Contemporary Advances in Physics, XXVIII
The Nucleus, Third Part *

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This article deals first with the newer knowledge of alpha-particle emission: that common and striking form of radioactivity, in which massive atom-nuclei disintegrate of themselves, emitting helium nuclei (alpha-particles) and also corpuscles of energy in the form of gamma-rays or high-frequency light. There follows a description of the contemporary picture of the atom-nucleus, in which this appears as a very small region of space containing various charged particles, surrounded by a potential-barrier; and the charged particles within, or those approaching from without, are by the doctrine of quantum mechanics sometimes capable of traversing the barrier even when they do not have sufficient energy to surmount it. The exponential law of radioactivity—to wit, the fact that the choice between disintegration and survival, for any nucleus at any moment, seems to be altogether a matter of pure chance—then appears not as a singularity of nuclei, but as a manifestation of the general principle of quantum mechanics: the principle that the underlying laws of nature are laws of probability. Moreover it is evident that transmutation of nuclei by impinging charged particles, instead of beginning suddenly at a high critical value of the energy of these particles, should increase very gradually and smoothly with increasing energy, and might be observed with energy-values so low as to be incomprehensible otherwise; and this agrees with experience.

Diversity of Energies in Alpha-Particle Emission

On EVERYONE who studied radioactivity some twenty years ago, there was impressed a certain theorem, an attractively simple statement about the energy of alpha-particles: it was asserted that all of these which are emitted by a single radioactive substance come forth from their parent atoms with a single kinetic energy and a single speed. When beams of these corpuscles were defined by slits and deflected by fields for the purpose of measuring charge-to-mass ratio, nothing clearly contradicting this assertion was observed: the velocity-spectrum of the deflected corpuscles appeared to consist of a single line. In studying the progression of alpha-particles across dense matter, it was indeed observed that not all of those proceeding from a single substance had sensibly the same range. It is, however, to be expected that if two particles should start with equal energy into a sheet of (let us say) air or mica, they would usually traverse unequal distances before being stopped; for the stopping of either would

* This is the second and concluding section of "The Nucleus, Third Part," begun in the July, 1934 *Technical Journal.*

"The Nucleus, First Part" was published in the July, 1933 issue of the *Bell Sys. Tech. Jour.* (**12**, pp. 288–330), and "The Nucleus, Second Part" in the January, 1934 issue (**13**, pp. 102–158).

be brought about by its encounters with atoms and the electrons which atoms contain; and there would be statistical variations between the numbers of atoms and electrons which different particles would encounter after plunging into such a sheet. The probable effect of these variations can be computed; and it was shown at an early date that for at least some of the substances emitting alpha-rays—α-emitters—the diversity in ranges of the particles is no broader than should be expected. The curve of distribution-in-range of an α-ray beam often consists of a single peak or hump, and the shape and breadth of the hump are consistent with the assumption that it is due entirely to the "straggling" (the name applied to the statistical variations aforesaid) of particles all possessed initially of a single speed.

The vanishing of this beautiful but too-simple theorem from physics is due to experiments of three types. First, it was found that when all of the well-known α-particles of about 8.6 cm. range from ThC′ were completely intercepted by a stratum of matter of rather more than 8.6-cm. A.E. (air-equivalent [12]), and the detecting apparatus was adjusted to a sensitiveness much greater than would have been tolerable for the main beam, a very few particles—a few millionths of the number in the 8.6-cm. flock—were still coming through. Some of these had ranges as great as 11.5 cm., immensely greater than could be ascribed to straggling. These are the "long-range" alpha-particles, other examples of which have been discovered with RaC′ and (very lately) with AcC′. Next the colossal new magnet at Bellevue near Paris was employed by Rosenblum for deflecting α-particles and observing their velocity-spectrum, and the unprecedented dispersion and resolving-power (to employ optical terms) of this superb apparatus disclosed that for several α-emitters (the list now comprises eight) the spectrum consists of two or several lines instead of only one. The "groups" of alpha-particles to which these lines bear witness lie closer to one another in energy than the aforesaid long-range particles lie to the medium-range ones, wherefore they are often said to constitute the "fine-structure" of the alpha-rays; but it is probable that a more significant basis for distinction lies in the fact that the long-range corpuscles are relatively scanty, while the various lines of a fine-structure system are not so greatly unequal in intensity.

[12] I recall that while the range of an alpha-particle of given speed depends on the density and nature of the substance which it is traversing, the student is usually dispensed from taking account of this by the fact that the investigators nearly always state, not the actual thickness of the actual matter which they used, but the thickness of air at a standard pressure and temperature (usually 760 mm. Hg and 15° C.) which would have the same effect.

Another great magnet of a peculiar and original construction, developed at the Cavendish, was then applied both to spectra displaying fine structure and to the spectrum of RaC′, with notable success; while the technique of determining distribution-in-range curves has been improved to such an extent that it now almost rivals the magnets in its capacity of distinguishing separate groups in an alpha-ray beam. The theorem of the unique speed is therefore like so many another theorem of physics; it was valid so long as the delicacy of the experimental methods was not refined beyond a certain point, its validity ceased as soon as that point was passed.

To enter now into detail:

The *long-range particles* were discovered by observing scintillations, a method of singular delicacy and value, but having great disadvantages: all the observations being ocular, it is wearisome and taxing, not every eye is capable of it, and there is no record left behind except in the observer's memory or notes. Tracks of some of these particles were later photographed in the Wilson chamber, but it is a long research to procure even a few hundred of such photographs, and yet even a few hundred are not sufficiently many for plotting a really good distribution-in-range curve (the disagreements between the early work with scintillations and the subsequent work with Wilson chambers are rather serious). The best available curve is that which Rutherford, Ward and Lewis obtained with the method of the "differential ion-chamber," of which the principle is as follows:

When an alpha-particle (or, for that matter, a proton) traverses a sheet of matter, its ionizing power or ionization per-unit-length-of-path—we may take one mm. as a convenient unit of air-equivalent—varies in a characteristic way with the length of path which the particle has yet to traverse before being stopped completely. The ionization-curve at first is nearly horizontal, then rises to a pretty sharp maximum, then falls rapidly to zero.[13] Suppose now that the particle traverses a pair of shallow ionization-chambers, each containing a gas of which the thickness amounts to not more than a few mm. of air-equivalent (the same for both) and the two separated by a metal wall of negligible air-equivalent. Suppose further that the metal wall is both the negative electrode of the one chamber and the positive electrode of the other, and that it is connected to the electrometer or other detecting device. The charge which is perceived is then the difference between the ionizations in the two chambers. If these are traversed by a particle which is yet far from the end of its range, the difference

[13] The curve for protons is exhibited in Fig. 7 of "The Nucleus, Second Part," p. 124.

will be small and perhaps imperceptible; if by a particle which is approaching its maximum ionizing-power, the difference will be appreciable and of one sign; if by a particle which is coming to the end of its range, the difference will be considerable and of the opposite sign. So the differential chamber and its detecting device (in these experiments, an oscillograph connected through an amplifier, reacting appreciably to the passage of a single particle) are sensitive above all to particles which are nearing the ends of their ranges; and if a small number of such corpuscles be mingled with even a much greater number of faster charged particles—be they alpha-particles, be they protons, be they the fast electrons produced by gamma-rays—this circumstance, which would cripple any other method, will be almost without effect on it.[14] If the readings of the electrometer are plotted against the air-equivalent of the thickness of matter between the source (of alpha-particles) and the chamber, the resulting curve should not be much distorted from the ideal distribution-in-range curve.

The curve obtained in this way for the long-range particles of RaC′ exhibits a notable peak at range 9.0 cm.; to one side thereof a very much lower hump at smaller range (7.8); to the other side a wavy curve with four distinct maxima, which Rutherford and his colleagues deem to be the superposition not of four peaks only, but of seven. I show this portion (Fig. 5) to illustrate the analysis of such a curve for groups. Even the tallest of the peaks just mentioned is a mere molehill compared to the mountain which the principal group of RaC′ —the 6.9-cm. α-particles which were formerly the only ones known— would form if it could be plotted on the same sheet of paper; for the abundances of the 7.8-cm., 9.0-cm. and 6.9-cm. groups stand to one another as 1 : 44 : 2,000,000.

These, however, are not the latest words concerning the α-spectrum of RaC′. The energies of these groups might be deduced from their ranges, but for this purpose it is necessary to use an empirical curve of energy *vs.* range which at the time of the foregoing spectrum-analysis had been extended only up to range 8.6 cm. It was desirable to measure the energies of some of these groups directly, not only for their intrinsic interest but in order to carry onward that empirical energy-*vs.*-range relation. Recourse must therefore be had to a deflection-method.

Now in the usual form of magnet employed in deflection-experiments the field pervades the whole of the space between the solid disc-shaped faces of two pole-pieces. Were the pole-pieces to be so hollowed out

[14] Also a proton near the end of its range can be distinguished from an alpha-particle near the end of *its* range, on account of the difference in maximum ionization-per-unit-length ("The Nucleus, Second Part," pp. 124–125).

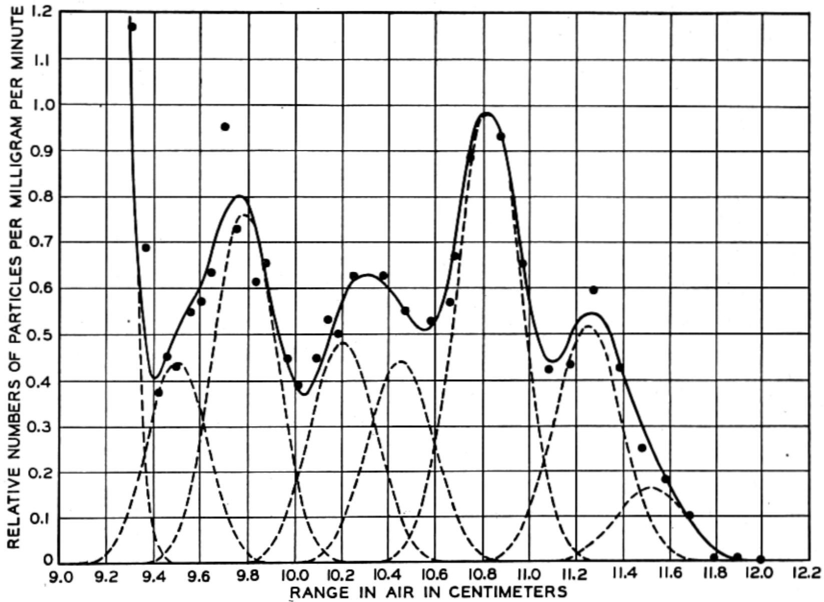

Fig. 5—Distribution-in-range of the long-range alpha-particles proceeding from RaC′, determined with the differential ionization-chamber (Rutherford Ward & Lewis; *Proc. Roy. Soc.*).

that their disc-shaped faces were reduced to narrow circular rings, there would be a great economy in magnetizable metal and a great reduction of weight and volume of the apparatus, as well as other advantages. This need not impair the availability of the magnet for analyzing a beam of α-particles, provided that the α-emitter can be located in the narrow annular space between the faces of the rings, and provided that the magnetization of the metal can be varied sufficiently widely. For then, for each group of α-particles there will be a certain value of the field-strength, whereby those particles which start out in directions nearly perpendicular to the field and tangent to the rings will be swept around in circular paths which are confined within that narrow annular space where alone the field exists. Somewhere in that space the detector should be placed; and the curve of its reading *vs.* field-strength H should show a peak for every group, and from the abscissa of the peak and the radius of the rings the energy of the group may readily be computed.

