Loud Speakers and Microphones*

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In ordinary radio broadcast of symphony music, the effort is to create the effect of taking the listener to the scene of the program, whereas in reproducing such music in a large hall before a large gathering the effect required is that of transporting the distant orchestra to the listeners. Lacking the visual diversion of watching the orchestra play, such an audience centers its interest more acutely in the music itself, thus requiring a high degree of perfection in the reproducing apparatus both as to quality and as to the illusion of localization of the various instruments. Principles of design of the loud speakers and microphones used in the Philadelphia-Washington experiment are treated at length in this paper.

AS EARLY as 1881 a large scale musical performance was reproduced by telephone instruments at the Paris Electrical Exhibition. Microphones were placed on the stage of the Grand Opera and connected by wires to head receivers at the exposition. It is interesting to note that separate channels were provided for each ear so as to give to the music perceived by the listener the "character of relief and localization." With head receivers it is necessary to generate enough sound of audible intensity to fill only a volume of space enclosed between the head receiver and the ear. As no amplifiers were available, the production of enough sound to fill a large auditorium would have been entirely outside the range of possibilities. With the advent of telephone amplifiers, microphone efficiency could be sacrificed to the interest of good quality where, as in the reproduction of music, this was of primary interest. When amplifiers of greater output power capacity were developed, loud speakers were introduced to convert a large part of the electrical power into sound so that it could be heard by an audience in a large auditorium. Improvements have been made in both microphones and loud speakers, resulting in very acceptable quality of reproduction of speech and music; as is found, for instance, in the better class of motion picture theaters.

In the reproduction, in a large hall, of the music of a symphony orchestra the approach to perfection that is needed to satisfy the habitual concert audience undoubtedly is closer than that demanded for any other type of musical performance. The interest of the listener here lies solely in the music. The reproduction therefore should be

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* Third paper in the Symposium on Wire Transmission of Symphonic Music and Its Reproduction in Auditory Perspective. Presented at Winter Convention of A. I. E. E., New York City, Jan. 23–26, 1934. Published in *Electrical Engineering*, January, 1934.

such as to give to a lover of symphonic music esthetic satisfaction at least as great as that which would be given by the orchestra itself playing in the same hall. This is more than a problem of instrument design, but this paper will be restricted to a discussion of the requirements that must be met by the loud speakers and microphones, and to a description of the principles of design of the instruments used in the transmission of the music of the Philadelphia Orchestra from Philadelphia to Constitution Hall in Washington. Some of the requirements are found in the results of measurements that have been made on the volume and frequency ranges of the music produced by the orchestra.

General Considerations

The acoustic powers delivered by the several instruments of a symphony orchestra, as well as by the orchestra as a whole, have been investigated by Sivian, Dunn, and White. Figure 1 was drawn on the basis of the values published by them.[1] The ordinates of the horizontal lines give the values of the peak powers within the octaves indicated by the positions of the lines. For a more exact interpretation of these values the reader is referred to the original paper, but the chart here given will serve to indicate the power that a loud speaker must be capable of delivering in the various frequency regions, if the reproduced music is to be as loud as that given by the orchestra itself. However, it was the plan in the Philadelphia-Washington experiment to reproduce the orchestra, when desired, at a level 8 or 10 db higher, so that with three channels each loud speaking system had to be able to deliver two or three times the powers indicated in Fig. 1. Sivian, Dunn, and White also found that for the whole frequency band the peak powers in some cases reached values as high as 65 watts. In order to go 8 db above this value, each channel would have to be capable of delivering in the neighborhood of 135 watts.

The chart (Fig. 1) shows that the orchestra delivers sound of comparable intensity throughout practically the whole audible range. Although it is conceivable that the ear would not be capable of detecting a change in quality if some of the higher or lower frequencies were suppressed, measurements published by W. B. Snow [2] show that for any change in quality in any of the instruments to be undetectable the frequency band should extend from about 40 to about 13,000 c.p.s. The necessary frequency ranges that must be transmitted to obviate noticeable change in quality for the different orchestral instruments are indicated in the chart of Fig. 2, which is taken from the paper by Snow.

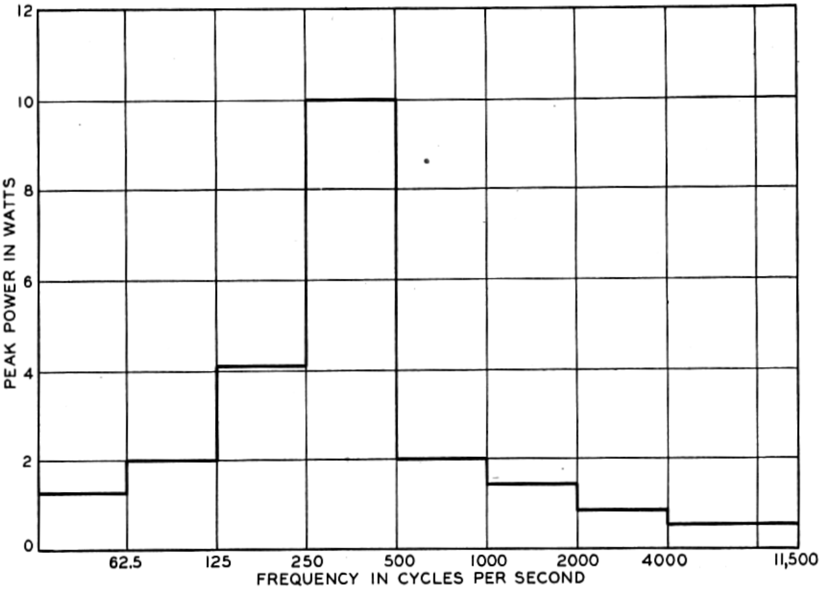

Fig. 1—Peak powers delivered by an orchestra within various frequency regions.

