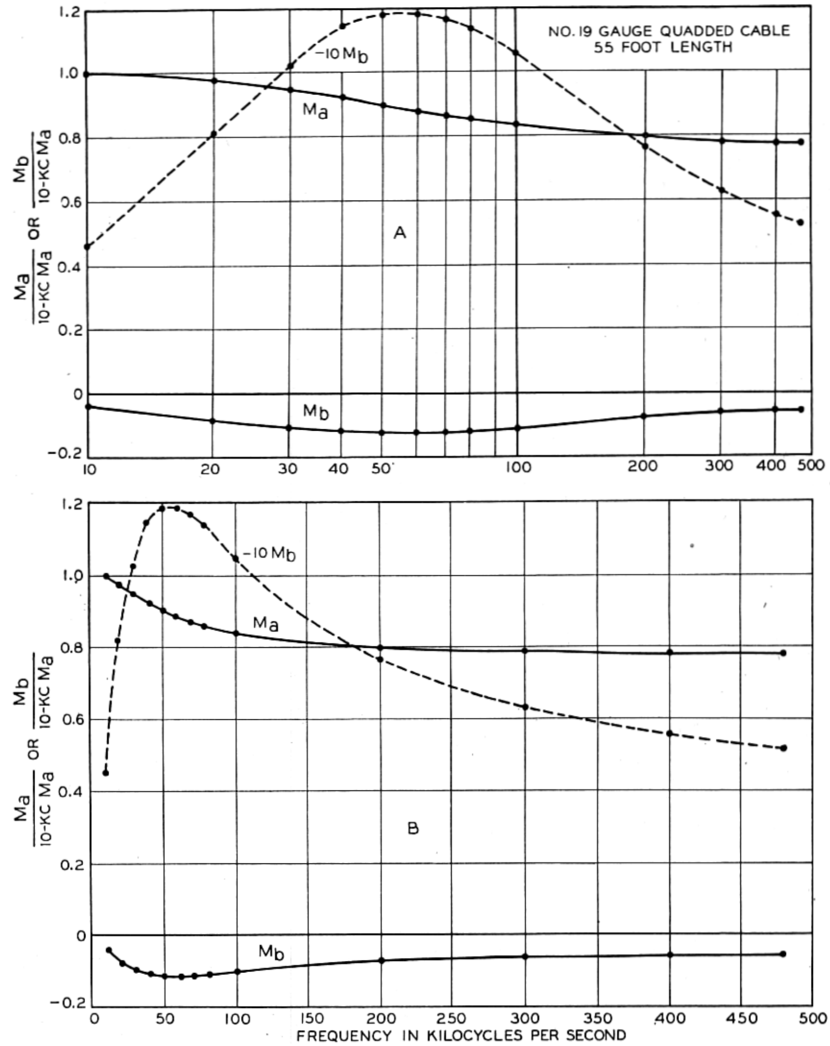

Fig. 10—Mutual inductance between cable pairs in terms of value for M_a at ten kilocycles.

in the same quad the average values at 10 kilocycles are 0.056 and − 0.0030 microhenries. For non-adjacent pairs the values are, of course, much smaller.

Acknowledgment

In this work the writers received much help from Ray S. Hoyt in the matter of general circuit theory. As previously noted, the accuracy of the measurements was established by the work of Sallie Pero Mead in developing a calculation formula for the case of straight wires.