Extraneous Frequencies Generated in Air Carrying Intense Sound Waves *

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The exact equation of the propagation of plane sound waves in air is not linear and consequently harmonics and combination tones are generated. The pressure of these extraneous frequencies in terms of the fundamental pressure, frequency, and distance from the source has been mathematically determined by Rayleigh, Lamb and others. These equations have been applied to an exponential horn.

Measurements of the second harmonic and combination tones have been made at various points within a long tube, and in front of an exponential horn. Measurements, in general, agree with theory, but the absolute values are lower than the calculated values.

RECENT developments in horn type loud speakers[1] for high quality reproduction of intense sounds necessitate a consideration of the more exact equations of wave motion if distortion due to the generation of extraneous frequencies in the air of the horn itself is to be avoided. Similar considerations may be of some importance in connection with the pick-up of intense sounds.

The propagation of waves of finite displacement has interested physicists for more than a century. In 1808 Poisson derived an equation which shows that, in general, a sound wave cannot be propagated without a change in form and consequent generation of additional frequencies. This distortion is caused by the non-linearity of air; that is, if equal positive and negative increments of pressure are impressed on a mass of air the changes in volume of the mass will not be equal; the volume change for the positive pressure will be less than the volume change for the equal negative pressure. An idea of the nature of the distortion can be obtained from the adiabatic curve AB for air as given in the familiar volume pressure indicator diagram (Fig. 1a). The undisturbed pressure and specific volume of air are indicated by point $P_0 V_0$. Any deviation from the tangent through this point causes distortion and consequent generation of extraneous frequencies. The theoretical magnitudes of the waves of extraneous frequencies are obtained from a solution of the exact differential equation of wave propagation in air. The solution shows that the pressure of the second harmonic frequency, which is generated in the air, increases with the frequency and the magnitude of the fundamental

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[1] E. C. Wente and A. L. Thuras, "Loud Speakers and Microphones," *Bell Sys. Tech. Jour.*, 13, 259 (1934).

pressure and also with the distance from the sound source. The solution also gives the magnitudes of the waves of sum and difference frequencies generated when two tones are simultaneously impressed

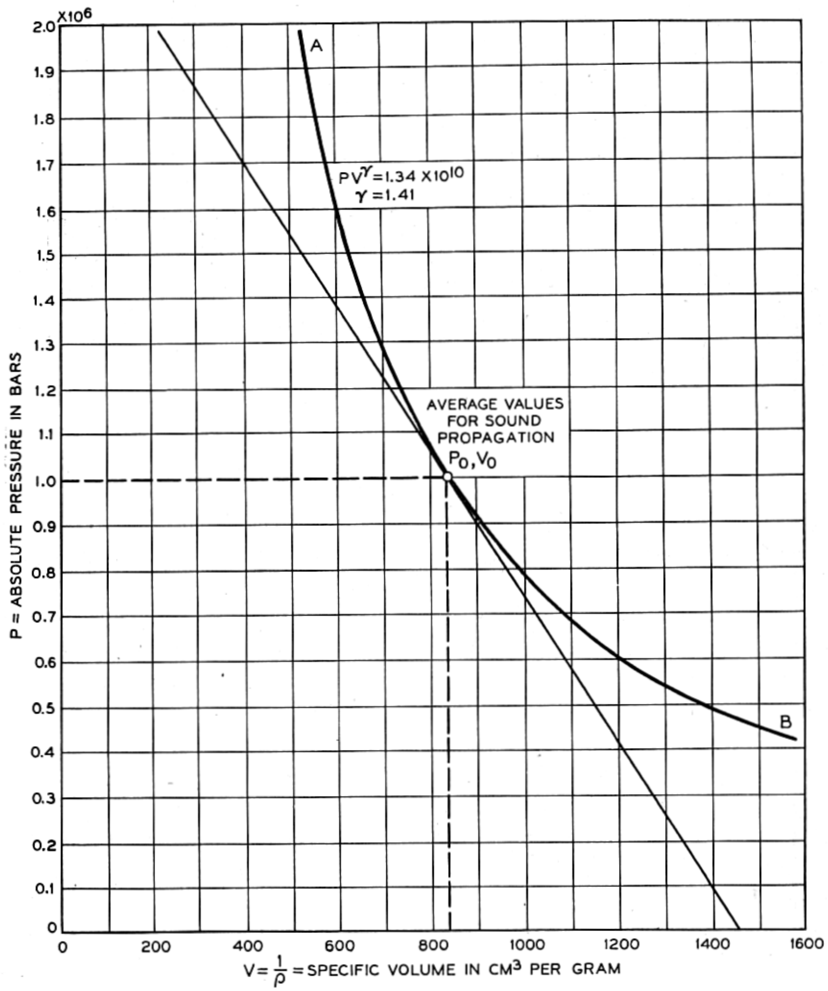

Fig. 1a—Adiabatic curve for air.

on the air; these magnitudes also increase with distance from the source and with the product of the fundamental pressures and, respectively, with the sum and difference frequencies.

