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High-Frequency Phenomena in Gases, First Part

By KARL K. DARROW

This is an account of the behavior of conducting gases subjected to high-frequency electrostatic fields—behavior which can be interpreted, in many cases with striking success, by supposing that the free electrons wandering in the gas are set into motion by the field, and oscillate and drift according to laws which can be derived from our knowledge of the response of free electrons to steady fields. When a constant magnetic field coexists with the high-frequency forces, the phenomena become more complicated, but remain predictable. There are also peculiar phenomena indicating that the electrons in a conductive gas have certain natural frequencies of oscillation. Applications are made to the absorption of radio-frequency waves in ionized gases.

IN this article I will describe some of the phenomena which are observed when the voltage across a region filled with gas is varying quite rapidly. Considered as a function of time, the voltage may be periodic, a sine-wave with uniform amplitude and of a frequency somewhere between a few thousands and a few hundreds of millions of cycles per second. It may be a succession of highly-damped wavetrains, each commencing with a rapid rise of voltage and continuing in oscillations of high frequency but swiftly declining amplitude, which die away into nothing and after an interval (it may be of a few hundredths or a few thousandths of a second) are followed by another train. It may be a brief irregular spasm of electromotive force, of which the highest value of the voltage is measured or merely guessed. In the gas itself there may be the phenomenon of sudden and violent breakdown; or the establishment of a self-sustaining luminous discharge, like in aspect to a glow or an arc; or merely a vibratory motion of electrons, freed by other agencies and set in motion by the oscillating field.

One might regard this as a subject which physicists had better leave alone, until they have full understanding of the seemingly much easier problems of discharges across a gas exposed to a constant voltage. So great are the apparent advantages of steady or "direct-current" discharges for the student, so great the apparent inconveniencies of high frequencies, that one might reasonably think it futile to assail the latter with weapons which have not yet overcome the former. Thus, in a self-sustaining glow maintained by a constant voltage, there is a peculiar distribution of space-charge, which distorts the field between the electrodes in a characteristic way, and manifests itself by a striking subdivision of the gas into zones of light and zones of darkness, differing much from one another in their electrical state as well as in their aspect. This distribution is not established instantly; would one not do better to examine it at leisure and understand it in full for each separate value of voltage, before beginning to study discharges in which it is continually changing over from the form appropriate to one voltage into the form appropriate to another? or discharges in which the voltage is varying so rapidly, that the gas is always in a state of transition, never even approaching the equilibrium appropriate to any steady value of voltage whatever?

Well, this is not necessarily to be supposed! it may be that when the voltage across the gas varies with extreme rapidity, the gas itself enters into a sort of equilibrium-state, perhaps even a more intelligible state than that which it attains when it is allowed an ample time to adjust itself to a constant value of voltage. The simplicity of certain empirical laws of the high-frequency glow suggests this, as also does the aspect of the glow. There is also the following argument, rather paradoxical in sound, perhaps, but forcible. A self-sustaining direct-current discharge, a glow or an arc, involves a steady outflow of electrons from its cathode. This outflow must be maintained by agents coming out of the gas itself—photons and positive ions and excited atoms, which are generated in the gas by the discharge and fall upon the cathode. Of the relative prominence of these agents, little is known—it forms one of the major problems of the steady discharge in gases; but at least it is certain, that they owe their existence and their effectiveness largely to the distribution of space-charge in the gas. The distribution of the space-charge therefore is controlled by the requirement (or seems controlled—one never knows what is cause and what is effect) that the electron-outflow from the cathode must be kept at a level suitably high. Now in the high-frequency discharge, the demand for electrons from the cathode is attenuated or abolished; witness the fact that such discharges may be maintained when the electrodes are separated by insulators from the gas! The peculiar disposition of the space-charge is therefore not demanded, at least not to so great an extent; conditions are intrinsically simpler.

This feature of high-frequency discharges—their competence to do without electrons from the cathode—requires attention. It is derived from a fundamental principle, which one is all too likely to forget if one has long been occupied with steady currents: the principle of Maxwell, that an electric field when varying in time is equivalent to a current. Let us apply this principle to a current in a circuit (Fig. 1) composed of a source E of electromotive force; wires leading (one of them through a galvanometer G) from the opposite poles of the source to electrodes A and C; and a gas between the electrodes. For convenience let us imagine the gas enclosed in a cylindrical vertical tube, the electrodes near its ends.

Take up first the direct-current discharge; say the cathode is above; consider any surface, S, cutting across the gas which the tube encloses. After the steady state is established, there will be positive ions crossing the surface on their way toward the cathode (upward)

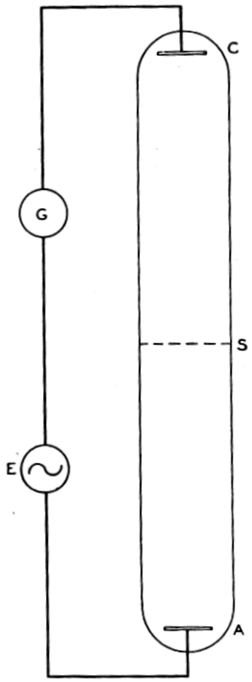

Fig. 1—Illustrating an internal-electrode tube for the transmission of high-frequency current through gas.

and negative ions crossing it on their way toward the anode (downward). We may speak of these, both classes, as ions going in the "right" direction. The total charge which they carry, as many of them as cross in unit time, will be the current across the surface —if there are no ions going in the "wrong" directions. But in a luminous discharge there are generally positive ions crossing toward the anode and negative ions crossing toward the cathode. The sum of the charges which they carry must be subtracted from the former sum; or, to express the idea better, they must be added with a negative sign. We have to form a fourfold sum: charges borne by positive ions going up and charges borne by negative ions going down, with *plus* sign; charges borne by negative ions going up and charges borne by positive ions going down, with *minus* sign. The terms of this fourfold sum may vary from one to another of the surfaces intersecting the tube (in general, they do) but the sum remains the same; it is the current through the gas. It is also the current flowing through the plane of the cathode-surface (it may then consist of two terms only, electrons emerging from the cathode metal and positive ions impinging on it; or it may involve the two additional terms, because of electrons falling upon the cathode and positive ions rebounding from it). It is also the current flowing through the wires, and the current measured by the galvanometer at G.

Now for a high-frequency discharge, this must be modified. The fourfold sum aforesaid is not the whole of the current. Or rather, we might call it the whole of the current, but then we should be compelled to say that the current across the various cross-sections of the circuit is not the same, and is not measured by the galvanometer. We do better to follow Maxwell, or rather the usage which developed out of Maxwell's theory: call it the "net convection current" and introduce the name "displacement current" for the quantity which must be introduced, in order to get a sum which is the same for all the cross-sections of the circuit and equal to the reading of the galvanometer. Nor is this "sum" to be obtained by simple addition. We are obliged to take into account another complication, from which direct-current discharges are free: the necessity of distinguishing between a current-component which is proportional to the voltage and a current-component which is proportional to the rate of change of the voltage; or, to take the simple case of a sinusoidal voltage, the current-components which are respectively "in phase" and "in quadrature" therewith. The displacement-current belongs entirely to the latter component, while the convection-current may belong partly to the one and partly to the other, as I will presently illustrate.

It is, then, *not* required that the net convection-current should be the same all through the gas; much less, that it should be the same as the current reported by the galvanometer. At sufficiently high frequencies, electrons and positive ions may oscillate in the interspace between anode and cathode without reaching either, if they start their independent careers far enough off from both. Can one attain a condition in which there is a convection-current of oscillating ions in the middle of the gas, but none whatever in the vicinity of the electrodes, and no electrons escape from the metal of the cathode into the gas? Without an impervious screen between the metal and the gas this would, I suppose, be impracticable; but if there be such a screen—if for instance the electrodes are outside of the glass-walled tube containing the gas, instead of being inside—it can be achieved. The ions in the imprisoned gas can be set into oscillatory motion by an alternating voltage applied to electrodes outside; indeed, a self-sustaining luminous glow-discharge can be maintained within the tube; the charged particles which vibrate in the shining gas never enter the metal parts of the circuit nor transfer their charges to these. Only by invoking Maxwell's concept of the displacement current are we able to contend that there is continuity of current-flow around the circuit and across the gap between the anode and the cathode.

The so-called "current through the gas"—i.e. the reading of the galvanometer in series with the circuit—is therefore a datum decidedly remote from the phenomena within the gas. Even in the simplest of all cases, that of a direct-current discharge, it merely gives the sum of the four terms aforesaid, not the value of any individually. In the much more intricate case of the high-frequency discharge, it is a combination of the four terms aforesaid and the displacement-current. This is probably why it figures so little in accounts of the high-frequency discharge.

If the current through the gas is hard to interpret when measured, the voltage across it may present an easier problem, or may on the other hand be unmeasurable altogether. The simplest case of all is that of a gas in contact with electrodes (as in the tube of Fig. 1) between the two of which an undamped sinusoidal voltage is applied. An alternating-current electrostatic voltmeter, of one type or another, shunted across the electrodes, will give the "effective" or "root-mean-square" value of the potential-difference between them. It is often the maximum value of the voltage which the experimenter wants, or thinks that he wants; to get it he must multiply the reading of the voltmeter by $\sqrt{2}$. This is not the proper factor unless the voltage is truly sinusoidal; the observer must find out about this (by using an oscillograph to make the waveform visible, or by hunting with a wavemeter for harmonics of the fundamental frequency) and use a different factor if the waveform is distinctly not a pure sine-function. If the electrodes are outside of the tube containing the gas, some part of the potential-gradient between them is located in the insulating walls, and does not act upon the gas. If the voltage is applied as a sequence of highly-damped short wavetrains with intervals between, the problem of determining its maximum value is a serious one. Perhaps the best available methods have never been applied in experiments in this field; at any rate, many a physicist has contented himself with measuring something which he thought to be proportional to the maximum voltage, without knowing what value to assign to the factor of proportionality.