Such a magnet was built after Cockcroft's design at the Cavendish; the radius of its rings is 40 cm., they are 5 cm. broad and 1 cm. apart (these figures are the dimensions of the annular space), and the field-strength was adjustable up to 10,000 (later 12,000) gauss which was sufficient for α-particles of energy up to and even beyond 10.6 MEV (millions of electron-volts) and range up to and even beyond 11.5 cm. Figure 6 exhibits the outward aspect, Fig. 7 the cross-section of this device (one sees how the armature is fully contained within the rings). The annular space and everything within it was evacuated (being walled in by the ring B seen in the figures); the detector—a simple ionization-chamber connected through a linear amplifier to an oscillograph—was set 180° around the annulus from the source. This device in the hands of Rutherford, Lewis and Bowden proved itself able to furnish even a better spectrum than the scheme of the differential ionization-chamber; all of the peaks indicated by the former curve were clearly separated, a hump which had suggested two groups was resolved into three maxima, and an extra group was discovered—*twelve* altogether! (Incidentally, the empirical energy-*vs.*-range curve of α-particles had previously been extended with the same device by Wynn-Williams and the rest, to energy = 10.6 MEV and range = 11.6 cm.)

The long-range spectrum of RaC′ is thus of no mean complexity. There will be occasion later for quoting its actual energy-values. Of the long-range spectrum of ThC′ there is relatively little to be said; evidently it has not been studied so intensively as the other, but it seems to be comparatively simple, for only two groups have been recognized. One of the groups of ThC′ has about the same range as the highest group of RaC′, so that between them they comprise the most energetic subatomic particles ever yet discovered (about 10.6 MEV) apart from those of the cosmic rays and those resulting from certain artificial transmutations. As for AcC′, it is one of the constituents of the mixture of radioactive bodies known as actinium active deposit, from which α-particles of a range of about 10 cm. have lately been observed in the Institut du Radium.

Thus far I have written as though RaC′ and ThC′ were isolable substances, of which one may obtain pure samples and analyze at leisure the α-rays thereof. The truth, however, is far otherwise; for the difficulties of making one radioactive substance practically free from others, serious in most cases, are utterly insuperable in these. Both RaC′ and ThC′ are so very ephemeral (their half-lives are too small to measure, and are guessed from the Geiger-Nuttall relation as 10^{-6} and 10^{-11} second respectively) that they can never be dissevered from their mother-elements RaC and ThC which are also α-emitters. Sometimes one finds the long-range particles designated as belonging to RaC or ThC, and indeed I have nowhere found stated any compelling reason for attributing them to the C′-elements rather than the C-

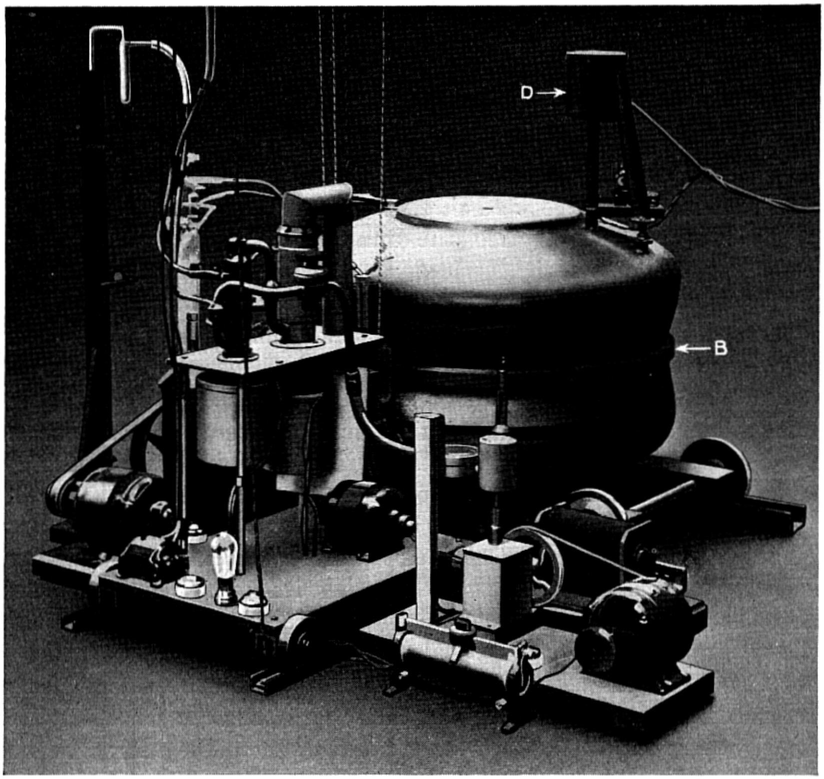

Fig. 6—Annular magnet employed for analysis of alpha-ray spectra. (After Rutherford, Wynn-Williams, Lewis & Bowden; *Proc. Roy. Soc.*).

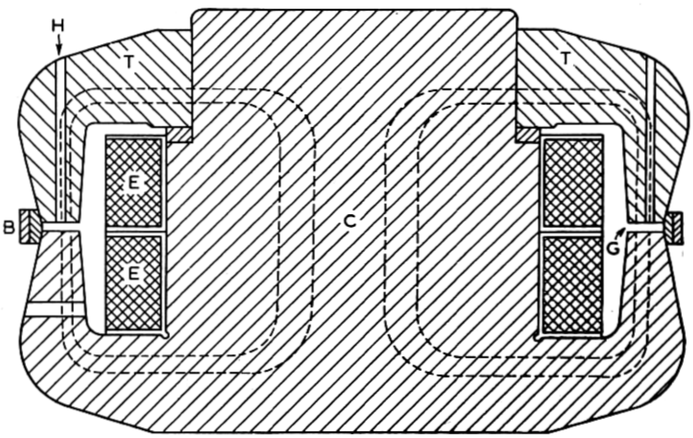

Fig. 7—Cross-section of the annular magnet used for analysis of alpha-ray spectra.

elements, apart from what is known about the correlated gamma-rays (see footnote 21).

Fine-structure was discovered, as I said before, with the great magnet of Bellevue. This has solid pole-pieces (75 cm. in diameter!) instead of rings; it is not necessary to adjust the field-strength step by step so as to bring group after group to a narrow detector; all the groups

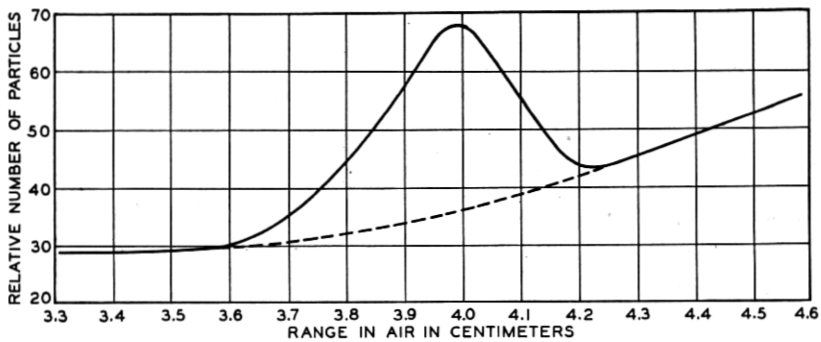

Fig. 8—Alpha-ray spectrum of RaC; peak observed with differential ionization-chamber, never before detected because of immensely greater number of particles in RaC′ peak just off the diagram to the right; asymmetry indicating fine-structure. (Rutherford, Ward & Wynn-Williams; *Proc. Roy. Soc.*).

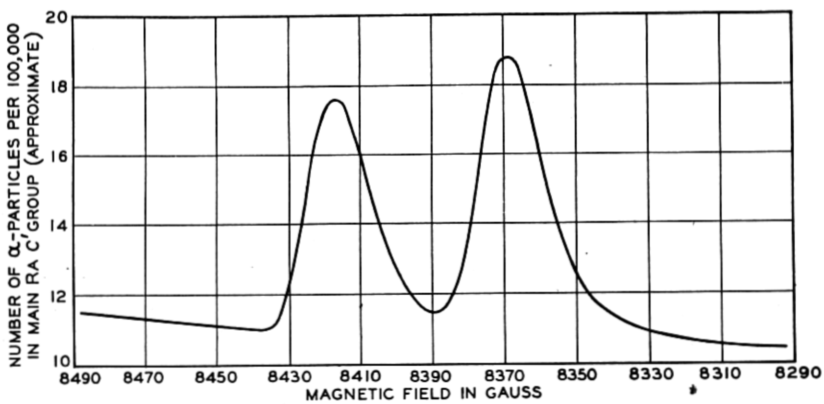

Fig. 9—Fine-structure of alpha-ray spectrum of RaC; as asymmetric peak of Fig. 8 resolved into two nearly equal peaks by annular magnet. (Rutherford, Wynn-Williams, Lewis & Bowden; *Proc. Roy. Soc.*).

of various speeds are deviated simultaneously in circular arcs each of its own particular radius, and simultaneously fall upon a photographic plate, producing what looks precisely like a line-spectrum in optics (Figs. 10, 11). The example in Fig. 10 relates to ThC, the earliest to be analyzed; four lines only are visible upon the reproduction, but some plates after long exposure have shown as many as six.* The fine-structure of AcC consists of a pair of lines, which were detected as peaks in the distribution-in-range curve obtained at the Cavendish with a differential ionization-chamber. Instead of showing this curve I have chosen the corresponding curve for RaC, albeit it shows only a single hump (Fig. 8).[15] The unsymmetrical shape of this hump, how-

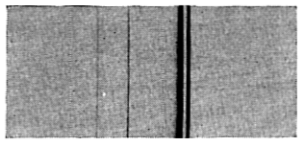

Fig. 10—Fine-structure of alpha-ray spectrum of ThC (not completely brought out in picture) obtained with Bellevue magnet. (S. Rosenblum.)