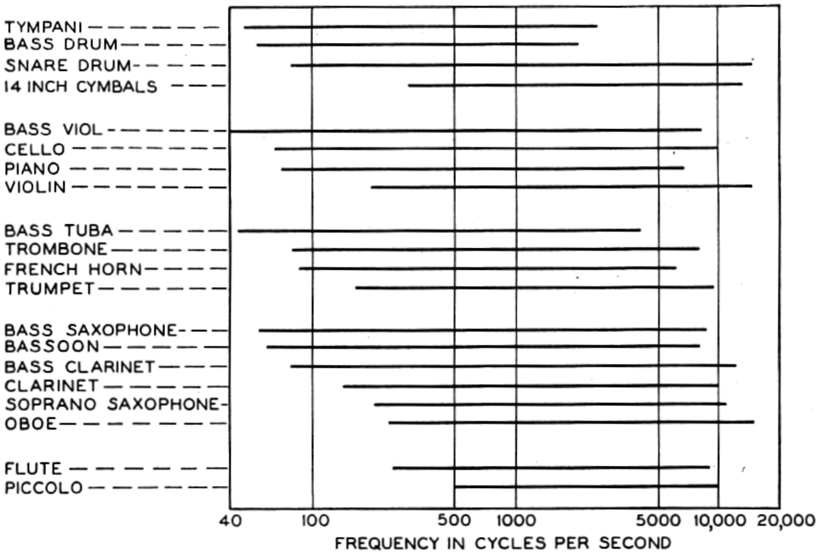

Fig. 2—Frequency transmission range required to produce no noticeable distortion for orchestral instruments.

Thus far only the sound generated by the orchestra itself has been considered. However, it is well known that the esthetic value of orchestral music in a concert hall is dependent to a very great extent upon the acoustic properties of the hall. At first thought one might be inclined to leave this out of account in considering the reproduction by a loud speaking system, as one should normally choose a hall known to have satisfactory acoustics for an actual orchestra. There would be no further problem in this if the orchestral instruments and the loud speaker radiated the sound uniformly in all directions, but some of the important instruments are quite directive; i.e., they radiate much the greater portion of their sound through a relatively small angle. As an example, a polar diagram giving the relative intensities of the sound radiated in various directions by the violin is given in Fig. 3, which is taken from a paper published by Backhaus.[3] The directional characteristics of some of the instruments is one of the chief reasons why the music from an orchestra does not sound the same in all parts of a concert hall. The music which we hear comes to us in part directly and in part indirectly; i.e., after one or more reflections from the walls. Both contribute to the esthetic value of the music. The ratio of the direct to the indirect sound, which has been designated by Hughes[4] as the *acoustic ratio*, is to a first approximation inversely proportional to the product of the reverberation time and the angle through which the sound is radiated.[5] For a steady tone by far the greater part of the intensity at a given point in a hall remote from the source is attributable to the indirect sound. However, inasmuch as many of the tones of a musical selection are

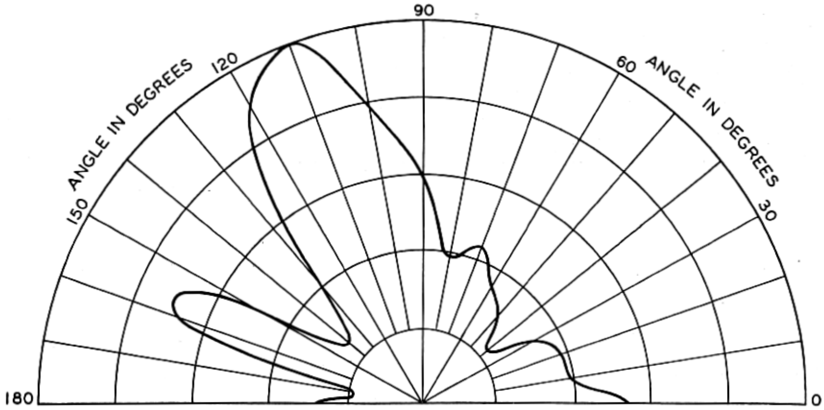

Fig. 3—Variation of intensity with direction of the sound radiated by a violin (660 c.p.s.).

of short duration, the direct sound is of great importance; it is this sound alone which enables us to localize the source. So far as this ratio is concerned, a decrease in the radiating angle of a loud speaker is equivalent to a reduction in the reverberation time of the hall. The effect on the music, however, is not entirely equivalent, for the rate of decay of sound in the room is unaltered by a change in directivity of the source, as this depends only on the reverberation time.

As already pointed out, some of the instruments of the orchestra are quite directive and others are nondirectional. In general, it may be said that the instruments of lower register are less directive than those of higher register. To have each instrument as reproduced by the loud speaker sound just as the instrument itself would sound in the same hall, the loud speaker would have to reproduce the music from each instrument with a directivity corresponding to that of the instrument itself. This manifestly is impossible. The best that can be hoped for is a compromise. Let the loud speaking system be designed so that it is nondirective for the lower frequencies, and at the higher frequencies it will radiate the sound through a larger angle than the most directive of the instruments and through a smaller angle than the least directive. Although this compromise means that the individual instruments will not sound exactly like the originals, it carries with it one advantage: At all the seats in the hall included in the radiating angle and at a given distance from the loud speaker the music may be heard to equal advantage, whereas with the orchestra itself the most desirable seats comprise only a certain portion of the hall. The optimum radiating angle is largely a matter of judgment; if it is too small the music will lack the spatial quality experienced at indoor concerts; if it is too large there will be a loss in definition.