The toughest problem of all, in respect of measuring current and voltage—or let me say, in respect of finding something significant to measure—is forced upon us by a form of discharge which Hittorf invented (this is probably a more suitable word than "discovered") in 1884. He wound a wire spirally around a tube containing air at low pressure, connected the spiral across a Leyden jar and the terminals of an induction-coil, and thus sent through the wire a sequence of current-pulses which were highly damped wavetrains; they incited a brilliant glow in the tube. Within the spiral, and therefore pervading the gas in the tube, there was of course a magnetic field parallel to the axis and alternating its direction with the alternations of the pulsing current. There was also a circular electric field due to the varying magnetic field, pointing alternately clockwise and counter-clockwise around the axis. Also there was a varying electric field due to the alternating potential-differences between the windings of the spiral. All of these three must have influenced the mobile ions of the gas! Often, as Thomson was later to stress, the discharge takes the form of a brilliant ring, thus suggesting that it is the second of the fields which dominates. But the problem of their relative responsibilities is a subtle and very difficult one; and the "ring discharge" is as troublesome to elucidate by theory, as it is easy to realize in practice.

I have written thus far as though we were concerned entirely with two sorts of stable conditions: the self-sustaining luminous high-frequency discharge, and the oscillations of ions in a gas where the supply of ions is maintained not by the alternating voltage but by some other agency acting independently. We are concerned with these, and with a third matter as well: the process of "breakdown," the sudden onset of the self-sustaining discharge which occurs when the amplitude of the high-frequency voltage across a gas hitherto tranquil is elevated past a critical value. This critical value or "breakdown-potential" is the most frequently measured of all the measurable qualities of the discharge; though strictly speaking it is scarcely a quality of the discharge, but rather of the tranquil gas of which it marks the oncoming transformation. Presumably it is preceded by an intermediate state, in which the oscillating voltage both displaces the ions in the gas, and gives them energy enough to form others; but of this we as yet know little.

I will arrange the optics much as I have previously arranged them in the description of direct-current discharges:[1] first, the observations on gases independently ionized and exposed to an oscillating voltage which displaces the independently-formed ions; then the observations on breakdown or transition, in which the oscillating voltage is raised to such a value that it initiates and maintains a self-sustaining discharge; and finally, the observations on the self-sustaining discharge itself. Beforehand, though, it will be convenient to derive some equations describing the presumable behavior of ions in gases exposed to alternating fields.

First, consider a charged particle free to move in a vacuum, and exposed to an alternating electric field. One might expect it to oscillate to and fro across a fixed point, like a simple harmonic vibrator. This however occurs only in a special case. In general, the particle will oscillate about a point which glides at uniform speed along the direction of the field. For, let us write down the equation for the acceleration of the particle, denoting by e its charge, by m its mass, by E_0 the amplitude and by ν the frequency of the field E, which last we suppose directed along the axis of x:

$$m(d^2x/dt^2) = eE = eE_0 \sin 2\pi\nu t. \tag{1}$$

Integrating once, we obtain for the speed of the ion,

$$dx/dt = \frac{1}{2\pi\nu}\frac{eE_0}{m}(1 - \cos 2\pi\nu t) + v_0, \tag{2}$$

and integrating a second time, we obtain for its displacement from the point which it occupied at $t = 0$:

$$x = \left(\frac{1}{2\pi\nu}\frac{eE_0}{m} + v_0\right)t - \frac{1}{4\pi^2\nu^2}\frac{eE_0}{m}\sin 2\pi\nu t. \tag{3}$$

The first term on the right of equation (3) represents the steady drift of the centre of oscillation, the second term the oscillation to and fro across this centre.

The speed of the steady drift is the coefficient of t in equation (3). Its value depends on that of the constant of integration v_0, which, as one sees from equation (2), is the speed (or rather, the x-component of the velocity) of the ion at the instant $t = 0$ when the electric field passes through zero. If this speed just happened to be equal to $eE_0/2\pi\nu m$, and directed in the sense opposite to that in which the increasing field was about to draw the ion, the particle would oscillate about a fixed point; the amplitude of its vibrations would amount to $eE_0/4\pi^2\nu^2 m$, its maximum kinetic energy to $e^2E_0^2/8\pi^2\nu^2 m$. But this is

[1] In a recent book, "Electrical Phenomena in Gases," abstracted in the July 1932 *Bell Sys. Tech. Jour.*, and to which I refer at various points in this article.

a very improbable event; it is difficult to conceive of any mechanism plausible or unplausible which would endow all the ions in a gas with such an initial speed. We can scarcely make use of any but the most general formulæ; among which I choose those for the drift-speed u and the maximum kinetic energy K_m of the ions:

$$\begin{aligned} u &= \frac{1}{2\pi\nu}\frac{eE_0}{m} + v_0, \\ K_m &= \frac{1}{2}m\left[v_0 + \frac{1}{\pi\nu}\frac{eE_0}{m}\right]^2. \end{aligned} \tag{4}$$

If we knew the distribution-in-speed of the ions at the instant $t = 0$, and they were in a vacuum (i.e., never collided with atoms), we could predict their future motions. But they are not in a vacuum, and we do not know their distribution-in-speed at $t = 0$. What use, then, can be made of the equations?

Little or nothing, I am afraid, of such definiteness as to permit of exact and verifiable predictions about high-frequency discharges! After all, it is of the essence of a discharge that the phenomena occur in a gas where molecules and ions make collisions with each other; inferences drawn from the assumption that ions never collide with molecules are not likely to be close to truth. Nevertheless we may make some deductions of general value.

Thus: the expressions for the amplitude of the vibration of the ion, the constant speed of the point about which it vibrates, and the maximum kinetic energy acquired by the ion, all of them decrease when m is increased. All of them either are inversely proportional to m, or else involve a term inversely proportional to m. Therefore all of them are much smaller for positive ions than for free electrons. If a gas contains ions of both these kinds, the free electrons have by far the most energy, oscillate the farthest and drift the fastest. On this account we may often pretend that all the ions in the gas are stationary, excepting only the free electrons; and I shall often make this pretence.

Again: the fact that the center of oscillation of each ion drifts (except in a special case, presumably very rare) suggests that even in a vacuum it is impossible to confine the ions to any restricted part of the region between the electrodes.

Finally: the value given in (4) for the maximum kinetic energy of the ions, with the particular choice of zero for the value of v_0, may be taken as an indication of the order of magnitude of the energy which an ion might acquire in a gas not so dense but that occasionally it might run without a collision for a time as long as the duration of one oscillation. But for more than the order-of-magnitude we should certainly not rely on it!

Since allowance should be made for the collisions of the ions with molecules of the gas, let us do what is customary in physics: rush to the extreme, exaggerate the effect instead of neglecting it, and suppose that collisions are so frequent that their net influence upon a wandering ion amounts to a force of the type called "viscous" or "frictional"—a force proportional and oppositely directed to the velocity of the particle. Into the equation of motion we introduce a new term, so that it takes the form:

$$m(d^2x/dt^2) + g(dx/dt) = eE = eE_0 \sin nt \tag{5}$$

g standing for the new "coefficient of friction," and n being written for $2\pi\nu$ so as to avoid constant repetition of the factor 2π. One integration is sufficient to bring out the important point; we obtain:

$$\frac{dx}{dt} = \frac{eg}{m^2n^2 + g^2} E - \frac{me}{m^2n^2 + g^2} \frac{dE}{dt}. \tag{6}$$

If now we multiply each member of the equation by Ne — N standing for the number of ions per unit volume—each becomes equal to the current-density borne by ions in the gas.

Notice now an important thing about this expression for current-density. It consists of two terms, one of which is proportional to the fieldstrength E, while the other is proportional to the time-derivative of the fieldstrength—two terms, therefore, of which one is in phase with E and the other in quadrature with E. The first we may set down in the following equation:

$$\textit{Current-density in phase with fieldstrength} = \frac{Ne^2g}{m^2n^2 + g^2} E = \sigma E \tag{7}$$

and we may apply the name "conductivity" to the coefficient σ. The second must be treated differently. By itself it is not the whole of the current-density in quadrature with the fieldstrength; there is the displacement-current also to be taken into account, equal to $(1/4\pi)(dE/dt)$. Then we must write:

Current-density in quadrature with fieldstrength

$$= \frac{1}{4\pi}\left(1 - \frac{4\pi Ne^2m}{m^2n^2 + g^2}\right)\frac{dE}{dt} = \frac{\epsilon}{4\pi}\frac{dE}{dt} \tag{8}$$

and we may apply the name "dielectric constant" to the coefficient ϵ.