Theory of Propagation of Plane Waves of Finite Amplitude

The derivation of the exact differential equation for sound wave propagation in air involves the continuity equation, Newton's force equation and the equation expressing the relation between pressure and specific volume in a gas. Since there may be some question as to the accurate definition of the density and force in the equation of motion a somewhat detailed discussion of this subject will be given.

Following Rayleigh,[2] let y and $y + (\partial y/\partial x)dx$ be the actual distances at time t from the plane $x = 0$ to neighboring layers of air whose undisturbed positions are defined by x and $x + dx$, respectively, Fig. 1b.

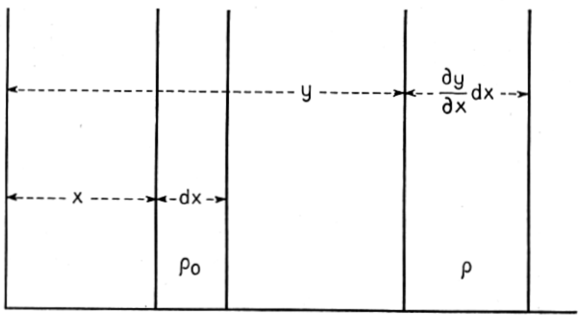

Fig. 1b.

The displacement corresponding to y is thus $\xi = y - x$ and the equation of continuity of the fluid is

$$\rho = \rho_0(\partial y/\partial x)^{-1} = \rho_0(1 + \partial\xi/\partial x)^{-1}, \qquad (1)$$

where ρ and ρ_0 are the densities of the fluid in the disturbed and undisturbed states, respectively. If the effect of viscosity is neglected the exact equation of motion of the element of mass $\rho(\partial y/\partial x)\cdot dx$ is

$$\frac{\partial^2 y}{\partial t^2}\cdot\rho\frac{\partial y}{\partial x}dx = \frac{\partial^2 y}{\partial t^2}\cdot\rho_0 dx = -\frac{\partial p}{\partial y}\cdot\frac{\partial y}{\partial x}dx,$$

or

$$\rho_0(\partial^2\xi/\partial t^2) = -\partial p/\partial x, \qquad (2)$$

p is the pressure at the point y (Fig. 1b) which moves with the air particle, not the pressure at a fixed point. Except for very large displacements these pressures are nearly the same. From equations (1) and (2)

$$\partial^2\xi/\partial t^2 = (dp/d\rho)\cdot(1 + \partial\xi/\partial x)^{-2}\cdot(\partial^2\xi/\partial x^2). \qquad (3)$$

[2] Lord Rayleigh, "Theory of Sound," 2nd Ed., Vol. II, p. 31.

By virtue of equation (1), equation (3) is linear in ξ only if $dp/d\rho = K\rho^{-2}$ or $dp/dv = -K$, where $v = 1/\rho$ = specific volume and K is a constant. This condition is not satisfied during any ordinary variations of state of a gas, but is approximately satisfied when the variations are very small. For isothermal changes we have $pv = p_0 v_0$ and for adiabatic changes:

$$p/p_0 = (v_0/v)^\gamma = (\rho/\rho_0)^\gamma, \tag{4}$$

where γ is the ratio of the specific heats and p_0 is the undisturbed atmospheric pressure. In either case, for very small variations, the pv curve is practically identical with the tangent to the curve, hence dp/dv is practically constant (Fig. 1a).

From equations (1), (3), (4) we obtain the exact equation of adiabatic plane wave motion in a non-viscous fluid:

$$\partial^2\xi/\partial t^2 = c^2(1 + \partial\xi/\partial x)^{-\gamma-1}(\partial^2\xi/\partial x^2), \tag{5}$$

where $c^2 = \gamma p_0/\rho_0$. This equation is given by Rayleigh.[2, 3] Rocard[4] was first to call attention to the generation of harmonics in the air within an exponential horn. His theoretical solution is based on a plane wave equation in which the term $\partial^2\xi/\partial t^2$ was replaced by

$$\frac{\partial^2\xi}{\partial t^2} + \frac{\partial\xi}{\partial t}\cdot\frac{\partial}{\partial x}\left(\frac{\partial\xi}{\partial t}\right). \tag{6}$$

In support of this substitution Rocard cites Riemann's[5] treatment of the problem. However, Riemann's analysis is based on the Eulerian form of the hydrodynamical equations whereas equation (5) is derived from the Lagrangian equations. (For a comparison of these systems of equations see Lamb.[6]) In the Lagrangian notation $\partial\xi/\partial t$ and $\partial^2\xi/\partial t^2$ are the *exact* values of the velocity and acceleration, respectively, of the particle whose displacement from its equilibrium position (x) is ξ. It is to be noted that in equation (2) the term ρ_0, or undisturbed density, does not represent an approximation.

