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Ultra-Short Wave Propagation *

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Part I of this paper first describes a method of measuring attenuation and field strength in the ultra-short wave range. A résumé of some of the quantitative experiments carried out in the range between 17 mc. (17 meters) and 80 mc. (3.75 m.) and with distances up to 100 km. is then given. Two cases are included: (1) "Optical" paths over sea-water and (2) "Non-optical" paths over level and hilly country. An outstanding result is that the absolute values of the fields measured were always less than the inverse distance value. Over sea-water, the fields decreased as the frequency increased from 34 mc. (8.7 m.) to 80 mc. (3.75 m.) while the opposite trend was found over land. As a rule, the signals received were very steady, but some evidence of slow fading was obtained for certain cases when the attenuation was much greater than that for free space.

Part II gives a discussion of reflection, diffraction and refraction as applied to ultra-short wave transmission. It is shown, (1) that regular reflection is of importance even in the case of fairly rough terrain, (2) that diffraction considerations are of prime importance in the case of non-optical paths, and (3) that refraction by the lower atmosphere can be taken into account by assuming a fictitious radius of the earth. This radius is ordinarily equal to about 4/3 the actual radius.

The experiments over sea-water are found to be consistent with the simple assumption of a direct and a reflected wave except for distances so great that the curvature of the earth requires a more fundamental solution. It is shown that the trend with frequency to be expected in the results for a non-optical path over land is the same as that actually observed, and that in one specific case, which is particularly amenable to calculation, the absolute values also check reasonably well. It is found both from experiment and from theory that non-optical paths do not suffer from so great a disadvantage as has usually been supposed.

Several trends with respect to frequency are pointed out, two of which, the "conductivity" and the "diffraction" trends, give decreased efficiency with increased frequency, and another of which, the "negative reflection" trend, gives increased efficiency with increased frequency under the conditions usually encountered.

The existence of optimum frequencies is pointed out, and it is emphasized that they depend on the topography of the particular paths, and that different paths may therefore have widely different optimum frequencies. Thus, among the particular cases mentioned, the lowest optimum values vary from frequencies which are well below the ultra-high frequency range up to 1200 mc. (25 cm.). For other paths the lowest optimum frequency may be still higher.

Introduction

WITH the extension of the radio frequency spectrum to higher and higher frequencies have come new problems, both of experiment and of theory, which require quantitative study for solution.

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The fundamental similarity of visual light and radio waves makes it obvious that somewhere between these regions a transition region must occur in which the apparently different phenomena merge into each other. In theoretical studies of this region it is necessary to use concepts borrowed from both the adjacent frequency ranges. A survey of a part of this field has now been in progress for some time and some of the results obtained to date are given in this paper and in a companion paper by Englund, Crawford and Mumford.

Since the Kennelly-Heaviside layers do not reflect ultra-short waves sufficiently to be a factor in the ordinary phenomena of this range, our interest is confined to the "ground" or direct wave. This term refers to any and all signals which arrive at the receiver except those which are affected by the upper atmosphere. It is otherwise non-committal as to the mechanism of transmission. The physical pictures of this mechanism which have been so useful in the case of long waves are of little help when the length of the wave is of the order of, or smaller than, the dimensions of irregularities of topography which it encounters. The well-known work of Abraham,[1] Zenneck,[2] Sommerfeld [3] and the more recent studies by Weyl,[4] Eckersley,[5] Strutt,[6] and Wise [7] apply to special cases of ultra-short wave propagation, but generally speaking help but little in the more numerous problems where irregularity of topography is the rule. Likewise, the important work of Watson [8] and of Van der Pol [8] may perhaps find application in the diffraction problems of ultra-short waves, but only to a limited extent.

It is obvious that rigorous solutions of problems in transmission over rough surfaces are out of the question, but progress can be made by way of the general concepts of reflection, diffraction and refraction. We shall endeavor to show that many phenomena observed can be

[1] Abraham, M., *Enz. d. math. Wissen.*, 5, Art. 18.

[2] Zenneck, J., "Über die Fortpflanzung ebenen elektromagnetischer Wellen langs einer ebenen Leiterflache und ihre Beziehung zur drahtlosen Telegraphie," *Ann. d. Phys.*, 4, 23, 846 (1907).

[3] Sommerfeld, Arnold, "Über die Ausbreitung der Wellen der Drahtlosen Telegraphie," *Ann. d. Phys.*, 4, 28, 665–736, Mar. 1909, and "Ausbreitung der Wellen in der drahtlosen Telegraphie. Einfluss der Bodenbeschaffenheit auf gerichtete und ungerichtete Wellenzuge," *Jahr. d. drahtlosen, Tel. u. Tel.*, 4, 157 (1911).

[4] Weyl, H., "Ausbreitung elektromagnetischer Wellen über einer ebenen Leiter," *Ann. d. Phys.*, 4, 60, 481–500 (1919).

[5] Eckersley, T. L., "Short-Wave Wireless Telegraphy," *Jour. I. E. E.*, 65, 600–644, June 1927.

[6] Strutt, M. J. O., "Strahlung von Antennen unter dem Einfluss der Erdbodeneigenschaften," *Ann. d. Phys.*, 5, 1, 721–772 (1929); 4, 1–16 (1930); 9, 67–91 (1931).

[7] Wise, W. Howard, "Asymptotic Dipole Radiation Formulas," *Bell Sys. Tech. Jour.*, 8, 662–671, Oct. 1929.

[8] Watson, G. N., "The Diffraction of Electric Waves by the Earth," *Proc. Roy. Soc.* (London), 95, 83–99, Oct. 7, 1918. Van der Pol, Balth., "On the Propagation of Electromagnetic Waves Around the Earth," *Phil. Mag.*, 6, 38, 365–380, Sept. 1919.

explained quantitatively in this way. Reflection, diffraction and refraction all play their parts.

On the experimental side, the longer distance ultra-short wave transmission studies described in the literature have been made almost exclusively with apparatus capable of making only qualitative measurements. In spite of this handicap, many valuable observations have been made.[9] The outstanding result of these has been the demonstration of the advantages of an "optical" path, or rather, one in which a straight line between the transmitting and receiving antennas is unbroken by the intervening terrain. In many cases, however, this advantage has been greatly over-emphasized.

As a basis for studying the relative importance of the various mechanisms that have been suggested, quantitative measurement must replace qualitative observation. Part I of this paper presents some of the results of an experimental study of the propagation of ultra-short waves, made with the objective of obtaining quantitative data of sufficient accuracy to serve as a basis for theoretical work. Part II discusses the theory of ultra-short wave transmission and analyzes some of the experimental results from that point of view.

Part I—Experiment

Equipment and Procedure

A considerable portion of the transmitting in connection with this survey was done with a 1000-watt transmitter located at Deal, N. J. In this transmitter the last stage employed four 1000-watt radiation-cooled tubes as an oscillator at 69 mc. The frequency was controlled by a 3833-kc. crystal oscillator acting through a chain of amplifiers and harmonic generators. A simple vertical half-wave antenna was used for most of the tests. It was located about 60 meters above ground and was driven through a long two-wire transmission line. The stability of this transmitter was a definite advantage and facilitated the taking of reliable data. Another transmitter of slightly higher power was employed for the lower frequency tests from Deal. Similar antennas were used.

For most of the over-water tests use was made of a mobile transmitter of some 100 watts output, while for some of the very short distance work, a simple portable oscillator using receiving tubes was employed. The radiator, a simple vertical antenna, was located on a wooden tripod on a bluff at Cliffwood Beach, N. J. This bluff over-

[9] On account of the extensiveness of these qualitative studies, no attempt is made to give a complete bibliography. A few articles giving results of especial interest in connection with the present paper are cited in the text.

indicated by the thermomilliammeter. So long as these were duplicated at both ends of the path, it was possible to determine the relative values of fields at the two ends, regardless of absolute errors. Investigation of the behavior of the meter and of the method in general, indicate that the absolute error itself is not large.

The map of Fig. 2 shows the locations of the transmitting and receiving sites used. The tests may be divided into two groups. Propagation over water was studied mainly with the transmitter

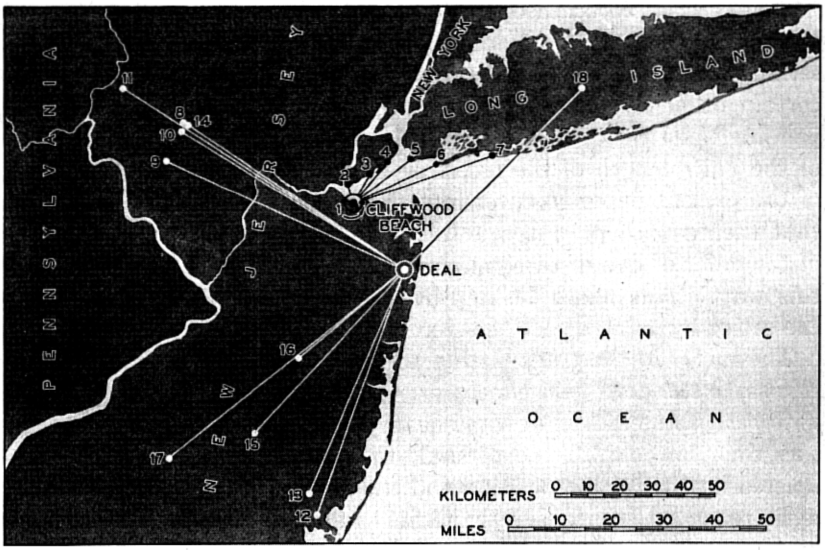


Fig. 2—Transmitting and receiving locations.

located on the bluff at Cliffwood Beach. Measurements on transmission over land were made from the transmitter at Deal. Lines radiating from these two points indicate the various transmission paths studied.