Say now that we can ionize a gas abundantly, as for instance by maintaining an intense direct-current discharge across it between auxiliary electrodes; and that we can apply a small oscillating voltage transversely, between a pair of electrodes facing one another like the plates of a condenser; and that we can measure, in the circuit containing this oscillating voltage and these condenser-plates, the components of current in phase and in quadrature with the field—then by equations (7) and (8), we can determine N and g. Or if we can measure either component by itself, and N in some independent way, we can determine g and test the equations by comparing its value with one derived from our knowledge of the flow of ions through gases in a steady field. I will describe how this is done, after giving equations for a more special and a more general case.

If the coefficient g is very large compared to mn—we shall later see that this is the same as saying: if an ion makes very many collisions with molecules during a single cycle of the applied field—the component of convection-current in quadrature with fieldstrength almost vanishes. The ions describe simple-harmonic vibrations about fixed centres (there is no possibility of a steady drift of the centre of vibration), the velocity being in phase with the field; the amplitude of vibration is easily shown to be given by this expression:

$$\text{Amplitude} = eE_0/2\pi\nu g = \mu E_0/2\pi\nu. \tag{9}$$

I take this opportunity of mentioning that the quantity e/g, the quotient by E of the drift-speed attained by the ion under a *steady* field of strength E, is called the "mobility" of the ion at the pressure in question, and is denoted by μ.

If the equation of motion of the ion includes in addition to the two terms in the left-hand member of equation (5) a third term fx (I have written it out with this additional term farther along in the article, as equation 26), the expressions for the current-components in phase and in quadrature with field strength involve the coefficient f. The formulae for σ and ϵ become the following:

$$\sigma = \frac{Ne^2g}{[(f/n) - mn]^2 + g^2} \qquad \epsilon = 1 - \frac{4\pi Ne^2[m - (f/n^2)]}{[(f/n) - mn]^2 + g^2}. \tag{10}$$

The term in question and its coefficient f would naturally be introduced if we were dealing with bound electrons, but it seems odd to postulate such a term in speaking of freely-moving electrons or ions; nevertheless there is reason to do so, as I will later mention.[2]

When a gas containing free electrons is traversed by electromagnetic waves, there is at every point an oscillating electric field having the frequency of the waves, and the electrons vibrate according to the

[2] There is also an allowance for polarization which should be made if extreme accuracy is desired. See Appleton & Chapman, *l.c. infra.*

foregoing laws. The waves are then absorbed—that is to say, their amplitude falls off exponentially as they progress—and their phase-velocity is altered. In the particular case in which g and therefore σ are negligibly small, there is no absorption, and the phase-velocity is altered in the ratio $1 : \sqrt{\epsilon}$—a fact which is described by saying that the index of refraction of the electron-populated gas is equal to $\sqrt{\epsilon}$. Since ϵ is less than unity according to equation (8), the waves go faster than they would if there were no free electrons roaming the gas. Formulae for index of refraction and absorption-coefficient, in the general case in which g and σ do not vanish, are given further along in this article (equations 15, 16). The transmission of radio waves through the atmosphere of the earth, in which free electrons are present in a concentration varying greatly with altitude, is much affected by this refraction.

Experiments on Oscillation of Electrons in Ionized Gases

There is a method which, in principle, is adequate for measuring the dielectric constant and the conductivity of an ionized gas subjected to high-frequency vibrations; adequate therefore, in principle, for testing the expressions (7) and (8), for evaluating the quantities g and N which figure in those expressions. I will describe the method, first in the form in which it was applied to ionized air by Szekely, with results which are of some interest in themselves but mainly serve to illustrate how great a gulf intervenes between "adequate in principle" and "adequate in practice."

The gas—air at some pressure of the order of a few hundredths of a millimeter of mercury—was contained in a long tube, having electrodes near its ends whereby a direct current could be passed through the tube, maintaining the air in a state of intense ionization. Inside the tube and near its middle were a pair of parallel plane electrodes, rectangular in shape (18 by 42 mm.) and 4.5 mm. apart; to these the high-frequency voltage was applied; I will speak of them hereafter as "the ionization-condenser." When the direct current was flowing, a self-sustaining glow-discharge existed in the tube, and the ionization-condenser was submerged in the negative glow thereof.

The ionization-condenser was shunted by an adjustable condenser outside of the tube, and the two of them by an inductance and an adjustable resistance; all this constituted a circuit—I will call it hereafter the "high-frequency circuit"—coupled at one place to a source of high-frequency E.M.F., at another place to a detector. With such an arrangement, when one varies the capacity of the adjustable condenser and leaves everything else unchanged, the current in the circuit (measured by the detector) passes through a maximum at a certain setting of the capacity. At the attainment of this maximum, the system is said to be in resonance.

In Szekely's experiments, the adjustable condenser was set to produce resonance in three different conditions: in case (*a*) the ionization-condenser was entirely disconnected; in case (*b*) it was connected, but there was no direct-current discharge flowing in the tube, and the air was not ionized; in case (*c*) it was connected, and the air was ionized by the direct-current discharge. The values C_a, C_b, C_c of the adjustable capacity were measured at resonance under the three conditions; their differences give the values of the capacity of

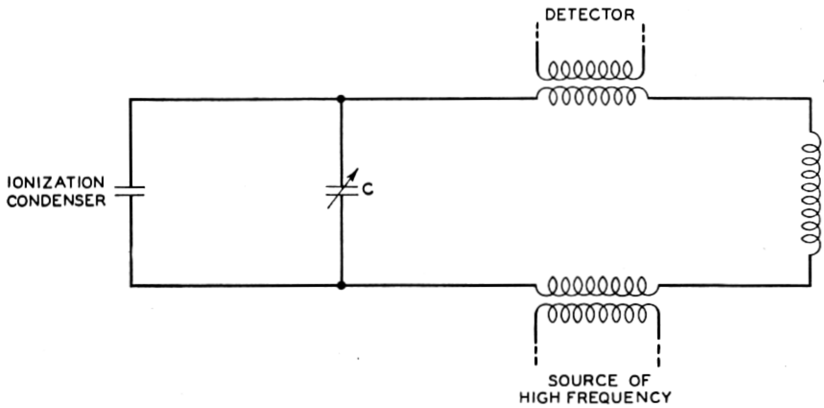

Fig. 2—Illustrating a method for measuring dielectric constant and conductivity of an ionized gas (between the plates of the "ionization-condenser").

the ionization-condenser (plus that of its leads, which apparently is a large part of the total) when the gas is ionized and when it is not. Out of these data one may calculate the dielectric constant of the ionized gas. As for the conductivity: the values i_{mb} and i_{mc} of the high-frequency current at resonance are different in cases (*b*) and (*c*), smaller in the latter. After measuring i_{mc}, one may return to condition (*b*)—shutting off the direct-current discharge—restore the resonance, and reduce i_{mb} to the value just found for i_{mc} by adding resistance to the circuit. From the amount ρ of added resistance which is necessary to achieve this, one may calculate the resistance between the condenser-plates when the air is ionized; and thence—taking account of the size and shape of the plates and the distance between them—the conductivity of the ion-populated air.

Everything thus is apparently provided for testing the expressions and determining the constants derived from the theory of electrons oscillating in the ionized gas. But, the results of the experiments were not agreeable. True, in one important respect there was concordance with theory. Szekely measured the resistance of the ionization-condenser (or, to express it better, the resistance of the ionized air between the plates) for various values of the direct current sustaining the ionization, and various frequencies ranging from one

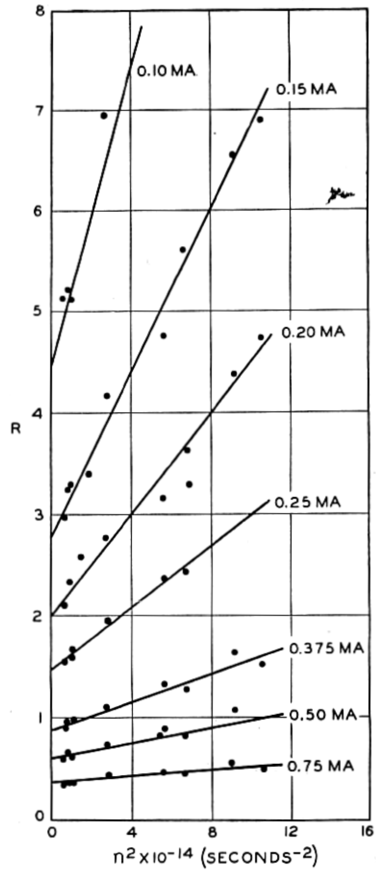

Fig. 3—High-frequency resistance of ionized rarefied air, plotted against square of frequency. (A. Szekely; *Annalen der Physik.*)

to five millions. Plotting against the square of the frequency those values of resistance which belonged to one and the same strength of direct current, she obtained ascending straight lines (Fig. 3). Now, this agrees with the equation (7) supplied by the theory, according to which the reciprocal of the conductivity of an ionized gas is a linear ascending function of frequency squared, if N be constant;

but the values computed from these curves for N and g are not very plausible, as I will later stress. What is much more serious: the capacity of the ionization-condenser is increased by ionizing the air; the dielectric constant of the ionized air is greater than unity, instead of less! The method therefore, adequate as it seems in principle, suffers from some defect or defects, which it is important to discover.