There is another respect in which the directivity of the source can greatly affect the tone quality. Most loud speakers radiate tones of low frequency through a relatively large angle, but as the frequency is increased this angle becomes smaller and smaller. Under this condition the relation between the intensities of the high and low frequency tones as received directly will be different for almost all parts of the hall. Hence, even with equalization by electrical networks, the reproduction at best can be good only at a few places in the hall. Therefore, the sound radiated not only should be contained within a certain solid angle, but the radiation throughout this angle should be uniform at all frequencies.

THE LOUD SPEAKER

At present two kinds of loud speakers are in wide commercial use, the direct radiating and the horn types. Each has its merits, but the latter was used in the Philadelphia-Washington experiment because it appears to have definite advantages where such large amounts of power are to be radiated. The horn type can be given the desired directive properties more readily, and higher values of efficiency throughout a wide frequency range are more easily realized. In consideration of the large power requirements, high efficiency is of special importance because it will keep to the lowest possible value the power capacity requirements of the amplifiers and because, with the heating proportional to one minus the efficiency, the danger of burning out the receiving units is reduced.

For efficiently radiating frequencies as low as 40 c.p.s., a horn of large dimensions is required. In order that the apparatus may not become too unwieldly the folded type of horn is preferable, but a large folded horn transmits high frequency tones very inefficiently. As actually used, therefore, the loud speaker was constructed in two units: one for the lower and the other for the higher frequencies, an electrical network being used to divide the current into two frequency bands, the point of division being about 300 c.p.s.

THE LOW FREQUENCY HORN

When moderate amounts of power are transmitted through a horn the sound waves will suffer very little distortion, but when the power per unit area becomes large, second-order effects, usually neglected in considering waves of small amplitude, must be taken into account. The transmission of waves of large amplitude through an exponential horn has been investigated theoretically by M. Y. Rocard.[6] His investigation shows that if W watts are transmitted through the throat of an exponential horn a second harmonic of intensity RW will be generated, where R is given by the relation

$$R = \frac{(\gamma + 1)^2 f^2 \times 10^7 W}{2\rho c^3 f_0^2 A}, \tag{1}$$

in which f is the frequency of the fundamental, f_0 the cut-off frequency of the horn, c the velocity of sound, ρ the density of air, and A the area of the throat of the horn, all expressed in c.g.s. units. It may be noted that the intensity of the harmonic increases with the ratio of the frequency to the cut-off frequency of the horn; this is another argument against attempting to cover too wide a range of frequencies with a single horn. In Fig. 1 it is shown that in the region of 200 c.p.s. the orchestra gives peak powers of about 10 watts. If, therefore, 30 watts be set as the limit of power that the horn is to deliver at 200 c.p.s., 32 c.p.s. as the cut-off frequency of the horn, and 30 db below the fundamental be assumed as the limit of tolerance of a second harmonic, from equation (1) a throat diameter of about 8 inches is determined.*

If the radiation resistance at the throat of a horn is not to vary appreciably with frequency, the mouth opening must be a substantial fraction of a wave-length. This condition calls for an unusually large horn if frequencies down to 40 c.p.s. and below are to be transmitted. However, the effect of variations in radiation resistance on sound output can be kept down to a relatively small value if the receiving unit is properly designed. This will be explained in the next section. The low frequency horn used in these reproductions has a mouth opening of about 25 square feet. As computed from well-known formulas [7] for the exponential horn the impedance of this horn

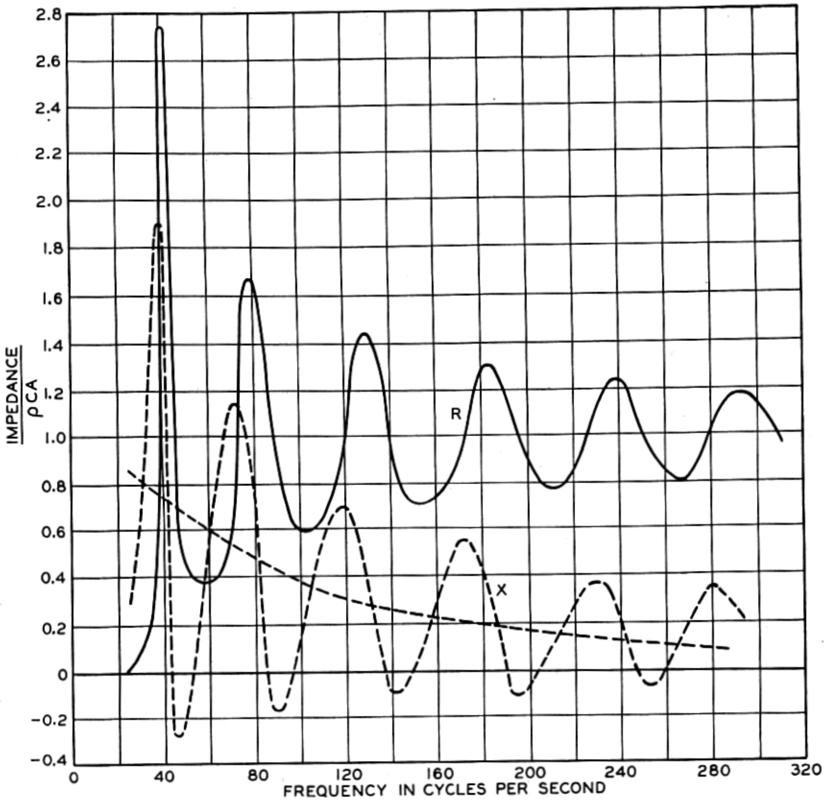

Fig. 4—Radiation resistance and reactance of low frequency horn.