Transmission Over Sea Water

For the measurements on propagation over water at 34, 51 and 80 mc., the receiving antenna was located at the water's edge, except for a few special tests. The height of its midpoint was varied up to a maximum of about twelve meters above sea level. The data presented in Fig. 3 show the results with the maximum elevations and vertical polarization (vertical electric field).

This figure shows that the received field was below the inverse distance field that would result from radiation in free space.[13] The field strength is more nearly inversely proportional to the second than to the first power of the distance as may be seen by comparison with the light dashed line in Fig. 3.

In addition to the measurements taken on the ground, measurements on the highest frequency, 80 mc., were made with the receiver in an airplane.[14] The results are discussed later in connection with Fig. 11.

The effect of altitude was determined at two distances, 77 and 142 kilometers, using the Deal transmitter at 69 and 17 mc. The results

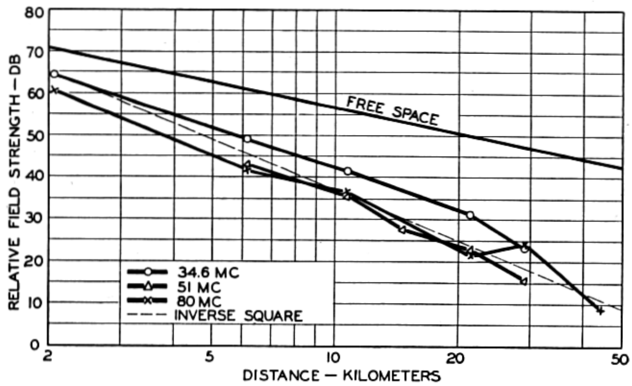


Fig. 3—Field strength as a function of distance for transmission over salt water from Cliffwood Beach.

are shown in Figs. 4 and 5. The increase of signal with elevation was much greater on the higher frequency than on the lower frequency. It is interesting to note, however, that if the field were plotted against altitude in *wave-lengths* the slopes would be approximately the same for the two frequencies. Significance should not be attached to the ratio of the field obtained on one frequency to that obtained on the other.

Transmission Over Land

The transmitters located at Deal were employed for studying the propagation of waves of 17, 34 and 69 mc. over various types of terrain. The transmission paths are shown by the lines radiating from Deal on the map of Fig. 2. Three types of paths are represented. The best for ultra-short wave work was found to be that with the other terminal on high ground, such as is found to the northwest. Another type, not so favorable to the transmission of ultra-short waves, but typical of flat country, could be studied by locating the receiving

[13] In free space, the field produced by a given current in a doublet is one half as great as that produced by the same current and doublet when located at and perpendicular to the surface of a perfect conductor.

[14] These measurements were possible through the cooperation of Mr. F. M. Ryan.

terminal to the south or southwest. Here the intervening ground is fairly level, and there are no high hills that can be used for the receiving terminal. The third type of path is mostly over water to points on

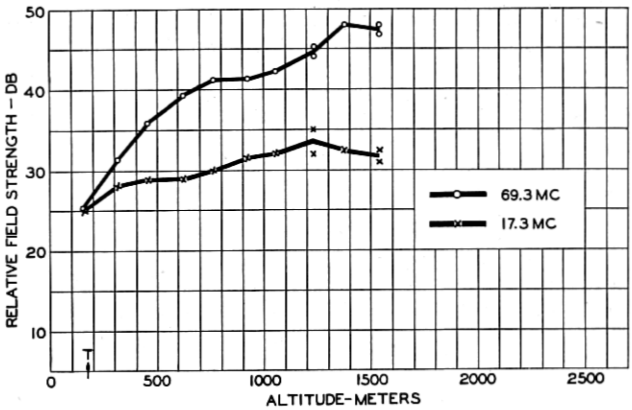


Fig. 4—Field strength as a function of receiver altitude at a distance of 77 km. The path was mostly over water. The arrow T shows the altitude at which the line of sight, neglecting refraction, becomes tangent to the earth's surface.

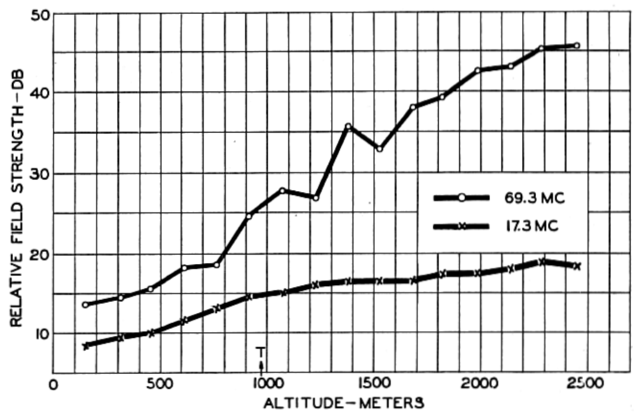


Fig. 5—Field strength as a function of receiver altitude at a distance of 141 km. The path was mostly over water. The arrow T shows the altitude at which the line of sight, neglecting refraction, becomes tangent to the earth's surface.

Long Island. Typical profiles of over-land paths are shown in Fig. 6.

The experimental results of transmission over these paths, together with some of their characteristics, are given in the table of Fig. 7. In the last three columns is given the received field in decibels below the free space value. At 69 mc. the best paths, 8 and 9, gave values which were 15 and 13 db below the inverse distance amplitude. The former gave 32 db at 17 mc. and the latter gave 28 db at 34 mc. In general, the highest frequency showed the smallest attenuation over land.

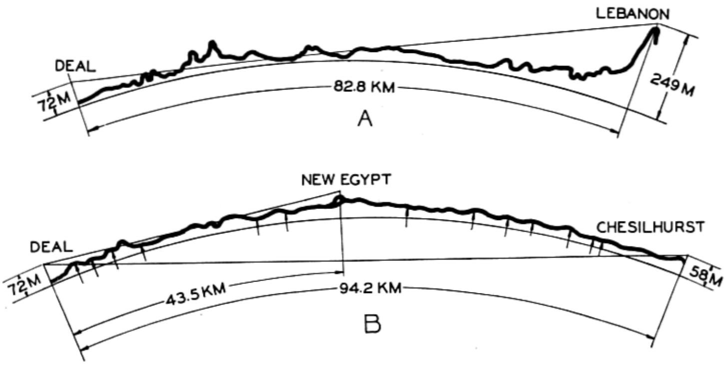


Fig. 6—Profiles of typical overland paths: *A*, path No. 8, over hilly country, with receiving location not masked by nearby hills; *B*, paths Nos. 16 and 17, over level country.

NO.	RECEIVING LOCATION	LAT W	LONG. N	ELEVATION m.	DISTANCE Km.	RECEIVED FIELD DB BELOW FREE SPACE VALUE		
						17.3mc	34.6mc	69.3mc
	HILLY COUNTRY OPEN SITE							
8	LEBANON 1	74°-51′-0′	40°-39′-9′	238	82.8	32.5		15.1
9	CHERRYVILLE	74°-53′-3′	40°-33′-4′	165	96.6		28.5	13.2
	HILLY COUNTRY MASKED SITE							
10	LEBANON 2	74°-51′-0′	40°-38′-5′	119	81.3	45.0	35.5	40.0
11	MONTANA	75°-4′-2′	40°-45′-3′	342	104.5	43.5	34.5	32.5
	LEVEL COUNTRY							
12	TUCKERTON 1	74°-22′-5′	39°-35′-2′	24	80.6	50.0	35.5	24.5
13	TUCKERTON 2	74°-23′-5′	39°-38′-5′	27	77.3	47.5	41.0	36.0
14	LEBANON 3	74°-49′-8′	40°-39′-2′	110	81.3	40.5	37.5	30.5
15	APPLE PIE HILL	74°-35′-5′	39°-48′-5′	63	69.2	40.0	35.0	27.7
16	NEW EGYPT	74°-25′-7′	40°-0′-9′	61	43.5			27.1
17	CHESILHURST	75°-53′-9′	39°-44′-2′	46	94.2			48.3
	OVER WATER							
18	HALF HOLLOW HILLS	73°-23′-3′	40°-47′-1′	73	81.3			31.5
6	ROCKAWAY BEACH	73°-54′-0′	40°-34′-0′	0	34.8		29.3	30.5
5	NORTONS POINT	74°-1′-0′	40°-34′-5′	0	35.1			28.4

Fig. 7—Table of data taken with transmitter at Deal.

It should be pointed out that these measurements are not independent of the local receiving conditions. The proximity of the ground has the effect of making the vertical directive characteristic far different from that of the same antenna in free space. In all cases the field increased as the receiving antenna was raised up to the maximum height available (12 m.). This effect of the ground was therefore more detrimental when the longer waves were used, since the antenna could not then be raised to corresponding heights. Even taking this into account, the over-land transmission paths of these tests favor the shorter wave-lengths. A theoretical reason for this will be given later.