A rigorous solution of (5) for the displacement ξ as an explicit function of x and t has not been obtained. As a first approximation to equation (5) we take

$$\frac{\partial^2\xi}{\partial t^2} = c^2\frac{\partial^2\xi}{\partial x^2} - (\gamma + 1)c^2\frac{\partial\xi}{\partial x}\cdot\frac{\partial^2\xi}{\partial x^2}. \tag{7}$$

[3] Lamb, "Dynamical Theory of Sound," 2nd Ed., p. 182.
[4] Y. Rocard, "Sur la propagation des ondes sonores d'amplitude finie," *Comptes rendus*, **196**, 161 (1933).
[5] Riemann, "Ueber die Fortpflanzung ebener Luftwellen von endlicher Schwingungsweitte," *Göttingen Abhandlungen*, No. 8, 1860.
[6] Lamb, "Hydrodynamics," 6th Ed., Chapter I.

This approximation restricts the dilatation $\partial\xi/\partial x$ to values small compared with unity or the excess pressure to values small compared with γp_0, but the restriction need not be as severe as would be required for the linear approximation:

$$\partial^2\xi/\partial t^2 = c^2\partial^2\xi/\partial x^2.$$

By a method of successive approximations, carried to the second approximation, Lamb [3] derives the solution of (7):

$$\xi = a \cos \omega(t - x/c) + \frac{\gamma + 1}{8}\frac{\omega^2}{c^2} a^2x[1 - \cos 2\omega(t - x/c)] \quad (8)$$

corresponding to a motion $\xi = a \cos \omega t$ imposed on the air at $x = 0$, and assuming complete absence of reflection. By virtue of (4) and (1) we have for the pressure:

$$p = p_0(1 - \gamma\partial\xi/\partial x + \cdots).$$

Neglecting the terms of small amplitude, in the region where $4\pi x$ is large compared with the wave-length λ we have

$$p = P_{dc} + p_1 + p_2, \quad (9)$$

where

$$P_{dc} = p_0 - \gamma p_0\cdot((\gamma + 1)/8)\cdot(\omega^2/c^2)a^2,$$
$$p_1 = - \gamma p_0\cdot(\omega a/c) \sin \omega(t - x/c),$$
$$p_2 = \gamma p_0\cdot((\gamma + 1)/4)\cdot(\omega^3/c^3)\cdot a^2x \sin 2\omega(t - x/c),$$

or

$$p_1 = 2^{\frac{1}{2}}P_1 \cos [\omega(t - x/c) + \pi/2],$$
$$p_2 = 2^{\frac{1}{2}}P_2 \cos [2\omega(t - x/c) - \pi/2], \quad (10)$$

where

$$P_1 = \gamma p_0\omega a/2^{\frac{1}{2}}c, \quad (11)$$

$$P_2 = \frac{\gamma + 1}{2(2)^{\frac{1}{2}}} \cdot \frac{P_1^2}{\gamma p_0} \cdot \frac{\omega x}{c}. \quad (12)$$

P_1 and P_2 are thus the r.m.s. fundamental and second harmonic pressures, respectively.

Lamb [3] also gives a solution of (7) for the case when the forced motion at $x = 0$ is: $\xi = \xi_A \cos \omega_A t + \xi_B \cos \omega_B t$. In addition to the two fundamentals and two second harmonics, the pressure now includes components whose frequencies are, respectively, the sum and difference of the two primary frequencies:

$$p_s = 2^{\frac{1}{2}}P_s \cos [(\omega_A + \omega_B)(t - x/c) - \pi/2],$$
$$p_d = 2^{\frac{1}{2}}P_d \cos [(\omega_A - \omega_B)(t - x/c) + \pi/2],$$

where

$$P_s = \frac{\gamma + 1}{2(2)^{\frac{1}{2}}} \cdot \frac{P_A P_B}{\gamma p_0} \cdot \frac{(\omega_A + \omega_B)x}{c}, \tag{13}$$

$$P_d = \frac{\gamma + 1}{2(2)^{\frac{1}{2}}} \cdot \frac{P_A P_B}{\gamma p_0} \cdot \frac{(\omega_A - \omega_B)x}{c}, \tag{14}$$

and P_A, P_B are the two r.m.s. fundamental pressures.