One of these defects was recognized towards 1925 by Appleton, who in previous work by the same sort of method had obtained the same bothersome result: when the air was ionized, the capacity of the ionization-condenser usually went up instead of down (although he, and van der Pol before him, did have the satisfaction of observing a decline of the capacity, when the strength of the direct current and hence the degree of ionization were relatively low). This however he explained by taking into account the space-charge sheaths of positive ions which form upon plates immersed in a strongly-ionized gas, or for that matter upon the walls of the tube containing the gas, if they are allowed to assume the potential which they naturally seek.[3]

If each of the two plates of the ionization-condenser is overspread by a positive-ion sheath of thickness x (x being less than half the distance d between the two plates, otherwise the sheaths would overlap) the system behaves not like a single condenser but like two in series. The capacity of each of the two, according to Appleton and Childs, varies inversely as x; their formula for each is $AK/3\pi x$, K standing for what they define as "the effective dielectric constant of the sheath" and A for the area of the plate; it differs from the customary formula for the capacity of a plane condenser—$AK/4\pi x$—because of the distribution of charge in the volume between the plate and the outer edge of the sheath. The capacity of the two in series is $AK/6\pi x$. If x is sufficiently small, this will be considerably larger than the capacity $A/4\pi d$ of the ionization-condenser as it was when the gas was not yet ionized; and the change in capacity occurring when ionization is started will consist chiefly of the substitution of the value $AK/6\pi x$, for the value $A/4\pi d$—even the influence of the layer of ionized gas between the outer surfaces of the sheaths will be minor.

If these ideas are correct, then when x is small the change of capacity of the ionization-condenser should vary inversely as x, provided we can vary x without altering K. Now when the potential V of the plates relative to the bulk of the ionized gas is varied, and the intensity of the ionization in the gas remains the same, the thickness of the sheath varies as $V^{3/4}$ while the current-strength across it remains

[3] "Electrical Phenomena in Gases," pp. 355–371.

unchanged.[4] If K remains the same, then the increment of capacity due to the ionization should vary as $V^{3/4}$; its logarithm should be a linear function of the logarithm of V, the line having slope $-3/4$. This inference was tested by Appleton and Childs, with the favorable result displayed in Fig. 4.

The space-charge sheaths thus furnish an explanation for the unexpected sign of the change in capacity occurring when the gas is suddenly filled with ions—an explanation releasing us from having to assume that the dielectric constant of the gas rises above unity, in contradiction to the simple theory.

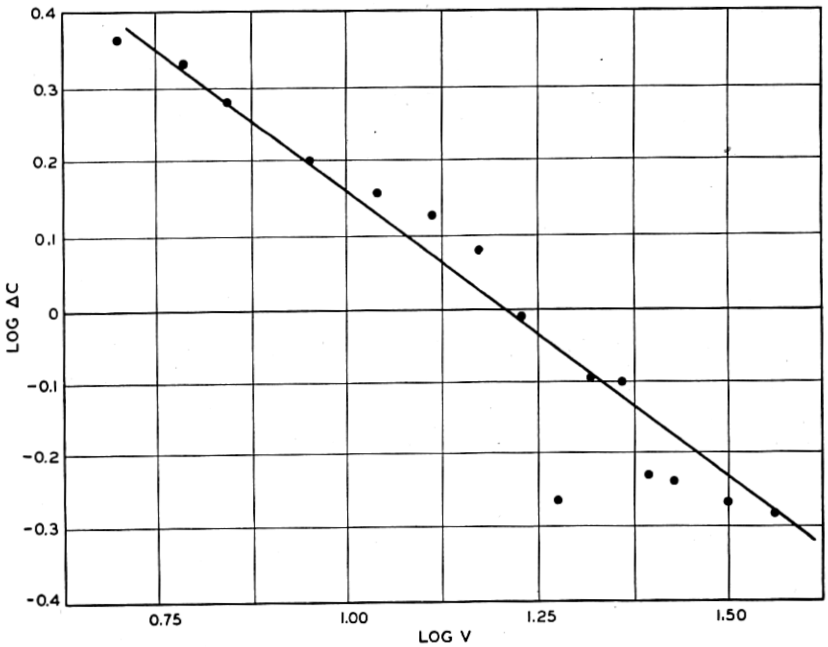

Fig. 4—Alteration in capacity of a stratum of ionized gas, ascribed to the formation of positive-ion space-charge-sheaths along the surfaces of the electrodes bounding the stratum. The slope of the line is $-\frac{3}{4}$. (E. V. Appleton & E. C. Childs; *Philosophical Magazine*.)

Could the sheaths be eliminated? If x is allowed to increase indefinitely, several things happen; in particular, the assumption that the thickness varies as $V^{3/4}$ and the assumption that $K/6\pi x$ is much larger than $1/4\pi d$ depart further and further from the truth, and

[4] "Electrical Phenomena in Gases," equation (182), page 360. The equation is the familiar "three-halves-power equation," which derives its usual name from the relation between voltage and current-density with which we are not here concerned. It is valid only if the ions mostly cross the sheath without collisions, an important restriction.

when x passes $d/2$ the sheaths overlap and the conditions change entirely. Other things being equal, x increases as the number of ions per unit volume of the gas outside the sheath decreases; and it is actually found that as the direct current maintaining the ionization is reduced, the increase of capacity due to the ionization is also reduced, passes through zero, and becomes a diminution as the simple theory indicates.

One might on the other hand get rid of the sheaths altogether, by adjusting the (mean) potential of the plates to so low a value that x vanishes, instead of leaving it to seek its own level. This has recently been done by Childs, though for some reason he chose to study not the dielectric constant, but the conductivity of the ionized air. A galvanometer was coupled into the high-frequency circuit, which was tuned to resonance when the mean potential of the plates had been so adjusted that the visible sheaths had just vanished. The reading of the galvanometer was taken; the current maintaining the ionization (it was 300-cycle A.C., instead of D.C.) was discontinued, and various high resistances were connected in parallel with the condenser plates, until the galvanometer resumed its former reading; the value of resistance then existing was taken as that of the ionized air between the condenser-plates. The edge-correction of the condenser was determined by filling the tube with alcohol of known conductivity; it was found that the conductance of the condenser had to be multiplied by 0.57 to be converted into conductivity of the medium between the plates. The values of conductivity (I will presently state the conditions more precisely) were of the order of 10^{-14} E.M.U.

A value of conductivity by itself, obtained at a single frequency, is theoretically the value of a combination of N and g; to determine either, without the aid of a simultaneous measurement of dielectric constant, one must have an independent value of the other. Childs evaluated N by the Langmuir probe-method.[5] Working with air at 1 mm. pressure and a frequency about one million, for three values of the 300-cycle A.C. maintaining the ionization (5 ma., 10 ma., 15 ma.) he obtained three values of the electron-concentration N (they were 9.6, 24 and 36 times 10^7 per cc.), and substituted them, not into the general equation (7) for σ, but into an approximate form thereof:

$$\sigma = Ne^2/g, \tag{11}$$

believing g to be so large that mn is small by comparison. The values of g thus obtained were 2.5, 3.1 and 2.5 times 10^{-18}. Despite

[5] "Electrical Phenomena in Gases," pp. 351–352.

the large differences between the values of N, those of g show no trend, which is as it should be.

Another way of determining g without bothering about N has just been carried into effect by Appleton and Chapman. Returning to equation (7): one readily sees that, if it be possible to vary g without altering N, the conductivity should pass through a maximum when $g = mn$. Now according to the simple interpretation the coefficient g, being the measure of a sort of friction which the electrons suffer when travelling through a gas, should increase with the pressure. If therefore one can vary the pressure of the gas sufficiently while applying the same high frequency, and maintaining the same degree of ionization, one should find the conductivity passing through a maximum. Moreover, anything proportional to the conductivity of the gas should pass through a maximum, anything inversely proportional to the conductivity should pass through a minimum. This conveniently makes it unnecessary to measure the conductivity (or anything else) absolutely; it is sufficient to choose something which varies (say) inversely with conductivity, plot it as function of pressure, and locate the minimum of the curve. For the pressure corresponding to this minimum, the value of g is equal to the product of the mass of the electron by the applied frequency.

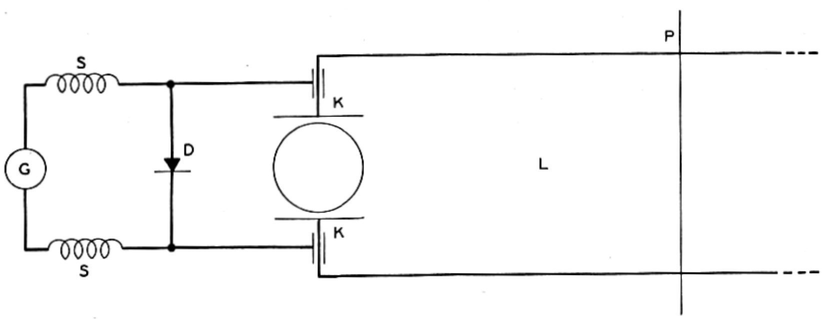

Fig. 5—Illustrating another scheme for estimating the dielectric constant and conductivity of an ionized gas (in the tube between the plates K), with Lecher wires and movable bridge P replacing the adjustable condenser of Fig. 1. (E. V. Appleton & F. W. Chapman; *Proc. Phys. Soc. London.*)