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* Since the original publication of this paper, experimental data have been obtained which indicate a second harmonic generation in horns 6 or more db below the value shown by Rocard's equation.[9]

with a throat diameter of 8 inches is shown in Fig. 4. These curves were computed under the assumption that the mouth of the horn is surrounded by a plane baffle of infinite extent, a condition closely approximated if the horn rests on a stage floor.

LOW FREQUENCY RECEIVING UNIT

When a moving coil receiving unit, coupled to a horn, is connected to an amplifier having an output resistance equal to $n - 1$ times the damped resistance R of the driving coil, it can easily be shown that the sound power output is

$$P = \frac{\left(\dfrac{EBLT}{nR}\right)^2 r \times 10^{-9}}{\left[T^2 r + \dfrac{B^2L^2 \times 10^{-9}}{nR}\right]^2 + [x_d + T^2 x]^2} \text{ watts,} \tag{2}$$

where E is the open circuit voltage of the amplifier, L the length of wire in the receiver coil, T the ratio of the area of the diaphragm to the throat area of the horn, $r + jx$ the throat impedance of the horn, and x_d the mechanical reactance of the diaphragm and coil, the mechanical resistance of which is assumed to be negligibly small. From Fig. 4 it may be seen that the mean value of x increases as the frequency decreases to a value below 40 c.p.s., and that x is smaller than r except at the very lowest frequencies. If, therefore, the stiffness of the diaphragm be adjusted so that x_d is equal to T^2 times the mean value of x at 40 c.p.s., the second term in the denominator may be neglected without much error because it will have but little effect upon the sound output except at the higher frequencies, where the mass reactance of the coil and diaphragm may have to be taken into account.

If minimum variations in sound output are desired for variations in r,

$$\frac{B^2L^2 10^{-9}}{nRT^2} = r_0, \tag{3}$$

where r_0 is equal to the geometric mean value of r, which is approximately equal to $A\rho c$.

If α is the ratio of the resistance at any frequency to the mean value, and if the second term in the denominator is neglected, equation (2) becomes

$$P = \frac{E^2}{nR}\frac{\alpha}{(1 + \alpha)^2}\cdot \tag{4}$$

In Fig. 4 it is shown that above 35 c.p.s. α has extreme values of 2.75 and 0.36, at which points there will be minimum values in P, but these minimum values will not lie more than 1 db below the maximum values. Hence, if the receiver satisfies the condition of equation (3), the extreme variations in the sound output will not exceed 1 db, although the horn resistance varies by a factor of 7.5. Also it may be stated here that when the condition of equation (3) is satisfied the horn is terminated at the throat end by a resistance equal to the surge resistance of the horn. Thus equation (3) establishes a condition of minimum values in the transient oscillations of the horn.

The mean motional impedance of the loud speaker is $\frac{B^2L^2 \times 10^{-9}}{T^2r_0}$, which, from equation (3), is equal to nR. The condition of equation (3) therefore specifies that the efficiency of the loud speaker shall be $\frac{n}{n+1}\cdot$ The maximum power that an amplifier can deliver without introducing harmonics exceeding a specified value is a function of the impedance into which it operates. Therefore, to obtain the maximum acoustic power for a specified harmonic content, the load impedance should have the value for which the product of the loud speaker efficiency and the power capacity of the amplifier has a maximum value. This optimum value of load impedance for the amplifier and loud speaker used in the Philadelphia-Washington experiments was found to be about 2.25 times the output impedance of the amplifier; the corresponding value of n then is 2.6 and the required efficiency 72 per cent. For best operating condition a definite value of receiver efficiency thus is specified.

The receiver may be made to satisfy the foregoing conditions regardless of the value of T, the ratio of diaphragm area to throat area. The area of the diaphragm has, however, a definite relation to the maximum power that the receiver can deliver at the low frequencies. The peak power delivered by the receiver is equal to $T^2\alpha r_0\xi^2\omega^2 \times 10^{-7}$ peak watts where ξ is the maximum amplitude of motion of the diaphragm. Figure 1 shows that in the region lying between 40 and 60 c.p.s., peak powers reach a value of from 1 to 2 watts. However, the low frequency tones of an orchestra are undesirably weak and may advantageously be reproduced at a relatively higher level. Therefore it was decided to construct the loud speaker to be able to deliver 25 watts in this region.

As the coil moves out of its normal position in the air gap, the force factor varies. Harmonics thus will be generated, the intensities of which increase with increasing amplitude. A limit to the maximum value of the amplitude ξ thus is set by the harmonic distortion that one is willing to tolerate. In this receiver the maximum value of ξ

was taken equal to 0.060 in. Figure 4 shows that $\alpha\omega^2$ has a minimum value at about 50 cycles, where α is equal to about 0.4. These values give a ratio of 4.5 for T.

Inasmuch as $R = \dfrac{\sigma L^2}{v}$, where σ is the resistivity of the wire used for the coil and v the volume of the coil, from equation (3) is obtained

$$B^2 v = n\sigma T^2 r_0 10^9. \tag{5}$$

The first member gives the total magnetic energy that must be set up in the region occupied by the driving coil. This value is fixed by the fact that all factors in the second member are specified. The same performance is obtained with a small coil and high flux density as with a large coil and low flux density, provided B^2v is held fixed, but the coil in any case should not be made so small that it will be incapable of radiating the heat generated within it without danger of overheating, nor so large that the mass reactance of the coil will reduce the efficiency at the higher frequencies.