In one direction from Deal, S. 50° 46′ W., measurements were made on 69 mc. at numerous places along the beam of a directive antenna, up to a distance of about 95 km. The profile of this path along the straight line to the most distant point, Chesilhurst, is shown in Fig. 6-*B*. Displacements of intermediate points from this line are negligible

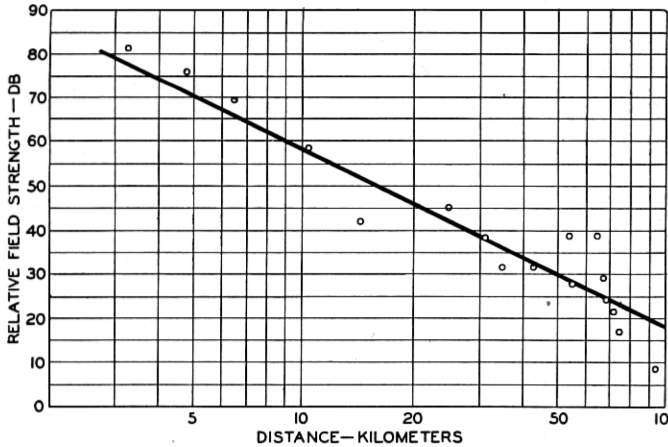


Fig. 8—Field strength as a function of distance for transmission over level country, along the profile of Fig. 6-*B*.

except in the case of New Egypt. Here a slight displacement was made in order to use a favorable receiving site for more extensive measurements. The profile in this neighborhood is superposed on the main profile. The various receiving points are shown by small arrows.

The received field is plotted as a function of distance in Fig. 8. For comparison purposes a straight line representing the inverse square law is drawn. This represents the general trend very well.

Transmission along this path is of particular interest since it represents conditions to be expected over flat land. The profile in Fig. 6-*B* shows that if the immediate neighborhood of terminal points be left out of consideration, the maximum difference in elevation along the path is only 45 meters. This path probably represents a spherical earth as well as any of similar length that exists in this part of the country.

Stability of Signals

Speaking generally, the signals received in ultra-short wave transmission vary little, if at all. In this respect they are in marked contrast with signals of lower frequencies in the transmission of which the Kennelly-Heaviside layer is involved. In this work, definite indications of fading have been found only in the case of paths in which the attenuation in excess of that represented by the inverse distance formula has been in the order of 30 to 40 db. The variations were in the order of one or two decibels, and the period was a few seconds. This may have been due to variable atmospheric refraction. On the other hand, it is not inconceivable that it may have been due to reflection from clouds. It is, of course, easy to show that there is so little moisture in clouds that reflections must be extremely weak. But we have to explain coefficients of reflection in the order of only 0.01. This is plausible since we are concerned with reflection from the cloud at near-grazing incidence for which the coefficient tends to be unity regardless of the difference in dielectric constant. Further investigation is needed along these lines.

Part II—Theory

Before entering into a quantitative explanation of some of the results which have been presented, it may be well to direct attention to certain ways in which the present problem is related to the familiar concepts of optical reflection, diffraction and refraction.

Reflection

Reflection constants are readily calculated in the case of smooth surfaces such as still water. Having obtained these, the resultant amplitude at the receiver can be calculated for different ground constants. (See Appendix I.)

Even if the surface is rough, it is to be expected that an ultra-short radio wave may be reflected regularly from a body of water. The existence of regular reflection is less obvious when transmission occurs over rolling land. In the first case we have the most simple conditions since the surface waves on the water are irregularities of a single general type and range of dimensions. They are merely deviations from a plane, or rather from a sphere. But in the second case, the irregularities of the land are of all forms and dimensions and the existence of regular reflection cannot be granted without consideration.

In most of the cases of radio propagation now being considered, we are concerned with near-grazing incidence since both transmitter and receiver are located near the ground and are separated horizontally by a comparatively large distance. That regular reflection may occur under such circumstances, even over irregular ground, can be shown by a simple optical experiment. A moderately rough piece of paper, such as a sheet of bond or any other paper without gloss is employed. The paper on which this is printed is rather too smooth to give a striking result, but it may be used. If the reader will focus his eye on some distant object which shows up with contrast against the sky, and if he will then hold the paper about a foot from the eye so that the line of sight is parallel and very close to the plane of the paper, it will be seen that the rough sheet has become a surface with a high gloss. It is helpful to bend the paper slightly so as to produce a cylindrical surface having elements parallel to the line of sight. Images of

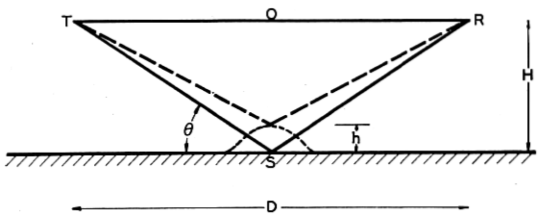


Fig. 9

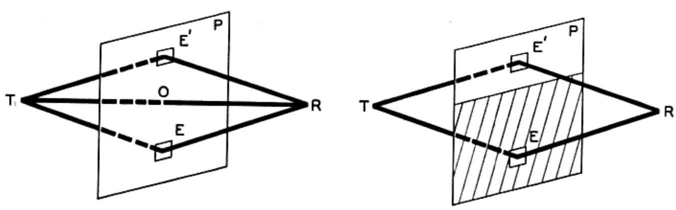
distant objects can be seen clearly in such a paper mirror and considerable detail can be obtained provided that the angle of incidence differs from 90° by something less than one degree. It is to be remembered that in most of the optical paths encountered in ultra-short wave propagation, we are concerned with angles which are as near to grazing as this is.

The reason for this reflection from a rough surface is readily explained on the basis of Huyghens' principle. The situation is represented in Fig. 9. Let us suppose that the general level of the rough surface is below the line of sight *TOR* by a distance H. H is assumed small compared with D, the length of the path. As a result of variations in H due to the ruggedness of the terrain there will be corresponding variations in the total length of the optical path *TSR*. Reflections will be approximately regular, however, if these variations in *TSR* are small enough in comparison with half a wave-length. In Fig. 9, a change in level, h, is represented at S, the dotted line representing an irregularity which has been added. These assumptions lead readily to the requirement for regular reflections: h, the height of the hill, should be small compared with $\lambda D/8H$, which equals $\lambda/4\theta$, where $\pi/2 - \theta$ is the angle of incidence. This relation expresses the fact that the regularity of reflection from a given rough surface can be improved either by increasing the wave-length or by decreasing the angle θ.

While these considerations show the reasonableness of regularity of reflection, they do not enable us to calculate the value of the coefficient. In the over-land tests which we have described, the amplitude of the coefficient of reflection would have been very near to unity and its phase angle would have been very near to 180° if the ground had been smooth. In the absence of data on the reflection from rough surfaces, we have used these same values although it is apparent that the coefficient will be less than unity due to scattering and increased penetration. The fact that a fairly good quantitative check has been obtained experimentally indicates that this assumption is reasonable. The check is somewhat better when the magnitude of the reflection coefficient is somewhat reduced (Fig. 16).

Diffraction

In ultra-short wave propagation, the effect of an obstacle, such as a hill, can be visualized best by considering it from the point of view of this same principle of Huyghens. Fig. 10-A represents this. A wave



A Fig. 10 *B*

originates at T and travels unobstructed to R, passing through the plane P. It is, of course, incorrect to say that the effect travels exclusively along the line TOR. Consideration must be given to other paths such as TER, and the effect of the latter can be neglected only in case the path length TER exceeds TOR by many wave-lengths; or more properly, a region about E can be neglected only in case the phases of the components transmitted through the elements within it (*e.g.*, along TER) are such as to cause destructive interference among themselves.

When a hill is interposed as shown in Fig. 10-*B*, elements such as *E*, below the profile of the hill, are prevented from contributing to the signal at *R*, while elements such as *E′*, above the profile, contribute as before. This is the simple concept as used in optics and will be used without essential modification in the explanation of non-rectilinear radio transmission.

Refraction

Besides reflection and diffraction, a third optical concept, atmospheric refraction, must be considered in this study.[15] It is a well-known fact that a star, appearing to be exactly on the horizon, is really 35 minutes below it. It is obvious that the "image" of an ultra-short wave transmitting antenna will be elevated above its true direction by this same means. The only question is whether the effect is appreciable or not. The answer, obtained theoretically, is that refraction must be taken into account. Unfortunately, we so far do not have quantitative measurements which show the effect of refraction of ultra-short waves in an unmistakable way. Those that we do have, however, appear to be consistent with expectations based on the theory which will now be presented.

The physical picture to be assumed is one in which the dielectric constant of the atmosphere decreases with height above sea level and is not a function of horizontal dimensions. In other words, the phase velocity of a wave in this medium becomes greater as the distance from the center of the earth increases. In the case of ultra-short waves, we are almost always interested in waves traveling in a substantially horizontal direction. The wave-front, therefore, lies in a plane which is nearly vertical and since the upper portions travel faster than the lower, there is a tendency for the ray to bend slowly back toward the earth.

This phenomenon, in its general aspects, is the same as that which is commonly assumed to explain the bending of longer waves about the earth. There is an important difference, however, in regard to the part of the atmosphere which is important. In the case of these longer waves (for example, one having a wave-length of 15 meters or a frequency of 20 mc.), the ionization in the atmosphere 100 to 400 km. above the earth is the cause of the refraction which makes long distance signaling possible. In the case of ultra-short waves, however (for example, one having a wave-length of 1.5 meters or a frequency of 200 mc.), this upper region is of no importance but it is the region

[15] Jouaust (*L'Onde Electrique*, 9, 5–17, Jan. 1930) has pointed out the importance of refraction in the propagation of ultra-short waves. The authors believe, however, that he has overemphasized its importance.

below one kilometer or so, where the ionization is negligible, that is essential.