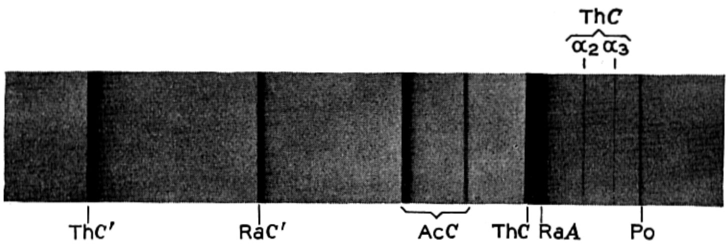

Fig. 11—Alpha-ray spectra of several elements (those of Po and AcC shifted slightly to the right with respect to the rest). (S. Rosenblum, *Origine des rayons gamma*, Hermann & Cie.).

ever, implies that it really consists of a pair of overlapping peaks; and so it does; for when the Cavendish magnet was applied to an α-ray beam from this element, the curve of detector-reading *vs.* field-strength displayed two equal peaks quite sharply separate (Fig. 9).

According to Rosenblum's latest census (February 2, 1934) there are now eight known examples of fine-structure: from the radium series, Ra (two groups), RaC (2); from the thorium series, RdTh (2), ThC (6); from the actinium series, RdAc (no fewer than eleven groups, the richest case of all!), AcX (3), An (3), AcC (2). According to Lewis and Wynn-Williams, there are (or were, in the spring of 1932)

* I am indebted to Dr. Rosenblum for a print from which Fig. 10 was made.

[15] This curve was the first to disclose the α-rays of RaC, previously known only by inference (though it was very compelling inference). Being of somewhat lesser range than the much more numerous (to be precise, 3000 times as numerous) α-particles emanating from the RaC′ atoms with which RaC is always inevitably mingled, they were completely hidden from observation by any method known before the use of the differential chamber and the powerful magnet.

at least five cases in which the distribution-in-range curve obtained with the differential chamber shows a single symmetrical peak suggesting only one group: from the radium series, Rn and RaA; from the thorium series, Tn and ThA; from the actinium series, AcA. Altogether there are twenty-three [16] known alpha-emitters, so that nearly half of the total remain to be investigated to this end. It may be significant that out of the four known alpha-emitters having odd atomic number, the high proportion of three at least is known to display fine-structure (the fourth, Pa, being as yet uninvestigated).

Interrelations of Alpha-Ray Spectra and Gamma-Ray Spectra

Evidently, if two atoms of the same radioactive substance were to emit alpha-particles of different speeds, there would be three obvious possibilities. The resultant nuclei might be different: in this case we should expect (though not with certainty) that they would be the starting-points of different radioactive series, and we should speak of "branching." The initial nuclei might have been different, in which case it would have been improper to speak of them as belonging to the same substance. Finally one at least of the two atoms might also emit gamma-ray photons, of energies complementary to those of the alpha-particles, in such a way that the total amount of energy released by the one atom would be the same as the total amount released by the other.

The first of these possibilities is not to be excluded *a priori* (since there are known cases of branching, though in them the alternative is between emission of an alpha-particle and emission of an electron) and neither is the second. The third, however, is the most agreeable, since if realized it allows us to believe that in the transformation of radium (to take one example) every radium nucleus is like every other before its change begins and every resulting (radon) nucleus is like every other after its change is over. Now alpha-ray emission and gamma-ray emission often occur together, which suggests that often the third possibility is the one which is realized; but this cannot be proved without measuring the energies or the wave-lengths of the gamma-rays.

The simplest cases are those in which the alpha-ray spectrum consists of two lines only. Here and always, there is an inconvenient complication at the start: when an alpha-particle is emitted, the residual nucleus recoils, and it is the sum of the kinetic energies of

[16] Not including Sm and other elements of atomic number lower than 81. The rest are depicted (together with the beta-ray emitters of atomic numbers 81 and greater) in Fig. 21.

the two (not that of the alpha-particle alone!) which must be taken into account.[17] Denote by U_1 and U_2 the values of this sum for the faster and for the slower alpha-particles. Does the gamma-ray spectrum then consist of a single line of which the photon-energy $h\nu$ is equal to $(U_1 - U_2)$?

In the case of AcC, the difference $(U_1 - U_2)$ is 0.35 or 0.36 MEV. There is an intense gamma-ray line proceeding from actinium active deposit (comprising AcC), and the energy of its photons is concordant. In the case of RaC, the difference $(U_1 - U_2)$ is only 0.04 MEV, and the search for so relatively soft a radiation of photons is difficult. In the case of Ra the conditions are more favorable, and here the history is worth retelling. Long before the earliest analysis of alpha-ray spectra, radium was known to emit feeble gamma-rays of photon-energy about 0.19 MEV. An estimate of their intensity was made in 1932 by Stahel; he concluded that the photons are less than one-tenth as numerous as the alpha-particles already known. Search was thereupon made by Rosenblum for fine-structure in the alpha-ray spectrum of radium. Two lines appeared on the plate after five minutes' exposure: they were due to groups proceeding one from radium and the other from its daughter-element radon. On plates exposed for hours there appeared yet another line. The values of U_1 and U_2 being computed for this and for the stronger radium group, the difference was found to be close to 0.185 MEV, with a sufficient latitude to be concordant with the estimate for the photons.

As the number of alpha-ray lines increases beyond two, the prospects rapidly become formidable; for a spectrum of n such lines suggests n possible states of the residual nucleus, and every one of these might "combine" (in the technical sense of the word) with every one below it in the energy-scale, making a total of $n(n - 1)/2$ gamma-ray lines to be expected. Even so, anyone acquainted only with optical spectra might think it no difficult matter to photograph (say) the gamma-ray spectrum of ThC, and see whether it consists in just 15 lines in just the right places to correspond with the six alpha-particle groups. But one does not photograph gamma-ray spectra—one photographs the beta-ray spectra of the electrons ejected by the gamma-rays from atoms, and tries to deduce the photon-energies $h\nu$ from the electron-energies.[18] The atoms may be those of the radioactive substance

[17] By multiplying the kinetic energy of the alpha-particle by the factor $(1 + m/M)$, where m stands for the mass of the alpha-particle and M for that of the recoiling nucleus. This point was overlooked by a number of people before it was noticed by Feather.

[18] I have dealt with this procedure at length in the article "Radioactivity," No. XII of this series (*Bell Sys. Tech. Jour.*, **6**, 55–99, 1927).

itself (either the very ones which are emitting the gamma-rays, in which case the rays are said to undergo "internal conversion," or their neighbours) or they may be those of other elements mixed with the radioactive substances, or those of nearby solids or gases on which the gamma-rays fall. Each gamma-ray line is responsible for several different beta-ray lines, a circumstance which makes the analysis more difficult at the beginning though it makes the inference more reliable in the end. There may be gamma-rays having nothing to do with alpha-particle emission, and there may be gamma-rays from several different radioactive substances inextricably mixed up together, so that the problem of analyzing the spectrum of one transformation is preceded (or, more truly, accompanied) by that of distinguishing it from the intermingled spectra of others.[19] The experimental errors in the estimates of U-values and $h\nu$-values may be so large that apparent agreements are actually unreliable. Altogether, the comparison of a rich alpha-ray spectrum with a rich gamma-ray spectrum is an exceedingly intricate business, the outcome of which is not to be summarized in a few sentences. To give a mere notion of the sort of conclusion which is reached, I quote some lines from Rutherford, Lewis and Bowden, in their comparison of the thirteen-line alpha-ray spectrum and the very rich gamma-ray spectrum of RaC′:

"When we consider in broad terms the data which have been presented, there can be no doubt that there is a high correlation between the alpha-particle levels which have been observed and the emission of gamma-rays. In more important cases the numerical agreement is well within the experimental error of measurement, while the relation between the intensity of the alpha-ray groups and the gamma-rays associated with them is of the right order of magnitude to be expected on general theoretical grounds. In other cases the agreement is very uncertain, and more definite information on the gamma-rays is required to make the deductions trustworthy. It is unfortunate that we have been unable to detect the alpha-particle groups corresponding to certain postulated levels [*i.e.* postulated from the classification of the gamma-ray lines] . . . ."

Thus it appears that there are excellent agreements between $h\nu$-values and $(U_i - U_j)$ values, and yet nothing approaching a perfect one-to-one-correspondence. Nevertheless, the general programme is fixed: to assume that each nucleus possesses a system of stationary

[19] It is interesting to notice that after the $h\nu$-values of certain gamma-rays emitted from mixtures of ThC and ThC″ had been found to agree with values of $(U_i - U_j)$ taken from the alpha-ray spectrum of ThC, these gamma-rays were proved to proceed from ThC in its transformation into ThC″, whereas till then they had been supposed to proceed from ThC″ in its transformation into ThD (Meitner & Philipp, Ellis).

states and energy-levels, to assume further that $h\nu$-values and $(U_i - U_j)$-values are alike the differences between these energy-levels, and to ascribe apparent defects of correlation to special circumstances by virtue of which certain gamma-rays and certain alpha-rays are too feeble to be detected. Should these ideas prove untenable, we shall probably have to suppose that the nucleus is even more different from the extra-nuclear world than we have hitherto admitted.