This receiver unit, when constructed according to the above principles and when connected to an amplifier and a horn in the specified manner, should be capable of delivering power 3 or 4 times that delivered by the orchestra in the frequency region lying between 35 and 400 c.p.s., with an efficiency of about 70 per cent, and with a variation in sound output for a given input power to the amplifier of not more than 1 db throughout this range.

THE HIGH FREQUENCY HORN

It is well known that a tapered horn of the ordinary type has a directivity which varies with frequency. Sound of low frequency is projected through a relatively large angle. As the frequency is increased this angle decreases progressively until, at frequencies for which the wave-length is small compared with the diameter of the mouth opening, the sound beam is confined to a very narrow angle about the axis of the horn.

If we had a spherical source of sound (i.e., a source consisting of a sphere, the surface of which has a radial vibratory motion equal in phase and amplitude at every point of the surface), sound would be radiated uniformly outward in all directions; or, if we had only a portion of a spherical surface over which the motion is radial and uniform, uniform sound radiation still would prevail throughout the solid angle subtended at the center of curvature by this portion of the sphere, provided its dimensions were large compared with the wave-length. Throughout this region the sound would appear to originate at the center of curvature. Hence, for the ideal distribution of a spherical source within a region to be defined by a certain solid angle, it is necessary and sufficient that the radial motion be the same in amplitude and phase over the part of a spherical surface intercepted by the angle and having its center of curvature at the vertex and located at a sufficient distance from the vertex to make its dimensions large compared with the wave-length. If, further, these conditions are satisfied for this surface at all frequencies, all points lying within the solid angle will receive sound of the same wave form. A horn was designed to meet these requirements for the high frequency band.

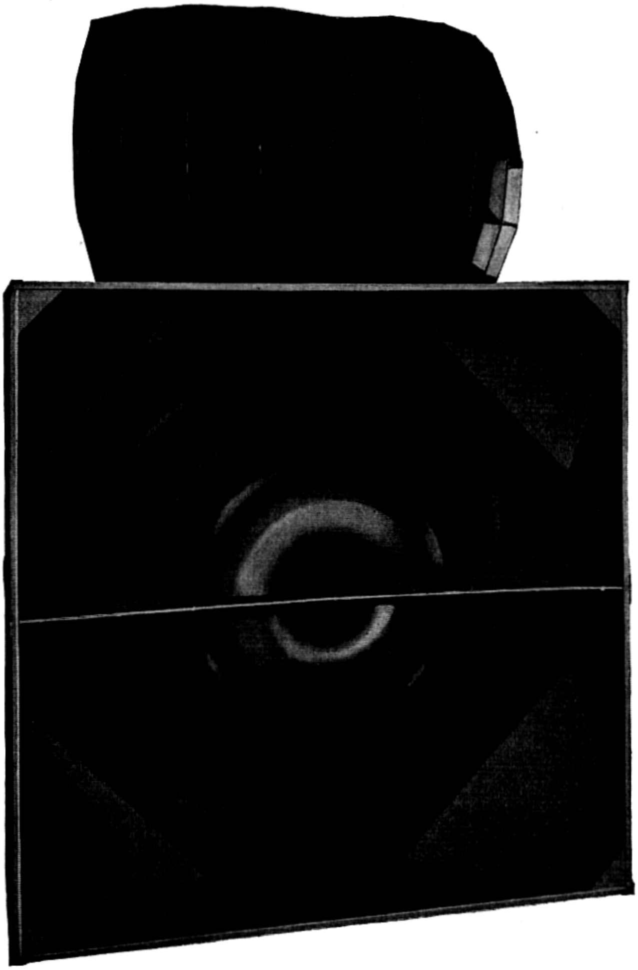

Fig. 5—Special loud speaker developed for auditory perspective experiment.

The horn, shown in the upper part of Fig. 5, comprises several separate channels, each of which has substantially an exponential taper. Toward the narrow ends these channels are brought together with their axes parallel, and are terminated into a single tapered tube which at its other end connects to the receiver unit. Sound from the latter is transmitted along the single tube as a plane wave and is divided equally among the several channels. If the channels have the same taper, the speed of propagation of sound in them is the same. The large ends are so proportioned and placed that the particle motion of the air will be in phase and equal over the mouth of the horn. This design gives a true spherical wave front at the mouth of the horn at all frequencies for which the transverse dimensions of the mouth opening are a large fraction of a wave-length.

As the frequency is increased, the ratio of wave-length to transverse width of the channels becomes less, and the sound will be confined more and more to the immediate neighborhood of the axis of each channel. The sound then will not be distributed uniformly over the mouth opening of the horn, but each channel will act as an independent horn. To have a true shperical wave front up to the highest frequencies, the horn would have to be divided into a sufficient number of channels to make the transverse dimension of each channel small compared with the wave-length up to the highest frequencies. If it is desired to transmit up to 15,000 c.p.s., it is not very practical to subdivide the horn to that extent. Both the cost of construction and the losses in the horn would be high if designed to transmit also frequencies as low as 200 c.p.s., as is the case under consideration. However, it is not important that at very high frequencies a spherical wave front be established over the whole mouth of the horn. For this frequency region it is perfectly satisfactory to have each channel act as an independent horn, provided that the construction of the horn is such that the direction of the sound waves coming from the channels is normal to the spherical wave front.

The angle through which sound is projected by this horn is about 60 degrees, both in the vertical and in the horizontal direction. For reproducing the orchestra two of these horns, each with a receiving unit, were used. They were arranged so that a horizontal angle of 120 degrees and a vertical angle of 60 degrees were covered. These angular extensions were sufficient to cover most of the seats in the hall with the loud speaker on the stage. The vertical angle determines to a large extent the ratio of the direct to the indirect sound transmitted to the audience. The vertical angle of 60 degrees was chosen purely on the basis of judgment as to what this ratio should be for the most pleasing results.