The radius of curvature of a ray traveling horizontally in the lower atmosphere can readily be calculated if it is known how the refractive index, n, varies from point to point. If H is the altitude above sea-level, the radius of curvature of the ray is simply

$$\rho = -\frac{n}{dn/dH}.$$

But since $n = \sqrt{\epsilon}$, where ϵ is the dielectric constant, the radius of curvature is

$$\rho = -\frac{2}{d\epsilon/dH},$$

provided n is not very different from unity.

In Appendix II the estimation of this radius of curvature is discussed in some detail. While some of the data upon which such a calculation can be based are rather uncertain, it appears that a good first approximation is obtained by assuming the radius of curvature, ρ, of the refracted ray to be four times the radius of the earth, r_0. As pointed out in the appendix, this varies to some extent with weather, and even as an average value, it may have to be changed when more reliable data on dielectric constants become available.

On first consideration of the ways in which refraction can be taken into account, it appears that the attempt must complicate an already involved situation. Fortunately, however, refraction is much simpler to calculate than diffraction or reflection. The method is presented rigorously in Appendix III. At this point we shall merely state the result and show its plausibility.

In ultra-short wave work we are almost always concerned with propagation in a nearly horizontal direction. The curvature of the ray is $1/\rho$, while that of the earth is $1/r_0$. We are interested, however, in the relative curvature, which we shall call $1/r_e$. If, instead of using simple rectangular coordinates, we transform to a coordinate system in which the ray is a straight line, the curvature of the earth will become $1/r_e$, which is $1/r_0 - 1/\rho$. The equivalent radius of the earth would be

$$r_e = r_0\left(\frac{1}{1 - r_0/\rho}\right),$$

and is therefore greater than the actual radius of the earth by the factor $\frac{1}{1 - 1/4}$ which is 1.33. This fictitious radius is therefore

8500 km. instead of 6370 km. Since in the new system of coordinates, the ray is straight, the new equivalent dielectric is to be assumed constant and equal substantially to unity.

Refraction can therefore be taken into account as follows: In making calculations, we start with the topographical features of the path and construct an equivalent profile [16] of some sort plotted from known elevations of points along the path. If refraction were to be neglected, the actual radius of the earth would be used. To take refraction into account, the process is exactly the same except that the fictitious radius $r_e (= 1.33 r_0)$ is now used. Reflection and diffraction calculations are then based on this equivalent profile, in which account has already been taken of refraction by means of the fictitious radius.

It follows from the discussion given in Appendix III, that this transformation is not limited to optical paths. The discussion applies to the amplitude of the disturbance set up at one point due to a radiating source at any other point, whether that source be an actual antenna or one of the elementary reradiating oscillators of Huyghens. Under all circumstances where Huyghens' principle applies, the signal is passed on from one intermediate plane to another by the repeated application of the principle. Since this transformation is justified for determining the effect that any elementary oscillator at one point produces at a second nearby point it is justified for the process as a whole provided only that the line connecting the two points is inclined to the horizontal by only a small angle.

Optical Path Transmission

Let us now consider the application of these concepts to the case of transmission along an optical path. It has been pointed out that in many cases we would expect to find a well-defined reflected wave superposed on the direct wave. The two will, therefore, interfere constructively or destructively depending on phase relations. In other words, a set of Lloyd's fringes will be set up.

The airplane measurements over New York Bay gave direct evidence of the existence of these fringes. In order to check this quantitatively, the data are presented in Fig. 11-*A*. Vertical polarization was used.

[16] The elevations above sea-level involved are so small compared with the distances along the surface of the earth that they cannot be plotted on the same scale. This difficulty can be overcome within limits by increasing the scale used in plotting elevations, and at the same time decreasing the scale used in plotting the radius of the earth by the same ratio. When this is done, a line which in the actual case is straight remains approximately so even with these distorted scales. This gives a general picture of the profile but due to the slight curvature introduced, all distances involved in the calculations of this paper have been determined analytically. The scales of the profiles shown have thus been altered by a factor of about 50.

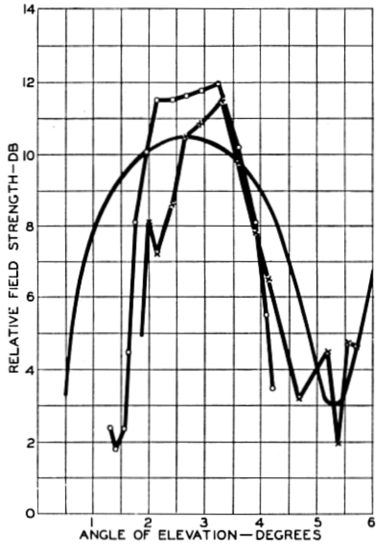


Fig. 11-*A*—Field strength as a function of the angle of elevation of the receiver, for transmission over salt water at 80 mc. The two experimental curves are from data taken with the receiver in an airplane flying at two constant altitudes. The smooth curve is theoretical.

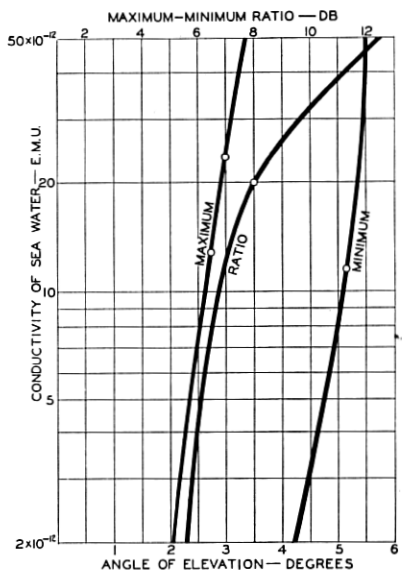


Fig. 11-*B*—Theoretical angles of elevation of maximum and minimum, and magnitude of their ratio, as functions of the conductivity of the water for a dielectric constant of 80, and a frequency of 80 mc. The experimental points shown (from Fig. 11-*A*) indicate a conductivity of about 17×10^{-12} e.m.u.

Since the altitude of the transmitting antenna was small compared with that of the airplane, we would expect, on the basis of the optical picture, that the field received would depend on the distance and the angle of elevation of the plane as seen at the transmitter. In the figure, the distance has been eliminated by recourse to the inverse distance law, which applies to the separate component waves. The result has been plotted for two elevations with varying distance. The peaks and troughs of the Lloyd's fringes are fairly well indicated.[17]

While little weight can be given to the absolute values as measured in the airplane,[18] it is of interest to estimate the conductivity of the

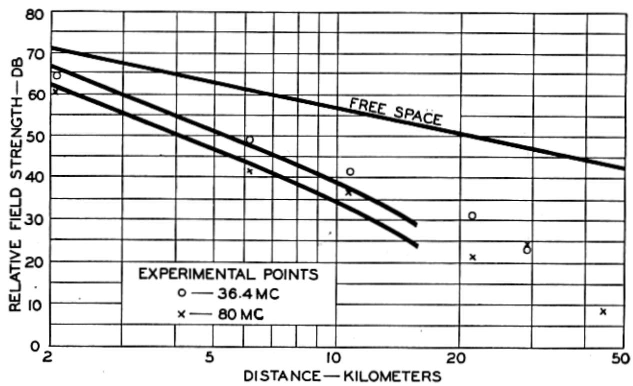


Fig. 12—Theoretical characteristics for transmission over salt water ($\sigma = 1.5 \times 10^{-11}$ e.m.u., $\epsilon = 80$ e.s.u.) on the basis of simple optical reflection. Upper curve: 36.4 mc. Lower curve: 80 mc.

water from the relative values. Fig. 11-*B* shows the theoretical location of maxima, minima and their ratio as functions of the conductivity of sea water. Four experimental values have been plotted. Their average indicates a conductivity of 1.7×10^{-11}. This, at least, has the correct order of magnitude, but the experimental data are too inaccurate to justify much faith in the numerical value otherwise. The important point is that the field pattern is qualitatively what would be expected. The theoretical characteristic for this value is also plotted in Fig. 11-*A*.

Turning now to the more accurate data taken on the ground (already presented in connection with Fig. 3), theoretical curves have been fitted to the data in Fig. 12. In the experiment, the antennas were

[17] Similar fringes were obtained over land by Englund, Crawford and Mumford.

[18] Because of the irregular shape of the airplane, the orientation with respect to the line of sight affects the gain of the receiving antenna. Each of the two curves has been plotted from data taken with approximately constant orientation of the airplane.

some 25 and 6 meters above sea level and under this condition the effect of earth curvature cannot be neglected in the calculation except for paths less than a kilometer in length. This curvature has been taken into account here to the extent of replacing the curved surface by a plane which is tangent to the earth at the point where the reflected ray of geometric optics touches the earth. This is justified for short optical paths but cannot be used at the longer distances when the receiver nearly disappears from the view of the transmitter.

Fig. 12 shows the theoretical curve for vertical polarization based on a conductivity of 1.5×10^{-11} e.m.u. and a dielectric constant of 80 e.s.u. Other values of conductivity give the same type of curve but the best fit to the experimental data is obtained by this curve. The dielectric constant was chosen equal to that which has been found to hold for fresh water throughout this frequency range.[19]

The agreement between the experimental and theoretical curves is reasonably satisfactory. By varying one constant, the conductivity, it has been possible to check approximately the absolute attenuation at two distances and two frequencies.