If we extend Lamb's method of solution of equation (7) to the third approximation and again consider the case $\xi = a \cos \omega t$ at $x = 0$ we find that in the region $kx \gg 1$, the r.m.s. third harmonic pressure is

$$P_3 = \frac{3}{16}\left(\frac{\gamma + 1}{\gamma p_0} \cdot \frac{\omega x}{c}\right)^2 P_1^3. \tag{15}$$

In the case of the greatest r.m.s. fundamental pressure used in the experiments, $P_1 = 8000$ bars at 600 c.p.s., equation (15) indicates that the third harmonic at 400 cm from the source is about 10 db below the second harmonic.

An approximate correction for the effect of attenuation in a tube caused by viscosity and heat conduction can be obtained by assuming that each of the extraneous frequencies and the fundamental is attenuated as if it were the only wave present. Thus the r.m.s. value of the fundamental at any point x is assumed to be

$$P_1 = P_0 e^{-\alpha_1 x} \qquad \text{or} \qquad dP_1/dx = -\alpha_1 P_1,$$

where P_0 is the r.m.s. value of the fundamental at the point $x = 0$ and α_1 is the measured attenuation factor for the fundamental. If P_2 is the r.m.s. value of the second harmonic at the point x and α_2 is the measured attenuation factor for the second harmonic in the absence of the fundamental, we have by using equation (12):

$$dP_2/dx = KP_1^2 - \alpha_2 P_2 = KP_0^2 e^{-2\alpha_1 x} - \alpha_2 P_2, \tag{16}$$

where

$$K = \frac{\gamma + 1}{2(2)^{\frac{1}{2}}} \cdot \frac{1}{\gamma p_0} \cdot \frac{\omega}{c}.$$

When $\alpha_1 = 0$ and $\alpha_2 = 0$, equation (16) is equivalent to (12). The solution of (16) which is consistent with the fact that the second harmonic vanishes at $x = 0$ is

$$P_2 = [KP_0^2/(2\alpha_1 - \alpha_2)][e^{-\alpha_2 x} - e^{-2\alpha_1 x}].$$

Hence

$$\frac{P_2}{P_1} = \frac{\gamma + 1}{2(2)^{\frac{1}{2}}} \cdot \frac{P_0}{\gamma p_0} \cdot \frac{\omega}{c} x \cdot R, \tag{17}$$

where

$$R = \frac{e^{-(\alpha_2 - \alpha_1)x} - e^{-2\alpha_1 x}}{(2\alpha_1 - \alpha_2)x} = 1 - \frac{\alpha_2 x}{2} + \cdots.$$

For the tube used in these experiments α_2 may be taken as $2^{\frac{1}{2}}\alpha_1$ which gives

$$R = 1 - \alpha_1 x/2^{\frac{1}{2}} + \cdots.$$

Similar correction factors were derived for the other extraneous frequencies measured in the experiments.

Measurements of Plane Waves of Finite Amplitude in a Tube

The experimental work consisted of measuring the second harmonic generated along a tube. Measurements were also made of the sum and difference tones when two fundamental frequencies of equal pressure were simultaneously impressed on the air of the tube. For the fundamental pressures and distances used in the experiments, the magnitudes of the other harmonics and higher order sum and difference frequencies were probably small. For high fundamental frequencies and pressures, however, these other tones are important, since they increase more rapidly with frequency and pressure than the second harmonic; for instance, the third harmonic pressure increases as the square of the fundamental frequency and as the third power of the fundamental pressure.

A sinusoidal displacement, uniform over the cross section, was impressed on the air at one end of a long tube. The tube had an inside diameter of 3.8 cm and was 1566 cm long. Measurements were made in the first 705 cm only and the remainder of the tube was used for obtaining a non-reflective termination.

A search transmitter, comprising a small tube of 0.08 cm inside diameter and 7.5 cm long, coupled to a small condenser microphone,[7] was used for the measurements. Attenuation in this search tube was sufficiently high to prevent either overloading the microphone or altering the sound wave propagated within the long tube. The search transmitter was connected to a stage of amplification so operated as to preclude non-linear distortion. This was followed by a band-pass filter which selected the frequency desired in the measurements. The filter was terminated by a measuring circuit consisting of a high-gain amplifier and a vacuum tube voltmeter. A diagram of the arrangement is shown in Fig. 2.