The High Frequency Receiving Unit

In the design of the low frequency receiver one of the main objectives was to reduce to a minimum the variations in sound transmission resulting from variations in the throat impedance of the horn. However, the high frequency horn readily can be made of a size such that the throat resistance has relatively small variations within the transmitting region. On the other hand, whereas the diameter of the diaphragm of the low frequency unit is only a small fraction of the wave-length, that of the high frequency unit must be several wave-lengths at the higher frequencies in order to be capable of generating the desired amount of sound. Unless special provisions are made there will be a loss in efficiency because of differences in phase of the sound passing to the horn from various parts of the diaphragm. The high frequency receiver therefore was constructed so that the sound generated by the diaphragm passes through several annular channels. There are enough of these channels to make the distance from any part of the diaphragm to the nearest channel a small fraction of a wave-length. These channels are so proportional that the sound waves coming through them have an amplitude and phase relation such that a substantially plane wave is formed at the throat of the horn.

In the appendix it is shown that, for the higher frequencies where the impedance of the horn may be taken as equal to ρc times the throat area and for the type of structure adopted, the radiation resistance is equal to

$$\rho c a T^2 \left[\frac{1}{k^2 h^2 T^2 + k^2 l^2 \cot^2 kl} \right] \tag{6}$$

and the reactance

$$-j \frac{\rho c a}{kh} T \left[1 - \frac{1}{kl \cot kl + \left(\frac{hT}{l} \right)^2 kl \tan kl} \right], \tag{7}$$

where a is the area of the throat of the horn, T the ratio of the area of the diaphragm to the throat area, $k = \omega/c$, and the other designations are those indicated in Fig. 11. At the lower frequencies the resistance is $T^2 r$ and the reactance $T^2 x$, where r and x are, respectively, the resistance and reactance of the throat of the horn.

Equation 6 shows that at a given frequency, other conditions remaining the same, the radiation resistance will have a maximum value when l is approximately equal to $\pi/2k = c/4f$. In Fig. 6 the resistances as computed from equation (6) are plotted as a function of frequency for several values of h/w. It is seen from these curves that the resistance at the higher frequencies is determined very largely by the relation of h/w but is independent of it at the lower frequencies, where it is equal to ρcaT^2. At the lower frequencies where the mechanical impedance of the diaphragm is negligible, the efficiency, as was the case for the low frequency receiver, depends

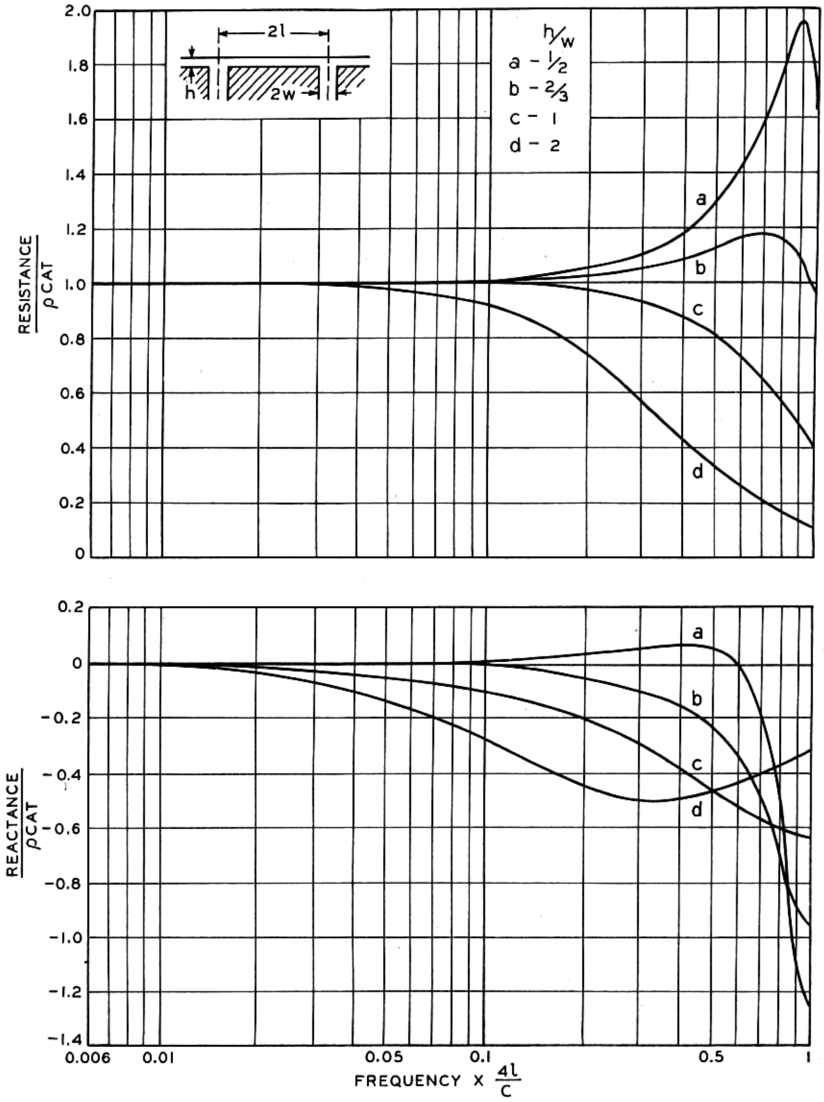

Fig. 6—Load impedance of speaker diaphragm.