The conductivity (1.5×10^{-11}) is lower than that measured for sea water at low frequencies. For this reason, it was considered desirable to check the low frequency conductivity in this part of the bay since it may have been reduced by the fresh water emptied into the bay by numerous nearby streams. A number of samples were taken from different points between the transmitting and the receiving locations at both low and high tide. The values varied between 2.9×10^{-11} and 3.7×10^{-11} e.m.u., with an average of about 3.3×10^{-11}. This is more than twice the value of 1.5×10^{-11} indicated by the optical calculations. A sample of undiluted ocean water taken at the same time had a conductivity of 4.3×10^{-11}.

This agreement of experiment with simple optical theory does not prove that the assumed picture of a direct and a reflected wave is complete. It is to be pointed out that a rigorous solution (as opposed to the simple reflection picture), might require an appreciably different conductivity. Mr. C. B. Feldman of these Laboratories has made some short distance experiments over smooth land.[20] Using fre-

[19] Since the writing of this paper, an article by R. T. Lattey and W. G. Davies on "The Influence of Electrolytes on the Dielectric Constant of Water" has appeared (*Phil. Mag.*, 12, 1111–1136, Dec. 1931). Their results indicate that the dielectric constant is materially increased by salt in the water. Their experiments were made for solutions that were very much more dilute than sea water. This, together with the fact that the effect of a combination of solutions was not determined, makes it impossible to estimate the dielectric constant of sea water from their results with a reasonable degree of certainty.

[20] A paper covering this work will appear later: "The Optical Behavior of the Ground for Short Radio Waves," C. B. Feldman.

quencies in the short wave range he found that the simple optical picture cannot always explain the results obtained with vertical polarization. With horizontal polarization, however, satisfactory agreement was obtained. The propriety of the simple optical picture is therefore much clearer for horizontal than for vertical polarization.

Reasons have been given in an earlier section for expecting regularity of reflection even in the case of rugged land, if the incidence is near enough to grazing. It was also shown that there probably exists an effective coefficient of reflection which is actually near to − 1 for both polarizations. At the receiver the phase relation between the direct and reflected waves, and hence the field, thus depend only on the path

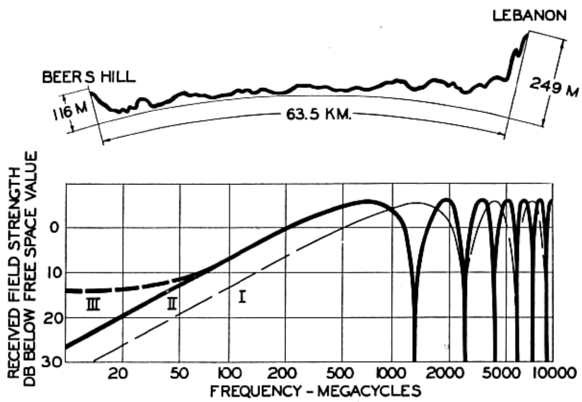


Fig. 13—Above: Profile of "optical" path between Beer's Hill and Lebanon. Below: Calculated frequency characteristics for this path.

Curve I, reflection only (coefficient, − 1).
Curve II, refraction and reflection (coefficient, − 1).
Curve III, refraction and reflection (coefficient − 0.8).

difference measured in wave-lengths. A set of interference fringes will therefore be set up, and the received signal at any given point will be a function of the frequency.

Making these assumptions as to reflection and taking refraction into account, it is interesting to calculate the frequency characteristic of a typical path. For this purpose, we may choose the path from Beer's Hill to Lebanon, which is discussed by Englund, Crawford and Mumford. The characteristic which would be obtained from the foregoing considerations of reflection and refraction is shown in Fig. 13. The light curve shows the frequency characteristic that results by neglecting refraction. It can be seen that below about 500 mc. the expected gain due to refraction is about five db., a gain which is by no means inconsiderable. For 70 mc. the field strength indicated by the curve is in fair agreement with measurements made over this path by Englund, Crawford and Mumford.

The effect of reducing the reflection coefficient to -0.8 is to raise the low frequency end of the curve, to reduce the maxima to 5.1 db. and to raise the minima to -14 db. This is shown by the dashed curve of Fig. 13.

Another point in connection with the solid curve in Fig. 13 is of interest. At 715 mc. (42 cm.) the path difference is half of a wavelength and the two components now add in phase. This is the optimum phase relation since it gives the largest possible resultant. Hence 715 mc. is an optimum frequency for this particular path on these assumptions and a field 6 db above the inverse distance value would be expected. Even at one third this frequency, 240 mc. (126 cm.), fields equal to the inverse distance value might be expected. For higher frequencies many maxima and minima are indicated.

Since the lowest optimum frequency depends on the difference between the path lengths of the direct and reflected components, it should be possible to obtain much lower optimum frequencies by picking paths in which the terminals are located very much higher than the valley between them. Thus, optical paths more than one hundred miles long may be found in California for which the lowest optimum frequencies may be considerably less than 30 mc. (10 m.).

Error in the assumption of a phase shift of 180° would change the frequency at which maximum and minimum fields occur, and failure to obtain a reflection coefficient of unity might materially reduce the difference between the received field and the free space value.

The profile shown in Fig. 14 is used to illustrate the effects of change in polarization and ground constants as indicated by calculations based on simple optical theory. In the computations indicated by the various frequency characteristics of this figure, the same profile has always been used, but two different sets of ground constants, and both horizontal and vertical polarizations, have been employed. The curves are self-explanatory. It is especially to be noted that for horizontal polarization the field decreases with decrease in frequency and is nearly the same for land as for sea-water, *i.e.*, it is nearly independent of conductivity and dielectric constant. For vertical polarization this trend is reversed for frequencies such that the conduction currents are large compared with the displacement currents. In this example, this occurs in the neighborhood of 60 mc. in the case of sea-water and 5 mc. in the case of "average" land. Thus for vertical polarization there exists a "poorest" frequency separating the excellent transmission at very low frequencies, where there is no phase shift due either to reflection or to path difference, from the excellent transmission at very high frequencies (*e.g.*, 2000 mc.) where large phase shifts due to these two causes nullify each other.

In those cases in which calculations of this sort indicate a very weak resultant field, these estimates may be considerably in error due to neglect of terms which are usually unimportant.

It may be of interest to note that two of the experiments described have given an inverse square of distance variation. In both cases the antennas were near the surface of the earth. It can easily be shown that this should be expected when total reflection occurs with reversal

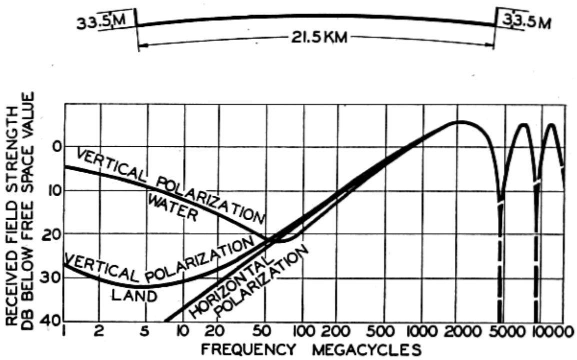


Fig. 14—Above: Profile of a hypothetical path. Below: Calculated frequency characteristics for various conditions. Curves are shown for vertical polarization over sea water ($\sigma = 20 \times 10^{-12}$ e.m.u., $\epsilon = 80$ e.s.u.), for vertical polarization over land ($\sigma = 5 \times 10^{-14}$ e.m.u., $\epsilon = 15$ e.s.u.), and for horizontal polarization over either (ground constants not important in this case).

of phase provided that the difference in path length is smaller than one sixth of a wave-length. Thus, in Fig. 9 the signal received at R will tend to be zero or very small, except as the phase relation is altered by the difference in the path lengths TOR and TSR. The corresponding phase difference in radians is $4\pi H^2/D\lambda$, if H is small. Since the differences of two vectors of equal magnitude are equal to the product of their phase difference, if small, and their magnitude, the resultant field is equal to $4\pi KH^2/\lambda D^2$. One of the inverse distance factors is due to the phase angle and the other is due to the fact that the amplitude, K/D, of the direct wave itself varies inversely with the distance. Under these conditions, therefore, the signal would vary inversely as the square of the distance, D, directly as the square of elevation, H, and inversely as the wave-length. Qualitatively, at least, all of these tendencies have been observed experimentally.

Even with vertical polarization, the reflection coefficient is also approximately -1 for transmission over smooth land with near-grazing incidence. The same inverse square tendency is therefore to be expected with vertical polarization under these conditions.

Non-Optical Paths

We shall now discuss one type of non-optical path which is of interest both because it occurs frequently and because on the basis of the assumptions made it is amenable to approximate calculation. It is represented in simplified form in Fig. 15.