[7] H. C. Harrison and P. B. Flanders, "An Efficient Miniature Condenser Microphone System," *Bell Sys. Tech. Jour.*, **11**, 451 (1932).

To obtain reliable measurements throughout the length of the test tube it was necessary to reduce standing waves to a negligible magnitude. This was accomplished by laying a strip of felt in the last 761 cm of the tube, terminating the end with an acoustic resistance approximately equal to the characteristic impedance of the tube, and carefully sealing up all joints along the tube. The pressure variation in the standing wave was ± 0.3 db, which corresponds to a reflection coefficient of 0.035.

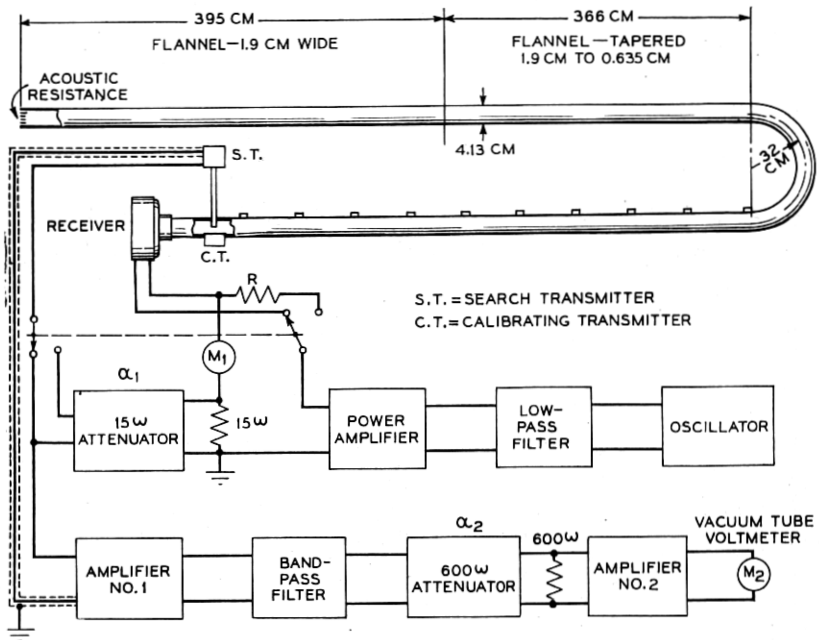

Fig. 2—Apparatus for measuring extraneous frequencies generated in air carrying intense sound waves. *R*, resistance substitute for receiver.

The oscillator current was supplied to the loud speaker through a low-pass filter and the measured harmonic content was found to be 73 db below the fundamental. Pressure measurements in the tube close to the loud speaker indicated that the harmonics generated in the measuring circuit and loud speaker were more than 50 db below the fundamental pressure at 2000 bars.

A calibration of the search transmitter was obtained by comparison with a small condenser microphone whose diaphragm was exposed directly to the sound wave at the same position on the test tube, see Fig. 2. The calibrating microphone (*C.T.*, Fig. 2) had been previously calibrated by a thermophone.[8] The measuring circuit following the search transmitter was calibrated for each frequency measured by introducing the oscillator current into the search transmitter circuit through the attenuator (α_1, Fig. 2).

The ratio of the pressure of the frequency generated along the tube to the fundamental pressure was measured by the attenuator α_2, Fig. 2, at various holes along the tube in which the search transmitter was inserted. If an appreciable fraction of the harmonic generated along the tube is reflected at the end of the tube, the magnitude of the reflected component near the source may be comparable with or larger than the harmonic generated between the source and the point in question. This was found to be the case when several measurements were made near the source over a distance covering a wave-length. The measured variation in the total pressure of the harmonic over this distance was ± 2.5 db whereas the variation for a fundamental of this frequency, as previously stated, was ± 0.3 db. Therefore the measurements close to the receiver may be inaccurate.

Figure 3 shows the measured pressure ratio of the generated second harmonic to the fundamental along the tube and the theoretical curve

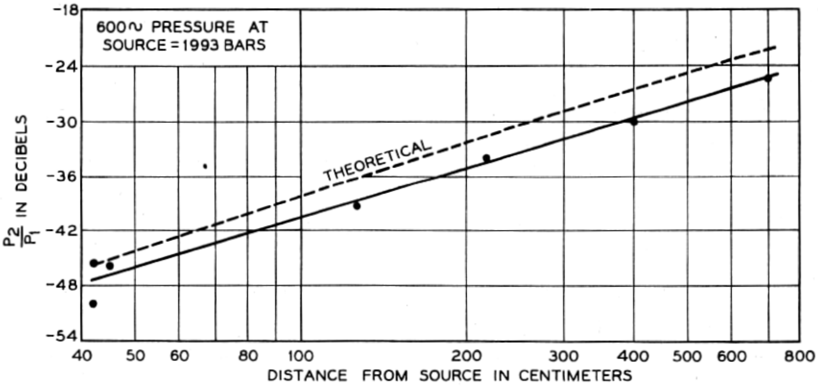

Fig. 3—Magnitude of 2nd harmonic *vs.* distance from source.