The apparatus and the circuit of Appleton and Chapman are shown in Fig. 5; as one notices, the plates of the ionization-condenser are now outside the tube and the ionized gas, and the high-frequency circuit now comprises long parallel wires, known (after the man who first used them as portions of high-frequency circuits) as Lecher wires. The current through the galvanometer to the left (shunted by a galena detector) is the measured quantity; the change which occurs in the reading of the galvanometer, when the discharge and the ionization commence in the tube (resonance being restored in the circuit, by shifting the "bridge" across the wires), is the quantity which supposedly varies inversely as conductivity. In the curves of Fig. 6, one sees the minima. Such curves were plotted for four frequencies—340, 500, 550 and 625 millions—and the maxima occurred at the pressures 0.08, 0.11, 0.12 and 0.15 mm. Hg. The values of mn corresponding were 1.9, 2.8, 3.1 and 3.5 times 10^{-18}; these by the theory are the values of g for the corresponding pressures. They are roughly proportional to the pressure, as they should be. (Inci-

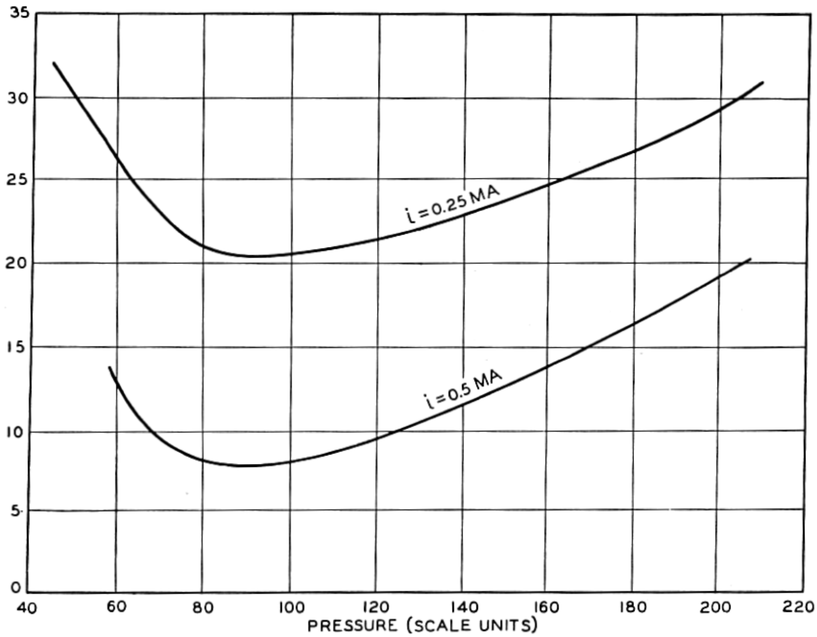

Fig. 6—Evidence of a maximum of conductivity occurring when the frequency is related in a particular way to the frictional coefficient g. (Appleton & Chapman.)

dentally, the resonance-frequency of the system was shifted, when the ionization commenced, in the proper sense—the sense corresponding to a dropping of the dielectric constant below unity.)

Before considering further data, let us compare these values of g with those deduced from observation (or theory) of electrons drifting through the same or a similar gas under a constant field. In such a condition, they attain a "terminal drift-speed" u given by the equation,

$$gu = eF, \tag{12}$$

F standing for the fieldstrength. Despite the aspect of the equation, u is not proportional to F; the coefficient g must be regarded as a function of F (over wide ranges of fieldstrength it is proportional to the square root thereof). Though this is a fact of experience, it will be helpful to develop the theory to some extent.

Picture the drifting of electrons through a gas in the customary (though far too primitive) way. Imagine the gas as a congregation of elastic spheres, with which the electrons make elastic impacts. When these latter enter the region where the constant and uniform field is applied, they are speeded up in the direction of the field; but owing to the deflections and the losses of energy which they suffer at their impacts, their velocities become and remain almost isotropic, and their average energy approaches but does not surpass a certain limiting value determined by the fieldstrength. After they have progressed sufficiently far, they form an "electron-gas" mingled with the atoms of the material gas; this electron-gas drifts slowly along towards the positive electrode, but its individual corpuscles have (as a rule) random velocities tremendously in excess of that comparatively modest drift-speed, just as the molecules of the air have random velocities many times greater than the speed of the wind. Let me denote the drift-speed by u, the mean speed of the random motions of the electrons by ω. Now it may easily be shown [6] that if the simple picture of the molecules as big elastic spheres and the electrons as little ones is acceptable, u and ω and the mean-free-path of the electrons l are related by the equation:

$$u = \theta e E l / m\omega, \tag{13}$$

θ here standing for a numerical factor not very different from unity, the exact value of which depends on the underlying assumptions made in the statistical analysis of the motions of molecules and electrons. The value 0.8 for θ is probably as good as any, but it would be pointless to spend time deciding between different values, since the molecules cannot be represented exactly as elastic spheres, and the degree of their deviation from this simple model would affect the quantity θ.

Now comparing the last two equations, we find:

$$\begin{aligned} g &= m\omega / \theta l \\ &= m / \theta\tau = mZ / \theta, \end{aligned} \tag{14}$$

τ and Z here representing respectively the mean duration of the free flight of an electron between consecutive impacts, and the number of impacts made by an electron in unit time. (I introduce these

[6] "Electrical Phenomena in Gases," pp. 174–175.

symbols because they are used in some of the original articles, not because they add anything to the fundamental ideas.)

The value of ω may be determined by the Langmuir probe-method; values for l, the mean-free-path of the electrons, are supplied by various methods of varying reliability. Childs, in his experiment which I have quoted, obtained with the probe the three values 1.3, 1.5, 1.7 times 10^8 (cm./sec.) for ω, these corresponding to the three cited values of the current maintaining the ionization; combining these with estimates [7] of l, he obtained for $m\omega/l$ the values 3.2, 3.8, $4.3 \cdot 10^{-18}$. Comparing these with the values found for g, and remembering the manifold chances of faults in the assumptions, one is favorably impressed with the agreement.

Could we not evaluate g directly, by measuring the drift-speed u of the electrons exposed to a certain constant fieldstrength F, and forming the ratio of eF to u to which, according to equation (12), g is equal? Here we must be careful. According to equation (13), g depends on the vivacity of the random motions of the molecules, whereof ω is the mean speed; now, ω depends on the fieldstrength: we must select such a value of F, that the random agitation of the electrons shall be the same as it is in the actual gas on which the high-frequency field is imposed. Now this actual gas is subjected to a constant field, that which maintains the ionization. It is natural and simple to assume, that this field controls the value of ω, the effect of the high-frequency field on the mean speed of random agitation of the electrons being presumably slight. The measurement of the drift-speed should therefore be made in the very gas under the very same constant field, in which the high-frequency phenomena are observed.

Let me denote by i the steady current along the length of the tube, due to the constant field; by N, the number of electrons per unit volume; by u, their drift-speed; by A, the cross-sectional area of the tube. It can then be readily seen that i is equal to $NeuA$.[8] The method is consequently simple in principle; in practice, the chief difficulty apparently is that N is not the same near the walls of the tube as along its axis, so that probe-measurements should be made at a number of distances from the axis and the results averaged. Appleton and Chapman determined N at a single point of the tube; the value of the ratio eF/u came out equal to $2.3 \cdot 10^{-18}$—again, a remark-

[7] Apparently he multiplied by $4\sqrt{2}$ the value of the mean-free-path of molecules of nitrogen, this being the simple gas presumably most like air; at any rate he used the value .036 cm.

[8] *Electrical Phenomena in Gases*, p. 207, pp. 232–233; on the two latter pages I describe Killian's application of the method to mercury vapor.

able agreement with the values deduced from the high-frequency phenomena![9]

(It is rather surprising to realize that in such discharges, we must not conceive the electrons as moving with a steady velocity along the axis of the tube, on which a sinusoidal oscillation parallel to the field is superposed. If we could follow the wanderings of an individual electron, the sinusoidal oscillation would be as little obvious as the steady drift. Only the rapid zigzag motions of the corpuscles would be conspicuous; what I have been calling a vibratory motion is, in truth, only a slight bias of these random flights, just as the apparent steady drift is itself a slight bias.)

Another experiment, capable in principle of testing the expressions for dielectric constant and conductivity and of giving the values of N and g, consists in sending electromagnetic waves of the frequency desired across a stratum of ionized gas, and measuring their index of refraction and their absorption in the stratum. The former of these two has not, so far as I know, been measured; but apparatus for determining the second (it is that of Hasselbeck's experiment) is shown in Fig. 7: one sees the paraffin lenses which convert a diverging beam of Hertzian waves into a parallel beam which is sent through the ionized gas, and others which reconvert the parallel beam into one which converges upon a bolometer. Part of the beam is reflected from a semi-transparent mirror onto another bolometer, so that the ratio of the intensities of the waves before and after the passage through the gas can be determined without regard to fluctuations.

The formulae for refractive index n_0 (I add the subscript because of having already used n, the conventional symbol, for another purpose) and absorption-coefficient k are familiar to everyone who has studied the theory of absorption and reflection of light by metals, for in the classical theory of metals such a substance is conceived exactly as we are now conceiving an electron-populated gas. We have:[10]

$$n_0^2 - k^2 = \epsilon, \qquad n_0 k = 2\pi\sigma/n, \tag{15}$$

and putting for ϵ and σ the expressions (7, 8), we get:

$$n_0^2 - k^2 = 1 - \frac{4\pi N e^2 m}{m^2 n^2 + g^2}, \qquad n_0\, k = \frac{2\pi}{n}\,\frac{Ne^2 g}{m^2 n^2 + g^2}, \tag{16}$$

solving which equations for k, and putting $m/\theta\tau$ for g (according to equation 14) and n_1^2 for the combination $4\pi Ne^2/m$ (we shall meet it

[9] Similar observations by Jonescu and Mihul on air and on hydrogen, subjected in some cases to magnetic field, have recently been published, but with scant detail.