T and R are located on opposite sides of a hill, M, and the distances TM and TR are great compared with the altitudes involved. The low land on both sides of the hill is comparatively flat, though not

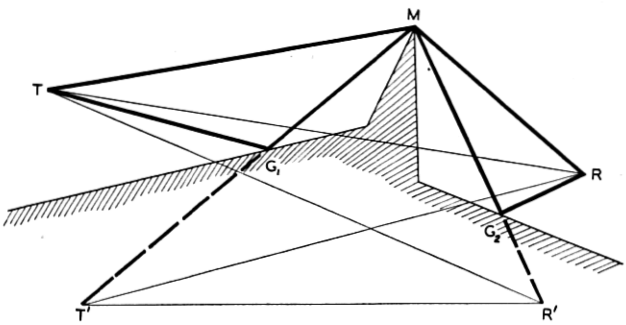


Fig. 15

necessarily coplanar. As previously discussed, the magnitude of the coefficient of reflection to be expected will be close to unity [21] for many conditions likely to be met and the phase change will be not much different from 180°. In other words, the wave reflected from the ground between T and M will appear to have come from a negative virtual image, T'. The disturbance above the mountain, M, will be made up of two components corresponding to the antenna and its negative image. In passing from the region above M to the receiver, R, each of these components is broken down into two new components due to reflection between M and R. One of these proceeds directly to the receiving antenna. The other proceeds indirectly, being reflected by the intervening ground; it may be thought of as traveling

[21] An exception of this occurs in the case of vertical polarization over surfaces having appreciable conductivity, such as sea-water. Recent experimental work not described in this paper indicates that the assumption is incorrect over land at frequencies considerably higher than those of the present experiment. In such cases the theory is still tenable if appropriate constants are used.

to the virtual image of the receiving antenna, R', with a phase change of 180° due to reflection between M and R.

The received field is therefore propagated in four ways: (1) directly from T to R by diffraction at M represented by TMR, (2) by reflection at G_1 and diffraction at M represented by TG_1MR, (3) by diffraction at M and reflection at G_2 represented by TMG_2R, and (4) by reflection at G_1, diffraction at M and a second reflection at G_2 represented by TG_1MG_2R. The amplitudes and phases of these four components can be calculated by usual methods of diffraction (see Appendix IV) by assuming the components to travel from the real transmitting antenna or its virtual image, to the real receiving antenna or its virtual image. The ratio of the received field to the free space field may then be calculated by combining the four components as follows:

$$\begin{aligned} E/E_0 = {} & C_1 \exp[-j(\eta_1 + \zeta_1)] \\ & + C_2K_1 \exp[-j(\eta_2 + \zeta_2 - \varphi_1)] \\ & + C_3K_2 \exp[-j(\eta_3 + \zeta_3 - \varphi_2)] \\ & + C_4K_1K_2 \exp[-j(\eta_4 + \zeta_4 - \varphi_1 - \varphi_2)], \end{aligned}$$

where the C's are the ratios of the field strengths with and without diffraction, the η's are the phase lags introduced by diffraction and the ζ's are the phase lags due to path lengths TR, $T'R$, TR' and $T'R'$, while the K's are magnitudes of the reflection coefficients and the φ's are the phase advances at reflection.

It is true that actual conditions will seldom be as simple as these. The valleys will not be flat. There will often be more than one hill and it may be impossible to represent the obstructions accurately by the single straight edge, M, which we shall assume. It will often be possible, however, to choose equivalent planes and straight edges in such a way as to justify some confidence in the results.

On these assumptions the frequency characteristic of the transmission path from Deal to Lebanon has been calculated (Fig. 16). By actual measurement it has been found that the attenuation over this path at 17 mc. (17 meters) was more than that at 69 mc. (4 meters). This characteristic of poorer transmission on the longer wave-lengths is the opposite of what would have been expected either on the basis of diffraction alone or by analogy with the trend observed on lower frequencies. The calculations show, however, that this is the characteristic that we should expect on the theory outlined. In view of uncertainties in the reflection coefficients and errors of measurement, the agreement of the absolute values calculated and measured is as good as should be expected. An improvement in this agreement is obtained by assuming a reflection coefficient of − 0.8. The resulting curve is shown by the broken line in Fig. 16. In the case of the optical path of Fig. 13 reflection coefficients of − 1 and − 0.8 agree equally well with the experimental point. (In Fig. 16 a correction has been applied to the experimental data eliminating the effect of local reflections at the receiver.)

This curve brings out the important fact that even for non-optical paths, one may expect to find optimum frequencies. On this particular

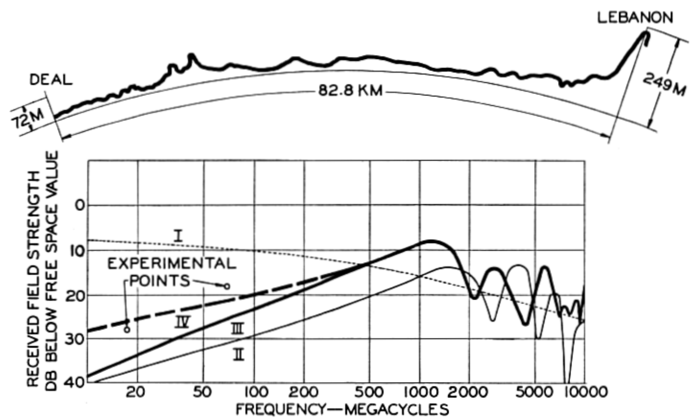


Fig. 16—Above: Profile of "non-optical" path between Deal and Lebanon. Below: Frequency characteristics for this path calculated on various assumptions.

Curve I takes only the shadow effect (diffraction) into account. Note that the experimental points fall far below it.

Curve II is calculated on the basis of diffraction and reflection (coefficient − 1.0). Note that this gives a better check with experiment, but values are too low.

Curve III adds to II a correction for refraction.

Curve IV assumes diffraction, refraction and a reflection coefficient of − 0.8. It checks the experimental points to within experimental error.

The original experimental data have been corrected to eliminate the effect of ground reflection near the receiver. The transmitter, being above level ground, needed no such correction.

path the simple assumptions give 1200 mc. (25 cm.) for the lowest of these. On other paths which have been calculated, optimum frequencies would be expected in the range between 1 and 10 meters.

It is fully realized that the details of these curves will probably not be found experimentally. We do not as yet have sufficient experience to pick the simple picture that will in effect represent a complicated topography and transmission mechanism, and it is obvious that this may never be possible. It is encouraging, however, that the limited number of measurements which have already been made experimentally, agree reasonably well with the theory proposed.

Discussion of Certain Trends with Respect to Frequency

It may be helpful, in recapitulating, to consider the different trends which ultra-short wave transmission shows with respect to frequency, and to mention their relationships to the phenomena of the ground wave at lower frequencies.

The simplest trend is that to be found in free space, that is, in cases for which the effect of the ground is negligible. Changes, if any, in transmission efficiency with frequency are then due to the air itself. Such evidence as there is on this point indicates that the assumption of absorption by the air is unnecessary within the range of our experiments, and in fact this is to be expected on theoretical grounds. The "free space" trend therefore gives merely a horizontal line. In Figs. 13 and 14 the high frequency portion of the curve oscillates about this line and would approach it if reflections from the earth were decreased in strength.

In general, however, the effect of the earth will alter this trend in such a way as to give a variation with respect to frequency. Perhaps the most familiar variation is the loss of efficiency in going to high frequencies when vertical polarization is used. The decrease is due to conduction losses in the ground. This "conductivity trend" appears, for example, in the work of Sommerfeld and of Zenneck. Many experimental observations have been made of it at broadcast frequencies for distances up to a few hundred miles over paths which are obviously "non-optical." We have seen it here in the optical path tests made over sea-water, Fig. 3. It appears also in the low frequency end of the "vertical polarization over water" curve in Fig. 14.

In several cases we have noted that 69 megacycles was more efficiently transmitted over land than 17 megacycles. This is opposite to the conductivity trend and appears to have a very different cause. For both optical and non-optical paths it is believed to be associated with a phase change at reflection of 180°, and the effect is most pronounced when reflection occurs without appreciable loss of amplitude. This "negative reflection" trend is exemplified on the one hand by the very poor transmission with very low frequencies when horizontal polarization is used, and, on the other, by the excellent transmission at 75 mc. (4 meters) between Beer's Hill and Lebanon (Fig. 13). In the latter case the difference in path lengths of direct and reflected waves was not negligible compared with a half wave-length, and it is a phase shift due to this cause which apparently prevents destructive interference. This negative reflection trend also appeared in the non-optical paths over level land (Fig. 8). It is affected not only by the negative reflection in the neighborhood of the antennas (negative image), but also by negative reflection all along the path. (The term "negative reflection" is used here even in the non-optical case, since when we visualize the process in terms of Huyghen's principle, it is apparent that this case is merely a succession of optical paths.) At higher frequencies this characteristic will cease to rise steadily and at least in the case of simple optical paths will oscillate up and down instead. The rising trend at lower frequencies, however, is found so often that it deserves special mention. It is illustrated by the rising curve and the experimental points shown in Fig. 16.

A fourth trend is due to diffraction and it is in the same direction as the conductivity trend. Long waves bend more easily about obstacles than do the short; the obstacle may be a mountain or it may be the ever-present bulge of the earth. This type of characteristic is indicated in Fig. 16 in the high frequency part of the calculated curve, but in our experiments we have so far not had conditions in which its effect could with certainty be separated from the opposite "negative reflection" trend. The reason for this is that the diffraction trend does not predominate except with frequencies which are sufficiently high. In tests from Deal to Lebanon (Fig. 16) it appears that frequencies greater than 1200 megacycles might have to be used in order to separate these effects clearly. This is a point of great importance in view of the wide-spread belief that ultra-short waves suffer most in transmission because of the failure of the waves to bend around obstacles. Except when high mountains or very short waves are involved, the loss in transmission is more likely to be due to reflection.