calculated from equation (17). Each of the three experimental points plotted at about 45 cm from the receiver is the average pressure ratio for a series of readings taken over a distance of a wave-length along the tube.

The measured and theoretical pressure ratios of the second harmonic to the fundamental are shown as a function of frequency and pressure in Figs. 4 and 5.

[8] L. J. Sivian, "Absolute Calibration of Condenser Transmitter," *Bell Sys. Tech. Jour.*, **10**, Jan. (1931).

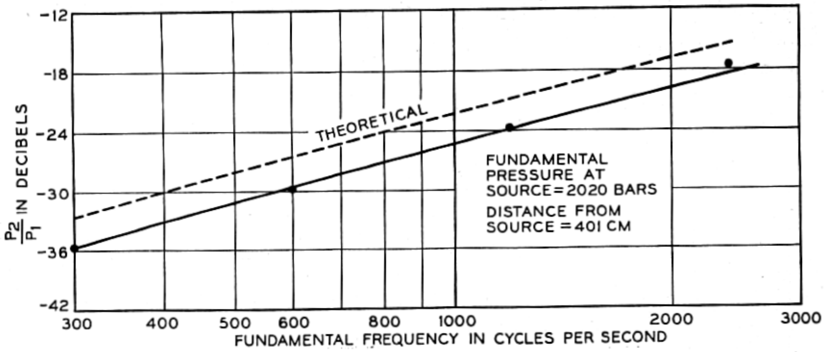

Fig. 4—Variation of 2nd harmonic magnitude with frequency of fundamental.

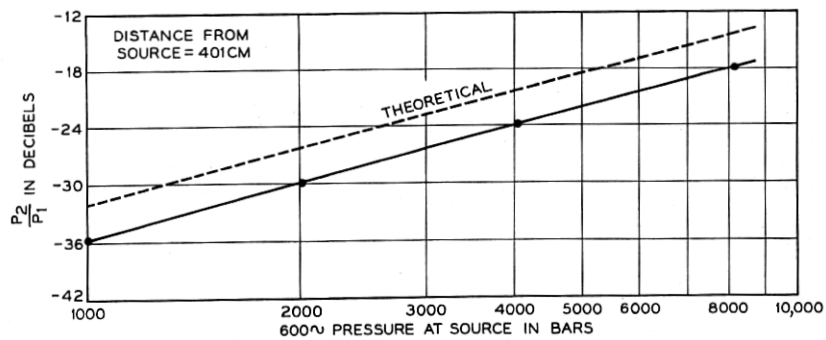

Fig. 5—Variation of 2nd harmonic pressure with fundamental pressure.

Figure 6 shows the magnitude of the first order sum and difference frequencies along the tube when two frequencies of equal pressure are simultaneously impressed on the air in the tube.

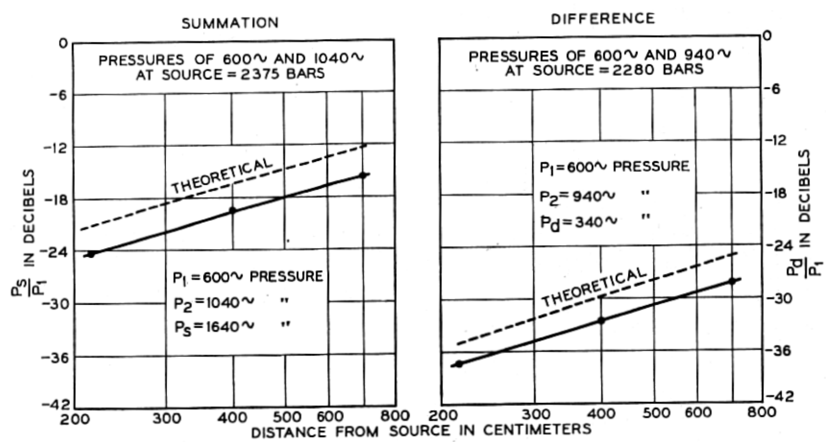

Fig. 6—Magnitude of summation and difference frequencies *vs.* distance from source.