Now arises the important question: when alpha-particles and photons both are emitted in the course of the complete transformation of one nucleus into another, which comes first? Despite the immeasurable shortness of the times which are involved, this is in principle an answerable question. For as I have mentioned already, gamma-rays are detected and their photon-energies are measured by examining the spectrum of the electrons which they eject from the orbital electron-layers of atoms, chiefly from the layers surrounding those very nuclei whence the photons themselves proceed. Now if the photons come before the alpha-particles, say for example in the transformation of ThC into ThC″, these electrons will come from the electron-layers of ThC atoms; in the contrary case, from the layers of ThC″. It is possible to distinguish from which they do come, even when the energy of the photons is not independently known and must itself be derived from the same data.[20]

The classical and crucial experiment of this type was performed about ten years ago by Meitner, and it proved that the gamma-rays emitted during the transformation of RdAc into AcX and during that of AcX into An spring forth *after* the alpha-particle has departed and the nucleus has become that of the daughter-element. These are two of the cases in which the alpha-ray spectrum exhibits fine-structure; and it is now generally supposed that the rule extends to all such cases. The stationary states or energy-levels deduced from the $(U_i - U_j)$-values and the $h\nu$-values then would pertain to the "final" or daughter nucleus. In the instances where all the alpha-particle groups except the main one are designated "long-range groups"—RaC′ and ThC′ (the quotation above from Rutherford, Lewis and Bowden refers to the former of these)—Gamow argues that the gamma-rays are emitted before the alpha-particles; the energy-levels deduced from the alpha-ray and the gamma-ray spectra would then pertain to

[20] See the previously-cited article "Radioactivity," pp. 94–96. I mention in passing that sometimes the "internal conversion" of photons whereby electrons are ejected is apparently so much the rule, that no appreciable fraction of the gamma-rays of some particular energy (or energies) escape from the atoms at all; in which cases it becomes expedient to speak not of gamma-rays at all, but of an immediate transfer of energy from the nucleus to the orbital electrons (a policy which may be applied to all cases of internal conversion).

the "initial" or mother nucleus. It is not clear from the literature whether this hypothesis has been fully tested in the manner of Meitner's tests aforesaid, but presumably it was adopted in calculating the $h\nu$-values from the electron-energies, so that the agreements between $h\nu$ and $(U_i - U_j)$ support it.[21]

The search for interrelations among the energy-levels, the different $h\nu$-values and the different U-values belonging to individual transformations has of course already begun. Rutherford and Ellis find that the frequencies of many of the lines in the gamma-ray spectrum of RaC′ can be fitted by assigning various integer values to p and q and constant values to E_1 and E_2 in the formula $pE_1 + qE_2$; while H. A. Wilson finds that if the U-values or the $h\nu$-values are added together in pairs, an amazing number of the pairs are equal to integer multiples (the integer multipliers ranging from 16 to 54) of the amount 0.385 MEV—this even if the two members of a pair are taken from different spectra!

The Quantum-Mechanical Theory and the Crater Model of the Nucleus[22]

Anyone who is acquainted with the contemporary atom-model in its present or in its earlier stages, with its congeries of charged particles revolving in or jumping between definitely-prescribed and quantized orbits, governed by attractions and repulsions both classical and unimaginable—any such person will probably be looking for a nucleus-model of the same variety but built on a very much smaller scale,

[21] I learn by letter from Dr. Ellis that in the case of RaC′, some at least of the gamma-rays which agree with the $(U_i\text{-}U_j)$-values of the long-range alpha-particles are definitely proved in this fashion to proceed from nuclei of atomic number 84 (that of RaC′) as distinguished from 83, 82 or 81; the proof is especially strong for the most intense gamma-ray, of photon-energy 0.607 MEV. Perhaps this is the most powerful evidence that the long-range particles come from RaC′ rather than RaC.

The half-periods of RaC′ and ThC′ are exceedingly short, 10^{-6} sec. and 10^{-11} sec. respectively; had it been otherwise, objection might be made to Gamow's contention on the ground that atoms in states, from which they are liable to depart by emitting radiation, generally do depart from those states and emit that radiation within a period of the order of 10^{-7} or 10^{-8} sec. There is no *a priori* certainty that this principle applies to nuclei, but if it does it may suffice to explain why the long-range particles are observed only from these very short-lived nuclei, why they are so scanty even in these cases (nearly always the photon *is* emitted before the alpha-particle gets ready to leave, so that the latter nearly always leaves with low energy instead of high), and why the fine-structure of other alpha-ray spectra is related to energy-levels of the final instead of the initial nucleus (Gamow). Incidentally it strengthens the case for ascribing the long-range particles to the C′-products instead of the C-products.

[22] Quantum mechanics was first applied to the nucleus-model here to be described, independently and almost simultaneously, by Gurney and Condon and by Gamow. Rather than by all three names together, it seems preferable to denote this model thus interpreted by a neutral descriptive term, such as "crater model" (an allusion to the aspect of the graph obtained when the curve of Fig. 12 is coupled with its mirror-image in the *yz*-plane).

with protons and possibly neutrons figuring among the revolving particles. He will be looking too far into the future, and will be disappointed with the present. The present nucleus-model consists of little more than a single curve—a curve which, moreover, relates only to the fringe of the nucleus and to the region surrounding it, and for want of knowledge is not extended into the central region or nucleus proper where the constituent particles must be. The theory which it serves is a theory not of the nucleus as a stable system of corpuscles, but of the escape of some from among these corpuscles and the entry of new ones—a theory professing to deal only with the entry and the escape, not at all with the events succeeding the one or preceding the other.

The curve purports to portray the electrostatic potential, as function of r the distance measured from the centre of the nucleus, from $r = \infty$ inward to a minimum distance which is indeed very small even in the atomic scale—10^{-13} cm. or less—but still definitely not zero, since the components of the nucleus must be presumed to be normally at distances yet smaller. When it is plotted as in Fig. 12, its ordinate

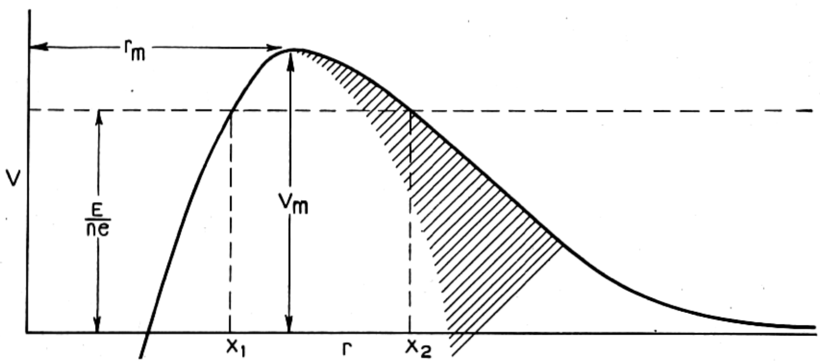

Fig. 12—Nuclear potential-curve postulated for explaining transmutation (without allowance for resonance) and radioactivity.

at any r is a measure of the amount of kinetic energy which a positively-charged particle approaching the nucleus must sacrifice—i.e. which must be converted into potential energy—in order to come from infinity to r. Traced from infinity inward, the curve must follow at first the function *const.*/r, corresponding to the inverse-square law of force; for it is known, both from experiments on alpha-particle scattering (which supplied the foundation for the contemporary atom-model) and from the successes of the theory of atomic spectra, that beyond a certain distance a nucleus is surrounded by an inverse-square force-field. This is the obstacle, or at any rate, a part of the obstacle, which an oncoming proton or deuton or alpha-particle must overcome in order to reach the nucleus and achieve transmutation. One may picture it as a hill, up which the ball must roll to reach the castle at the top—and *down* which the ball will roll if it starts from the top, shooting outward towards infinity as the fast-flying alpha-particle.

Now to assume the inverse-square force as prevailing all the way inward to $r = 0$ would be to postulate a point-nucleus without room for parts or structure, surrounded by a hill of infinite height which no approaching positive particle could climb; all of which is inadmissible. Departure from the inverse-square law is actually shown by some experiments on scattering of alpha-particles which pass very close to the nucleus, and these indications are to be heeded in tracing that part of the curve of Fig. 12 which lies to the right of the maximum; but the maximum itself and the sharp descent to its left are dictated by no such observations, and to postulate them is to make the theory which is now to find its employment and its test. That there should be such a maximum and such a descent is of course the most natural supposition to make. If there are several particles of positive charge which stay for a finite time within the nucleus, there must be something which restrains them from flying away. This something must either be an agency of a type as yet unknown, or else be described by a potential-curve with a maximum at what we may henceforward call the boundary of the nucleus; and the latter assumption is to be preferred till proved unusable.

Applying classical ideas to this "model" (if the word be not considered too presumptuous) of a nucleus, one is led at once to two predictions, which may be sharply formulated if we adopt symbols such as V_m and r_m for the two parameters indicated on Fig. 12, *viz.* the "height of the potential-barrier" and the "radius of the potential-barrier" as they are commonly called, the latter being also called the "radius of the nucleus." These are:

1. If the nucleus emits a particle of positive charge $+ 2e$, the kinetic energy with which this particle is endowed when it completes its escape cannot be less than $2eV_m$; consequently, when it is observed that atoms of a certain element emit alpha-particles with kinetic energy K_0, the height of the potential-barrier for that element cannot surpass $K_0/2e$; consequently, when the force-field about the nuclei of such atoms is explored by the classical method of studying the scattering of alpha-particles projected against a sheet of that element, it must be found that the region of repulsive force, and *a fortiori* the

inverse-square field, do not extend far enough inward for the integral to surpass the value $K_0/2e$.

2. When particles of charge ne (n = any integer) are projected at a sheet of any specific element, they cannot enter the nuclei at all unless their kinetic energy exceeds the critical value neV_m; and if the curve of number-entering-nuclei *vs.* kinetic-energy can in any way be deduced from any experiments, it should rise fairly sharply from the axis of kinetic energy at this critical abscissa.

(It will have been noticed that I expressed both of these predictions as though the escape or the entry of a particle made no difference to the height of the potential-barrier, which is the universal practice. This is obviously too crude an assumption; the error in it must be graver the smaller the atomic number of the element, therefore graver in theorizing about the transmutation of light elements than in theorizing about the radioactivity of heavy ones; it must be rectified in future.)

The former of these predictions can be sharply and unquestionably tested; and it proves to be wrong. Uranium I. emits alpha-particles of kinetic energy K_0 equal to 4 MEV; but Rutherford suspected from scattering-experiments on other heavy elements, and subsequently proved by such experiments upon uranium itself, that the inverse-square force-field extends so far inwards as to involve a height of potential barrier at least twice as great as $K_0/2e$; so that an emerging alpha-particle should possess at least 8 MEV of kinetic energy derived from coasting down the hill, and even this is merely a lower limit to the estimate, since the hill may be higher and the particle might come with some excess of energy over its brow!