When reflection of vertically polarized waves takes place from a very good conductor, there is no change of phase at reflection, the "negative reflection" mechanism is therefore absent, and the tendency is toward reinforcement rather than cancellation. Physically these conditions can be found in the case of transmission over sea water for frequencies less than 5 mc. In this case, as shown in an as yet unpublished study, the diffraction trend has definitely been found experimentally and checked quantitatively with theory.

Optimum Frequencies

In the preceding pages calculations have been made for various types of path. Both for optical paths and for non-optical paths these have pointed to certain frequencies which, from the transmission standpoint, give most efficient results. The value of this optimum frequency depends almost entirely upon the topography of the path and therefore changes from path to path.[22] At the same time there are certain frequencies which give results which are poorer than those obtained with higher or lower frequencies. It is obviously desirable to avoid these in practice. In general, it seems important that in making a choice of frequency, the particular path should be considered by itself in order to insure that maximum transmission efficiency, or at least the best compromise with apparatus difficulties, will be obtained.

Acknowledgment

The experiments described in this paper have been possible only through the assistance of many members of the Bell Telephone Laboratories and we wish to make acknowledgment of this cooperation. We also wish to express our appreciation of the support and encouragement given in the course of this work by Dr. W. Wilson.

Appendix I—Reflection Calculations

The ratio of the resultant of the direct and reflected waves to the direct wave is

$$\sqrt{1 + K^2 - 2K \cos \gamma} = \sqrt{(1 - K)^2 + 4K \sin^2 (\gamma/2)},$$

where K is the ratio of the amplitude of the reflected wave to that of the direct wave and $\gamma \pm \pi$ is their phase difference.

$$\gamma = \psi - \Delta,$$

where Δ is 2π times the path difference in wave-lengths and

$$\varphi = \psi \pm \pi$$

is the phase advance at reflection. The convention here used for phase change at reflection is the change in phase of the vertical component in the case of vertical polarization, and the change in phase of the horizontal component in the case of horizontal polarization. In the case of vertical polarization this is different from the convention

[22] Beverage, Peterson and Hansell ("Application of Frequencies above 30,000 kc. to Communication Problems," *Proc. I. R. E.*, 19, 1313–1333, August 1931) found that a maximum range was obtained with a frequency of 35 mc. in some tests made over sea water. This maximum, if not due to peculiarities of the apparatus, it would seem, must be a function of the heights of transmitting and receiving antennas above sea level and above local ground.

used in optics.

$$K = \sqrt{\frac{1-\alpha}{1+\alpha}}, \quad 1 - K = \alpha\left[1 - \frac{\alpha}{2} + \frac{\alpha^2}{2} - \frac{3}{8}\alpha^3 + \cdots\right],$$

$$\psi = \tan^{-1}\beta = \beta\left[1 - \frac{\beta^2}{3} + \frac{\beta^4}{5} - \frac{\beta^6}{7} + \cdots\right],$$

$$\alpha = \frac{a \sin \xi}{1 + c \sin^2 \xi},$$

$$\beta = \frac{b \sin \xi}{1 - c \sin^2 \xi},$$

$$\xi = \frac{\pi}{2} - \theta,$$

where θ is the angle of incidence and for vertical polarization,[23]

$$a = \frac{\sqrt{2}}{s}[\epsilon\sqrt{s+r} + q\sqrt{s-r}],$$

$$b = \frac{\sqrt{2}}{s}[q\sqrt{s+r} - \epsilon\sqrt{s-r}],$$

$$c = \frac{1}{s}(\epsilon^2 + q^2);$$

while for horizontal polarization,

$$a = \frac{\sqrt{2}}{s}\sqrt{s+r},$$

$$b = -\frac{\sqrt{2}}{s}\sqrt{s-r},$$

$$c = \frac{1}{s},$$

where

$$q = 2\sigma/f,$$

$$r = \epsilon - \cos^2 \xi,$$

$$s = \sqrt{r^2 + q^2},$$

f is the frequency in cycles per second, and ϵ and σ are the dielectric constant and conductivity respectively, both in electrostatic units.[24]

[23] Vertical polarization refers to vertical electric field. Horizontal polarization refers to horizontal electric field. This is different from the concepts of optics.

[24] If σ is expressed in electromagnetic units, $q = 2\sigma V^2/f$, where V is the velocity of light (3×10^{10}).

For angles near grazing incidence, both $(1 - K)$ and ψ are proportional to ξ.

$$1 - K = a\xi, \qquad [\xi \to 0],$$
$$\psi = b\xi, \qquad [\xi \to 0],$$

where a and b are now both independent of ξ. If $K = 1$, the ratio of the resultant of the direct and reflected waves to the direct wave becomes $2 \sin (\gamma/2)$. If in addition γ is small, this ratio becomes simply γ.

For angles near normal incidence, both K and ψ are independent of ξ.

$$K = \sqrt{\frac{1 + c - a}{1 + c + a}}, \qquad [\xi \to \pi/2],$$

$$\psi = \tan^{-1}\left(\frac{b}{1 - c}\right), \qquad [\xi \to \pi/2],$$

where a, b, and c are now independent of ξ.

For good conductivity, $q(= 2\sigma/f) \gg \epsilon > r(= \epsilon - \cos^2 \xi)$; $a = b = \sqrt{2q}$; $c = q$ for vertical polarization. For horizontal polarization $a = -b = \sqrt{2/s}$, $c = 1/q$.

For poor conductivity, $q(= 2\sigma/f) \ll r(= \epsilon - \cos^2 \xi) < \epsilon$; $a = 2\epsilon/\sqrt{r}$, $b = 0$; $c = \epsilon^2/r$; $\psi = 0$ when $\xi < \cot^{-1}\sqrt{\epsilon}$, and $\psi = \pi$ when $\xi > \cot^{-1}\sqrt{\epsilon}$ for vertical polarization. For horizontal polarization $a = 2/\sqrt{r}$, $b = 0$, $c = 1/r$, $\psi = 0$.

Appendix II—Refractive Index and Curvature of Rays

The dielectric constant, ϵ, of dry air is given by the expression

$$\epsilon - 1 = 210 \times 10^{-6} p/K,$$

where p is the pressure in millimeters of mercury and K is the temperature in degrees absolute.

When water is present, however, an appreciable change is produced in the dielectric constant and doubtful points arise. Such, for example, are the effect of association of water molecules with each other or with other molecules, and the effect of adsorption on the surface of the plate of the test condenser. The work of Zahn[25] seems to have clarified

[25] *Phys. Rev.*, 27, 329, March, 1926.

the situation for pure water vapor. He showed that in his own experiments the anomalies which appeared at the lower temperatures were probably due to adsorption and not to association, as Jona[26] had assumed, and he states that his results are consistent with those measured by Jona at higher temperatures.

For pure water vapor, we may use the following formula which has been based on Zahn's data:

$$\epsilon - 1 = 1800 \times 10^{-6} \frac{p}{K}\left(1 + \frac{200}{K}\right).$$

Even though the separate values for water and for air may be considered to be known with sufficient accuracy, it does not follow that a mixture of the two will necessarily follow the usual additivity law for mixtures of gases. According to this law the values of $\epsilon - 1$ for the several components may be added to give the $\epsilon - 1$ for the mixture. Delcelier, Guinchant and Hirsch[27] gave some data for moist air taken as a preliminary to a more thorough study. They interpreted their results as denying this law for a mixture of water and air. Their experiments were carried out at 15° C. and at 25° C. It should be noted that this is the temperature range in which Zahn found anomalous behavior due to adsorption. It is therefore natural to suppose that this same spurious effect may have been present in the work of Delcelier, Guinchant and Hirsch.

It seems, therefore, that the law of additivity has at least not been disproved for this particular mixture and that we can do no better for the present than to assume that it does hold. We shall therefore proceed on this basis.

In obtaining the derivative with respect to height $d\epsilon/dH$, it must be remembered that ϵ is a function of the partial pressures of dry air and water vapor, and of the temperature. All of these vary with H. The values of significance are those occurring in the first kilometer or so above the ground. The conditions actually observed are variable and we have therefore chosen to use average values as given by Humphreys,[28] obtaining the rates of change from the values that he gives for 0.0 and 0.5 km. above sea-level.

The following table summarizes the results obtained:

[26] M. Jona, *Phys. Zeit.*, 20, 14 (1919).
[27] *L'Onde Electrique*, May 1926, p. 211 et seq.
[28] "Physics of the Air," McGraw-Hill, 1929, p. 55 and p. 74.

Condition	Radius of Curvature of Ray $= \rho$	$\frac{\rho}{r_0}$ [29]	Equiv. Earth Radius Without Refrac., r_e	$\frac{r_e}{r_0}$
Average summer (average moisture)	23,800 km.	3.74	8,650	1.36
Same, without moisture	31,600	4.95	7,950	1.25
Average winter	26,500	4.15	8,420	1.32
Same without moisture	29,300	4.61	8,100	1.27
Annual Average Used in Computations		4.0	8,500	1.33

r_0 = radius of the earth = 6370 km.

Appendix III—Effect of Refraction

In the following it will be shown that the transformation given in the text gives the proper path for the ray and the proper phase relations. The latter are more conveniently treated by determining the "phase time," or the time required for a given phase to traverse the path. As a matter of fact the ray paths and phase times are not exactly the same in the two constructions but it will be shown that for one distribution of refractive index, n, which closely resembles that actually encountered, the error is negligibly small.

As indicated, this analysis is based on the customary ray treatment of refraction through a medium with a continuously varying refractive index. This simple ray theory is known not to be exact but in the present case we shall always be dealing with very small gradients, a condition in which the error becomes very small.