EXPONENTIAL HORN THEORY

The second harmonic generated in any short section of a horn is approximately the same as that generated in a tube of area equal to the mean area of the section of the horn. Therefore, from the tube equation and the expression for the change in pressure due to the divergence of the horn the magnitude of the generated second harmonic pressure at any point along the horn can be obtained.

The r.m.s. value of a small excess pressure in an exponential horn of section $S = S_0 e^{mx}$ is attenuated according to the law

$$P = P_t e^{-mx/2} \qquad \text{or} \qquad dP/dx = -mP/2,$$

where P_t is the r.m.s. pressure in the throat of the horn (at $x = 0$). The index of taper m is equal to $4\pi f_c/c$ where f_c is the cut-off frequency of the horn.

From equation (12), the rate at which the r.m.s. value of the second harmonic increases along a tube is

$$\frac{dP_2}{dx} = \frac{\gamma + 1}{2(2)^{\frac{1}{2}}}\frac{P_1^2}{\gamma p_0} \cdot \frac{\omega}{c} = KP_1^2.$$

If it is assumed that the same expression represents the rate of *generation* of second harmonic along a horn, and that both the fundamental and second harmonic diverge in the same manner, the complete differential equation for P_2 becomes

$$\frac{dP_2}{dx} = KP_1^2 - \frac{m}{2}P_2 = KP_{1t}^2 e^{-mx} - \frac{m}{2}P_2,$$

where P_1 and P_{1t} are, respectively, the r.m.s. fundamental pressures at the point in question (x) and in the throat of the horn. The solution consistent with the condition $P_2 = 0$ at $x = 0$ is

$$P_2 = [KP_{1t}^2/(m/2)][e^{-mx/2} - e^{-mx}]. \tag{18}$$

Since $P_1 = P_{1t} e^{-mx/2}$, the ratio of the second harmonic pressure to the fundamental pressure at any point x in the horn is thus

$$\frac{P_2}{P_1} = KP_{1t} \cdot \frac{1 - e^{-mx/2}}{m/2} = \frac{\gamma + 1}{2\sqrt{2}} \cdot \frac{P_{1t}}{\gamma p_0} \cdot \frac{\omega}{c} x', \qquad (19)\ *$$

* According to this equation the magnitude of the second harmonic pressure generated in the air of a horn is 6 db lower than that given by Rocard's equation previously published in the paper "Loud Speakers and Microphones" (Reference 1), page 264, equation 1.

where

$$x' = (1 - e^{-mx/2})/(m/2). \tag{20}$$

Equations (19) and (20) indicate that the second harmonic at the mouth of an exponential horn of length x is equivalent to the second harmonic at the end of a tube of length x' and of uniform section, equal to the area of the throat of the horn. Thus the second harmonic in the mouth of a horn having an index of taper $m = 0.075$ cm^{-1}, cut-off frequency 200 c.p.s. and length 78 cm is equal to the second harmonic at the end of a straight tube 25 cm long and of the same diameter as the throat of the horn.

Exponential Horn Measurements

Measurements of the output of a horn attached to a moving coil receiver were made in an acoustically damped room. The horn had a throat diameter of 3.8 cm, a length of 78 cm and a cut-off frequency of 200 c.p.s. The diaphragm of the loud speaker was coupled to the throat of the horn through a straight tube 13 cm long. A filtered single frequency tone was impressed on the loud speaker and the sound was picked up by a small microphone in front of the horn. The fundamental and second harmonic voltages from the microphone amplifier were separated by means of a band-pass filter and measured.

The approximate acoustic power at the throat of the horn was calculated from the known efficiency of the loud speaker and the electrical voltage and current supplied.

The measured and calculated ratios of the second harmonic pressure to the fundamental pressure at the mouth of the horn, including the effects of generation of second harmonic in both the horn and the straight tube coupling the receiver and the throat of the horn, are shown in Fig. 7 in terms of the sound output in watts. See equations

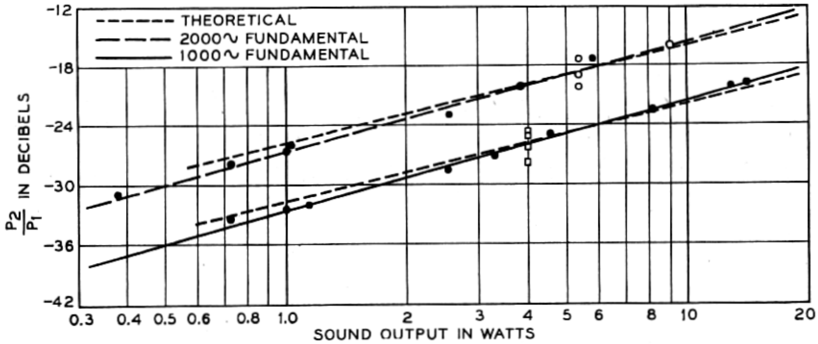

Fig. 7—2nd harmonic generated in an exponential horn *vs.* sound output.