The second prediction is not so readily tested. If all of the charged particles (protons or deutons or alpha-particles, say) projected at the postulated sheet of matter were directed straight towards the centres of nuclei, and arrived at the potential-hills without suffering any prior loss of energy elsewhere, the fraction entering through the potential barriers would rise suddenly from zero to unity as the kinetic energy K of the particles was raised to neV_m, and any phenomenon depending solely upon entry would make its advent suddenly if at all. Unfortunately this does not occur in any experiment now possible or likely ever to become possible. If the sheet of matter is a monatomic layer, most of the oncoming particles will be going towards the gaps between the nuclei, and the initial directions of the rest will be pointed towards all parts of the cross-sections of the nuclei, only an infinitesimal fraction going straight toward the centres. Designate by p the perpendicular distance from a centre to the line-of-initial-motion of an oncoming particle; it is evident that the minimum kinetic energy permitting of entry will increase with p, starting from neV_m and rising to infinity as p rises from 0 to r_m. The relation between fraction-of-particles-entering-nuclei—call it P_e—and kinetic energy K could be calculated, given specific assumptions about the values of V_m and r_m and the trend of the potential-curve. Without undertaking the calculation, it is easy to see that the vertical rise of what I will hereafter call the "ideal" curve—the curve of probability-of-entry-at-central-impact *vs.* K—will be distorted into a bending slope, starting, however, at the same critical abscissa neV_m. If the sheet of matter is a thick layer, there will of course be a much greater fraction of the impinging particles of which the initial paths point straight toward some nucleus or other, but the fraction achieving entry will not be raised in the same ratio, for the particles going toward nuclei embedded deep in the layer will lose some or the whole of their velocity in passing through the intervening matter.[23] This also will contribute to converting the vertical rise into a gradual bend. Still it does not seem possible that if the ideal curve had such a shape, the experimental ones could rise with so extreme a gradualness as does the one of Fig. 17 or those of Figs. 16 and 17 in the Second Part; for these suggest no sudden beginning at all, but rather they have the characteristic aspect of curves asymptotic to the axis of abscissas, as if their apparent starting-points could be pushed indefinitely closer to the origin by pushing up indefinitely the sensitiveness of the apparatus. Neither does it seem possible that V_m can be so low as their starting-points imply.

There is, however, another difficulty: these curves refer not directly to P_e, but to number of transmutations, or to be precise (for precision is essential in these matters) to the number of particles producing transmutations involving the ejection of fragments having certain ranges. Call this number P_t. It is easiest to conduct the argument as though P_t were proportional to P_e—as if an observable transmutation could result only from the entry of a particle through the potential-barrier of a nucleus, and as if the number of transmutations of any special type were strictly proportional to the number of entries, the factor of proportionality being independent of K. Yet few assumptions are less plausible. It is far more reasonable to suppose that the probability of a particle bringing about a transmutation when it enters a nucleus is not invariably unity, but is instead some function $f_1(K)$. It is reasonable also to suppose that a particle passing close to the potential-barrier but not traversing it may yet be able to touch off an internal explosion or eruption leading to a transmutation. Denote

[23] Contrast the two curves of Fig. 17.

by $f_2(K)(1 - P_e(K))$ the number of cases in which this happens. The least which we can take for granted is some general relation of the form,

$$P_t = f_1(K)\cdot P_e(K) + f_2(K)(1 - P_e(K)), \tag{20}$$

and the variations of f_1 and f_2 may contribute still further to blotting out all signs of the hypothetical vertical rise in the ideal curve. Moreover, f_2 might be appreciable at values of K smaller than neV_m, thus blotting out every sign of the critical energy-value at which entry commences.

Thus with regard to the second prediction, the situation is this: the experimental curves of number-of-observed-transmutations *vs.* kinetic-energy-of-impinging-particles rise so smoothly and so gradually from the axis as to give not the slightest support to the idea that entry into the nucleus commences suddenly at a critical value of K; moreover, transmutation commences to be appreciable—for several elements, at least—when K is still so small that K/ne is only a small fraction of the least value which can reasonably [24] be ascribed to V_m, in view of what we know from alpha-particle-scattering about the circumnuclear fields of these or similar elements. This again might be due to the hypothetical effect to which the term $f_2(K)$ in the equation alludes, but it seems far too prominent for that! With it is to be linked the fact that alpha-particles emerge from nuclei with kinetic energy less than $2eV_m$. The potential-hill seems not to be so high either for entering or for emerging particles, as it is for those which only skirt its slopes!

Now if in theorizing about potential-hills and particles we substitute quantum mechanics for classical mechanics, these phenomena cease to be things contrary to expectation, and become instead the very things to be expected.

This is one of the situations—regrettably frequent in the present-day theoretical physics—where neither pictures nor words are adequate. The nearest description which can be made with words is probably somewhat as follows: We set out to ascertain whether a particle of charge ne and kinetic energy K, coming from infinity straight toward the nucleus (I simplify the problem as much as possible) will surmount the potential-hill of height V_m. Were we to conceive it as a particle conforming to classical mechanics, we should arrive at the answers: yes, if $K \geqq neV_m$—no, if $K < neV_m$. But we are to turn away from

[24] It is true that the elements of which the circumnuclear fields have been most carefully explored by alpha-particles are not in general the same as those for which transmutability has been observed down to very low values of K; but boron and carbon figure on both the lists, Riezler having studied the scattering of alpha-particles by these (*Proc. Roy. Soc.*, **134**, 154–170, 1932).

the particle for awhile, and to conceive a train of waves advancing from infinity towards the nucleus. The phase-speed and the frequency of this wave-train are prescribed by definite rules making them dependent upon E, and the train is governed by a prescribed wave-equation in which figures the function $V(r)$ of Fig. 12. On solving this equation in the prescribed fashion we find that it requires the wave-train to continue (though reduced in amplitude) past the top of the hill if K is greater than neV_m. This is partially satisfactory, for the particle when it is reintroduced is to be associated with the waves, and everything would be spoiled if the particle could go where the waves cannot. But also, the equation requires the wave-train to continue past the top of the hill when K is less than neV_m. True, it does not wholly pass; there is a reflected as well as a transmitted beam, and the ratio of reflected to incident amplitude goes very rapidly up towards unity and the ratio of transmitted to incident amplitude goes very rapidly down toward zero as K drops downward from the value neV_m. All the same there *is* this wave-train beyond the hill with an amplitude greater than zero; and the association of particles with waves is apparently spoiled, for the waves can go where the particle cannot.

At this point, however, it is the rule of theoretical physicists to give the precedence to the waves, and declare that *where the waves go there the particle must go also, whether it can* (by classical mechanics) *or cannot.* Since some of the waves are beyond the hill, the particle also must be able to traverse the hill, even though its kinetic energy is insufficient for it to climb to the top. But since the waves beyond the hill have a smaller amplitude than those coming up from infinity, it is not certain that the particle will pass through, but merely possible. The chance or probability of its passing through is determined chiefly (not fully) by the ratio of the squared-amplitudes of the waves on the two sides of the hill, and this is what must be computed by quantum-mechanics. How the particle gets over or through the hill—where and what it is and how it is moving while it is getting through—these are questions which the theorist usually declares to be unanswerable in principle, and having so declared, he does not attempt to visualize this part of the process.

Into Fig. 12 the diagonal lines have been introduced in a crude attempt to make graphic as much as possible of the theory. The length of the sloping line drawn from any point P of the curve is meant as a sort of inverse suggestion of the chance which a particle of charge $+ne$ has of entering the nucleus if its energy E is equal to ne times the ordinate of P: the longer the line, the less the chance of entry! (This energy E will be the same as the initial kinetic energy K already so often mentioned, which the particle has before it starts to climb the hill.) Perhaps it is not too fanciful to think of these lines as the posts of a fence standing up vertically from the curve, the varying height of which is a rough indication of the varying difficulties which particles of various energies have in getting through.

To predict successfully how the height of this metaphorical fence, the probability of transmission or of penetration, varies with V or E would be a magnificent triumph of nuclear theory, but it is vain to hope for such a success in the immediate future. Almost certainly the top of the fence curves much more rapidly upward than the drawing suggests, and also there is good reason to think that there may be gaps in the fence somewhere like those depicted in Fig. 13, where the

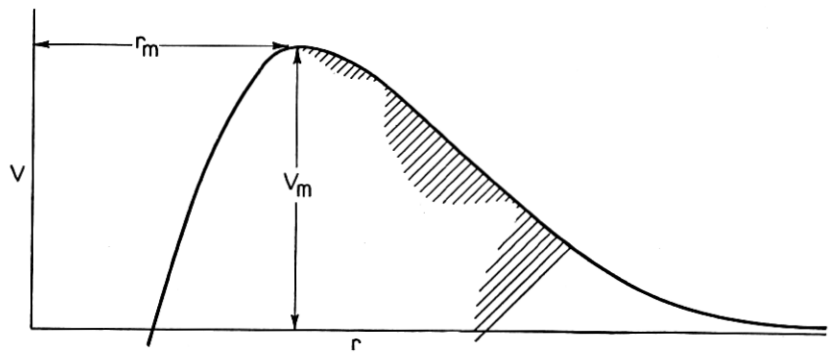

Fig. 13—Nuclear potential-curve postulated for explaining transmutation, with allowance for "resonance."

probability of penetration rises to values remarkably near to unity. Such at least are the features of certain one-dimensional potential-fields (do not forget that Figs. 12 and 13 refer to three-dimensional potential-fields having spherical symmetry!) which have isolated potential-hills or hills-adjoined-by-valleys.

Three of these cases are displayed in Figs. 14, 15, 16. Take the first for definiteness. One plunges *in medias res* by writing down at once Schroedinger's wave-equation:

$$d^2\Psi/dx^2 + (8\pi^2 m/h^2)[E - neV(x)]\Psi = 0, \qquad (21)$$

in which $V(x)$ stands for the potential-function exhibited in the figure,[25] while the meanings of ne, m and E have probably already been

[25] I deviate from the otherwise-universal usage of employing V for the potential energy of the particle, for the reason that the latter depends on the charge of the particle, while the potential-function is supposed (no doubt inaccurately) not to depend on it.

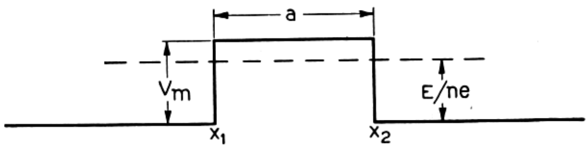

Fig. 14—Illustrating an artificial case of a potential-curve with a single square-topped potential-hill.

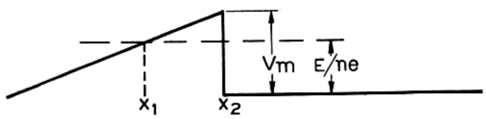

Fig. 15—Illustrating an artificial case of a potential-curve with a pointed hill.