upon the value of B^2v where v is the volume of the coil, but at the higher frequencies the efficiency decreases with increasing mass of the coil. It is advantageous, therefore, to keep v small and to make B as large as is practically possible. Values were selected to give the receiver an efficiency of 55 percent at the lower frequencies. For these conditions the relative sound power output was computed by equation (2) on the assumption that the receiver was connected to an amplifier having an output impedance equal to 0.45 times that of the receiver at the lower frequencies. Figure 7 shows the values so obtained. Corresponding values obtained experimentally when the receiver was connected to the horn previously described are shown in Figs. 8 and 9, where the sizes of the rooms in which the values were obtained were, respectively, 5000 and 100,000 cubic feet. Both of these curves differ considerably from the computed curve, particularly as regards loss at high frequencies. The curve of Fig. 8 shows less, and that of Fig. 9 more, loss at high frequencies. The computed curve, however, refers to the total sound output, whereas the measured curves give average values of sound intensity in a certain part of the room, values dependent upon the acoustic characteristics of the room.

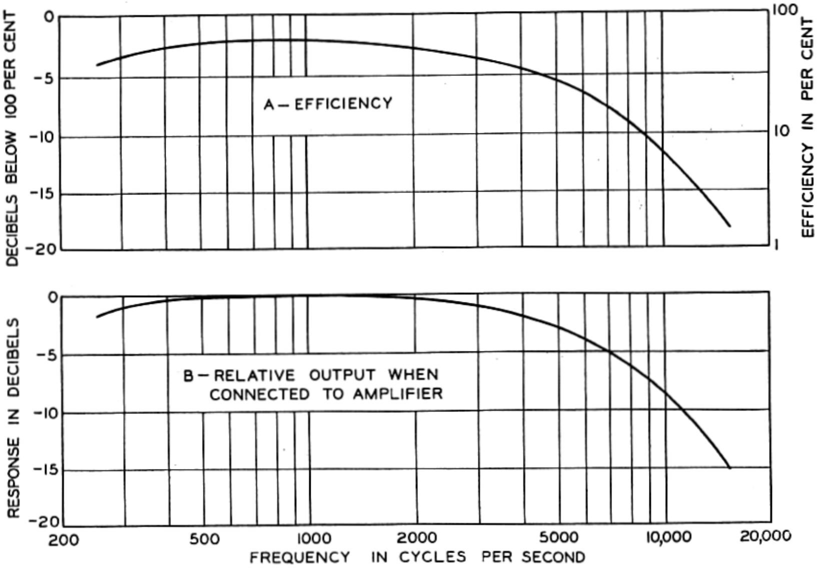

Fig. 7—Relative computed sound output of high frequency receiver.

The number of high frequency receivers that must be used for each transmitting channel is governed largely by the amount of power that the system is to deliver before harmonics of an objectionable intensity

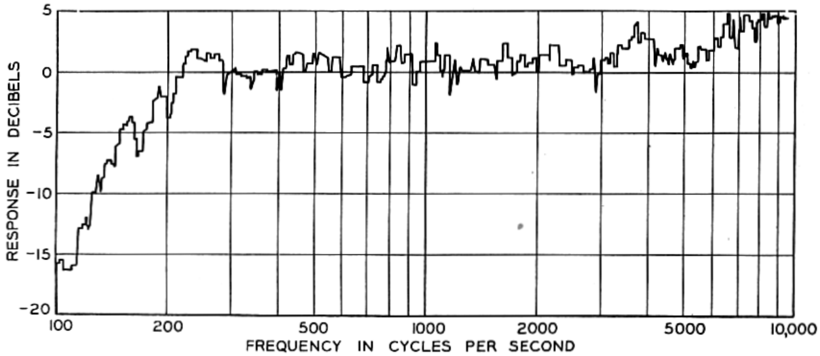

Fig. 8—Output-frequency characteristic of high frequency receiver as measured in a small room.

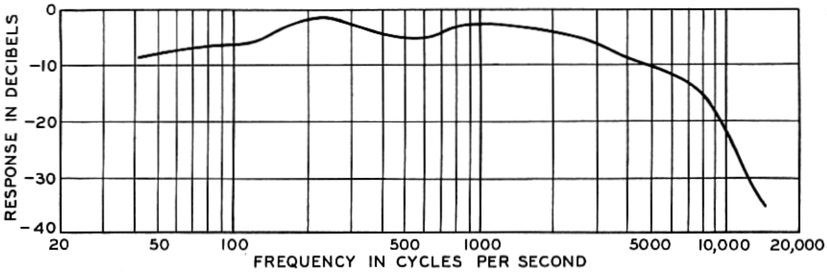

Fig. 9—Output-frequency characteristic of combined low and high frequency receivers as measured in a large room.

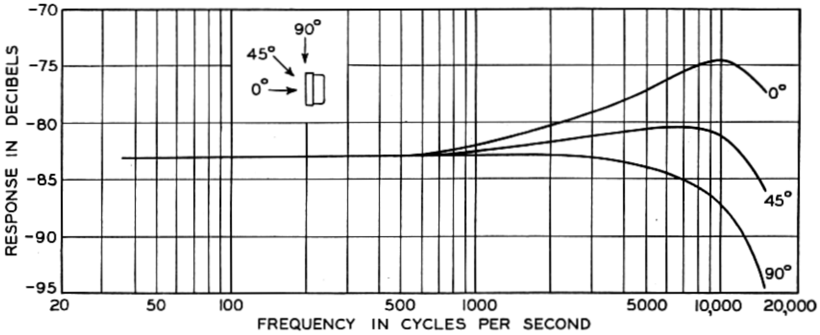

Fig. 10—Output-frequency characteristic of moving coil microphone.

are introduced. The generation of harmonics in a horn when transmitting waves of large amplitude already has been discussed. Let it suffice here to say that, for a given percentage harmonic distortion, the power that can be transmitted through the horn is proportional to the area of the throat and inversely proportional to the square of the ratio of the frequency to the cut-off frequency.