(17) and (19). The attenuation due to viscosity and heat loss in the horn has been neglected.

A number of measurements at various microphone positions in front of the horn, shown by the dotted circles, indicate the difficulty of obtaining accurate results in a room. The average of the measurements at a number of random positions in front of the horn at a constant sound power output and the measurements at a single position for various sound outputs gives the plotted curve which is probably not greatly different from that which would be obtained in open air.

Figure 8 shows the measured and calculated ratios of the second harmonic pressure to the fundamental pressure for various fundamental frequencies.

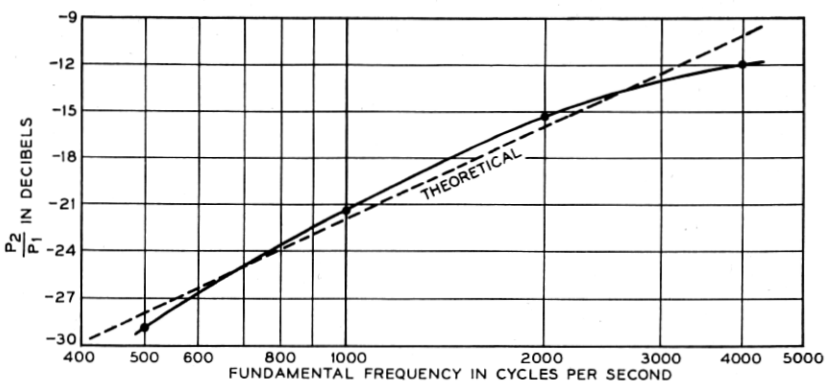

Fig. 8—2nd harmonic generated in an exponential horn *vs.* fundamental frequency (sound output = 10 watts).

Demonstration of Extraneous Frequencies

Almost two hundred years ago Sorge, a German organist, and Tartini, an Italian violinist, discovered independently, apart from all theory, that the union of two loud independent tones produced a difference tone. That this is not entirely a subjective tone produced by the ear but is actually present in the air was demonstrated by others some years later by the use of a tuned resonator. With modern apparatus consisting of power amplifiers, oscillators and tuned electro-mechanical vibrators it is a relatively simple matter to show not only the difference tone but the summation and harmonic tones as well.

An exponential horn was attached to the open end of the 1566 cm tube previously described but with the damping material removed. A moving coil microphone placed in front of the horn picked up the complex tone produced when two equal pure tones of 600 and 940 c.p.s. were impressed on the air. The microphone voltage was amplified and impressed on six torsional vibrators tuned to 340, 600, 940, 1200, 1540 and 1880 c.p.s. Spherical mirrors attached to the vibrators produced on a screen bands of light the amplitudes of which were approximately proportional to the relative pressures of the various frequencies in the complex tone.

For the higher power inputs to the loud speaker used in the experiment, the presence of the sum and difference frequencies and the harmonic frequencies was easily observed. At these power outputs the quality of the sound was very disagreeable and the fundamental tones could hardly be distinguished.

Conclusion

The theoretical and experimental determinations of the extraneous frequency waves generated in the air within a tube are in good agreement as regards the variation in magnitude with frequency, distance from the source and magnitude of the primary tones. The magnitude of the second harmonic is very nearly proportional to the distance from the source, to the fundamental frequency and to the square of the amplitude of the fundamental pressure. When there are two primary tones, the extraneous frequencies generated in the air include, as well as the harmonics of the primary tones, frequencies which are, respectively, the sum and the difference of the primary frequencies and also other higher order tones. The magnitudes of the summation and difference tones are very nearly proportional to the distance from the source, to the product of the magnitudes of the two primary pressures, and in each case, to the frequency of the particular combination tone. As regards the absolute magnitudes of the generated tones in the tube all of the measured values are about 3 db lower than the theoretical values.

Good agreement was obtained also between the experimental and theoretical determinations of the second harmonic generated in the air within an exponential horn, as regards proportionality to the fundamental frequency and power and also as to absolute magnitude. In fact the agreement in absolute magnitude was closer for the horn than for the tube, but not much significance should be attached to this fact, as the horn theory is developed from the theoretical solution for the tube and the horn measurements are known to be less reliable than the tube measurements.