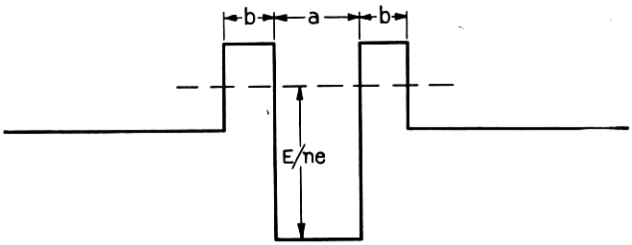

Fig. 16—Illustrating an artificial case of a potential-curve with a valley between two hills.

guessed by the reader—they are constants to which any values may be assigned, and the eventual result of the mathematical operations is going to be taken as referring to a stream of particles of charge ne, mass m and energy E.

The problem is stated as that of finding a solution of (21) for whatever value is chosen for E—a solution everywhere single-valued, bounded, continuous, and possessed of a continuous first derivative, such being the general requirement in quantum mechanics. Not, however, *any* solution possessing these qualities, but a solution apt to the physical situation. On the right of the hill, it must specify a wave-train (I) going from right to left; for we are interested in the adventures of particles coming from the right toward the hill. But on the right of the hill, it must also be capable of specifying a wave-train (II) going from left to right, for some or all of the particles may be reflected from the hillside. On the left of the hill it must be capable of specifying a wave-train (III) going from right to left, for some or all of the particles may traverse the hill and continue on their way.

So far as the region to the right of the hill ($x > x_2$) is concerned, a solution having all of these qualities is the following:

$$\Psi = A_1 e^{-ikx\sqrt{E}} + A_2 e^{+ikx\sqrt{E}}, \qquad k = \sqrt{\frac{8\pi^2 m}{h^2}}, \tag{22}$$

in which the two terms stand for wave-trains I and II, and A_1 and A_2 are adjustable constants. So far as the region to the left of the hill ($x < x_1$) is concerned, a solution having all of the required qualities is the following:

$$\Psi = C_1 e^{-ikx\sqrt{E}}. \tag{23}$$

It stands for wave-train III and C_1 is an adjustable constant. As I have already said, our "intuition" based on notions of what particles should do, expects C_1 to vanish and A_2 to become equal to A_1 when E is less than neV_m, but the solution of (21) does not consent to these limitations. Our intuition also expects that when E is less than neV_m nothing will happen in the region comprised within the hill ($x_1 < x < x_2$), but here again the solution of (21) does not conform with it. For in this region comprised within the hill, the solution must take the form:

$$\Psi = B_1 e^{-kx\sqrt{neV_m - E}} + B_2 e^{+kx\sqrt{neV_m - E}}, \tag{24}$$

which looks at first glance like (22) but is essentially different, since the exponents are real and not imaginary, and the terms do not represent progressive waves. The five coefficients—A_1, A_2, B_1, B_2, C_1—must now be mutually adjusted so that at the sides of the hill ($x = x_1$ and $x = x_2$) the expressions (22) and (23) and (24) flow smoothly each into the next, with no discontinuity either of ordinate or of slope. This imposes four conditions on the five coefficients, and therefore fixes the relative values of all of them—in other words, determines them completely except for a common arbitrary factor which corresponds to the intensity of the incident beam, and is irrelevant to the course of the argument.

In particular, this requirement of continuity imposed by the fundamental principles of quantum mechanics upon the acceptable solution of (21) fixes the ratio of the amplitudes C_1 and A_1 of "transmitted" and "incident" wave-train. From Gurney and Condon I quote an approximate formula [26] for the ratio of the squares of these amplitudes, denoting them on the left by the customary symbols:

$$\frac{(\Psi\Psi^*)_{\text{trans.}}}{(\Psi\Psi^*)_{\text{inc.}}} = \phi(E)\ \text{exp.}\left[-\ (4\pi a/h)\sqrt{2m(NeV_m - E)}\right], \qquad \phi(E) = 16(E/neV_m)(1 - E/neV_m). \tag{25}$$

[26] Exact formula given by E. U. Condon, *Reviews of Modern Physics*, **3**, 57 (1931).

This expression does not vanish suddenly as soon as E drops below neV_m, but falls away continuously—and very rapidly, it must be admitted, owing to the exponential factor—as E diminishes from neV_m on downwards. Its value depends on a, the breadth of the hill (Fig. 14) in such a way that the broader or thicker the hill of given height the less the amplitude of the transmitted waves: the thicker the hill, the more nearly it comes to fulfilling the classical quality of being a perfect obstacle to particles having insufficient energy to climb it!

I rewrite (25) in the equivalent form,

$$\frac{(\Psi\Psi^*)_{\text{trans.}}}{(\Psi\Psi^*)_{\text{inc.}}} = \phi(E) \exp. [- (4\pi/h) \int_{x_1}^{x_2} \sqrt{2m(NeV_m - E)}dx, \quad (26)$$

the integral in the exponent being taken "through the hill" from x_1 to x_2. This form is generalizable. Take the case of Fig. 14: the ratio of the squared-amplitudes of transmitted and incident wave-trains is given, according to Fowler and Nordheim, by an expression which is of the type (26), except that $\phi(E)$ is a somewhat different function (it is $4[(E/NeV_m)(1 - E/NeV_m)]^{1/2}$). The distance from x_1 to x_2, over which the integration is carried, obviously depends on E in this and every other case but the particular one of Fig. 14. Take finally the general case of a rounded hump, such as appears in Fig. 12. According to Gamow, a formula of type (26) is approximately—not exactly—valid for every such case, $\phi(E)$ being given by him as simply the number 4 when the hill descends to the same level on both sides as in Fig. 14; while in the general case where the potential-curve approaches different asymptotes at $-\infty$ and $+\infty$—say zero at the latter, V_r at the former—the factor $\phi(E)$ assumes the form $4[E/(E - neV_r)]^{1/2}$. Now E was the kinetic energy of the particles at infinity in the direction whence they come, and $(E - neV_r)$ will be the kinetic energy of the particles at infinity in the direction whither they are going. We have been denoting the first of these quantities by K; denote the second by K_r, and the corresponding velocities by v and v_r. Then $\phi(E)$ can be written as $4v/v_r$.

The question must now be answered: what is the actual relation between the ratio $(\Psi\Psi^*)_{\text{trans.}}/(\Psi\Psi^*)_{\text{inc.}}$, and the probability that a particle will traverse the hill? In associating waves with corpuscles, it is the rule to postulate that the square of the amplitude of the waves at any point is proportional to the number-per-unit-volume of corpuscles in the vicinity of that point. If one prefers to think of a single particle instead of a great multitude, one may say that the square of the amplitude of the waves at any point is proportional to the probability of the particle being at that point. Let us hold, however, to the picture of a dense stream of corpuscles approaching the potential-hill—say that of Fig. 14—from the right, and a much weaker stream receding from it on the left. If there are s times as many corpuscles per-unit-volume on the right as on the left, then there cannot be a steady flow unless only one out of s incident particles traverses the hill. The reciprocal of s is the fraction of particles getting through the hill, or the probability of a single particle getting through; and it is also the ratio $(\Psi\Psi^*)_{\text{trans.}}/(\Psi\Psi^*)_{\text{inc.}}$. This statement, however, is too narrow, being valid for the case where the speeds v and v_r of the particles on the two sides of the hill are the same. In the general case, we have:

Probability of transmission or penetration

$$= (v_r/v)(\Psi\Psi^*)_{\text{trans.}}/(\Psi\Psi^*)_{\text{inc.}}$$

$$= (v_r/v)\cdot\phi(E)\cdot\exp.\left[-\ (2\pi/h)\int_{x_1}^{x_2} dx\sqrt{2m(E - neV_m)}\right]. \quad (27)$$

In Gamow's approximation the product of the first two factors has the pleasantly simple constant value of 4. In the approximations of Gurney and Condon and of Fowler and Nordheim for the cases of Figs. 14 and 15, the product is some function of E which the reader can construct from the foregoing equations. In all these cases, however, it is the exponential factor which dominates the trend of either member of (27) considered as function of E.

Now immediately one sees, that *if* transmutation is due to the penetration of a charged particle through a potential-hill or potential-barrier surrounding a nucleus—and *if* this penetration is governed by laws of quantum mechanics as illustrated in the one-dimensional cases—*then* when the number of observed transmutations is plotted against the kinetic energy K of the impinging particles, the curve should be expected to rise with a gradual smooth upward curvature from the axis of K; and there should be no critical minimum value of K for the advent of the phenomenon, but rather the beginning of perceptible transmutation should be observed at progressively lower and lower energy-values, as the sensitiveness of the detecting-apparatus is improved; and it may well be that transmutation can be detected when K is still so low, that the quotient of K by the charge of the particle is far smaller than any reasonable guess that can be made of the height of the barrier. All these are features of such curves as those of Fig. 17, or Figs. 16 and 17 of the Second Part. The adoption of quantum mechanics permits us to accept these features without ascribing them to the hypothetical functions denoted by f_1 and f_2 in equation

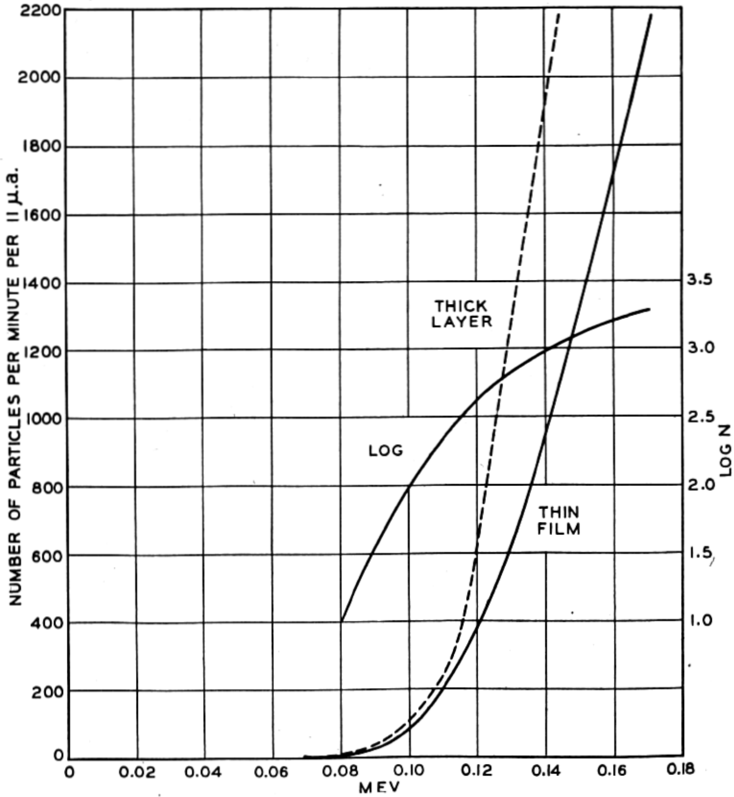

Fig. 17—Transmutation of boron by impact of protons: rate of observed transmutation as function of K, for a very thin film and for a thick layer. (Oliphant & Rutherford, *Proc. Roy. Soc.*).