[10] See for instance P. Drude, "Treatise on Optics" (page 361 of the English translation).

later in another meaning), and α for the ratio n_1/n, we obtain:

$$k^2 = \frac{1}{2(1 + n^2\theta^2\tau^2)} \{\sqrt{[1 + n^2\theta^2\tau^2(1 - \alpha^2)]^2 + n^2\theta^2\tau^2\alpha^4} + [1 + n^2\theta^2\tau^2(1 - \alpha^2)]\}. \quad (17)$$

By measuring k with waves of a single frequency, and measuring in addition either of the two quantities N and g, it is possible to evaluate the other of the two by means of this equation; provided the equation is correct, a presumption which should be tested by making the measurements at several different frequencies.

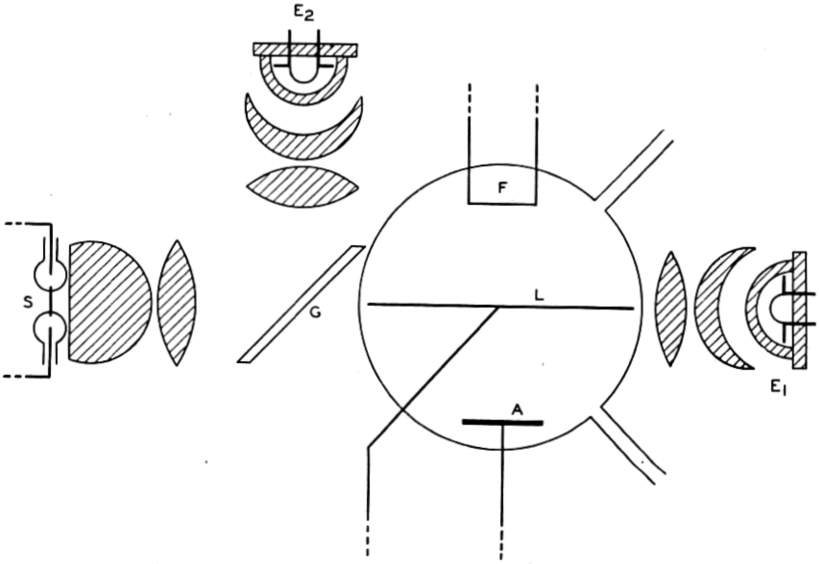

Fig. 7—Apparatus for measuring the absorption of Hertzian waves by ionized gas. S, source of waves; G, semitransparent mirror; E_1, E_2, receivers; A, F, anode and cathode of ionizing discharge; L, wire probe. (W. Hasselbeck; *Annalen der Physik.*)

Dänzer sent 4-cm. waves (of frequency $7.5 \cdot 10^9$)through a stratum of neon-helium mixture excited by a low-frequency discharge; measured the absorption, and measured also g by the method later employed by Appleton and Chapman. For N he then computed the value $1.3 \cdot 10^{12}$ (electrons per cc.)—a value of which, in the lack of further knowledge concerning electrons in this gas (he does not even state the pressure) one can only say that it is probably of the right order of magnitude. Much more extensive was the work of Hasselbeck, who used various frequencies ranging from 4.8 to 1.44 times 10^9, and introduced a probe into the gas in order to measure N and the mean speed ω of the electrons by Langmuir's method—it was a wire stretching clear across the globular tube, being made of this form so that it might give average values appropriate to the whole of the discharge. The best of his data refer to neon-helium mixture at various pressures of the order of a few tenths of a millimeter; there are data also for argon and nitrogen.

From equation (16), k should be proportional to N so long as n_0 does not depart too much from 1 and other things remain unchanged. Hasselbeck varied N by varying the current maintaining the ionization, measured it with the probe and measured the absorption. So long as the pressure of the gas was below a certain value, of slightly over 0.1 mm. Hg, the curves of k vs N were indeed straight lines rising from the origin (Fig. 8), though at higher pressures they were concave-upward. As for the curves of absorption *vs* frequency, some display

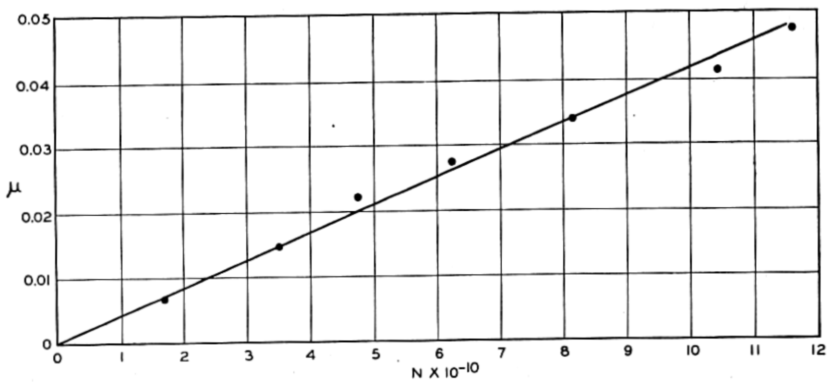

Fig. 8—Showing the proportionality of absorption of Hertzian waves to number of free electrons per cc. of absorbing gas (neon-helium mixture, pressure 0.18 mm. Hg). (W. Hasselbeck; *Annalen der Physik.*)

maxima in the range over which they are plotted, some do not; but it is possible to suppose that each is characterized by a single maximum, which moves toward higher frequencies when the pressure or the ionization is augmented, and in some of Hasselbeck's experiments happened to be out of the range.

The comparison of the absorption-*vs.*-frequency curves with so complicated an expression as that of equation (17) is no simple matter, nor is Hasselbeck's description of his procedure entirely clear. It appears, however, that the curves obtained in neon-helium mixture at a pressure of 0.87 mm. Hg depart but little from the form of the expression on the right of (17), and that what departure there is may not be significant. However the curves obtained at the much higher pressure of 21 mm. Hg differ very seriously from the theoretical form.

Reverting to the question of testing equation (8) for the dielectric constant, that is, for the current in quadrature with the voltage: one might try the test with a gas-free space, populated by electrons derived from a hot filament or from some other source. This in fact has been tried, the electrons being shot across the interspace between the plates of the "ionization-condenser," in directions nearly parallel to the plane of these plates; from the (controllable) speed of these electrons, and from the charge which they bear per unit time to a collector located beyond the condenser, one may estimate the number-per-unit-volume heretofore denoted by N.

Since the density of the gas may easily be made so low that few electrons collide with even one atom between the condenser-plates, one expects the coefficient g to vanish. Strictly speaking, it does; and yet there is in effect a component of current in phase with voltage. The electrons, being pulled aside by the transverse high-frequency field as they traverse the condenser, acquire kinetic energy; and this absorption of energy from the high-frequency circuit produces the same reaction, and is measured in the same way, as an ordinary conductance. Benner, who developed these ideas, derived (with certain simplifying assumptions) these expressions for the coefficients ϵ and σ of a cloud of streaming electrons which individually take the time T to cross the condenser:

$$\epsilon = 1 - 4\pi \frac{Ne^2}{mn^2}\left(1 - \frac{\sin nT}{nT}\right), \qquad \sigma = \frac{Ne^2}{mn^2T}(1 - \cos nT), \quad (18)$$

and they have been tested with satisfactory results by Jonescu and Mihul.

We turn now to the possibility for which preparation was made in equations (10); that an electron in an ionized gas may experience, in addition to the force due to the applied field (eE) and the quasi frictional force due to the gas ($g\dot{x}$) yet a third force proportional to its distance from some fixed point. One is accustomed to postulate this last for electrons bound to molecules or atoms. (Such electrons, by the way, contribute to the current-component in quadrature with field strength, so that the dielectric constant of a gas in absence of ionization is not quite unity as I have been writing). But to imagine such a force acting on free electrons must seem strange—as if one were denying them the quality of freedom. Still, for a cloud of electrons mingled with positive ions there is such a force, and therefore a "natural frequency"; and another natural frequency in addition, if there happens to be a magnetic field.

Considering the former first: suppose that initially we have a uniform distribution of electrons, N per unit volume; and that at a certain moment it is suddenly distorted, by shifting every particle a distance ξ parallel to the x-direction, this distance being a function of the original position x of the particle. Fix attention on a column of unit cross-section; initially, between two planes x and $x + dx$, there were Ndx electrons; they suddenly move over and occupy the space between the planes $x + \xi$ and $x + dx + \xi + (d\xi/dx)dx$, so that the density between these two planes, or let me say the number of electrons per unit volume at $x + \xi$, is given by the formula:

$$N'_{x+\xi} = N\left(1 - \frac{d\xi}{dx}\right). \tag{19}$$

We now introduce Poisson's relation between net density ρ of electric charge and space-derivatives of field strength. As by assumption all shifts of electrons are parallel to the x-direction, so also is the field-vector; we put X for its magnitude, and write Poisson's equation thus:

$$dX/dx = 4\pi\rho. \tag{20}$$

If the electrons were the only charged particles, we should have to put $N'e$ for ρ; and there would be a field of strength different from zero and varying from place to place, even when the distribution of the electrons was uniform. If however there is also a uniform distribution of positive ions, N per unit volume, and this is not affected when that of the electrons becomes non-uniform, then for ρ we need set only the second term on the right-hand side of (19), multiplied by e; we get

$$dX/dx = -4\pi Ne(d\xi/dx), \tag{21}$$

and integrating, with the boundary-condition [11] $X = 0$ at $\xi = 0$,

$$X = -4\pi Ne\xi, \tag{22}$$

so that an electron shifted from its original location, by virtue of such a mass-distortion of the formerly uniform distribution, is indeed subjected to a restoring force proportionate to its shift.