Inasmuch as the moving coil microphones used for the transmission of music in acoustic perspective have been described previously [8] they will not be discussed here at length. Their frequency response characteristic as measured in an open sound field for several different angles of incidence of the sound wave on the diaphragm are shown in Fig. 10 where it is seen that the response at the higher frequencies becomes less as the angle of incidence is increased. In general, this is not a desirable property, but with the instruments as used in this experiment the sound observed as coming from each loud speaker is mainly that which is picked up directly in front of each microphone; sound waves incident at a large angle do not contribute much.

At certain times the sound delivered by the orchestra is of very low intensity. Therefore it is important that the microphones have a sensitivity as great as possible, so that the resistance and amplifier noises may readily be kept down to a relatively low value. At 1,000 c.p.s. these microphones, without an amplifier, will deliver to a transmission line 0.05 microwatt when actuated by a sound wave having an intensity of 1 microwatt per square centimeter. This sensitivity is believed to be greater than that of microphones of other types having comparable frequency response characteristics, with the possible exception of the carbon microphone.

APPENDIX

Load Impedance of a Diaphragm Near a Parallel Wall with Slot Openings

First assume a diaphragm and a parallel wall of infinite extent separated by a distance h, and that the wall is slotted by a series of equally spaced openings as shown in Fig. 11. From symmetry it is known that when the diaphragm vibrates there will be no flow perpendicular to the plane of the paper or across the planes indicated by the dotted lines. Therefore only one portion of unit width, such as *abcdef* need be considered. Let the x and y reference axes be located as shown. If the general field equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + k^2 \varphi = 0 \qquad (8)$$

is applied when the diaphragm has a normal velocity equal $\dot{\xi}e^{i\omega t}$ the following boundary conditions are obtained:

When
$$x = 0, \quad \partial\varphi/\partial x = -\dot{\xi},$$
$$x = h, \quad \partial\varphi/\partial x = 0,$$
$$y = 0, \quad \partial\varphi/\partial y = 0,$$

and when $y = l$, the pressure is equal to the product of acoustic impedance and volume velocity or

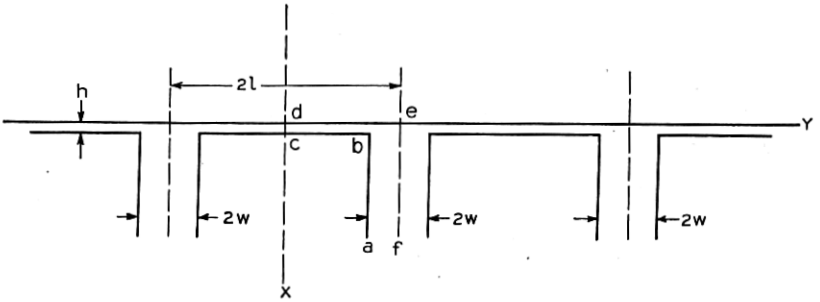

Fig. 11—Schematic diagram of diaphragm and parallel slotted wall of infinite length.

$$\frac{\rho}{h}\int_0^h \left(\frac{\partial\varphi}{dt}\right)_{y=l} dx = \frac{c\rho}{w}\int_0^h \left(-\frac{\partial\varphi}{\partial y}\right)_{y=l} dx$$

where φ is the velocity potential, $k = \omega/c$, and c is the velocity of sound.

The appropriate solution of equation (8) then is

$$\varphi = \frac{\dot{\xi}}{k}\left[\frac{\cos ky}{kh\left(\cos kl + i\dfrac{h}{w}\sin kl\right)} - \frac{\cos k(x-h)}{\sin kh}\right].$$

The average reacting force per unit area of the diaphragm is

$$\frac{ik\rho c}{l}\int_0^l (\varphi)_{x=0}\, dy$$

Thus, for the impedance per unit area, which is equal to the force divided by the velocity, is obtained

$$\frac{\rho c l}{w}\left\{\left[\frac{\sin^2 kl}{k^2 l^2}\,\frac{1}{\cos^2 kl + \left(\dfrac{h}{w}\right)^2 \sin^2 kl}\right] - j\frac{w}{h}\left[\frac{\dfrac{kh\cos kh}{\sin kh}\,kl - \dfrac{\sin kl\cos kl}{\cos^2 kl + \left(\dfrac{h}{w}\right)^2\sin^2 kl}}{k^2 l^2}\right]\right\} \equiv r' + jx'.$$

In all practical types of loud speakers $kh \cos kh/\sin kh$ would be very nearly equal to 1; then

$$r' = \frac{\rho c l}{w}\left[\frac{1}{k^2l^2\left(\left(\dfrac{h}{w}\right)^2 + \cot^2 kl\right)},\right]$$

$$x' = -\frac{\rho c l}{h}\left[\frac{kl - \dfrac{1}{\cot kl + \left(\dfrac{h}{w}\right)^2 \tan kl}}{k^2l^2}\right].$$

If the total area of the diaphragm is A and that of the corresponding channels a, then $A/a = l/w$, approximately, and the total impedance becomes

$$r = \frac{\rho c A^2}{a}\cdot\frac{1}{\left(\dfrac{kh}{a}\right)^2 A^2 + k^2l^2 \cot^2 kl},$$

$$x = -j\frac{\rho c A}{kh}\left[1 - \frac{1}{kl \cot kl + \left(\dfrac{h}{l}\dfrac{A}{a}\right)^2 kl \tan kl}\right].$$

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