(20), though it does not rule out the possibility that these functions may have influence upon the curves.

But not all of the curves of probability-of-transmutation versus K are of the simple type of Fig. 17. There are also some which show distinctly-marked peaks superimposed upon the gradual upward sweep; that of Fig. 18 for example, which relates to the transmutation of beryllium by impact of alpha-particles with emission of neutrons, presumably by the process

$$ {}_2He^4 + {}_4Be^9 = {}_6C^{12} + {}_0n^1 $$

and that of Fig. 19, which relates to the presumptive process,

$$ {}_2He^4 + {}_{13}Al^{27} = {}_{14}Si^{30} + {}_1H^1. $$

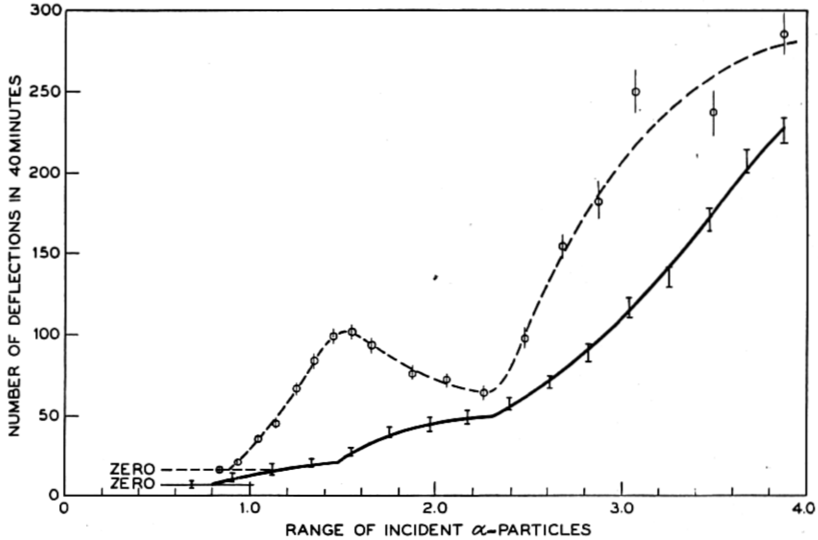

Fig. 18—Transmutation of beryllium by impact of alpha-particles, with production of neutrons; rate of observed transmutation as function of K, for a very thin film and for a thick layer (dashed and full curves respectively), illustrating resonance. (Chadwick; *Proc. Roy. Soc.*).

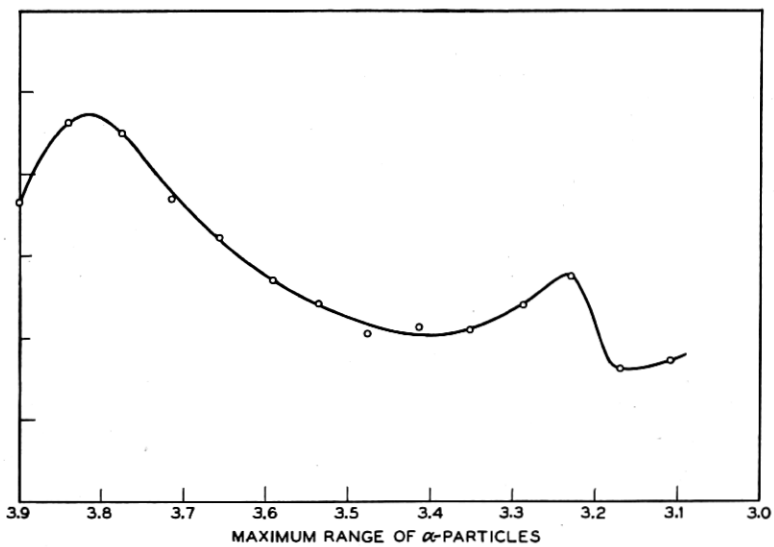

Fig. 19—Transmutation of aluminium by impact of alpha-particles, with production of protons; rate of observed transmutation as function of residual range of alpha-particles, illustrating resonance. (Chadwick & Constable; *Proc. Roy. Soc.*).

In Fig. 19 the abscissa is not K, but a quantity (the range of the impinging alpha-particles) which increases more rapidly than K; but this does not affect the meaning of the peaks. Moreover, there is abundant indication that quantities of such curves are simply waiting for someone to take the data and plot them; for this is the phenomenon of "resonance" to which many pages [27] were devoted in the Second Part, and which has chiefly been observed by the other methods there described, but should always manifest itself in this way when the proper experiments are performed.

If we wish to interpret this without letting go of the classical theory, we must say that either or both of the functions f_1 and f_2 have maxima at certain values of K. But here again, the adoption of quantum mechanics may make this step superfluous. For consider the one-dimensional potential-distribution of Fig. 16, a valley between two hills, with energy-values reckoned from the bottom of the valley. If the wave-equation be solved for this potential-distribution and for any such value of particle-energy E as the dashed line of Fig. 16 indicates—such a value, that according to classical theory a particle possessing it might either be always within the valley or always beyond either hill, but never could pass from one of these three zones to another—a curious result is found. For the solutions which the laws of quantum mechanics demand and accept, the ratio of squared-amplitude $\Psi\Psi^*$ within the valley to squared-amplitude $\Psi\Psi^*$ beyond either hill is usually low, but for certain discrete values of E it attains high maxima!

Now the three-dimensional nucleus-model of which I am speaking resembles this case more than it does the other one-dimensional cases of Figs. 14 and 15, because it consists of a potential-valley surrounded on all sides by a potential-hill. One may therefore expect the probability of entry or penetration to pass through maxima such as are symbolized by the dips in the "fence" of Fig. 13, entailing maxima in the curve of probability-of-transmutation P_t plotted as function of K. Such is the quantum-mechanical explanation of the phenomenon of "resonance," which indeed derives its name from this theory; for the values of K or E at which the maxima occur are those for which the amplitude of the oscillations of the Ψ-function in the valley within the barrier are singularly great.

One wants next to know what quantitative successes have been achieved in predicting or explaining such things as the actual locations of the resonance-maxima, or the precise trend of the curve of P_t-vs-K

[27] "Nucleus, Part II," pp. 148–153; more fully treated in *Rev. Sci. Inst.*, **5**, 66–77 (Feb. 1934).

as it rises away from the axis of abscissæ. Here it must be admitted that almost everything remains to be done. The locations of the resonance-maxima must be expected to depend upon the details of the potential-distribution within the valley, of which there is as yet no notion. The precise trend of P_t as actually observed cannot be the same as that of (27) however good the theory may be, first because not all of the impacts are central (page 596), then because in most (not quite all) of the experiments the bombarded substance is in a thick layer instead of a thin film (page 597), and finally because of the functions f_1 and f_2 of equation (20). Much as we should like for simplicity to put these functions equal to unity and zero respectively, and even though quantum mechanics has removed some of the obstacles to doing so, yet we are obliged to take them into account—the most striking and cogent reason being that with a variety of elements, P_t is not the same for impact of protons as for impact of deutons, though *ne* is the same for both! [28]

The situation being such, one cannot ask as yet for accurate statements about the values of r_m and V_m, the constants of the "crater model" exhibited in Figs. 12 and 13. These must wait upon a thoroughgoing fitting of the theory to the experimental curves of P_t-vs-K, involving a decision as to the magnitude of f_1. The values of V_m for several of the lighter elements have been estimated from the data on transmutation, but the procedure of arriving at the estimates

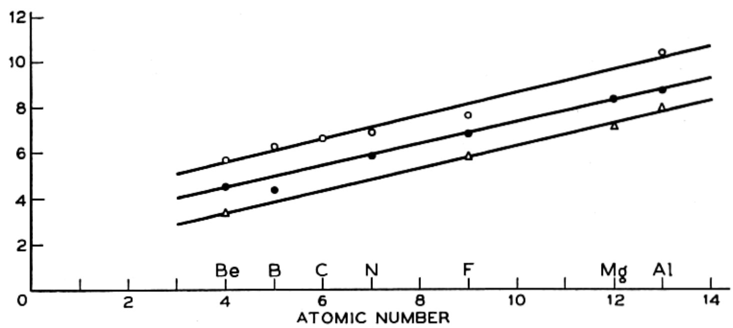

Fig. 20—Resonance-levels and (estimated) heights of potential-barriers for some of the lighter elements, deduced from observations of transmutation. (Pollard; *Phys. Rev.*)

has not (so far as I know) been published. I reproduce as Fig. 20 a graph of Pollard's, the circles along the uppermost line showing the estimated values of eV_m and the crosses along the other two lines

[28] Also it has been said that the shapes of the best experimental curves of P_t-vs-K imply that as K is increased, f_1 increases at first and then becomes constant; but there is a great lack of published theory on these matters.

showing the values of eV at which resonance occurs. The linear trends suggest that these may be properties of the nucleus which are susceptible of simple interpretations. There are also the estimates of V_m made from observations on alpha-particle scattering, most of which are merely minimum-admissible-values below which V_m cannot lie, while a few are more definite. As for r_m, there is at any rate nothing