A good summary of the relations which we shall use is given in "The Propagation of Radio Waves," by P. O. Pedersen on pp. 154 and 155. The nomenclature is indicated in Fig. 17.

The length of the element of path, bb', equals

$$\frac{rd\theta}{\sin\varphi} \tag{1}$$

and the time required for the wave to traverse it is

$$dt = \frac{rd\theta}{v\sin\varphi} = \frac{nr}{c}\frac{d\theta}{\sin\varphi} \tag{2}$$

[29] It is inferred from a statement made by Jouaust, *Proc. I. R. E.*, Vol. 19, p. 487, Mar. 1931, that for his experiments between France and Corsica ρ/r_0 would have to be 5 or less in order for the direct ray to be unobstructed. Our figure of 4 therefore would indicate that his is an "optical" path. We do not believe, however, that an optical path is a necessary or sufficient condition for strong signals, although it certainly does help to make them probable.

(c is velocity of light and n the refractive index, which is assumed approximately equal to one at point a). But by Pedersen's equation 9′

$$nr \sin \varphi = r_0 \cos \psi. \tag{3}$$

Hence

$$dt = \frac{n^2 r^2}{r_0 c \cos \psi} d\theta. \tag{4}$$

Now, Eccles [30] has shown that when the dielectric constant varies with r (distance to center of earth) as follows:

$$n = \left(\frac{r_0}{r}\right)^{s+1}, \tag{5}$$

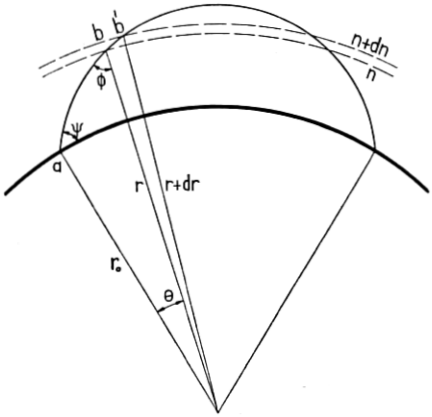


Fig. 17

the solution for the path of the ray is

$$r_0^s \cos (s\theta - \psi) = r^s \cos \psi. \tag{6}$$

Here s is a constant and r_0 is the distance to the center of the earth from a, the arbitrarily fixed point of reference in the path. r_0 is therefore not very different in our case from the radius of the earth.

By combining (5) with (6) we find that

$$nr = \frac{r_0 \cos \psi}{\cos (s\theta - \psi)}, \tag{7}$$

which when substituted in (4) gives

$$dt = \frac{r_0 \cos \psi}{c} \cdot \frac{d\theta}{\cos^2 (s\theta - \psi)}, \tag{8}$$

[30] *Electrician,* 71, 969–970 (1913).

integrating which from $\theta = 0$ to θ we obtain

$$t - t_0 = \frac{r_0 \cos \psi}{cs}[\tan (s\theta - \psi) + \tan \psi]. \tag{9}$$

If s in (5) is made somewhat less than one in absolute value and is negative, we obtain a fairly good approximation of the distribution actually encountered. The exponent $s + 1$ has then a small positive value and from (5)

$$\rho = -\frac{1}{dn/dr} = \frac{r}{n(s + 1)} \cdot \tag{10}$$

Since n is very close to unity and since we may assume $\rho/r = 4.0$ (Appendix II), we find that $s = -0.75$. This value will be used later.

Consider now a second series of values in equation (9), t', r_0', s', etc. which represent another situation which we shall define as follows:

$s' = -1$ (*i.e.*, constant index and no bending of the rays)

$\psi' = \psi$ (*i.e.*, no change in the initial direction of the ray)

$s'\theta' = s\theta$ and $r_0' = -\frac{r_0}{s}$,

so that $r_0'\theta' = r_0\theta$, that is, the peripheral distance traveled is the same in the two cases although the radius of the earth has been increased from r_0 to $r_0(-1/s)$.

By substituting these new primed values for the unprimed values in equation (9), we obtain

$$t' - t_0' = \frac{r_0 \cos \psi}{cs}[\tan (s\theta - \psi) + \tan \psi], \tag{11}$$

which is identical with $(t - t_0)$ in (9). Note that the only assumption that has been made, limiting the generality of this equivalence, is the special distribution assumed in (5). The phase time is therefore unaltered by this substitution.

We have yet to prove, however, that in these two cases, rays leaving at the same angle, $(\psi = \psi')$, and describing angles θ and θ' at the real and fictitious centers of the earth, will have the same increase in elevation above sea level. If this can be shown, the equivalence will have been completely established.

The increases in elevation of the ray above that of the starting point is found with the help of (6) to be as follows for the two cases:

$$r - r_0 = r_0([1 + L]^{1/s} - 1) \qquad \text{(real case)}, \tag{12}$$

$$(r' - r_0') = -\frac{r_0}{s}([1 + L]^{-1} - 1) \quad \text{(fictitious case)}, \tag{13}$$

where L is defined by the equation

$$(1 + L) = \left(\frac{r}{r_0}\right)^s = \frac{\cos (s\theta - \psi)}{\cos \psi}. \tag{14}$$

L is small compared with unity in the cases that we are considering. By expanding each and subtracting, the error caused by assuming that $(r - r_0)$ equals $(r' - r_0')$ is found to be

$$\frac{r_0 L^2}{2s^2}(1 + s) + \text{higher order terms in } L. \tag{15}$$

We have found above that s is approximately equal to -0.75. Taking $r_0 = 6370$ km. and remembering that we are ordinarily not concerned with rays farther above the earth than, say, 5 km., we have from (14)

$$\frac{r}{r_0} = (1 + L)^{1/s} \leq \frac{6375}{6370},$$

whence $L \geq -0.0006$.

Substituting these values in (15) we find that the error in height is less than 50 cm. This is negligible in the altitude of 5 km. which was assumed and we may consider the equivalence to be proved.

Appendix IV—Diffraction Calculations

The method of Huyghens applied to optical diffraction past a straight edge results in the following expression for the received field, E, in terms of Fresnel integrals.

$$\frac{E}{E_0} = a - jb = C \exp(-j\eta),$$

where

$$a = \frac{1}{\sqrt{2}} \int_v^\infty \cos \frac{\pi v^2}{2}\, dv,$$

$$b = \frac{1}{\sqrt{2}} \int_v^\infty \sin \frac{\pi v^2}{2}\, dv,$$

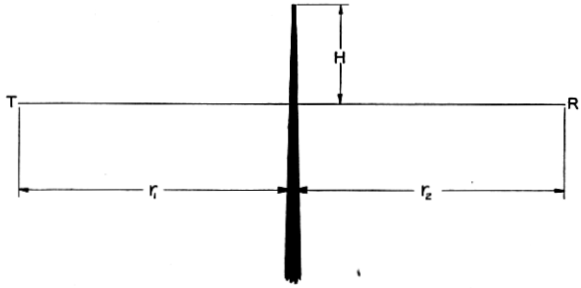


Fig. 18

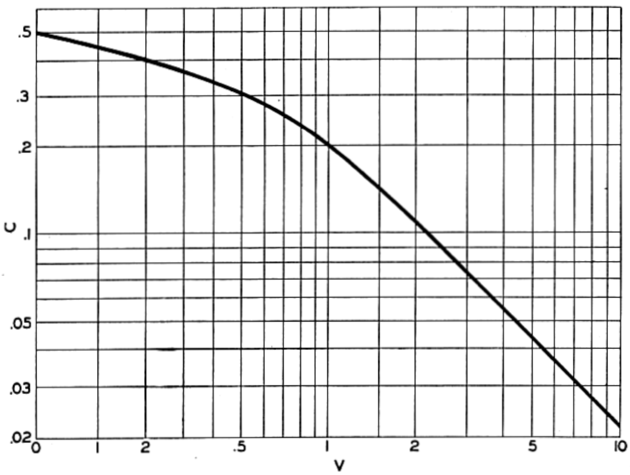


Fig. 19

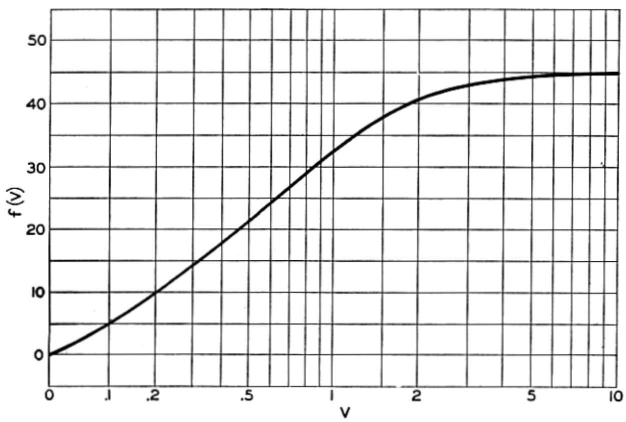


Fig. 20

and

$$v = H\sqrt{\frac{2}{\lambda}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}.$$

E_0 is the field with straight edge removed, a and b are Fresnel integrals, H is the height of the obstruction above the straight line from transmitter to receiver, r_1 and r_2 are the distances from the obstruction to the transmitter and receiver respectively (Fig. 18).

To facilitate calculation the value of C has been plotted in Fig. 19. η may be expressed as follows,

$$\eta = f(v) + \pi v^2/2,$$

where $f(v)$ is the function plotted in Fig. 20.