Putting $m(d^2\xi/dt^2)/e$ for X in equation (22), we get:

$$d^2\xi/dt^2 = -(4\pi Ne^2/m)\xi = -n_1^2\xi, \tag{23}$$

showing that there is a tendency to oscillations—"plasma-electron oscillations," as Tonks and Langmuir call them—of frequency ν_1 thus given:

$$\nu_1 = n_1/2\pi = \sqrt{Ne^2/\pi m} = 8980\sqrt{N}. \tag{24}$$

[11] This seems to be demanded by symmetry if ξ is a sinusoidal function of x: otherwise the case is more obscure.

This expression is strictly valid only when the gas does not interfere at all with the motion of the electrons; otherwise the frequency is reduced, in the same way as the natural frequency of a pendulum or a circuit is lowered by damping. The equation of motion of the electrons in a high-frequency field is as follows:

$$m(d^2x/dt^2) + g(dx/dt) + fx = eE \sin nt; \qquad f = 4\pi Ne^2. \quad (25)$$

The solution has already been indicated (equations 10).

Attempts to discover this natural frequency have been made in two ways: by examining curves of σ or ϵ or other correlated quantities plotted against frequency or against degree of ionization, and by searching for electromagnetic waves due to oscillations arising of themselves in highly-ionized gases.

The most thorough experiments by the former way are due to Tonks. He placed a tube containing a mercury arc between condenser-plates attached to long parallel wires, these being crossed by a movable bridge including a thermocouple, and coupled to an oscillating circuit. After establishing fixed values of the frequency and the current-strength in the latter circuit, he shifted the bridge until the thermocouple reported a maximum of current, and measured this maximum value; it and the shift of the bridge (the zero from which this latter is measured is unimportant) were plotted as functions of the current-strength in the mercury arc, which controls the number of free electrons per cc. A natural frequency is indicated by a minimum in the former of the curves, and in the latter curve a peculiar crinkle, similar to that which appears in a dispersion-curve in the neighborhood of an absorption-frequency, and in the curve with black dots in Fig. 9.

Embarrassingly it turned out that there were two, and indeed sometimes three, minima in the one curve and crinkles in the other. To these the corresponding values of N were correlated, being obtained by the Langmuir probe-method. The comparison with theory may then be made in either of two ways: by putting the value of the applied high frequency into equation (24), computing N, and comparing it with the observed values of N at the minima; or alternatively, by putting the observed values of N into equation (24), computing ν_1, and comparing its values with that of the applied frequency ν. As an example of the result of the first procedure: in one experiment, the applied frequency was $1.59 \cdot 10^8$; this should coincide with the plasma-electron frequency, resonance should occur, when $N = 0.63 \cdot 10^9$ electrons per cc.; the values of N at the two minima which were observed were 0.27 and 0.86 times 10^9. The latter is illustrated by a graph in Tonks' article; it turns out that when the comparison is made at various frequencies between $1.59 \cdot 10^8$ and $3.66 \cdot 10^8$, the curves of ν_1 *vs.* N agree only fairly with those of ν *vs.* N; where ν_1 varies as the square root of N, ν varies as the powers 0.42 and 0.45 for the two minima regularly appearing.

Another instance of a crinkle in a curve of the second type aforesaid —a curve in which the shift of the bridge across the Lecher wires, necessary to restore resonance in the circuit which includes the ionization-condenser, is plotted against N or something varying with

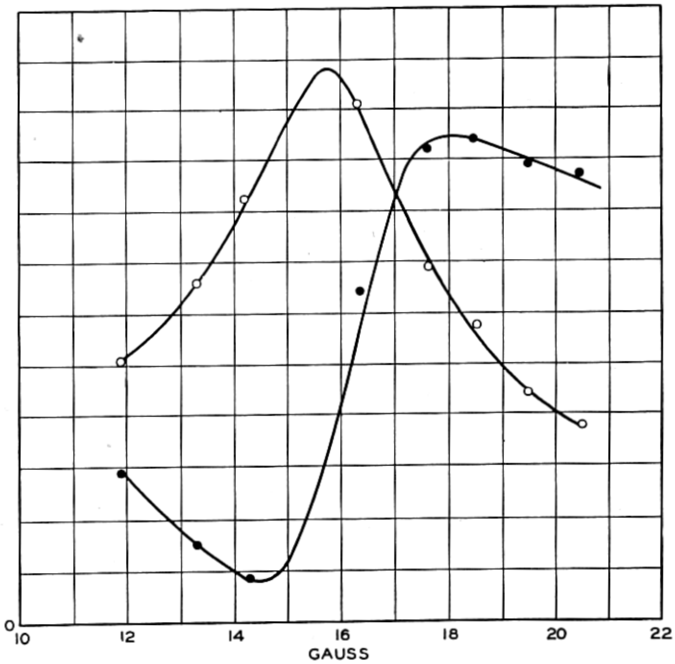

Fig. 9—Illustrating variation of dielectric constant (curve with full circles) and conductance (curve with hollow circles) of ionized gas in the neighborhood of a natural frequency, here due to a constant magnetic field. (S. Benner; *Naturwissenschaften.*)

N—has been published by Appleton and Chapman. It refers to ionized air; the applied frequency is $3.75 \cdot 10^8$, the value of N is $1.62 \cdot 10^9$, and these two stand to one another in substantially the relation of equation (24). Yet other instances have been published by H. Gutton, and these offer difficulties.

Gutton in his later work (with ionized hydrogen at the low pressure of 0.0004 mm. Hg) followed the method which I have just described, measuring the shift y of the Lecher bridge required to restore resonance, and the mean-square current I^2 traversing the Lecher bridge at resonance. He did not measure N, but something probably (though not certainly) proportional to N, the factor of proportionality not known: the direct current i flowing across the gas between two probes inserted on opposite sides of the tube and maintained at a constant P.D. Plotting y and I^2 against i, he observed a crinkle in the former curve, a minimum in the latter—evidence of a natural frequency at a particular value of electron-concentration.

In his earlier work (which I mention because the curves are often reproduced) Gutton connected the plates of the ionization-condenser to one another through a thermocouple, and to this circuit coupled an oscillator of which the frequency could be varied. For each of a number of values of the ionizing-current in the discharge-tube he varied the frequency until resonance was declared by a maximum of the mean square of the current in the thermocouple; he measured this maximum I^2 and the wave-length λ of the oscillations. Not having any quantitative measure of the ionization against which to plot I^2 and λ, he plotted one against the other—I^2 as ordinate, λ as abscissa—and obtained curves of the curious appearance shown in Fig. 10, in which the arrow indicates the sense in which the ionization increases along each curve. The start is made from the wave-length (408 cm.) at which the system is in resonance when the gas is not ionized. At low pressure, the curve bends first to the left and downward; this signifies that as the ionization increases the dielectric constant of the gas is falling and the conductivity rising, as by the theory they should. Then at an unknown but seemingly sharply-marked value of ionization, the curve bends sharply to the right; and this signifies the same as the crinkle and the minimum in the other more-fully-comprehended curves which I have been discussing.

It is still dubious whether the natural frequency so revealed is that which the foregoing theory predicts. Neither in his earlier nor in his later experiments did Gutton measure N (the estimate of its value which he once makes is derived in an indirect and fallible way). Measuring the values of i at which the resonance appeared in his later work, and comparing them with the corresponding values of the frequency, he found ν^2 proportional to $i^{3/4}$; the range of frequencies was comparatively small (not quite 4 : 1) but if the result is certain and i is truly proportional to N, equation (24) is contradicted. Appleton and Chapman testify that they found the resonance-effect observed by Gutton, but at values of N (which they measured by the Langmuir method) entirely too great to permit of regarding it as the plasma-electron frequency, from which they believe it to be distinct.

As for the search for spontaneous oscillations probably having the proper plasma-electron frequency, it is at present even less advanced.

Electromagnetic waves of high frequency often emanate from gases intensely ionized by a flow of direct current, and their wave-lengths have several times been measured, but for one reason or another the comparison with equation (24) was not and cannot be made. In a mercury-vapor arc Tonks and Langmuir detected oscillations of frequencies nearly as high as 10^9, accompanied by others ranging downwards as low as 10^6; the former, they believe, were plasma-electron vibrations; but unluckily they were unable to estimate N with any degree of exactness.