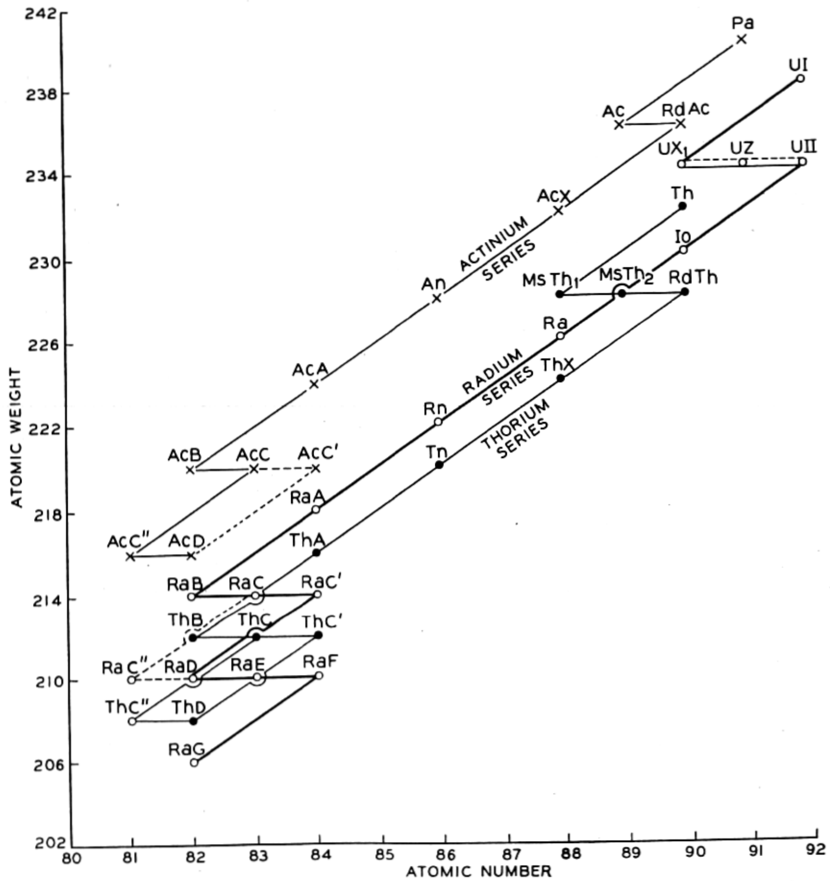

Fig. 21—Genealogies of the radioactive elements. (The actinium series is plotted some distance above the others for legibility, but almost certainly An should lie one unit below Tn, the rest correspondingly).

to indicate that we must make it higher than the values—a few times 10^{-13} cm.—which many reasons impel us to assign to the dimensions of nuclei.

Thus the quantum-mechanical theory of transmutation is as yet in a primitive state, and indeed not advanced enough (in my estimation) to be considered fully proved by its own successes. Quantum mechanics has, however, many other buttresses, quite sufficiently many to allow us to take it for granted; and in this particular field it has the prestige of prophetic powers. Until the experiments of Cockcroft and Walton, no one had ever effected transmutation except with alpha-particles of charge $2e$ and energy K amounting to several millions of electron-volts. Now Cockcroft and Walton say that they were encouraged to build the elaborate apparatus necessary for trying it with protons of energy much less than one million electron-volts, by Gamow's inference that particles of charge $+e$ should have a very much greater chance of penetrating through a potential-hill and into a nucleus, than particles of equal kinetic energy and only twice the charge—the inference from the fact that ne occurs in the exponent of the exponential function appearing in equation (23) and others like it. Moreover, the phenomenon of resonance was predicted by Gurney (and mentioned by Fowler and Wilson, who, however, apparently did not believe that it could ever be observed) before it was discovered in the experiments of Pose.

The merits of the crater model with the quantum-mechanical theory have, however, not yet been fully presented, for I have left to the last their application to radioactivity.

One of the principal features of radioactivity—both the "induced" variety described in the early part of this article, and the "standard" variety known these thirty-five years—is the exponential decline or decay of the intensity, hence of the quantity of any radioactive substance, as time goes on. This signifies that the average future duration, reckoned from any instant of time, of all the atoms surviving unchanged at that instant, is the same whichever instant be chosen—or, that the probability that an atom, not yet transformed at instant t_0, shall undergo its transformation within (say) a second of time beginning at t_0, has the same value however long the atom may have existed up to this arbitrarily-chosen-moment t_0.

All this is commonly expressed by saying that radioactive transformations obey the laws of chance. I quote (not for the first time) a passage from Poincaré, which illustrates how this had to be interpreted before the advent of quantum mechanics; I take the liberty of writing "nucleus" where he wrote "atom":

". . . If we reflect on the form of the exponential law, we see that it is a statistical law; we recognize the imprint of chance. In this case of radioactivity, the influence of chance is not due to haphazard encounters between atoms or other haphazard external agencies. The causes of the transmutation, I mean the immediate cause as well as the underlying one (*la cause occasionnelle aussi bien que la cause profonde*) are to be found in the interior of the atom [read, in the nucleus!]; for otherwise, external circumstances would affect the coefficient in the exponent. . . . The chance which governs these transmutations is therefore internal; that is to say, the nucleus of the radioactive substance is a world, and a world subject to chance. But, take heed! to say 'chance' is the same as to say 'large numbers'—a world built of a small number of parts will obey laws which are more or less complicated, but not statistical. Hence the nucleus must be a complicated world." [29]

Well! the advent of quantum mechanics has made unnecessary the conclusion which Poincaré was obliged to draw; for according to this doctrine, the statistical law is characteristic as much of a single particle confronted with a potential-hill, as of the greatest conceivable number of particles mixed up together. It must be admitted that Poincaré's conclusion is probably right enough for the radioactive nuclei of which he knew, all of which must be conceived to comprise several hundreds of particles, protons and electrons and neutrons and the like; but it is not enforced by the reason which he gives, if quantum mechanics is valid. For reversing the argument of previous pages: if in the valley-enclosed-by-hills which is illustrated (for the oversimplified one-dimensional case) by Fig. 16, we postulate a particle and the waves associated with that particle, then the quantum-mechanical boundary-conditions require waves beyond the hills as well, and the coexistence of waves without and within implies a tendency—a tendency governed by the "laws of chance," a probability—for the particle to escape from within to without. As soon as the physicist has successfully made the effort of consenting to quantum mechanics, he is dispensed from the further effort of contriving nuclear models with special features to account for the law of decay of radioactive substances.

Like the probability of entry, the probability of escape of the particle from the confined valley is governed by the ratio of the squared amplitudes $\Psi\Psi^*$ within the valley and beyond the hill (the latter in the numerator). It thus is governed by the exponential function,

$$\exp. [- (4\pi/h) \int \sqrt{2m(NeV_m - E)}dx],$$

of which we have already made the acquaintance, multiplied by a

[29] H. Poincaré: "Dernières Pensées," pp. 204–205 (he credits Debierne with the idea).

factor $\phi(E)$ which itself may be a function of E the energy of the particle, but is of secondary importance. Such a formula, in the case of penetration from without, represented the probability of entry for a single approach of the particle to the hill. This suggests that we should deem it in this case as representing the probability of escape for a single approach of the particle from the depths of the valley to the inner side of the hill. Suppose the valley to be of breadth a, the particle to be bumping back and forth in it with speed v_i: the number of approaches of the particle per unit time to the hill will be equal to v_i/a. If the bottom of the valley is at the same level as the axis of abscissæ in Fig. 16, v_i is equal to $\sqrt{2E/m}$; if the valley is deeper, v_i is greater. We deduce for the mean sojourn of the particle within the valley, which is the reciprocal of the probability-of-escape-per-unit-time, the expression:

$$T \geqq [(v_i/a)\phi(E)]^{-1} \exp. [+ (4\pi/h) \int \sqrt{2m(NeV_m - E)}dx]. \quad (28)$$

The aspect of this expression is far from encouraging to one who wishes for a striking quantitative test of the theory. Its value depends not merely on the breadth a assumed for the space within the potential-hill and the height V_m of the hillcrest, but on the details of the shape assumed for the potential-curve of Fig. 12 both within and without the crest; and since there is little or no independent knowledge of these qualities of the nucleus, they may be adjusted practically at will to fit any observed value of T whatever. Furthermore it was obtained by making certain crude assumptions and certain not very close approximations.

One essential test, however, can be applied to it, which it must pass; and pass it does. Let values of a and V_m, and a shape for the potential-hill of Fig. 12, be so chosen that for some particular radioactive element, RaA for instance, equation (28) agrees with experiment; which is to say, that when into the right-hand member of (28) is substituted for E the observed kinetic energy of the emerging alpha-particles, the value of this right-hand member becomes equal to the observed mean life of the element. Now let precisely the same values of a and V_m and the same shape of hill[30] be assumed for some other radioactive element, RaC′ for instance; in the right-hand member of (28), let the observed kinetic energy of the (main group of) alpha-particles for RaC′ be substituted for E; and let the value of T be computed. We

[30] Excepting that the two hills should be expected to slope off towards infinity in the manners of the two functions Z_1/r and Z_2/r, where Z_1e and Z_2e stand for the nuclear charges of the nuclei left behind after the alpha-particle departs, and are often (not always) different for two different radioactive substances.

should not expect a perfect agreement, since the two nuclei are not identical; but we should be disconcerted by a sharp disagreement, since both nuclei belong to elements of which the nuclear charges differ at most by only a few per cent and the nuclear masses by little more. A very great disagreement would in fact be gravely injurious to the theory. Making the test, Gurney and Condon found, however that there is no grave disagreement: the theory survives the test.

A very similar test was applied with greater minuteness by Gamow. For each element he assumed a potential-hill having a vertical rise on the inward side, and on the outward side a curved slope conforming exactly to the function $(Z - 2)/r$, where Z stands for the atomic number of the element before the alpha-particle quits it and consequently $(Z - 2)e$ stands for the charge of the residual nucleus. In other words, he postulated a classical inverse-square electrostatic field ("Coulomb field") from infinity inward to a distance r_0 from the centre of the nucleus, and at r_0 a discontinuous potential-fall. This is a potential-distribution distinguished by a single disposable constant, to wit, r_0; for V_m itself is determined by r_0 and Z.

Gamow proceeded to compute what value must be assigned to r_0 in order to achieve agreement between theory and experiment for each of the twenty-three alpha-emitters. Approximations must be made in carrying through the calculations; those which Gamow employed convert equation (28) into this:

$$\log_e T = -\log_e (h/4mr_0^2) + 8\pi^2 e^2 \sqrt{m}\,(Z - 2)/h\sqrt{2E} + (16\pi e\sqrt{m}/h)\sqrt{Z - 2}\sqrt{r_0}.$$

Putting for E the kinetic energy of the alpha-particles and for T the mean lives of the several elements, he evaluated r_0. Had the values proved very different for the various alpha-emitters, it would have spoken ill for the theory; but all the values were comprised between 9.5 and 6.3 times 10^{-13} cm. The order of magnitude is satisfactory; the differences between the several values are by no means disagreeably great; and there are even signs of a systematic upward trend of the values of r_0 with the atomic numbers of the nuclei. The quantum-mechanical theory and the crater model of the nucleus so pass their crucial test.