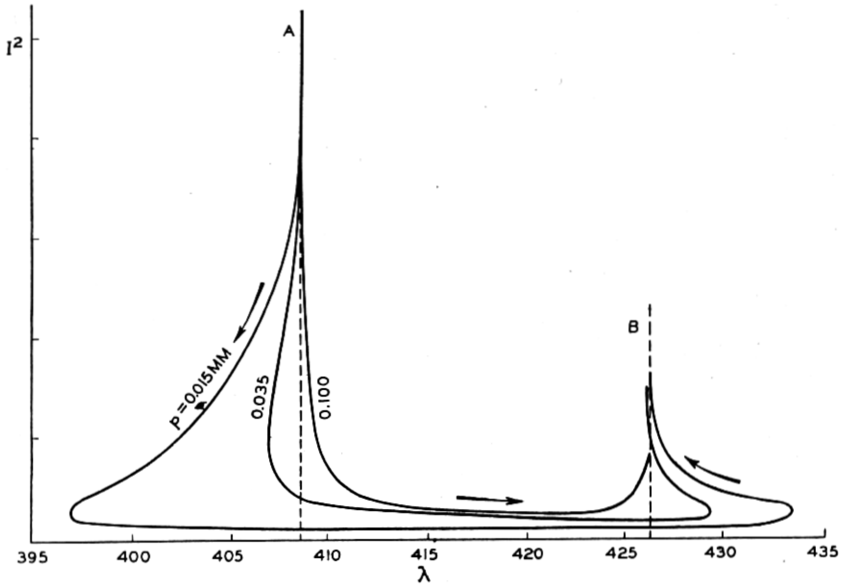

Fig. 10—Correlation of quantities serving as measures of high-frequency conductance and dielectric constant of rarefied ionized hydrogen. (H. Gutton; *Comptes Rendus.*)

If a region populated with free electrons is pervaded by a constant magnetic field, yet another natural frequency exists. Say, to begin with, that there is no electric field; then any moving electron, instead of continuing in a straight line, is constrained to describe a spiral path (the axis of the spiral being parallel to the magnetic field). The velocity of the electron affects the curvature of the helix, *but not the time of traversing a single winding* thereof. The reciprocal of this time, the number of windings described by any moving electron per second, is the "natural frequency" aforesaid, and is given by the formula:

$$\nu_H = n_H/2\pi = eH/2\pi mc, \qquad (26)$$

H standing as usual for the magnetic field strength. Since this frequency remains the same however much the speed of the electron may change, and therefore is the same for all the electrons in the region in question and for all values of electric field strength, we may expect it to be important when an ionized gas (or a volume containing free electrons but no atoms) is exposed to a high-frequency field; in curves of dielectric constant and conductance vs. frequency, we may look for peculiarities when $\nu = \nu_H$. The precise theory, I must add, is not simple when collisions of electrons with atoms must be taken into account (the fundamental equations were given long ago by Lorentz); but under certain restrictions—according to Appleton and Childs, the number of electrons per cc. and the number of collisions per cycle of the high frequency must be kept under certain limits—it leads to the inference that there should be a maximum of conductance at $\nu = \nu_H$.

This maximum is manifested by the sharp and striking minimum seen near the middle of the curve in Fig. 11. For obtaining these

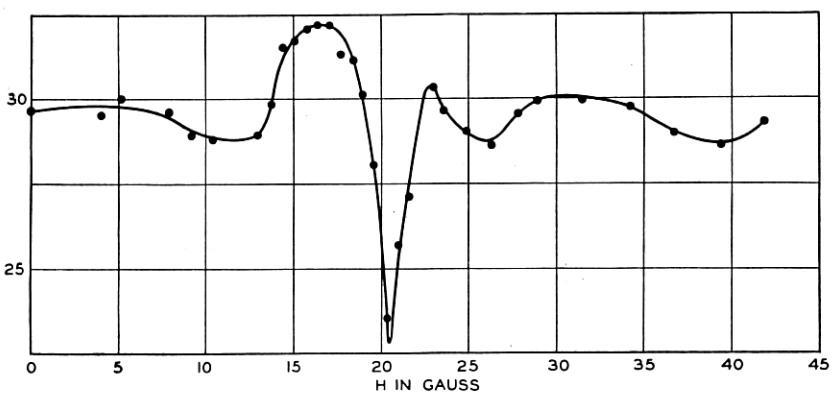

Fig. 11—Evidence of a natural frequency produced in an ionized gas by a constant magnetic field. (Appleton & Chapman.)

data, Appleton and Childs had an arrangement similar in the main to that of Fig. 5, excepting that the detecting galvanometer was coupled (through an amplifier) between the far ends of the wires of the bridge, and there was a magnetic field of adjustable strength parallel to the axis of the tube. Though I have spoken thus far of what should be observed when H is held constant and ν is varied through the value given by equation (26), it is more convenient in practice to hold the frequency constant and vary H through the corresponding value. The curve of Fig. 11 is accordingly a curve of galvanometer-reading vs. H; the remarkable minimum occurs at a value of H departing by less than 2 per cent from $2\pi mc\nu/e$. These data were obtained at frequency $5.46 \cdot 10^8$; others observed at $2.96 \cdot 10^8$ and $3.45 \cdot 10^8$ yielded agreements almost as good, or better.

Maxima of conductivity of ionized air have lately been found by Jonescu and Mihul, at the predicted values of field strength H, for various frequencies of the order 10^8. Earlier Benner, in a note deplorably brief, showed not only a curve of conductivity (or rather, of something proportional to conductance) displaying a maximum, but a curve of dielectric constant displaying a crinkle like that of a dispersion-curve in the neighborhood of a region of anomalous dispersion. These are the curves of Fig. 9, already introduced into this article to illustrate how these quantities vary in the vicinity of a natural frequency. The "ionization-condenser" of Benner's experiment consisted of the grid and plate of a triode, the space between them populated with electrons emitted from the filament; it is to be inferred that the tube contained some gas, but unfortunately nothing definite is said about the kind or amount. The maximum of the one curve and the crinkle in the other occurred at a value of H some twelve per cent higher than the predicted value aforesaid.

On applying a longitudinal magnetic field of about 21 gauss to the tube of ionized hydrogen with which he had observed the peculiar natural frequency mentioned above, H. Gutton found this one replaced by two, well marked and well separated, one being shifted toward higher frequencies from the original value and one toward lower. The same phenomenon has been observed by Tonks, in his studies of the natural frequencies which he identifies with the plasma-electron oscillations predicted by the theory culminating in equation (24). The doubling of the resonance is analogous to the Zeeman effect, and is amenable to theory.

The earth's magnetic field imprints a natural frequency upon the electrons populating the air, in particular the upper strata thereof; this affects the transmission of radio waves in curious ways, which were foretold in this journal seven years ago by Nichols and Schelleng, and in England by Appleton and Barnett.

In the second part of this article, I will treat of the conditions under which a high-frequency field may initiate and maintain a luminous discharge in a gas, and of the laws of these discharges.

References

The theory underlying equations (1) to (4) is well known; it is given in considerable detail by G. Mierdel, *Ann. d. Phys.*, **85**, 612–640 (1928), with some illustrative curves, and in several other papers. The theory underlying equations (5) to (10) apparently originated, like so much else in electronics, with H. A. Lorentz; it was developed with especial reference to the transmission of radio waves by W. H. Eccles, *Proc. Roy. Soc.*, **A 87**, 79–99 (1912); later enunciations, with various modifications, may be found in the papers of J. Salpeter, *Phys. ZS.*, **14**, 201–203 (1913); E. V. Appleton & F. W. Chapman, *Proc. Phys. Soc. Lond.*, **44**, 246–254 (1932), and in several papers cited in the next paragraph.

Experiments designed to measure dielectric constant and conductivity of ionized gas for high-frequency fields have been performed by A. Szekely, *Ann. d. Phys.* (5), **3**, 112–132 (1929); E. V. Appleton & E. C. Childs, *Phil. Mag.* (7), **10**, 969–994 (1930); Appleton & Chapman, *l.c. supra;* E. C. Childs, *Phil. Mag.* (7), **13**, 873–887 (1932); H. Gutton, *Annales de physique* (10), **13**, 62–129 (1930); H. Gutton & J. Clement, *C. R.*, **184**, 441–443, 676–678 (1927); V. Jonescu & C. Mihul, *C. R.*, **192**, 343–345 (1931.)

Experiments on the absorption of Hertzian waves by ionized gases: H. Dänzer, *Ann. d. Phys.* (5), **2**, 27–62 (1929); W. Hasselbeck, *ibid.* (5), **12**, 477–502 (1932).

Experiments on the reaction of electron-streams projected through very rarefied gas to a high-frequency field: L. Bergmann & W. Düring, *Ann. d. Phys.* (5), **1**. 1041–1068 (1929); S. Benner, *ibid.* (5), **3**, 993–996 (1929) (theory); V. Jonescu & C. Mihul, *C. R.*, **191**, 1436–1438 (1930).

Equation (24) for the so-called "plasma-electron" natural frequency appears to have been first derived by J. J. Thomson, and shortly afterward by L. Tonks & I. Langmuir, *Phys. Rev.* (2), **33**, 195–210, 990 (1929); see also Thomson, *Phil. Mag.* (7), **11**, 697–735 (1931). Experiments on this, and natural frequencies of possibly different origin: L. Tonks, *Phys. Rev.* (2), **37**, 1458–1483 (1931); H. Gutton, *Annales de physique* (10), **13**, 62–129 (1930); C. Gutton, *ibid.* (10), **14**, 5–14 (1930); Appleton & Chapman, *l.c. supra.*

Natural frequency or frequencies appearing when a constant magnetic field is imposed upon the gas: S. Benner, *Naturwiss.* **17**, 120–121 (1929); Appleton & Chapman, H. Gutton, L. Tonks, *ll.cc. supra.* The influence, upon the transmission of radio waves, of the magnetic field of the earth, was inferred independently in America by H. W. Nichols & J. C. Schelleng, *Bell Syst. Tech. Jl.*, **4**, 215–234 (1925) and in England by E. V. Appleton & M. A. F. Barnett, *Electrician*, **94**, 398 (1925).